

GUIDE TO THE USE OF ITEM NO. 76005. KINEMATIC AND DYNAMIC DATA FOR CRANK-ROCKER AND SLIDER-CRANK LINKAGES

1. INTRODUCTION

This Data Item is a Guide to the use of Item No. 76005 providing background information for the crank-rocker class of linkages over the complete range of possible plane four-bar linkages. The Item supports Data Item No. 76005[†] (Reference 1), a 'Four-bar linkages synthesiser and analyser for mechanical engineers', known as the ESDU Mechanisms Workbench[†].

The four-bar linkages used in this Item, and in Item 76005, are expressed in terms of three non-dimensional λ parameters, defined in Section 3.1, which describe the unique geometry of a four-bar linkage in terms of its 'four' link lengths. A further Data Item No. 81033 (Reference 2), presents an 'Atlas of performance curves for crank-rocker (and slider-crank) linkages' again over the complete range of possible four-bar plane linkages using the λ parameter presentation.

Data Item 76005 essentially consists of nine sets of transparent graphs or overlays, each set containing contours of properties relating to the behaviour of a four-bar linkage. A linkage having particular required properties can be found (if it exists) by viewing the contours for each of the appropriate properties through a stack of overlays. The properties considered include the ratios of link lengths, the total angle of swing of the rocker, the crank angle between dead-centres (average velocity or return ratio), rocker angle at extended dead centre, extreme transmission angles, maximum rocker velocity, maximum crank torque due to rocker inertia and maximum coupler end load. The maxima of the dynamic data are given with corresponding crank angles.

Table 1.1 associates the nine pack numbers of transparencies with the value of λ_3 (see Section 3.1 or Notation) and the mechanism type of each pack. Table 1.2 lists each of the transparencies A to H, together with a description of the contours, an illustration of the mechanism, its properties and the range of values of λ_3 .

TABLE 1.1

Pack Number	1	2	3	4	5	6	7	8	9
Value of λ_3	-1.0	-0.75	-0.5	-0.25	0	0.25	0.5	0.75	1.0
Mechanism type	Parallel-coupler	Crank-rocker							Slider-Crank

[†] This Item consists of two guides to the use of ESDU Item No. 76005 (this Item), 9 packs of transparencies, 4 register pins on a location card and 4 reusable tracing sheets located within a box and protective cover. The Item is used by overlaying transparencies each containing a property of a four-bar linkage and consequently cannot in its present form be accommodated in electronic format.

TABLE 1.2 List of Transparencies

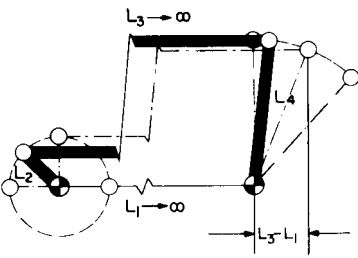
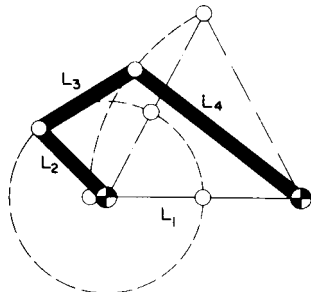
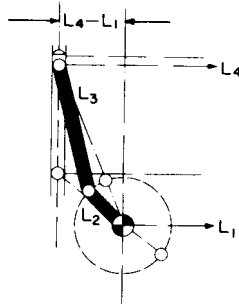
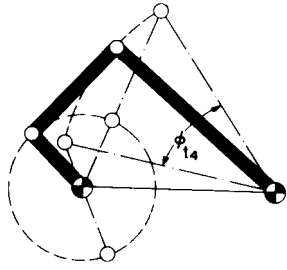
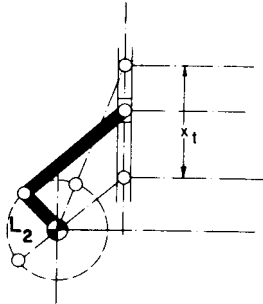
Description of Contours	Illustration	Properties	Value of λ_3	Transparency Pack Number	Property Identification Letter
Ratios of link lengths		L_4/L_2 and $(L_3 - L_1)/L_2$	-1.0	1	A
		L_2/L_1 , L_3/L_1 and L_4/L_1	-0.75 -0.5 -0.25 0 0.25 0.5 0.75	2 3 4 5 6 7 8	A
		L_3/L_2 and $(L_4 - L_1)/L_2$	1.0	9	A
Total angle of swing of rocker		ϕ_{t4}	-1.0 -0.75 -0.5 -0.25 0 0.25 0.5 0.75	1 2 3 4 5 6 7 8	B
Slider stroke ratio		$\frac{x_t}{L_2}$	1.0	9	B

TABLE 1.2 List of Transparencies (Continued)

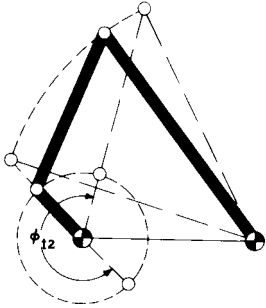
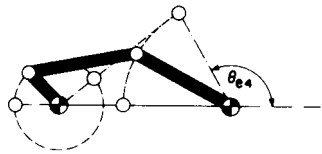
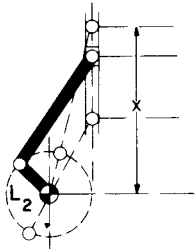
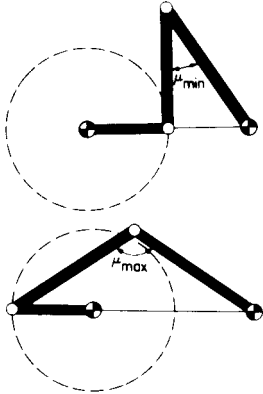
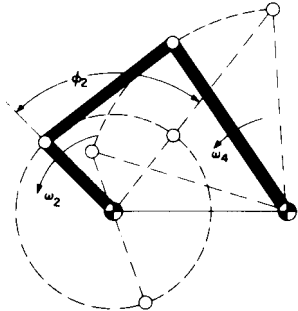
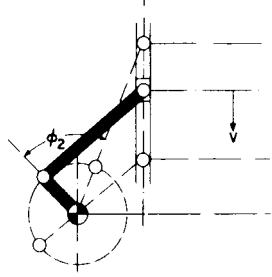
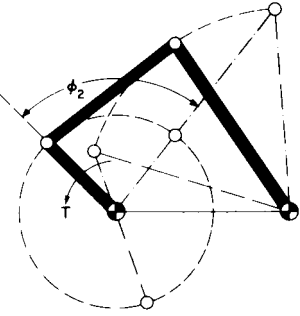
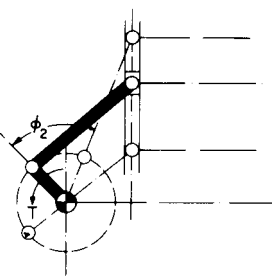
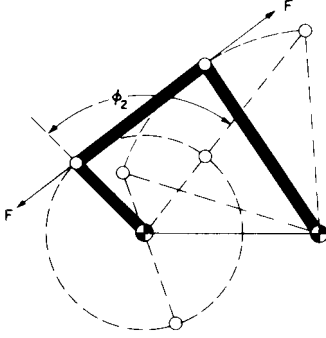
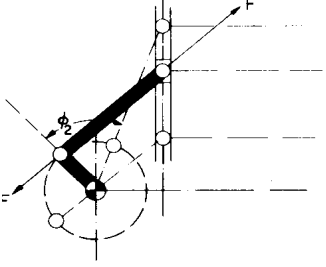
Description of Contours	Illustration	Properties	Value of λ_3	Transparency Pack Number	Property Identification Letter
Crank angle between dead-centres and return ratio		ϕ_{t2} and R	-1.0 -0.75 -0.5 -0.25 0 0.25 0.5 0.75 1.0	1 2 3 4 5 6 7 8 9	C
Rocker angle at extended dead-centre		θ_{e4}	-1.0 -0.75 -0.5 -0.25 0 0.25 0.5 0.75	1 2 3 4 5 6 7 8	D
Extension ratio		$\frac{X}{L_2}$	1.0	9	D
Extreme transmission angles		μ_{min} and μ_{max}	-1.0 -0.75 -0.5 -0.25 0 0.25 0.5 0.75 1.0	1 2 3 4 5 6 7 8 9	E
Maximum rocker velocity and corresponding crank angle		Ω and ϕ_2^{Ω}	-1.0 -0.75 -0.5 -0.25 0 0.25 0.5 0.75	1 2 3 4 5 6 7 8	F

TABLE 1.2 List of Transparencies (Continued)

Description of Contours	Illustration	Properties	Value of λ_3	Transparency Pack Number	Property Identification Letter
Maximum slider velocity and corresponding crank angle		v_{max} and ϕ_2^v	1.0	9	F
Maximum crank inertia torque and corresponding crank angle		T'_{max} and ϕ_2^T	-1.0 -0.75 -0.5 -0.25 0 0.25 0.5 0.75	1 2 3 4 5 6 7 8	G
		T'_{max} and ϕ_2^T	1.0	9	G
Maximum coupler end-load and corresponding crank angle		F'_{max} and ϕ_2^F	-1.0 -0.75 -0.5 -0.25 0 0.25 0.5 0.75	1 2 3 4 5 6 7 8	H
		F'_{max} and ϕ_2^F	1.0	9	H

2. NOTATION AND UNITS

In this notation units are given for both a familiar British system and for SI. It should be noted that the degree may be retained as a unit of angular position in SI.

		British	SI
a	acceleration of slider in slider-crank mechanism	in/s ²	m/s ²
c	diametral clearance in pivot	in	m
d	distance of coupler centre of gravity from crankpin	in	m
e	offset of slider-crank mechanism, given by $(L_4 - L_1)$	in	m
f	natural frequency of vibration of a link	c/s	Hz
F	coupler axial load (positive tensile as shown in Sketches 2.2 and 2.4)	lbf	N
F'	non-dimensional pin force coefficient in equations $F = (I\omega_2^2/L_1)F'$ and $F = m_s\omega_2^2L_2F'$		
I	effective moment of inertia of rocker and attached system about its fixed pivot (see Section 3.4)	[†] lbf in s ²	kg m ²
I_3	moment of inertia of coupler about its centre of gravity	[†] lbf in s ²	kg m ²
ΔI_3	difference between true and approximate coupler moment of inertia, $I_3 - I_{3e}$	[†] lbf in s ²	kg m ²
I_{3e}	moment of inertia of approximately equivalent system of two masses located at coupler pins (see Section 3.4)	[†] lbf in s ²	kg m ²
L_1	distance between fixed centres of crank and rocker	in	m
L_2	length of crank	in	m
L_3	length of coupler or connecting rod	in	m
L_4	length of rocker	in	m
m	mass of mechanism link and any attached mass	[†] lbf s ² /in	kg
m_s	effective mass of slider and any attached mass of slider-crank mechanism (see Section 3.4)	[†] lbf s ² /in	kg
P	coupler pin-force, positive inducing tension in and rotation of coupler in same sense as positive θ (see Sketch 3.9)	lbf	N
R	return ratio (see Section 3.2.2)		

For footnotes see end of Notation section.

T	crank torque due to rocker inertia (positive in sense illustrated in Sketches 2.2 and 2.4)	lbf in	N m
T'	non-dimensional crank torque coefficient in equations $T = I\omega_2^2 T'$ and $T = m_s \omega_2^2 L_2^2 T'$		
v	velocity of slider in slider-crank mechanism	in/s	m/s
x	distance of slider from extended-dead-centre position in slider-crank mechanism	in	m
X	distance of slider in extended-dead-centre position from normal to its line of motion through crank centre	in	m
α	angular acceleration (positive in sense illustrated in Sketches 2.2 and 2.4)	rad/s ²	rad/s ²
λ_1	linkage characteristic parameter (see Section 3.1) $= L_2/(L_3 + L_4 - L_1)$		
λ_2	linkage characteristic parameter (see Section 3.1) $= L_2/(L_1 - L_4 - L_3)$		
λ_3	linkage characteristic parameter (see Section 3.1) $= (L_4 - L_3)/L_1$		
θ	angle between link and line of centres (positive in sense illustrated in Sketches 2.2 and 2.4)	degree	degree
ϕ	angular rotation of link measured from extended-dead-centre position (positive in sense illustrated in Sketches 2.2 and 2.4)	degree	degree
μ	transmission angle, <i>i.e.</i> angle between coupler and rocker	degree	degree
ω	angular velocity (positive in sense illustrated in Sketches 2.2 and 2.4)	rad/s	rad/s
Ω	maximum velocity ratio, $(\omega_4/\omega_2)_{max}$		

Subscripts

1	appertains to frame
2	appertains to input crank
3	appertains to coupler (or connecting rod)
4	appertains to rocker (or slider)
<i>one</i>	appertains to loads due to inertia of rocker, neglecting coupler inertia

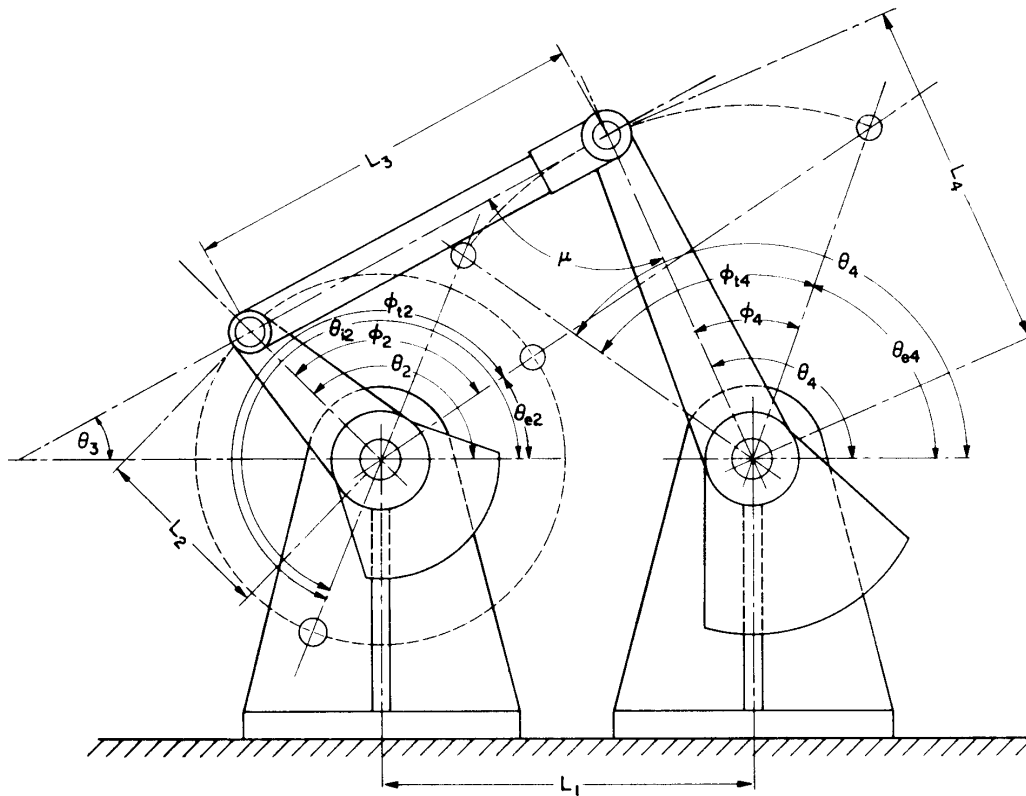
For footnotes see end of Notation section.

<i>two</i>	appertains to loads due to inertia of rocker and coupler (represented approximately by two masses located at coupler pins, connected rigidly together)
<i>three</i>	appertains to loads due to inertia of rocker and coupler
<i>backlash</i>	appertains to angular free play of link
<i>e</i>	appertains to extended-dead-centre position
<i>i</i>	appertains to contracted-dead-centre position
<i>max</i>	indicates maximum
<i>min</i>	indicates minimum
<i>n</i>	appertains to loads normal to coupler
<i>p</i>	appertains to loads parallel to coupler
<i>t</i>	appertains to difference between extended- and contracted-dead-centre positions

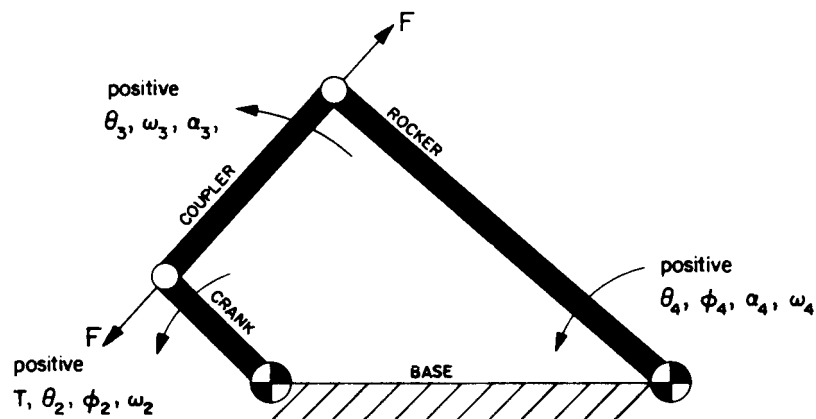
Superscripts

<i>F</i>	appertains to crank angle at which maximum coupler end-load occurs
<i>T</i>	appertains to crank angle at which maximum crank inertia-torque occurs
<i>V</i>	appertains to crank angle at which maximum slider velocity occurs
Ω	appertains to crank angle at which maximum velocity ratio, $(\omega_4/\omega_2)_{max}$, occurs

[†] A mass (or moment of inertia) value expressed in familiar British units as lb (or lb in²) has to be divided by 386 before it can be used in the dimensionless formulae in this Item. The resulting unit is the lbf s²/in (or lbf in s²). (A force of 1 lbf acting on a mass of 1 lb produces an acceleration of 386 in/s².)



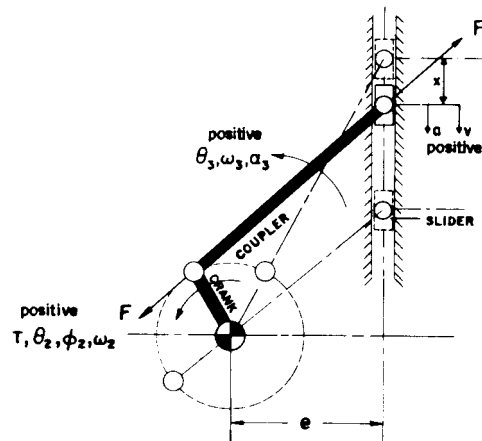
Sketch 2.1 Illustration of notation for crank-rocker mechanisms



Sketch 2.2 Crank-rocker mechanism sign convention for sense of rotation



Sketch 2.3 Illustration of notation for slider-crank mechanisms



Sketch 2.4 Slider-crank mechanism sign convention for sense of rotation

3. NOTES ON THE DATA AND THEIR USE

The data in this Item are presented in terms of contours of the required linkage property plotted in terms of linkage characteristic parameters λ_1 , λ_2 and λ_3 described in Section 3.1. The complete range of possible crank-rocker linkages is covered, with the exception of one class of double slider described in Section 4.5.

To use the data for synthesis, the contour for the chosen value (or maximum or minimum value) of each of the required properties may be traced on to one of the blank overlays provided. Alternatively, the need for tracing may be obviated by reading through a stack of transparencies directly. The required design is obtained from the intersections (where exact values are required) of the contours for two properties, or, more usually, from a point chosen within the area enclosed by contours of maximum or minimum acceptable values. A geometric design is thus obtained in terms of λ_1 , λ_2 and λ_3 . These parameters are converted into the requisite link lengths as explained in Section 3.1. This design procedure is illustrated in an example in Section 6.

3.1 Characteristic Parameters λ_1 , λ_2 and λ_3

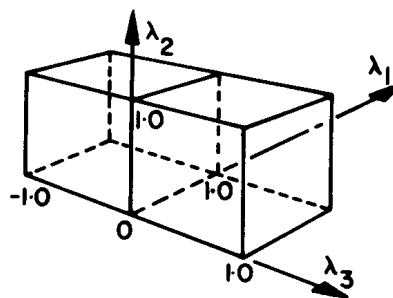
The characteristic parameters λ_1 , λ_2 and λ_3 are used as a convenient description of the geometry of the crank-rocker.

$$\lambda_1 = \frac{L_2}{L_3 + L_4 - L_1},$$

$$\lambda_2 = \frac{L_2}{L_1 - |L_4 - L_3|},$$

$$\lambda_3 = \frac{L_4 - L_3}{L_1}.$$

It is assumed that one of the link lengths (usually the centre-distance L_1) is known. The physical significance of the characteristic parameters may be observed from the “spectrum” of linkage mechanisms illustrated in Figures 1–9.



Sketch 3.1 The λ -cuboid

For crank-rockers, λ_1 always lies between 0 and 1.0, λ_2 between 0 and 1.0, and λ_3 between -1.0 and 1.0 . The range of the λ values is illustrated in cuboid form in Sketch 3.1. Values at or outside the cuboid boundaries, the significance of which is shown in Table 7.7, represent other classes of four-bar linkage, and impossible geometries.

Any combination of values of λ_1 , λ_2 and λ_3 which all lie between the limits represents a unique crank-rocker whose geometry can be found from the transparencies of Property A, or in exceptional cases analytically.

3.1.1 Evaluation of the link-lengths from the transparencies. Property A

The transparencies of Property A give contours of various ratios between the link lengths, in particular L_2/L_1 , L_3/L_1 and L_4/L_1 .

The boundaries of the diagrams, where $\lambda_3 = \pm 1.0$ and where $\lambda_1 = 0$, correspond with various types of mechanism which have some links of infinite length. These types of mechanism can only be defined in terms of other ratios.

Thus the offset swinging block mechanisms characterised by $\lambda_1 = 0$ are definable in terms of the ratio L_2/L_1 , and the ratio of the offset of the block to the distance between centres *i.e.* $(L_4 - L_3)/L_1$, or λ_3 , and these values may be obtained from the transparencies of Property A in packs 2–8.

The parallel-coupler mechanisms characterised by $\lambda_3 = -1.0$ are definable in terms of L_4/L_2 and $(L_3 - L_1)/L_2$ and the contours for these ratios are plotted in the transparency of Property A in pack 1.

Similarly the offset slider-cranks characterised by $\lambda_3 = 1.0$ are definable in terms of L_3/L_2 and the ratio of the offset of the slider to the crank length, which can be shown to be equivalent to $(L_4 - L_1)/L_2$. The contours for these ratios are plotted in the transparency of Property A in pack 9.

3.1.2 Analytical evaluation of link lengths

Analytical expressions for the link lengths are given in Table 7.1. These expressions are normally only required when exceptional precision is necessary, for use in computer-based procedures, or to avoid interpolation between charts for intermediate values of λ_3 .

3.2 Important Angles Subtended by the Links

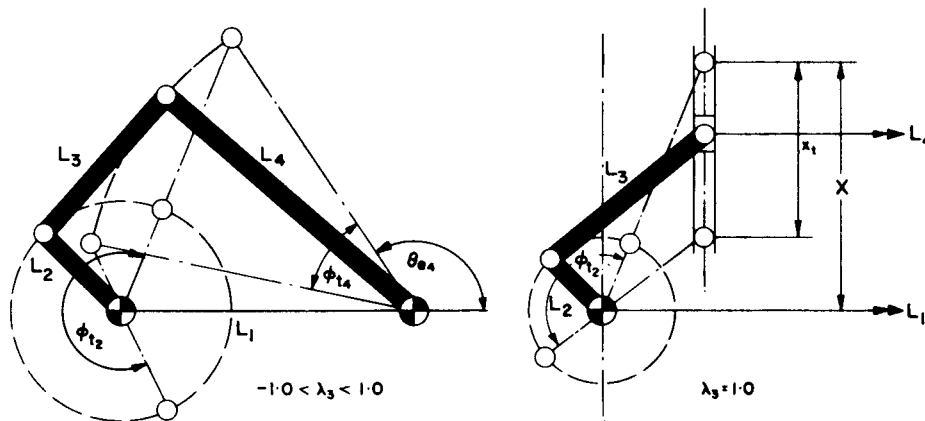
So that the angles discussed in Sections 3.2.1 to 3.2.4 are measured consistently from a fixed datum, it is necessary that all crank-rocker and slider-crank configurations are considered in the same way as that used in preparing the data. Sketches 2.1 and 2.3 show the orientation which must be adopted. The mechanism is always viewed with the base crank-pin on the left-hand side, and the rocker or slider above the base line. When $\lambda_1 = 0$, there exists a form of quick-return mechanism (see Figures 2–8) which again must be viewed with the base crank-pin on the left-hand side. In this case, the sign of λ_3 dictates the position of the swinging link with respect to the base line (see Table 7.6).

3.2.1 The angle ϕ_{t4} , total angle of swing of the rocker. Property B

Contours for the total angle of swing are given in the transparencies of Property B. This angle is usually the most important of the required output properties. When $\lambda_3 = 1.0$, ϕ_{t4} is always zero because L_4 and L_1 are infinite. However, in this case which applies to offset slider-cranks, the length of the slider stroke, x_t , is the corresponding significant property. Values of x_t/L_2 are, therefore, given in the transparency of Property B in pack 9. The angle ϕ_{t4} , and stroke, x_t , are shown in Sketch 3.2.

3.2.2 The crank angle between dead-centres, ϕ_{t2} , and return ratio R . Property C

Contours for values of the angle ϕ_{t2} are given in the transparencies of Property C. This angle may be regarded as a measure of the asymmetry of the rocker motion; when ϕ_{t2} , which is shown in Sketch 3.2, is 180° the forward and return strokes of the rocker are of equal duration. Values of ϕ_{t2} much greater or less than 180° give simple “quick-return” mechanisms.



Sketch 3.2

The degrees of asymmetry of the rocker motion are sometimes more conveniently described in terms of the return ratio. The return ratio, R , is the ratio to the remainder of the crank revolution of that part during which the rocker rotates in the opposite sense to the crank, defined by

$$R = \frac{360^\circ - \phi_{t2}}{\phi_{t2}}$$

and the inverse of this expression gives the mean velocity ratio. When the crank angular velocity is constant, the return ratio is equal to the average return angular velocity divided by the average approach velocity of the rocker or slider.

For parallel-coupler mechanisms, where $\lambda_3 = -1.0$, ϕ_{12} is 180° and R is unity for all cases.

3.2.3 The angle $\theta_{\rho 4}$, rocker angle at extended-dead-centre. Property D

Contours for this angle are given in the transparencies of Property D. This angle is often significant where space limitations restrict the outward swing of the rocker. It also affects the magnitude of the backlash, see Section 4.7.

It is occasionally required for converting rocker angles measured from the extended-dead-centre position datum to line-of-centres datum using the expression

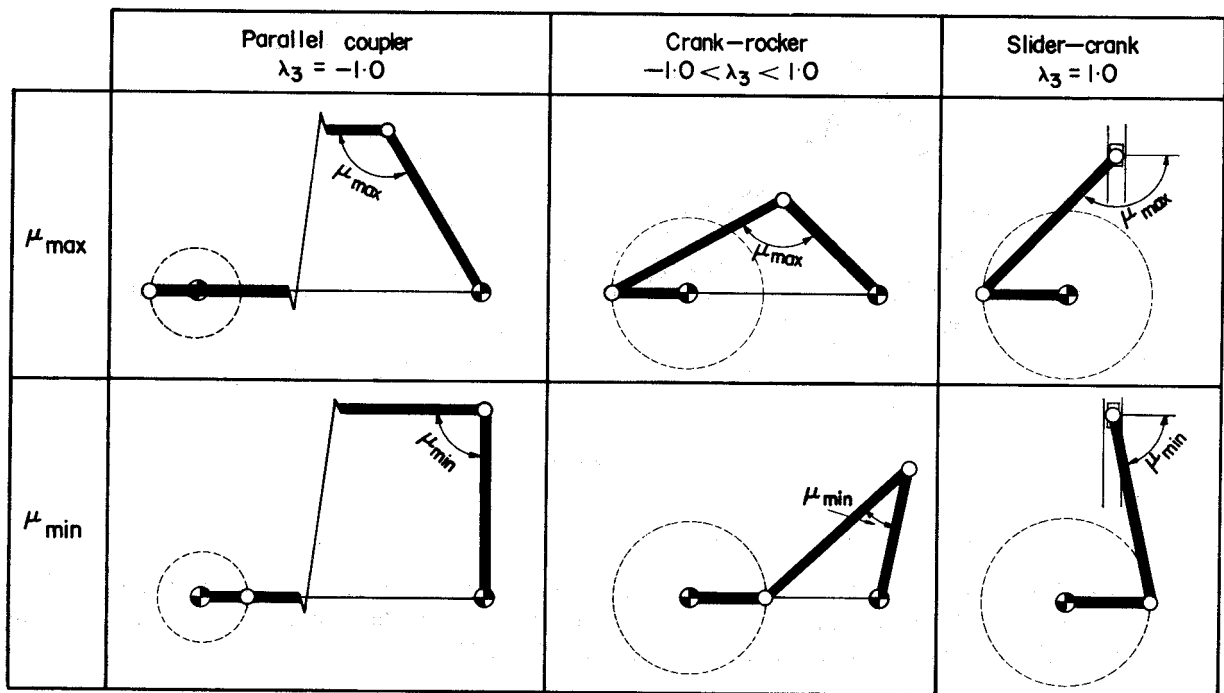
$$\theta_4 = \phi_4 + \theta_{e4}.$$

When $\lambda_3 = 1.0$, θ_{e4} is nominally 180° . However in this case, which applies to offset slider-crank, the corresponding significant property is extension, X . Contours of X/L_2 are therefore given in the transparency of Property D in pack 9. The angle, θ_{e4} , and extension, X , are shown in Sketch 3.2.

3.2.4 The angles μ_{min} and μ_{max} , extreme transmission angle. Property E

Contours of extreme transmission angle are given in the transparencies of Property E. The full, or critical, lines indicate which values of μ_{min} or μ_{max} are furthest removed from 90° . Transmission angle is a rough guide to the quality of the transmission of a linkage. To avoid jamming under starting or low-speed

operation, jerky and noisy operation with high loads, and excessive backlash, the transmission angle should deviate as little as possible from 90° during the cycle of the linkage.



Sketch 3.3

The effects of play in the joints or errors in the link lengths are particularly important under poor transmission angle conditions (see also Section 4.7). The extreme transmission angle occurs when the mechanism is in one of the two positions illustrated in Sketch 3.3.

For most normal applications in which there is little friction in the joints, experience suggests that it is satisfactory to work within the ranges of extreme transmission angles indicated in the following table, without further analysis.

Application	Extreme Transmission Angle
Low speeds, low loads (inertia forces and elastic deformations negligible)	$> 30^\circ$ $< 150^\circ$
Moderate speeds and/or loads (inertia forces and elastic deformations not negligible)	$> 45^\circ$ $< 135^\circ$
High speeds and/or loads (inertia forces and elastic deformations dominant considerations)	$> 60^\circ$ $< 120^\circ$

Satisfactory designs having extreme transmission angles outside these ranges are possible, but in such cases the crank-torque, pin-force and backlash conditions need to be considered in detail, see Sections 3.5 to 3.7 and 4.7. In some applications springs or coupled linkages can be used to overcome transmission angle problems.

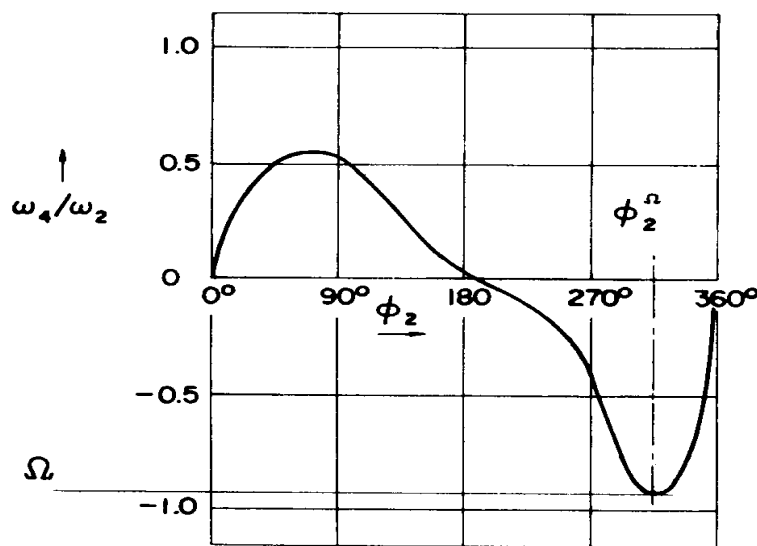
3.3 Maximum Rocker Velocity, $(\omega_4)_{max}$, and Corresponding Crank Angle, ϕ_2^Ω . Property F

Contours of maximum rocker velocity, and corresponding crank angle are given in the transparencies of Property F. The velocities are given in the non-dimensional form $(\omega_4/\omega_2)_{max}$ which is denoted by Ω .

The maximum velocity ratio is an appropriate transmission criterion when the predominant resistance to rocker motion is rocker torque that is either constant (e.g. dry friction), or proportional to rocker velocity (e.g. viscous drag). Thus, neglecting other types of rocker or coupler resistance such as inertia, for equilibrium

$$\text{crank torque} = (\omega_4/\omega_2) \cdot \text{rocker torque}.$$

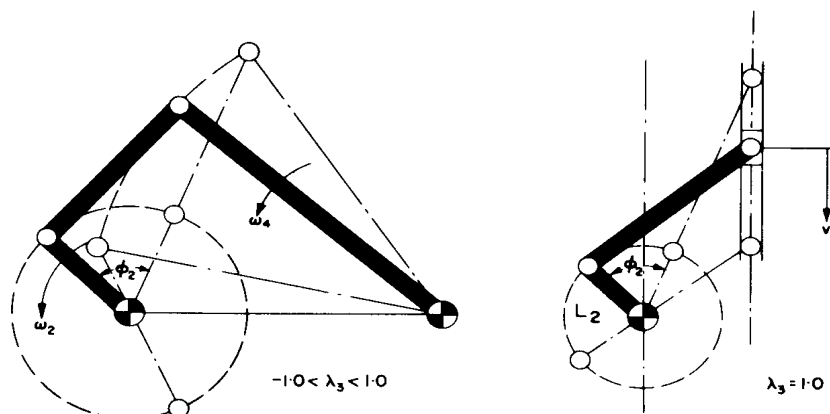
For constant crank angular velocity, the rocker angular velocity varies typically as shown in Sketch 3.4. The numerically greater value of the positive and the negative peaks is taken as Ω .



Sketch 3.4

When $\lambda_3 = 1.0$, ω_4 is always zero. However, in this case, applying to offset slider-cranks, the corresponding significant property is the slider linear velocity. Contours of $(v/L_2\omega_2)_{max}$ and corresponding crank angle, ϕ_2^V , are therefore given in the transparency of Property F in pack 9.

The crank and rocker angular velocities, ω_2 and ω_4 , the slider linear velocity, v , and the corresponding crank angles, ϕ_2 , are shown in Sketch 3.5.



Sketch 3.5

3.4 Effective Mass and Inertia

A proportion of the mass of the coupler must be included in the moment of inertia of the rocker or the mass of the slider when calculating inertia torques, coupler end-loads and pin-forces as described in Sections 3.5, 3.6 and 3.7. The distributed mass of the coupler can conveniently be assumed to be replaced by an approximately equivalent system consisting of two concentrated masses connected rigidly together, located at the coupler pins. Equations and methods for determining these equivalent masses are given here. The resultant torques and loads to which the mechanism is subjected may be calculated to three different degrees of precision (Idealisations One, Two and Three) using the expressions in Table 7.2.

3.4.1 Idealisation One

The inertia of the coupler is neglected. The loads are assumed to be due to rocker inertia only.

3.4.2 Idealisation Two

The inertia of the coupler is taken into account by an approximate dynamic equivalent system of two masses m_{23} and m_{34} located at the coupler pins. The masses are chosen such that their total mass m and the location of their mutual centre of gravity are the same as those of the real coupler; that is to say that

$$m_{23} = \frac{m_3(L_3 - d)}{L_3} \text{ and } m_{34} = \frac{m_3 d}{L_3}$$

where d is the distance of the coupler centre of gravity from the pin 23 (see Sketch 6.10).

If the total mass, location of the centre of gravity, and moment of inertia of the equivalent system are equal to those of the coupler, the equivalent system is exact. Otherwise, errors will occur whose magnitudes can be estimated from the equations for Idealisation Three in Table 7.2, see Section 3.4.3. For a coupler with its mass concentrated near the pins (which is usual) the errors are small.

3.4.3 Idealisation Three

The inertia of the coupler is taken fully into account by including the effect of the difference between the moment of inertia of the real coupler and that of the approximate system in Section 3.4.2; that is to say that

$$\Delta I_3 = I_3 - I_{3e} = I_3 - md(L_3 - d) .$$

3.5 Maximum Inertia Torque, T_{max} , and Corresponding Crank Angle, ϕ_2^T . Property G

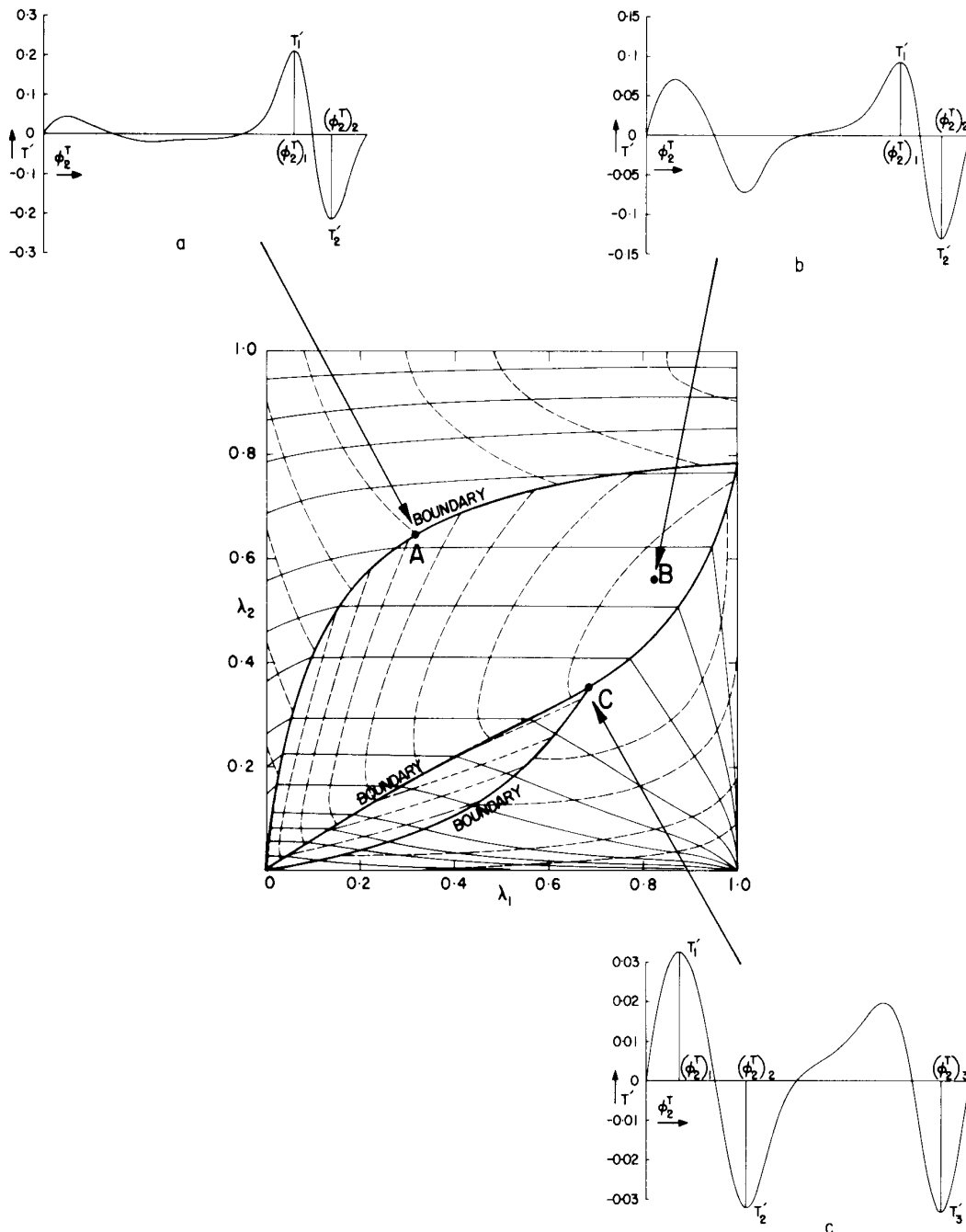
Contours of maximum crank torque due to rocker inertia, T'_{max} , and corresponding crank angle are given in the transparencies of Property G. The inertia torque, T , is the torque required at the crank to maintain constant crank velocity against inertial resistance of the rocker and any mass attached to it (see Section 3.4). Thus T_{max} is an appropriate criterion of transmission quality when (as is often the case) the predominant resistance to motion is effective rocker inertia. The effects of other forces and couples on the linkage (for example those due to friction in the joints, gravity, etc.) must be treated separately. The effect of the rotational inertia of the coupler is not completely accounted for in T_{max} which corresponds with Idealisation One. The magnitude of the resultant error may be estimated using Idealisations Two or Three in Table 7.2, according to the degree of precision required.

The contours are presented in terms of the non-dimensional coefficient T'_{max} which is equal to $(\omega_4 \alpha_4 / \omega_2^3)_{max}$.

For constant crank velocity ω_2 , $T = I\omega_2^2 T'$.

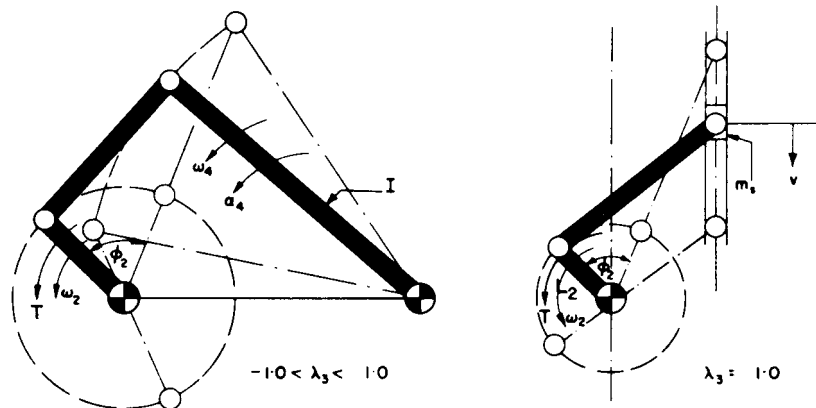
The sense of T , together with its related variables, is shown for crank-rockers and slider-cranks in Sketch 3.7.

For positive values of λ_3 , the contours are nested in regions as shown in Sketch 3.6. (Compare with the transparencies of Property G, packs 2, 3 and 4.) A change in sign or direction of the T'_{max} contours and an abrupt change in magnitude and direction of the ϕ_2^T contours occurs at the boundaries of the regions. These transitions are exemplified in Sketch 3.6 at point A which lies on the boundary of the upper region. From Sketch 3.6a it can be seen that on the boundary the peak positive torque T'_1 is equal in magnitude to the peak negative torque T'_2 , that is, $|T'_1| = |T'_2|$. Should the mechanism link lengths be changed, thereby altering the values of λ_1 and λ_2 to correspond with, say position B, $|T'_2|$ exceeds $|T'_1|$, and T'_2 occurs at a new angle $(\phi_2^T)_2$. See Sketch 3.6b.



Sketch 3.6

A further example at point C shows that here there are three values of T'_{max} indicated in Sketch 3.6c. Now $|T'_1| = |T'_2| = |T'_3|$ at angles $(\phi_2^T)_1$, $(\phi_2^T)_2$, and $(\phi_2^T)_3$ respectively.



Sketch 3.7

It should be noted that changes in the length L_4 have a very strong effect on the value of I , and consequently T , (unless the contribution of the rocker itself to I is small). Accordingly, caution should be exercised in designing for a minimum value of T if as a consequence L_4 is increased. See example, Section 6.1.5.

For $\lambda_3 = 1.0$, representing slider-crank mechanisms, T' is defined as $(av/L_2^2\omega_2^3)$ and contours of T'_{max} for this case are given in the transparency of Property G in pack 9.

For constant crank velocity ω_2 , $T = m_s\omega_2^2L_2^2T'$.

3.6 Maximum Coupler End-Load, F_{max} , and Corresponding Crank Angle, ϕ_2^T . Property H

Contours of maximum coupler end-load due to rocker inertia, F'_{max} , and corresponding crank angle are given in the transparencies of Property H. The end-load F is the force transmitted through the coupler to maintain constant crank angular velocity against the inertial resistance of the rocker and any mass attached to it (see Section 3.4). The value of F'_{max} is an alternative criterion to T'_{max} when coupler bearing loads or compressive loads are likely to be more critical in the design than crank torque. The effects of coupler inertia may be included as indicated in Section 3.4.

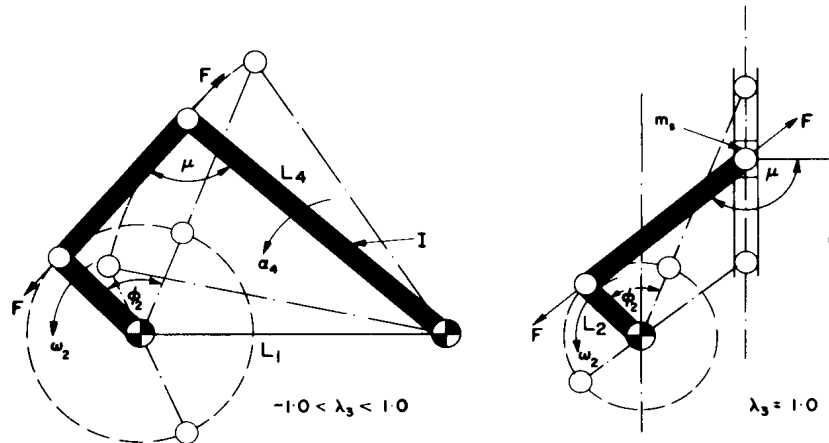
The contours are presented in terms of the non-dimensional coefficient F'_{max} which is given by the expression $(L_1\alpha_4/\omega_2^2L_4\sin\mu)_{max}$ for crank-rockers. When $\lambda_3 = -1.0$, representing parallel-coupler mechanisms, F'_{max} is equal to $(L_2\alpha_4/\omega_2^2L_4\sin\mu)_{max}$.

For constant crank velocity ω_2 , $F = (I\omega_2^2/L_1)F'$ for crank-rockers, and $F = (I\omega_2^2/L_2)F'$ for parallel-coupler mechanisms.

The sense of F , together with its related variables, is shown for crank-rockers and slider-cranks in Sketch 3.8.

The F'_{max} contours (transparencies of Property H, packs 6, 7 and 8) are nested in regions in a similar manner to that of the T'_{max} graphs. The description of the contour behaviour in the proximity of the region boundaries, given in Section 3.5, is applicable to F'_{max} and ϕ_2^F .

It should be noted that changes in the length L_4 have a very strong effect on the value of I and consequently F , (unless the contribution of the rocker itself to I is small). Accordingly, caution should be exercised in designing for a minimum value of F if as a consequence L_4 is increased. See example, Section 6.1.5.



Sketch 3.8

For $\lambda_3 = 1.0$, representing slider-crank mechanisms, F' is defined as $(a/\omega_2^2 L_2 \sin \mu)$ and contours of F'_{max} for this case are given in the transparency of Property H in pack 9.

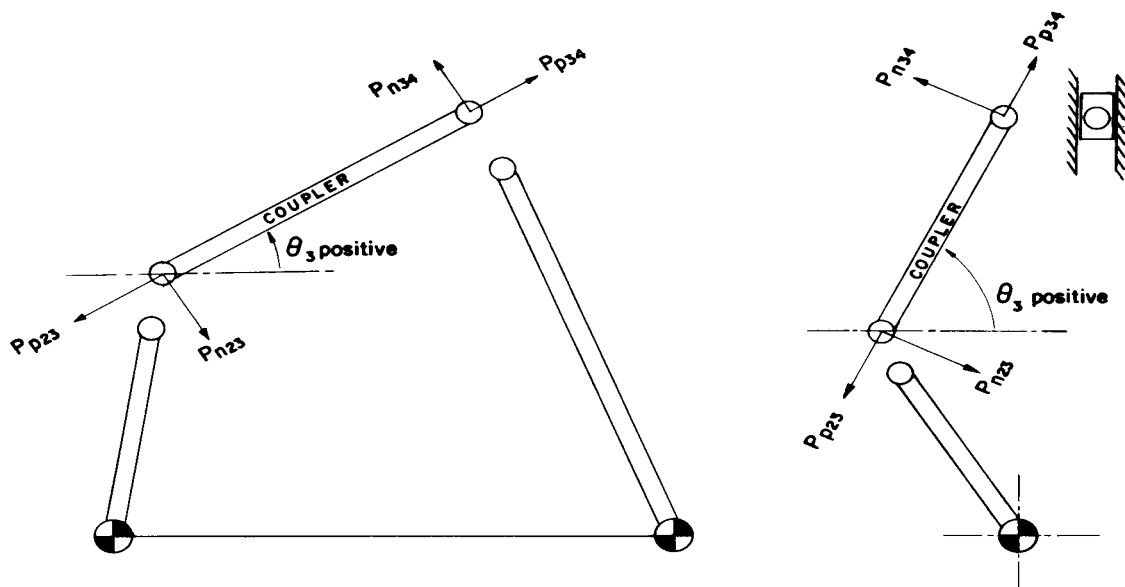
For constant crank velocity ω_2 , $F = m_3 \omega_2^2 L_2 F'$.

3.7 Maximum Coupler Pin-force

The coupler pin-forces can be assumed to be equal to the end-loads in the coupler if the inertia effects of the coupler are negligible compared with those of the rocker and the system to which it is attached.

The pin-forces can be calculated from the equations in Table 7.2. It should be noted that maximum end-load and maximum pin-force do not necessarily occur at the same crank angle if the contribution of coupler inertia to the pin-force is a significant proportion of the total.

The pin-forces normal to and parallel with the coupler are shown in a positive sense in Sketch 3.9.



Sketch 3.9

4. NOTES ON DESIGN

A detailed consideration of other than the kinematics and rigid-body dynamics of the linkage is beyond the scope of this Item. However in the following sections some general principles of design are briefly described. For further information see the References and Derivation, Section 5.

4.1 Link Materials

To minimise out-of-balance forces and at the same time to reduce errors due to deformations, it is desirable to use materials of low density and relatively high modulus for the coupler and rocker, particularly in high speed applications. The use of aluminium or magnesium alloys is sometimes justified on these grounds, subject to satisfactory fatigue life.

In the case of the crank, a flywheel effect is often required to minimise velocity variations so that a relatively high density material is appropriate.

4.2 Balancing

The need to balance the inertia forces and couples in the linkage should be considered in all but low speed applications. Complete balancing of the four-bar linkage is generally uneconomic and partial balancing is usually employed. Description of methods of balancing is beyond the scope of this Item, and users are referred to References 2, 3, 4 and 5.

4.3 Force Analysis

The need to predict the forces and bending moments in the linkage should be considered. Under low speed, high load, unfavourable transmission angle conditions, particularly when pin diameters are large and joint friction is high, the concepts of friction circle and friction axis explained in most elementary books on theory of machines are relevant provided that the friction torque is proportional to the transmitted load.

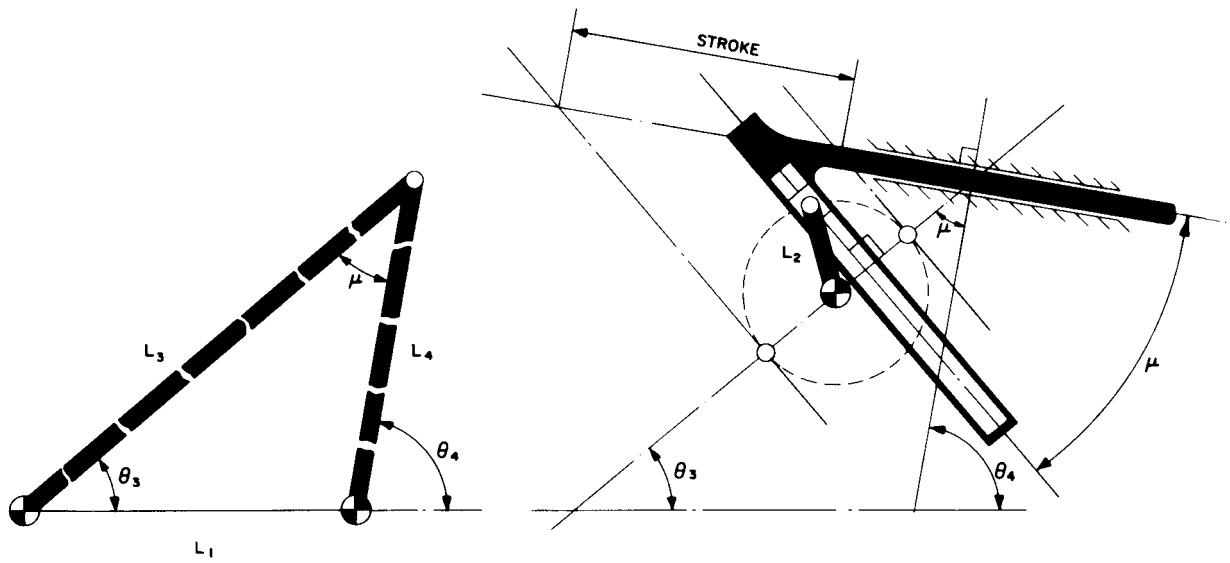
At high speed, when inertia loads predominate, see Sections 3.4 to 3.6.

4.4 Short Crank Lengths

When the required crank length is short compared with the size of the bearing needed, it is often advantageous to replace the crank by an eccentric as illustrated for low values of λ_2 in Figures 2 to 8.

4.5 Long Link Lengths

When the required link length is very long, excessive inertia may be avoided by replacing the link by a curved slide of radius equal to the link length. When one or more of the required link lengths is infinite ($\lambda_1 = 0$, or $\lambda_3 = \pm 1.0$) the crank-rocker becomes one kind or other of slider-crank. When λ_2 is zero and λ_1 is finite all link lengths except the crank are infinite, and the links are co-linear representing geometries with no practical applications (although for geometries closely approaching this condition there are applications for the high amplification of crank motion obtained, *e.g.* in Moiré Fringe measuring techniques). However a class of practical linkages does exist at the point where $\lambda_1 = 0$ and $\lambda_2 = 0$, which is not covered in this Item. In these cases all links except the crank are infinite, forming a triangle which, using the method of this Item, could be defined by λ_3 and $L_1/(L_3 + L_4)$. Consideration of the conditions at the joint between the coupler and rocker shows that the mechanism can be represented by a scotch yoke in which the slides are at an angle equal to that between the coupler and rocker, see Sketch 4.1. Analysis of this is simple enough not to warrant a set of graphs to cover this particular case.



Sketch 4.1

4.6 Vibration

Two types of vibration may occur which can be considered separately. Firstly the coupler may vibrate in bending under periodic end-loads due to rocker resistance. This can be avoided by making sure the natural frequency of the coupler in bending is well removed from low half-integer multiples of the crank speed, *i.e.* by avoiding the condition $f_3 \approx (1/2\pi)(n\omega_2/2)$ where n is an integer. The denominator of 2 in the right-hand side is due to the forcing load being longitudinal. Serious vibration is unlikely with n greater than 4. This mode of vibration is important only in configurations with long, flexible couplers.

Secondly the system of which the linkage is part may vibrate as a whole; in this mode the bending flexibilities of the crank and rocker and the torsional flexibility of the crank and rocker shafts may be important. The system is non-linear and possible loading conditions are complicated. Analysis of the conditions is beyond the scope of this Item, but for guidance the user is referred to Reference 3 where the need for information on the mean value and the range of values of linkage stiffness is explained. This information can be derived from References 1 and 2 but in the case of crank-rockers it may be necessary to take into account the part of the cycle when θ_2 is between 180° and 360° which is not considered explicitly in the references.

With either type of vibration the probability of excitation of resonances can be reduced by ensuring, through suitable geometric design, that higher harmonics of the rocker motion are minimised. The following conditions have been suggested in Derivation 7 for this purpose:

L_2/L_1 as small as possible (*i.e.* λ_2 as small as possible),

$L_3 = L_4$ (*i.e.* $\lambda_3 = 0$)

and $L_1^2 + L_2^2 = L_3^2 + L_4^2$.

These conditions coincide with the requirements for extreme transmission angle closest to 90° and a return ratio of unity.