

DESIGN OF DISC CAMS WITH VARIOUS FOLLOWERS: DERIVATION OF KINEMATIC EQUATIONS

1. NOTATION

Any coherent set of units may be used in conjunction with this Guide. The dimensional parameters in this notation have their units given for the SI system. All angles are measured positively in the anti-clockwise manner outlined in Data Item 95001 (Reference 8). Derivatives with respect to time are indicated with superior dot notation; derivatives with respect to input rotation, u , are indicated with prime notation.

		Units
A	calculation parameter, given by H/Φ for translating followers and Ψ/Φ for swinging followers	–
a	distance along X_m -axis from cam centre, O, to point of intersection, P, of contact normal with X_m -axis (see Sketch 3.1)	m
B	offset of cranked follower arm, measured in positive sense above X_m -axis	m
b, c, d, e	local length notation used in Sections 5 and 6	m
$\dot{b}$	linear velocity notation used in Section 5	m/s
D	distance from cam centre, O, to pivot point of swinging arm, measured in positive sense to right of cam centre	m
E	offset of follower line-of-action from cam centre, O, measured in positive sense to right of cam	m
F_E	external force acting on follower	N
$f\{\dots\}$	function of $\{\dots\}$	–
F_{xi}, F_{yi}	components of resultant external force acting on part i in x - and y -directions, respectively	N
g, h, i, j, l	local length notation used in Sections 3 to 6	m
H	total follower lift or fall associated with one motion segment	m
H_i	total follower lift or return associated with segment i	m
I_f	mass moment of inertia of follower system about follower pivot	kg m ²
k	constant of value +1 or –1 taking the sign of D	–
k_{SC}	linear spring rate	N/m

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k_{ST}	torsion spring rate	N m/rad
L	distance between follower arm pivot and roller centre	m
m	distance between velocity vectors v_p and v_{cr}	m
m_f	mass of follower	kg
n	local length notation used in Section 6	m
O	centre of cam rotation and origin of co-ordinate axis system	–
P	point at which contact normal cuts X_m -axis	–
P_C	contact force between follower and cam normal to cam profile	N
P_{ij}	force exerted by part i on part j	N
P_{xij}, P_{yij}	components of joint force between parts i and j , P_{ij} , in x- and y-directions, respectively	N
P_S	linear spring force	N
P_{SP}	linear spring preload	N
p, q, r, t	local length notation used in Sections 3 to 6	m
R_a	prime circle radius of cam	m
$(R_a)_i$	‘notional’ prime circle radius of cam used for pressure angle and curvature computation for segment i	m
R_b	reference circle radius of cam	m
$(R_b)_i$	‘notional’ reference circle radius of cam for segment i	m
R_f	radius of domed follower	m
R_g	radius of cutter or grinding wheel	m
R_r	radius of roller follower	m
S	point of contact between cam and follower	–
$\dot{s}$	linear velocity of contact point relative to cam	m/s
t	time	s
T_S	torsion spring moment	N m/rad
T_{SP}	torsion spring preload	N m/rad
T_{ij}	moment exerted by part i on part j	N m
u	normalised input (cam rotation)	–

u_v, w_v	value of u and w , respectively, at blending point	—
$\dot{u}$	normalised input velocity	—
v_{cr}	linear velocity of roller centre or of contact normal	m/s
v_d	linear velocity of point P along X_m -axis	m/s
v_p	instantaneous linear velocity of point P	m/s
v_t	tangential velocity of swinging arm at point S	m/s
w	normalised output displacement of follower	—
w'	non-dimensional output velocity of follower	—
w''	non-dimensional output acceleration of follower	—
X_c, Y_c	axes fixed in, and rotating with, cam with origin at point O	—
x_g, y_g	co-ordinates of cutter or grinding wheel centre in X_c, Y_c axis system	m
X_m, Y_m	axes fixed in machine, with origin at point O	—
x_p, y_p	co-ordinates of nominal point of contact in X_c, Y_c axis system	m
y_b	least height of reference point on follower above X_m -axis during complete cam cycle	m
y_m	height of centre of roller follower, centre of follower radius or point of intersection of flat-faced follower with line-of-action, above X_m -axis	m
$(y_m)_i$	displacement of reference point on follower from X_m -axis for segment i	m
$\dot{y}_m$	velocity of translating follower	m/s
$(y_r)_i$	least height of reference point on follower above X_m -axis for segment i	m
α	pressure angle	rad
α	constant pressure angle associated with translating flat-faced followers	rad
$(\alpha_d)_i$	pressure angle during dwell for domed or roller follower for segment i	rad
β	angle between line-of-action and X_c -axis	rad
$\dot{\beta}$	angular velocity of line-of-action relative to cam	rad/s

γ	angle between contact tangent and X_c -axis	rad
$\dot{\gamma}$	angular velocity of contact tangent	rad/s
η	local angle notation used in Section 6	rad
ρ	radius of curvature of pitch curve	m
ρ_p	radius of curvature of cam profile	m
Φ	cam angle of rotation associated with one motion segment	rad
Φ_i	total angle of cam rotation for segment i corresponding to lift H_i or Ψ_i as appropriate	
ϕ	cam angle of rotation, measured from X_m -axis to X_c -axis, in anti-clockwise sense	rad
$\dot{\phi}$	angular velocity of cam, rotating about point O	rad/s
$\ddot{\phi}$	angular acceleration of cam	rad/s ²
$(\phi_n)_i$	defined by Equations (B3.14) and (B4.12)	rad
$(\phi_0)_i$	cam angle of rotation at beginning of segment i	rad
Ψ	total angle of follower swing associated with one motion segment	rad
Ψ_i	total angle of follower swing associated with segment i	rad
ψ	angle between X_m -axis and follower arm	rad
$\dot{\psi}$	angular velocity of swinging arm	rad/s
$\ddot{\psi}$	angular acceleration of swinging arm	rad/s ²
ψ_b	least value of ψ from X_m -axis to follower during complete cam cycle	rad
$(\psi_m)_i$	displacement of reference point on follower from X_m -axis for segment i	rad
$(\psi_r)_i$	least angle of follower reference point from X_m -axis for segment i	rad
ω	angular velocity of cam	rad/s
ω_{fm}	angular velocity of follower relative to machine	rad/s
ω_{mc}	angular velocity of machine relative to cam	rad/s

Subscripts

- 0 used to indicate value at start of segment
- 1, 2, 3, ... denote segment numbers

If further explanation of the notation is required, refer to the notation and sketches of Data Item 95001.

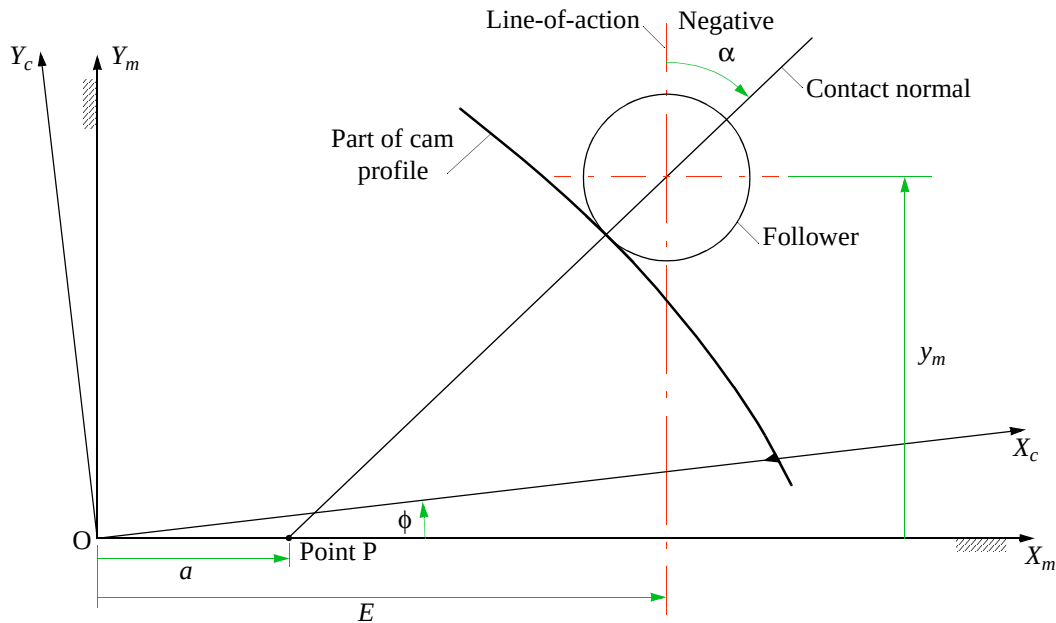
2. INTRODUCTORY NOTES

This Guide provides a record of the derivation of the basic kinematic relationships used in the design of disc cams. It was prepared in response to requests from industrial and academic users for the derivation of the equations contained in Data Item 95001, Reference 8 (which superseded Data Items 79007, 79021, 79023 and 80022).

Derivations of pressure angle, cam profile co-ordinates, cutter centre co-ordinates and cam profile curvature radii are given for the cases of a disc cam with a translating follower (domed, flat-faced or roller) or a disc cam with a swinging follower (domed, flat-faced or roller). If a domed follower is used the equations are the same as those for a roller follower except the follower radius should be used in place of the roller radius. The equations have been derived using conventional geometrical techniques to ensure that they are understood by a wide audience.

Although similar to a Data Item this guide is different in that it does not impart design guidance directly. It should be regarded as background information to the Data Items on cam design in the Mechanisms Series.

3. TRANSLATING ROLLER FOLLOWERS



Sketch 3.1

3.1 Pressure Angle

From Sketch 3.1 it can be seen that

$$\tan(-\alpha) = \frac{E - a}{y_m},$$

where from Equation (A1.3) in Appendix A

$$a = \dot{y}_m / \dot{\phi}$$

and from Equation (A2.4) in the same appendix

$$\frac{\dot{y}_m}{\dot{\phi}} = w' \frac{H}{\Phi} \equiv w' A,$$

so that

$$\tan(-\alpha) = \frac{E - w' A}{y_m},$$

which leads to a solution for α when the pressure angle is negative (all angles are measured positively in an anticlockwise sense, and the pressure angle is always measured from the line-of-action to the contact normal). As $\tan(-\alpha) = -\tan \alpha$, then

$$\alpha = \tan^{-1} \left[\frac{Aw' - E}{y_m} \right]. \quad (3.1)$$

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where $h = y_m \sin\left(\frac{\pi}{2} - \phi\right) = y_m \cos \phi$,

and $j = R_r \cos(\phi - \alpha)$.

3.3 Cutter Centre Co-ordinates

3.4 Pitch Profile and Cam Profile Curvature Radii



In order to derive the curvature radii, the concept of inversion of the machine is used; quite simply, this calls for interchanging the fixed and rotating axes sets so that the machine frame rotates about the stationary cam. The angular velocity of the contact tangent relative to the cam is $\dot{\gamma}$, and this is equal to the angular velocity of the contact normal, again relative to the cam. The angular velocity of the contact normal has two components, the first of which is the motion of the follower relative to the machine, ω_{fm} , and the second of which is the “rotation of the machine” relative to the cam, ω_{mc} .

Thus,
$$\dot{\gamma} = \omega_{fm} + \omega_{mc}. \quad (3.6)$$

The angular velocity of the follower, relative to the machine, is given by

$$\omega_{fm} = (v_{cr} - v_p) / m, \quad (3.7)$$

where m is the distance from the point of intersection of the contact normal with the X_m -axis to the roller centre. The terms v_{cr} and v_p are the velocities of the roller centre and point P respectively.

An infinitesimal increase in angle ϕ produces a corresponding infinitesimal decrease in angle β , so that

$$\delta\phi = -\delta\beta$$

and thus
$$\dot{\phi} = -\dot{\beta} \equiv -\omega_{mc}. \quad (3.8)$$

The component of velocity of the roller centre parallel to the contact tangent is

$$v_{cr} = -\dot{y}_m \sin \alpha,$$

which, substituting for $\dot{y}_m$ from Equation (A2.4), yields

$$v_{cr} = -\dot{\phi} w' \frac{H}{\Phi} \sin \alpha. \quad (3.9)$$

The component of velocity of point P parallel to the contact tangent is given by

$$v_p = -\dot{a} \cos \alpha,$$

which, substituting for $\dot{a}$ from Equation (A2.7), yields

$$v_p = -\dot{\phi} w'' \frac{H}{\Phi^2} \cos \alpha. \quad (3.10)$$

From Sketch 3.3, it can be seen that

$$y_m = m \cos(-\alpha)$$

so that
$$m = y_m \sec \alpha. \quad (3.11)$$

Substitution of Equations (3.9), (3.10) and (3.11) in (3.7) leads to

$$\omega_{fm} = \frac{-\dot{\phi} w' \frac{H}{\Phi} \sin \alpha + \dot{\phi} w'' \frac{H}{\Phi^2} \cos \alpha}{y_m \sec \alpha}. \quad (3.12)$$

The component of velocity of the contact due to “rotation of the machine” relative to the cam is given by ω_{mc} , where

$$\omega_{mc} = -\dot{\phi}$$

and substitution of this and Equation (3.12) in Equation (3.6) yields

$$\dot{\gamma} = \frac{-\dot{\phi} w' \frac{H}{\Phi} \sin \alpha + \dot{\phi} w'' \frac{H}{\Phi^2} \cos \alpha}{y_m \sec \alpha} - \dot{\phi}. \quad (3.13)$$

The angular velocity of the line of action relative to the cam is $\dot{\beta}$, and the linear velocity of the roller centre relative to the cam is $\dot{s}$, where

$$\dot{s} = m \dot{\beta}. \quad (3.14a)$$

Substituting Equations (3.8) and (3.11) into Equation (3.14a) yields

$$\dot{s} = -\dot{\phi} y_m \sec \alpha. \quad (3.14b)$$

Through the use of Equations (3.13), (3.14b) and (A3.1), the radius of curvature of the pitch curve can be written as

$$\rho = \frac{-\dot{\phi} y_m \sec \alpha}{\left[\frac{-\dot{\phi} w' \frac{H}{\Phi} \sin \alpha + \dot{\phi} w'' \frac{H}{\Phi^2} \cos \alpha}{y_m \sec \alpha} - \dot{\phi} \right]},$$

which is the same as

$$\rho = \frac{y_m^2}{\frac{H}{\Phi} \cos^2 \alpha \left(w' \sin \alpha - \frac{w''}{\Phi} \cos \alpha \right) + y_m \cos \alpha}. \quad (3.15)$$

The radius of curvature of the cam profile is given by

$$\rho_p = \rho - R_r. \quad (3.16)$$

$$\frac{\dot{\Psi}}{\dot{\Phi}} = w' \frac{\Psi}{\Phi} \equiv w' A$$
$$\tan \alpha = \frac{L(1 - w'A) + D \cos \psi}{D \sin \psi} \quad (4.2)$$
[illegible]

The distance of the point of contact, S from the origin, O, parallel to the X_c -axis is x_p and is given by the sum of the lengths p , q and r such that

Thus $x_p = D \cos \phi + L \cos (\psi - \phi) + R_r \sin (\phi - \psi - \alpha)$. (4.3)

The distance of the point of contact, S , from the origin, O , parallel to the Y_c -axis is y_p and is given by the sum of the lengths g , h and j such that

$$y_p = h - g - j,$$

where

$$g = D \sin \phi$$

$$(h - g) = L \cos\left(\psi - \phi - \frac{\pi}{2}\right) - D \sin \phi = L \sin(\psi - \phi) - D \sin \phi$$

and

$$j = R_r \cos(\pi + \phi - \psi - \alpha) = -R_r \cos(\phi - \psi - \alpha).$$

Thus

$$y_p = -D \sin \phi + L \sin(\psi - \phi) + R_r \cos(\phi - \psi - \alpha). \quad (4.4)$$

It is, however, necessary to take account of configurations where the follower pivot lies to the left of the cam centre (that is, when D is negative). This is accounted for by the inclusion of term k of value unity which adopts the sign of D . Equations (4.3) and (4.4) can be written generally as

$$x_p = D \cos \phi + L \cos(\psi - \phi) + k R_r \sin(\phi - \psi - \alpha) \quad (4.5)$$

and

$$y_p = -D \sin \phi + L \sin(\psi - \phi) + k R_r \cos(\phi - \psi - \alpha) \quad (4.6)$$

4.3 Cutter Centre Co-ordinates

Using a similar approach to that adopted to establish the cam profile co-ordinates, the cutter centre lies at distance x_g along the X_c -axis which is given by

$$x_g = x_p + t,$$

where

$$t = R_g \sin(\pi + \phi - \psi - \alpha) = -R_g \sin(\phi - \psi - \alpha)$$

and thus

$$x_g = D \cos \phi + L \cos(\psi - \phi) + k(R_r - R_g) \sin(\phi - \psi - \alpha). \quad (4.7)$$

Also,

$$y_g = y_p + l,$$

where

$$l = R_g \cos(\pi + \phi - \psi - \alpha) = -R_g \cos(\phi - \psi - \alpha)$$

and thus

$$y_g = -D \sin \phi + L \sin(\psi - \phi) + k(R_r - R_g) \cos(\phi - \psi - \alpha). \quad (4.8)$$

4.4 Pitch Profile and Cam Profile Curvature Radii

In a similar manner to that explained in Section 3.4, the angular velocity of the contact normal relative to the cam, $\dot{\gamma}$, is given by the difference between the velocities of the roller centre, v_{cr} , and instantaneous centre of rotation, v_p , divided by their separating distance, m , from all of which must be subtracted the rotational speed of the cam.

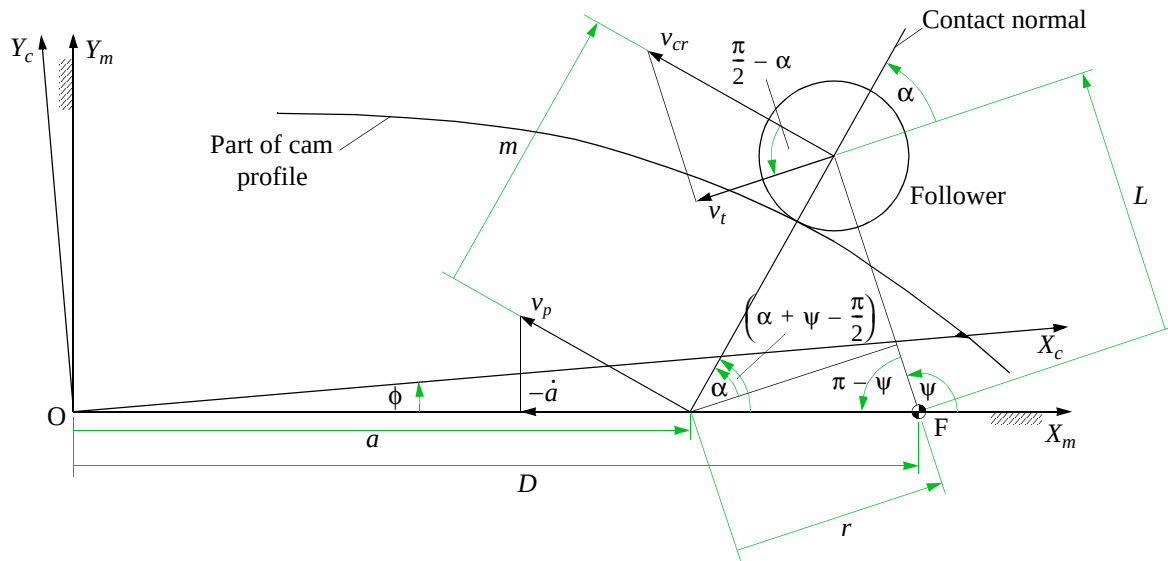
From Sketch 4.3,

$$v_t = \dot{\psi} L$$

$$v_{cr} = \dot{\psi} L \cos\left(\frac{\pi}{2} - \alpha\right) = \dot{\psi} L \sin \alpha. \quad (4.9)$$

Also

$$v_p = -\dot{a} \cos[\pi - (\psi + \alpha)] = \dot{a} \cos(\psi + \alpha). \quad (4.10)$$



Sketch 4.3

Also from the sketch,

$$r = (D - a) \sin(\pi - \psi) = (D - a) \sin \psi \quad (4.11)$$

and, because $m = r / \cos \alpha$, Equation (4.11) can be written as

$$m = (D - a) \sin \psi / \cos \alpha.$$

The angular velocity of the follower relative to the machine is given by

$$\omega_{fm} = (v_{cr} - v_p) / m,$$

which, on substitution of Equations (4.9) to (4.11), gives

$$\omega_{fm} = \frac{\dot{\psi} L \sin \alpha - \dot{a} \cos(\psi + \alpha)}{[(D - a) \sin \psi] / \cos \alpha}. \quad (4.12)$$

Use of Equations (3.6), (3.8) and (4.12) leads to

$$\dot{\gamma} = \frac{\dot{\psi} L \sin \alpha - \dot{a} \cos(\psi + \alpha)}{[(D - a) \sin \psi] / \cos \alpha} - \dot{\phi}$$

into which can be substituted Equations (A2.10) and (A2.13) to give

$$\dot{\gamma} = \frac{w' \frac{\Psi}{\Phi} \dot{\phi} L \sin \alpha \cos \alpha + D \frac{\Psi}{\Phi^2} w'' \frac{\dot{\phi}}{(1 - \dot{\psi}/\dot{\phi})^2} \cos(\psi + \alpha) \cos \alpha - \dot{\phi} (D - a) \sin \psi}{(D - a) \sin \psi}. \quad (4.13)$$

Now the angular velocity of the follower relative to the cam is given by $(\dot{\psi} - \dot{\phi})$ and thus the linear velocity of Point S is $\dot{s} = (\dot{\psi} - \dot{\phi})m$ which, by substituting for m , leads to

$$\dot{s} = (D - a)(\dot{\psi} - \dot{\phi}) \sin \psi / \cos \alpha. \quad (4.14)$$

Through the use of Equations (4.13), (4.14) and (A3.1), the radius of curvature of the pitch curve can be written

$$\rho = \frac{\left[\frac{(D - a)(\dot{\psi} - \dot{\phi}) \sin \psi}{\cos \alpha} \right]}{\left[\frac{w' \frac{\Psi}{\Phi} \dot{\phi} L \sin \alpha \cos \alpha + D \frac{\Psi}{\Phi^2} w'' \frac{\dot{\phi}}{(1 - \dot{\psi}/\dot{\phi})^2} \cos(\psi + \alpha) \cos \alpha - \dot{\phi}(D - a) \sin \psi}{(D - a) \sin \psi} \right]}.$$

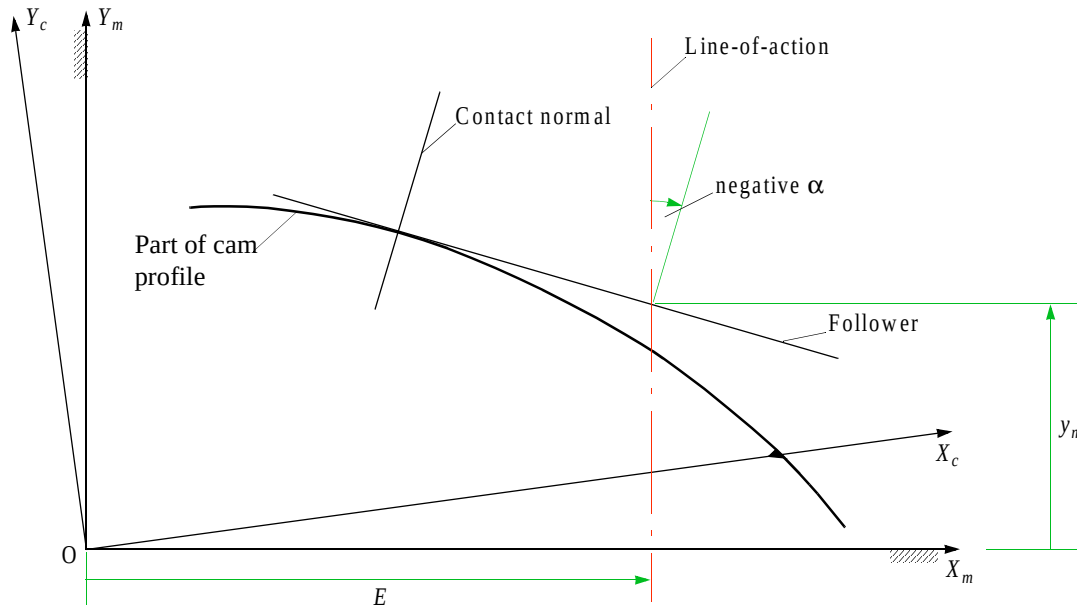
Using the substitution derived from $\dot{\psi}/\dot{\phi} = w'A$ and Equation (A1.2), i.e. $D - a = D/(1 - w'A)$, and also substituting $A = \Psi/\Phi$, the preceding equation can be written as

$$\rho = \frac{-D^2(1 - w'A) \frac{\sin^2 \psi}{\cos^2 \alpha}}{w'AL(1 - w'A)^2 \sin \alpha + D \frac{\Psi}{\Phi^2} w'' \cos(\alpha + \psi) - D(1 - w'A) \frac{\sin \psi}{\cos \alpha}}. \quad (4.15)$$

In order to take account of configurations where the follower pivot lies to the left of the cam centre (that is, when D is negative), the term k is introduced; k has a value of unity and adopts the sign of D . The general form of Equation (4.15) is thus

$$\rho = \frac{-k D^2(1 - w'A) \frac{\sin^2 \psi}{\cos^2 \alpha}}{w'AL(1 - w'A)^2 \sin \alpha + D \frac{\Psi}{\Phi^2} w'' \cos(\alpha + \psi) - D(1 - w'A) \frac{\sin \psi}{\cos \alpha}}. \quad (4.16)$$

5. TRANSLATING FLAT-FACED FOLLOWERS

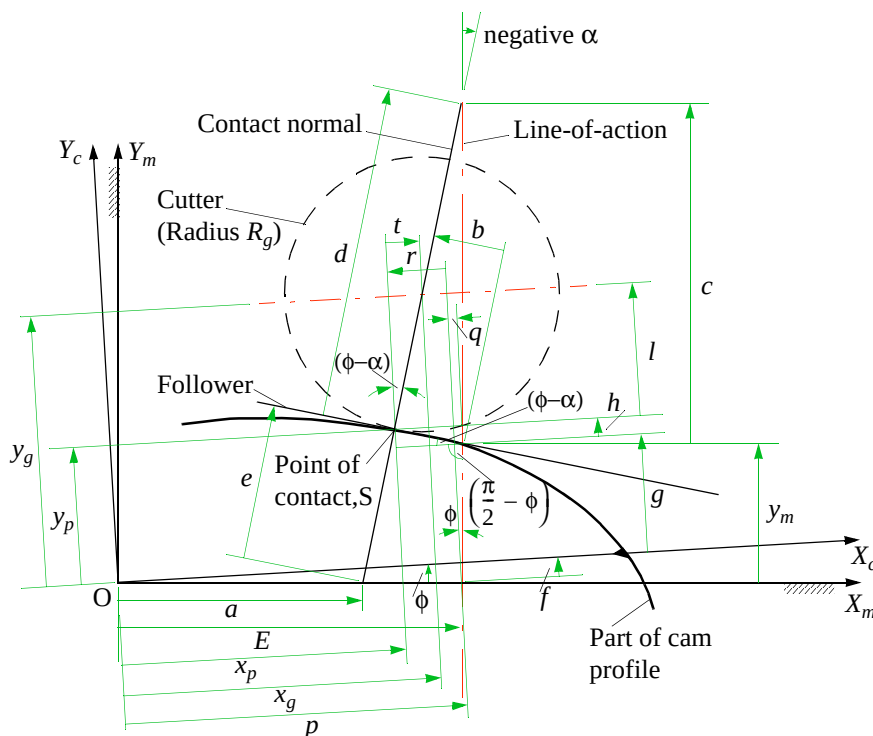


Sketch 5.1

5.1 Pressure Angle

The pressure angle relating to flat-faced followers remains constant throughout each revolution of the cam. The angle is usually chosen as zero but is shown with a finite and negative value in Sketch 5.1. To indicate that the angle is constant, it has been annotated as α . The offset, E_1 is also usually chosen as zero but in Sketch 5.1 it is shown as a positive value.

5.2 Cam Profile Co-ordinates



Sketch 5.2

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The distance of the point of contact, S, from the origin, O, parallel to the X_c -axis is x_p and is given by the sum of the lengths p and r such that

$$x_p = p - r \quad (\text{where } p = E \cos \phi) . \quad (5.1)$$

However, before obtaining an expression for r , it is necessary to obtain expressions for lengths b and c .

From the topmost triangle in Sketch 5.2,

$$\cos(-\alpha) = d/c \quad \text{and} \quad \sin(-\alpha) = b/c . \quad (5.2)$$

$$\text{Also} \quad \cos(-\alpha) = (y_m + c)/(e + d) \quad (5.3)$$

$$\text{and} \quad \sin(-\alpha) = (E - a)/(e + d) . \quad (5.4)$$

From Equations (5.2), by elimination of c ,

$$d = -b \cos \alpha / \sin \alpha$$

and from Equation (5.4)

$$-\sin \alpha = \frac{(E - a)}{\left[e - b \frac{\cos \alpha}{\sin \alpha} \right]} ,$$

which can be re-arranged to give

$$b = \frac{(E - a) + e \sin \alpha}{\cos \alpha} . \quad (5.5)$$

Equation (5.3) can be re-arranged to give

$$e = \frac{y_m + c}{\cos \alpha} - d$$

and substituting for d from the first of Equations (5.2), yields

$$e = \frac{y_m + c}{\cos \alpha} - c \cos \alpha .$$

Further, by substituting for c from the second of Equations (5.2), e can then be expressed as

$$e = \frac{y_m - (b/\sin \alpha)}{\cos \alpha} + b \frac{\cos \alpha}{\sin \alpha} ,$$

or

$$e = \frac{y_m \sin \alpha - b + b \cos^2 \alpha}{\cos \alpha \sin \alpha} . \quad (5.6)$$