

ANALYSIS OF GENEVA MECHANISMS: DERIVATION OF KINEMATIC AND KINETOSTATIC EQUATIONS

1. NOTATION AND UNITS

The units used in this Item are predominantly SI. All angles are measured in an anti-clockwise sense and the crank is considered to rotate in an anti-clockwise direction.

		Units
a_{2x}, a_{2y}	component of acceleration of mass centre of crank in x - and y -direction, respectively	m/s^2
a_{4x}, a_{4y}	component of acceleration of mass centre of wheel in x - and y -direction, respectively	m/s^2
C_x, C_y	constraint functions of Geneva mechanism in x - and y -direction, respectively	—
D_{in}	inner diameter of an internal Geneva wheel	m
D_{out}	outer diameter of a Geneva wheel	m
F_s	frame shaking force	N
F_x, F_y	component of shaking force in x - and y -direction, respectively	N
G	distance between crank shaft axis and wheel shaft axis	m
I_2	mass moment of inertia of crank about its centre of rotation	kg m^2
I_4	mass moment of inertia of wheel about its centre of rotation	kg m^2
L_s	length of slot	m
L_t	length of tip (between edge of slot and locking radius)	m
L_2	distance between crank shaft axis and centre of pin (or roller)	m
l_4	distance between wheel shaft axis and centre of pin (or roller)	m
m_2	mass of crank (including roller or pin)	kg
m_4	eccentric mass of wheel driven by mechanism	kg
N	number of slots	—
P_C	contact force between pin (or roller) and slot	N

P_{ij}	force exerted by Part i on Part j	N
P_{xij}, P_{yij}	component of force P_{ij} in x - and y -direction, respectively	N
R_L	radius of locking wheel	m
R_r	radius of pin (or roller)	m
r_2	distance from crank shaft axis to centre of mass of crank	m
r_4	distance from crank shaft axis to centre of eccentric mass of wheel driven by mechanism	m
T_2	total torque required to drive mechanism	N m
T_{Load}	external torque loading on wheel shaft (see Equations (6.7) and (7.7))	N m
T_ϕ	sum of driving torques due to external loading and Geneva wheel inertia	N m
T_ψ	torque acting on wheel (including its inertial properties)	N m
V_{ent}	lubricant entrainment velocity	m/s
$(V_{ent})_x, (V_{ent})_y$	component of entrainment velocity in x - and y -direction, respectively	m/s
$(V_r)_x, (V_r)_y$	component of velocity of roller (or pin) surface at contact point, relative to contact point, in x - and y -direction, respectively	m/s
$(V_w)_x, (V_w)_y$	component of velocity of wheel slot surface at contact point, relative to contact point, in x - and y -direction, respectively	m/s
X_m, Y_m	axes fixed to machine frame with origin at crank shaft axis	—
x_p, y_p	co-ordinates of contact point (relative to X_m, Y_m axes)	m
$\dot{x}_p, \dot{y}_p$	component of velocity of contact point, relative to fixed frame, in x - and y -direction, respectively	m/s
$\dot{x}_r, \dot{y}_r$	component of velocity of a point on roller (or pin) at contact point, relative to fixed frame, in x - and y -direction, respectively	m/s
$\dot{x}_w, \dot{y}_w$	component of velocity of a point on wheel slot surface at contact point, relative to fixed frame, in x - and y -direction, respectively	m/s
γ_2	angle between crank datum and line joining crank shaft axis to centre of mass of crank (see Sketch 6.1 or 7.1)	rad
γ_4	angle between wheel datum and line joining wheel shaft axis to centre of mass of wheel (see Sketch 6.1 or 7.1)	rad

ϕ	angle between X_m axis and crank datum	rad
ϕ_e	angle between X_m axis and crank datum at point of engagement between roller (or pin) and slot	rad
$\dot{\phi}$	angular velocity of crank	rad/s
$\ddot{\phi}$	angular acceleration of crank	rad/s ²
ψ	angle between X_m axis and centre line of slot	rad
ψ_e	angle between X_m axis and centre line of slot at point of engagement between roller (or pin) and slot	rad
$\dot{\psi}$	angular velocity of wheel	rad/s
$\ddot{\psi}$	angular acceleration of wheel	rad/s ²
ω_r	angular velocity of roller relative to crank	rad/s

Subscripts

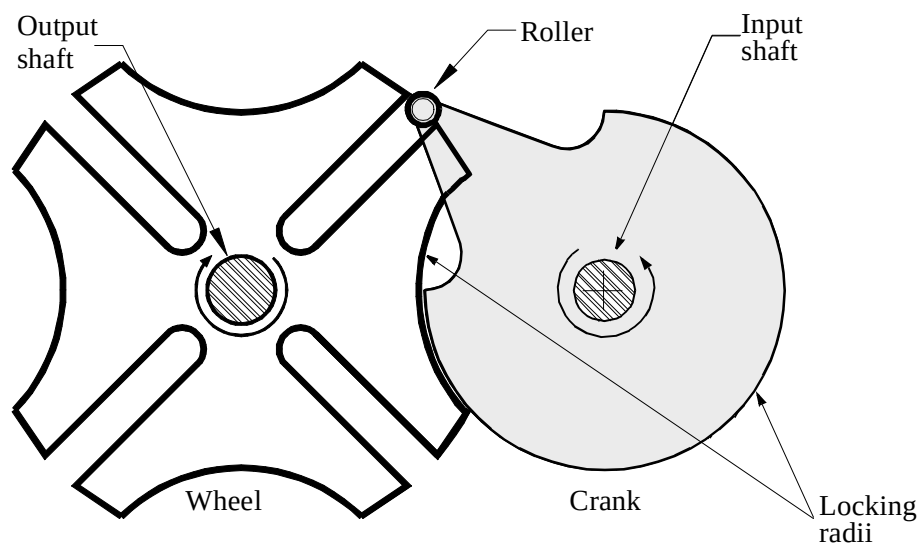
<i>case</i>	refers to wheel casing
<i>e</i>	refers to engagement
<i>ent</i>	refers to entrainment
<i>max</i>	refers to a maximum value
<i>min</i>	refers to a minimum value
<i>p</i>	refers to contact point
<i>r</i>	refers to roller (or pin)
<i>w</i>	refers to wheel

2. INTRODUCTION AND ASSUMPTIONS

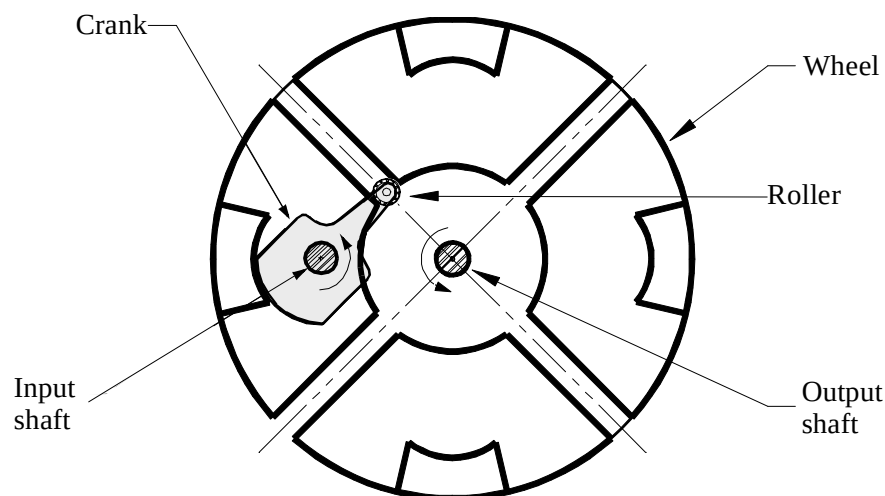
2.1 Introduction

This guide provides a record of the derivation of the basic relationships required during the kinematic and kinetostatic design and analysis of external and internal Geneva mechanisms. The final results are presented in Data Item 96011 and the equations are embodied in a computer program, ESDUpac A9611. The two mechanisms are illustrated in Sketches 2.1 and 2.2.

Although similar to a Data Item, this Item is different in that it does not impart design guidance directly. It should be regarded as background information to the Data Items on Geneva mechanism design in the Mechanisms Series.



Sketch 2.1 External Geneva mechanism



Sketch 2.2 Internal Geneva mechanism

2.2 Assumptions

When considering the kinematic relationships between the parts of a Geneva mechanism (Sections 3, 4, 5 and 8), it is assumed that:

- (a) in the mathematical model of a well-designed Geneva mechanism, the drive pin (usually a roller) enters the slot in the output wheel along the centre line of the slot, which in theory avoids impact forces occurring between the roller (or pin) and the sides of the slot,
- (b) the roller (where appropriate) is always in purely rolling contact with the sides of the slot, *i.e.* no slip takes place,
- and (c) the effect of the step change of acceleration (*i.e.* jerk) of both the wheel and roller (or pin), as the roller/pin enters the slot, is negligible.

When considering the kinetostatic relationships between the parts of a Geneva mechanism (Sections 6 and 7), it is assumed that:

- (d) the forces acting on the system are due to the inertia of the moving parts and to any external loads acting on the wheel,
- (e) the friction at all bearings, and between the slot and the pin (or roller) is negligible,
- (f) the force due to impact when the pin (or roller) enters the slot, and when the pin (or roller) moves from one side of the slot to the other, is negligible,
- (g) the mass moment of inertia of the roller is negligible
- and (h) all parts of the mechanism are stiff enough to be considered rigid bodies.

The convention adopted for forces and torques in this document is such that P_{x23} is the component of force in the x -direction exerted by Part 2 on Part 3, and so

$$P_{x23} = -P_{x32}.$$

The magnitude of the force exerted by Part 2 on Part 3 is given by

$$P_{23} = \sqrt{(P_{x23})^2 + (P_{y23})^2}.$$

Similarly, T_{34} is the moment (positive in an anti-clockwise sense) exerted by Part 3 on Part 4 and so

$$T_{34} = -T_{43}.$$

3. GEOMETRIC QUANTITIES

3.1 General Principle

Typically, there are two ways in which a Geneva mechanism is analyzed and designed. In the first, Case A, the centre distance between the crank and wheel shafts is specified and the dependent wheel and crank dimensions are evaluated. Alternatively, Case B, the dimensions of the wheel may be specified and the crank dimensions, together with the distance between the two shafts, become the dependent parameters to be determined.

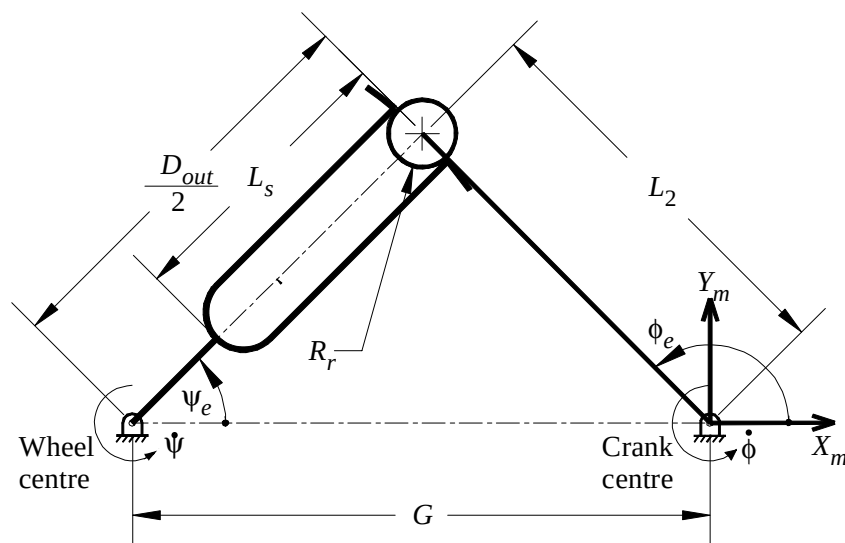
Sections 3.2.1 and 3.3.1 consider the evaluation of the appropriate dependent geometric quantities for the external and internal Geneva, respectively, for Case A, in which the centre distance is defined. Sections 3.2.2 and 3.3.2 consider the evaluation of the appropriate dependent geometric quantities, for the external and internal Geneva, respectively, for Case B, in which the dimensions of the wheel are defined. Note that the two sets of equations involve identical parameters, albeit developed in a different order, for both the external and internal Geneva mechanisms.

In the mathematical model of an idealised Geneva mechanism, the drive pin (usually a roller) enters the slot in the output wheel along the centre line of the slot. If this rule is applied then, at the point of engagement, the line joining the crank shaft axis to the centre of the roller is perpendicular to the axis of the slot (as shown in Sketches 3.1 and 3.4).

3.2 External Geneva Mechanism

3.2.1 Case A - centre distance specified

The output wheel of an external Geneva mechanism indexes in the opposite angular sense to the motion of the crank shaft. The dwell period is always longer than the period during which motion of the output shaft takes place. Sketch 3.1 shows a roller on the crank at the point of engagement with a slot on the wheel.



Sketch 3.1 External Geneva mechanism at point of engagement

The angle of the centre line of the slot on the output wheel, relative to the fixed machine frame, at the instant of engagement is given by

$$\psi_e = \frac{\pi}{N} . \quad (3.1)$$

Since the line joining the crank shaft axis and the centre of the pin (or roller) is perpendicular to the axis of the slot at the instant of engagement, the angle ϕ_e is given by

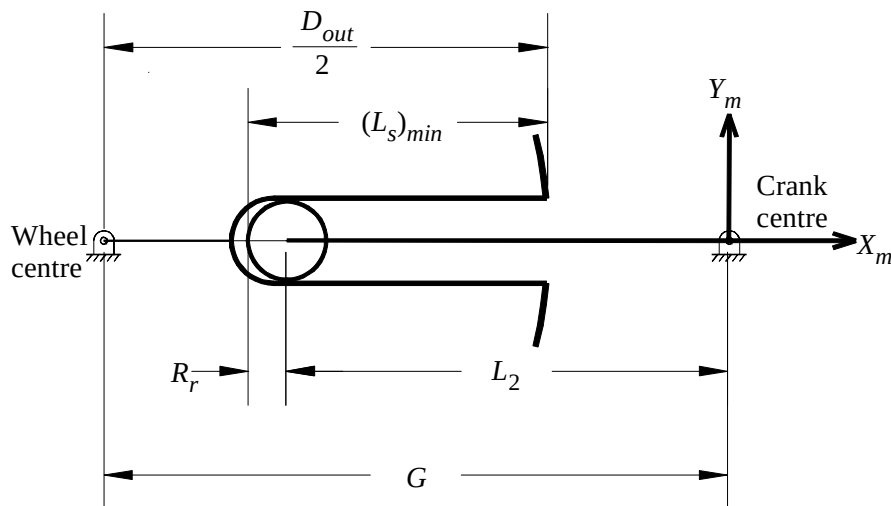
$$\phi_e = \psi_e + \frac{\pi}{2} . \quad (3.2)$$

From the specified centre distance, G , between the crank shaft axis and the wheel axis, the length of the crank is given by

$$L_2 = G \sin \psi_e \quad (3.3)$$

and the outer diameter of the wheel is given by

$$D_{out} = 2 \sqrt{G^2 - L_2^2} . \quad (3.4)$$



Sketch 3.2 External Geneva mechanism midway through period of engagement

From Sketch 3.2, it can be seen that (for a specified roller radius) the theoretical minimum length of the slot is

$$(L_s)_{min} = L_2 + R_r - \left(G - \frac{D_{out}}{2} \right) . \quad (3.5)$$

When the dimensions above have been specified, the size of the locking radius, R_L (see Sketch 3.3), is bounded. The upper limit, to avoid interference with the slots on the wheel, is then

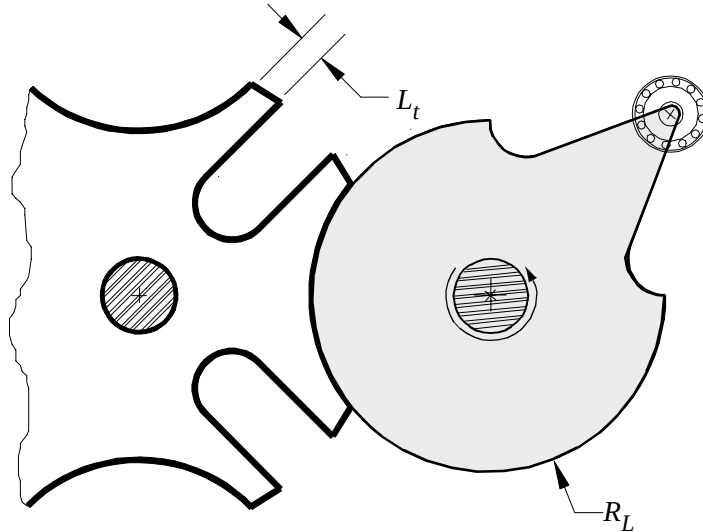
$$(R_L)_{max} = L_2 - R_r \quad (3.6)$$

and the lower limit, to ensure that the locking ring contacts the wheel, is

$$(R_L)_{min} = G - \frac{D_{out}}{2} . \quad (3.7)$$

When a value for the locking radius has been chosen, within the upper and lower limits found, the thickness of the wheel tip, L_t (the distance between the slot and the locking radius), as shown in Sketch 3.3, may be expressed as

$$L_t = L_2 - R_r - R_L . \quad (3.8)$$



Sketch 3.3 External Geneva mechanism showing locking radius

3.2.2 Case B - outer diameter of wheel specified

The output wheel of an external Geneva mechanism indexes in the opposite angular sense to the motion of the crank shaft. The dwell period is always longer than the period during which motion of the output shaft takes place. Referring to Sketch 3.1, the angle of the centre line of the slot on the output wheel, relative to the fixed machine frame, at the instant of engagement is

$$\psi_e = \frac{\pi}{N} . \quad (3.9)$$

Since the line joining the crank shaft axis and the centre of the pin (or roller) is perpendicular to the axis of the slot, the crank angle at engagement is given by

$$\phi_e = \psi_e + \frac{\pi}{2} . \quad (3.10)$$

From the defined diameter of the wheel, D_{out} , the length of the crank is given by

$$L_2 = \left(\frac{D_{out}}{2} \right) \tan \psi_e \quad (3.11)$$

and the distance between the crank shaft axis and the wheel axis, G , is

$$G = \sqrt{\frac{(D_{out})^2}{4} + L_2^2} . \quad (3.12)$$

From Sketch 3.2, it can be seen that, for a specified roller radius, the minimum length of the slot is given by

$$(L_s)_{min} = L_2 + R_r - \left(G - \frac{D_{out}}{2} \right) . \quad (3.13)$$

When the dimensions above have been specified, the size of the locking radius, R_L , is bounded. The upper limit, to avoid interference with the slots on the wheel, is

$$(R_L)_{max} = L_2 - R_r \quad (3.14)$$

and the lower limit, to ensure that the locking ring contacts the wheel is

$$(R_L)_{min} = G - \frac{D_{out}}{2} \quad (3.15)$$

When an appropriate value of locking radius has been chosen, within the upper and lower limits found, the thickness of the wheel tip, L_t (the distance between the slot and the locking radius), as shown in Sketch 3.3 may be expressed as

$$L_t = L_2 - R_r - R_L \quad (3.16)$$

3.3 Internal Geneva Mechanism

3.3.1 Case A – centre distance specified

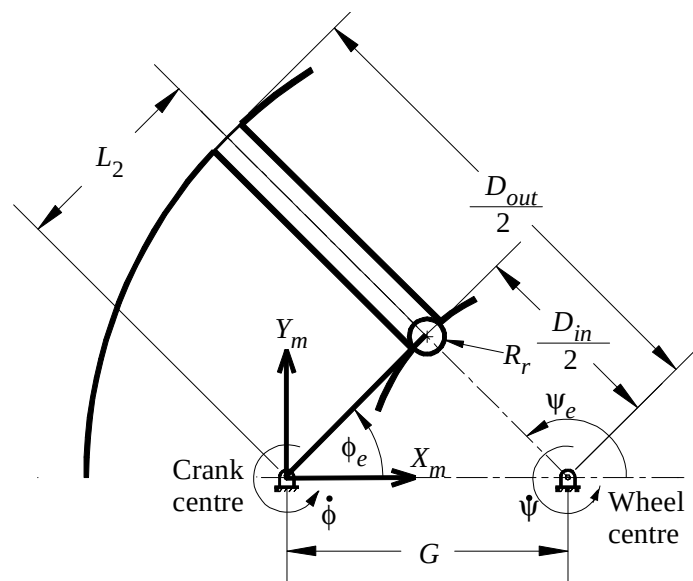
The output wheel of an internal Geneva mechanism indexes in the same angular sense as the crank. The dwell period is always shorter than the period during which motion of the output wheel takes place.

Referring to Sketch 3.4, the angle of the centre line of the slot on the output wheel, relative to the fixed machine frame, at the instant of engagement is

$$\psi_e = \pi - \frac{\pi}{N} \quad (3.17)$$

Since the line joining the crank shaft axis and the centre of the pin (or roller) is perpendicular to the axis of the slot, the corresponding angle between the X_m axis and the line joining the crank shaft axis and the centre of the pin (or roller) is

$$\phi_e = \psi_e - \frac{\pi}{2} \quad (3.18)$$



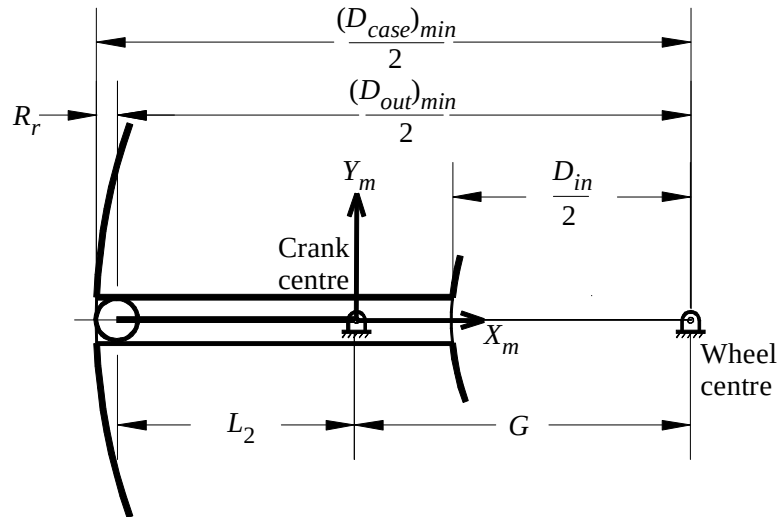
Sketch 3.4 Internal Geneva mechanism at point of engagement

From the defined centre distance, G , between the crank shaft axis and the wheel axis, the length of the crank is given by

$$L_2 = G \sin(\pi - \psi_e) = G \sin \psi_e \quad (3.19)$$

and the inner diameter of the wheel is

$$D_{in} = 2\sqrt{G^2 - L_2^2}. \quad (3.20)$$



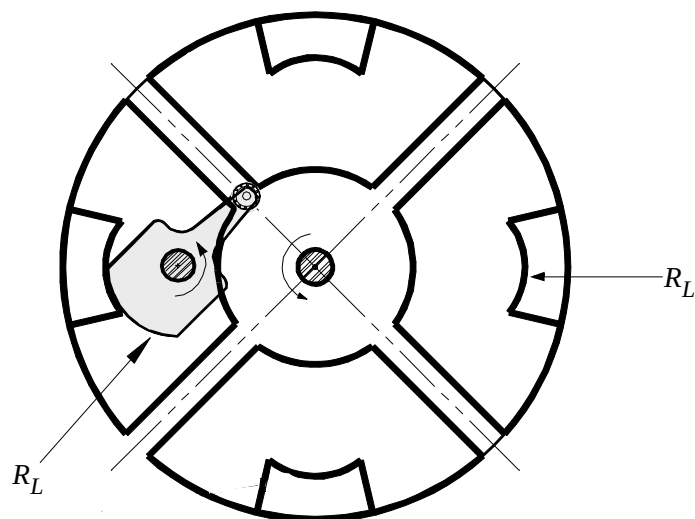
Sketch 3.5 Internal Geneva mechanism midway through period of engagement

From Sketch 3.5, it can be seen that (for a specified roller radius) the theoretical minimum outer diameter of the wheel is

$$(D_{out})_{min} = 2(G + L_2). \quad (3.21)$$

Note however, that (for a specified roller radius) the minimum inner diameter of a casing surrounding the Geneva wheel with no clearance is

$$(D_{case})_{min} = 2(G + L_2 + R_r). \quad (3.22)$$



Sketch 3.6 Internal Geneva mechanism showing locking radius

When the dimensions of the internal Geneva have been specified, the size of the locking radius, R_L , is bounded. The upper limit, to avoid exceeding the diameter of the output wheel, is found from

$$(R_L)_{max} = L_2 + R_r . \quad (3.23)$$

3.3.2 Case B - outer diameter of wheel specified

The output wheel of an internal Geneva mechanism indexes in the same angular sense as the crank. The dwell period is always shorter than the period during which motion of the output wheel takes place. Referring to Sketch 3.4, the angle of the axis of the slot on the output wheel, relative to the fixed machine frame at the instant of engagement is

$$\psi_e = \pi - \frac{\pi}{N} . \quad (3.24)$$

Since the line joining the crank shaft axis and the centre of the pin (or roller) is perpendicular to the centre line of the slot, the corresponding angle between the X_m centre line and the line joining the crank shaft axis and the centre of the pin (or roller) is given by

$$\phi_e = \psi_e - \frac{\pi}{2} . \quad (3.25)$$

From the defined inner diameter of the wheel, D_{in} , the length of the crank is given by

$$L_2 = \frac{D_{in}}{2} \tan(\pi - \psi_e) \quad (3.26)$$

and the distance between the crank shaft axis and the wheel axis, G , is

$$G = \sqrt{\frac{D_{in}^2}{4} + L_2^2} . \quad (3.27)$$

From Sketch 3.5, it can be seen that, for a specified roller radius, the theoretical minimum outer diameter of the wheel is

$$(D_{out})_{min} = 2(G + L_2) . \quad (3.28)$$

Note however, that (for a specified roller radius) the minimum inner diameter of a casing surrounding the Geneva wheel, with no clearance may be found from

$$(D_{case})_{min} = 2(G + L_2 + R_r) . \quad (3.29)$$

When the dimensions above have been specified, the size of the locking radius, R_L , is bounded. The upper limit, to avoid exceeding the diameter of the output wheel, is given by

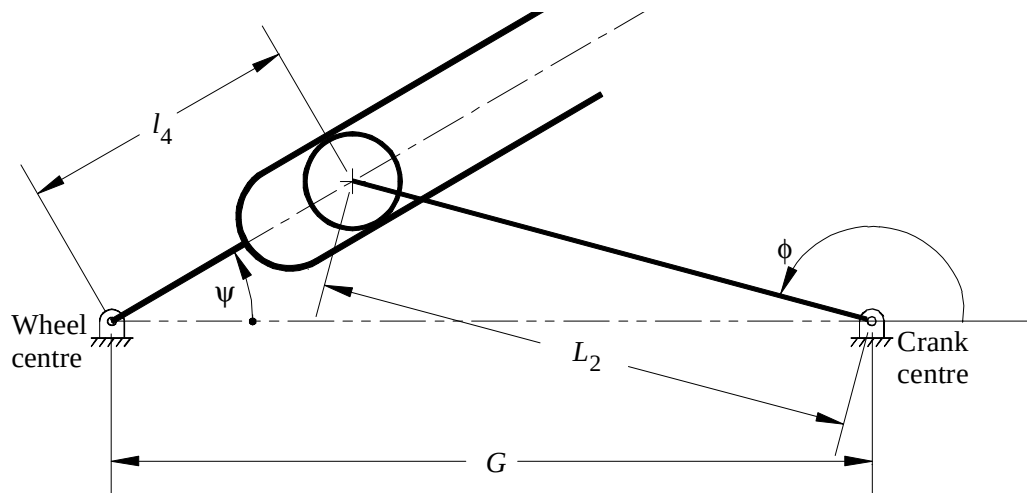
$$(R_L)_{max} = L_2 + R_r . \quad (3.30)$$

4. WHEEL MOTION - EXTERNAL GENEVA

4.1 Geometric Relationships During Motion

During the derivation of the equations for the wheel motion, it is assumed that the geometry of the mechanism is as described in Section 3 of this document. The motion of the wheel depends upon the location and shape of the individual slots, and alternatives to the 'standard' wheel motion presented in this Data Item have been successfully implemented. See References 2, 3 and 4.

It is necessary to derive equations for the angular displacement of the wheel during one complete rotation of the crank. The kinematic relationship between the geometric variables (L_2 , l_4 , G , ϕ and ψ) is defined by the functions C_x and C_y . A compatible kinematic relationship between the five geometric variables shown in Sketch 4.1 is defined by the (two) constraint equations which require both functions to be equal to zero. For the first phase of the crank rotation ($0 < \phi < \phi_e$), the locking ring is engaged and so the wheel does not move.



Sketch 4.1 External Geneva mechanism during wheel motion

For the period during which the pin (or roller) is engaged in the slot, the following constraint equations* can be obtained by inspection of Sketch 4.1.

$$C_x = l_4 \cos \psi - L_2 \cos \phi - G = 0 \quad (4.1a)$$

and
$$C_y = l_4 \sin \psi - L_2 \sin \phi = 0. \quad (4.1b)$$

Equations (4.1a) and (4.1b) are re-arranged to give

$$l_4 \cos \psi = L_2 \cos \phi + G \quad (4.2a)$$

and
$$l_4 \sin \psi = L_2 \sin \phi. \quad (4.2b)$$

Squaring both sides of Equations (4.2) and adding them together gives

$$l_4^2 (\sin^2 \psi + \cos^2 \psi) = L_2^2 \sin^2 \phi + (L_2 \cos \phi + G)^2, \quad (4.3)$$

* For further discussion of the 'constraint equation' technique used in the derivation of the kinematic equations in this section, see Reference 1.