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Guide to Items on struts, beams and shafts

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THE PREPARATION OF THIS DATA ITEM

The need to analyse the stresses in, and thereby to establish the strength of, components can arise in connection with a wide range of machine and structural designs and although the final products may vary widely, the data and methods of analysis that are applicable are often common. Accordingly, in order to show how much common data may be brought to bear in a different range of applications, ESDU has commissioned special Guides and these should be regarded as essential reading on the part of all who intend to obtain the maximum assistance from the "working tools" which are the essential content of the Series. These Guides are also intended to assist the application of reliable data and analytical methods in circumstances that hitherto may not have received the benefits of such treatments. ESDU commissions one, or more, experienced engineers in industry or in its own senior staff to prepare these Guides, often in consultation with leading practitioners in industry and elsewhere.

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It must be stressed that the understanding of the material presented in this Guide is essential to the proper understanding of the use of the data on the part of the user and to his deriving maximum assistance from the data in his professional work. ESDU is always ready to assist further in response to telephone, or other, queries and this service should be regarded as an integral part of the provisions made by ESDU along with the Data Items it issues.

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GUIDE TO ITEMS ON STRUTS, BEAMS AND SHAFTS

1. INTRODUCTION

This Item provides a guide to the use of the data available in the Stress and Strength Series on struts, beams and shafts. Many component parts of machines and structures can be idealised as beams or struts so that estimates of their stresses, deflections, slopes, *et cetera*, can be obtained. The methods of idealisation of real engineering components into such elements are dealt with in ESDU SS1, Section 7.

The general principles to be followed in the design and analysis of beams, struts and shafts are the same as those for any class of component. Guidance on the estimation of loads and the various failure modes to be considered is given in ESDU SS1, Sections 4 and 5. As noted, these components are particularly susceptible to failure by buckling, that is, by rapidly increasing deflection at certain load levels. Therefore, data on the effects of buckling and deflections are extensively covered in the Data Items described in this Guide.

2. BEAMS

2.1 Simple Straight Beams of Constant Section

The elastic analysis of structures and components that can be reasonably idealised as simple straight beams of constant section is well expounded in most textbooks on structures or elasticity, see Section 6. The stresses, deflections and slopes of such beams are all functions of the bending moment, M , acting in the beam, the second moment of area of the beam cross-section, I , the modulus of elasticity, E , of the beam material and, in the case of stresses, the distance, in the plane of bending, of the point under consideration from the neutral axis* of bending of the beam, $\bar{y}$. Under simple loading cases M is easily determined and for the more common cases the tables of Appendix C, which give expressions for M in terms of the beam proportions and loadings, are provided. The tables also provide expressions for the beam deflections. By means of super-position the bending moments and deflections of beams under more complex loadings can be built up from the tables. However, simple graphical or iterative methods are available if this procedure is too tedious. Calculation of I should present no problem since tables of values of I for the more common beam sections are provided in most textbooks together with methods of combining them for complex shapes. In extreme cases graphical methods of estimation of I are available and are illustrated in most textbooks, see Section 6.

2.2 Straight Stepped Beams

Estimation of the stresses in stepped beams presents no problem over and above those mentioned in Section 2.1. However, estimation of the slopes and deflections becomes increasingly complicated with increasing numbers of steps. ESDU 69017 deals with this problem and provides a quick and accurate method of determining the slopes and deflections at any chosen location along a beam. ESDU 69017 is directed primarily towards the analysis of circular shafts but the procedure is general and may be used for shafts or beams of any, or varied, cross-section.

* The neutral axis is the locus of the centroid of the beam cross-section and is the longitudinal axis about which the stresses induced by bending change from tension to compression. This simplified approach is invalid if the beam material properties in tension and compression differ.

2.3 Curved Beams Subjected to Bending in the Plane of Curvature

The elastic analysis of curved beams is more complex than that of straight beams. However, formulae that provide the required stresses are well known and are published in textbooks, see Section 6. The stresses in curved beams under pure bending are only significantly different from those calculated using straight beam theory if the ratio of the beam depth to its radius of curvature is high (*e.g.* if the mean radius of curvature is less than four times the depth of the beam, errors in excess of 10 per cent may result from the use of straight beam theory).

The straightforward elastic analysis of curved beams can, however, be seriously in error if the beam section has outstanding flanges such as those of an I-section beam. Under the action of bending the tips of the outstanding flanges can deflect and relieve some of the bending stresses imposed upon them. ESDU 71004 deals with this problem in detail and provides data whereby realistic beam stresses and deflections are obtained. Data on the secondary stresses acting across the beam flanges are also provided and should be taken into account when estimating the stresses in the beam.

2.4 Beams in which the Limit of Proportionality of the Material is Exceeded

The stresses in beams where, at some point, the limit of proportionality (see Appendix A) of the material has been exceeded will not vary linearly across the beam section. For some of the more common symmetrical-section beams this problem can be overcome by the introduction of form factors which provide a correction to simple engineers' bending theory discussed in Section 2.1. ESDU 01.06.00 defines the form factor and describes its use. Subsequent Items, Nos 01.06.01 to 03, provide values of the form factor pertinent to several common beam sections and illustrate their use with worked examples. Use of these Items eliminates the need for data that are often quoted on the flexural strength of a material. Such flexural strength data, which differ from the tensile strength data, are only applicable to sections similar to those tested and can be misleading if applied to other sections.

2.5 Allowable Stresses in Beams

Several methods are available for ensuring that a beam has a sufficient reserve of strength. However, care must be taken to ensure that the method chosen provides the true reserve of strength required.

2.5.1 Choice of an allowable stress that is below the limit of proportionality of the material

This is the simplest case because, below the limit of proportionality, the stresses in beams are linearly related to the applied loads and in consequence it is immaterial where the factor of safety is applied. However, it is advisable to apply a factor appropriate to the uncertainties of the part of the calculation in hand, *e.g.* the factor applied to the direct loads may well be different from that applied to the bending moments, and the stress calculation would be modified by another factor which is dependent upon the validity of the idealisations made (see Section 7 of ESDU SS1).

It should be noted that in some materials the limit of proportionality may be as low as 80 per cent of the 0.2 per cent proof stress (see Appendix A).

2.5.2 Choice of an allowable stress that is above the limit of proportionality of the material

Allowable stresses that are above the limit of proportionality of the material can be employed in two different ways. The simplest is to compare the factored stresses, or stresses derived from factored loads, with the allowable stress. However, this method assumes that the material stress-strain curve is linear above the calculated stress and up to the allowable stress. This is a conservative assumption and leads to the computation of stresses slightly higher than those that will exist. The more complex method is to calculate the loads required to produce the allowable stress, using the material properties appropriate to the allowable stress. The reserve capacity can then be determined from a comparison of the calculated and actual loads.

As was noted in Section 2.5.1 it is desirable to apply factors of safety to each part of the calculation that are appropriate to the degree of confidence in that portion of the calculation.

3. STRUTS

3.1 Elastic Buckling of Struts

A perfectly straight strut, subjected to a compressive load perfectly aligned along the neutral axis of bending sustains that load provided it is below a critical value. Below the critical, or buckling^{*}, load the strut remains undeflected from its original axis, and if deflected slightly by a lateral load it returns to its undeflected state on removal of that lateral load. Above the buckling load, provided there is no lateral restraint, the smallest lateral deflection precipitates collapse.

The buckling load corresponds to the condition where the bending moment in the strut increases without increasing the load. Therefore, before the buckling load is reached, the build-up of bending stresses in the strut will reach unacceptably high values. This is illustrated in Sketch 3.1, which is derived from ESDU 78030. The diagram illustrates the rate of increase of bending moment in the practical case of a real pin-ended strut with slight initial bow. It clearly shows that data on the bending moment in the strut under increasing end load are required so that the bending stresses can be determined and added to the direct stresses to obtain the maximum stress in the strut. If this check is not carried out then, at loads well below the Euler buckling load, the extreme fibres of the strut may yield and on removal of the load a permanent deformation will be retained. ESDU 78030 gives the required bending moments for many types of loading and includes a method that will allow calculation of the deflection, and hence the bending moment, under any combination of lateral in-plane loading together with initial eccentricities and any variations in the cross-section or material moduli along the length of the strut.

* The solution for the buckling load for a simple strut was first obtained by L. Euler in the eighteenth century and is often referred to as the Euler buckling load.