

THE STRENGTH OF STRUTS

1. NOTATION

A	cross-sectional area of strut	m^2	in^2
c	k^2/y	m	in
e	total eccentricity (including loading eccentricity, initial bow at centre of strut and manufacturing inaccuracies, see Figure)	m	in
E	modulus of elasticity in compression of strut material	N/m^2	lbf/in^2
E_t	tangent modulus in compression of strut material	N/m^2	lbf/in^2
f	average stress in strut at failure, P/A	N/m^2	lbf/in^2
f_{all}	allowable stress in most compressed fibre	N/m^2	lbf/in^2
f_n	stress at which $E_t = \frac{1}{2}E$ (see Item No. 76016 [†])	N/m^2	lbf/in^2
k	radius of gyration of section	m	in
l	actual length of strut	m	in
l'	equivalent length of strut	m	in
m	material characteristic (see Item No. 76016)		
P	failing end load	N	lbf
y	distance from neutral axis of section to most compressed fibre (y_p or y_n)	m	in
y_p, y_n	distance from neutral axis of section to extreme fibres either side of axis	m	in

Both SI and British units are quoted but any coherent system of units may be used.

[†] Data Item No. 76016 "Generalisation of smooth continuous stress-strain curves for metallic materials".

2. NOTES

The ratio f/f_n is plotted against $(l'/k)(f_n/E)^{1/2}$ for various values of m and f_{all}/f_n for eccentricity ratios of $e/c = 0, 0.05, 0.10, 0.20$ and 0.30 . The curves refer only to flexural failure in which the cross-section translates normal to the axis to which k applies. A computer program that calculates f from details of the strut section and material properties is provided in Data Item No. 90002[†]. The program also allows the calculation of the section properties and, if required, the investigation of local and/or inter-rivet buckling.

The total eccentricity, e , relates strictly to offset loading alone and is defined as shown in Sketch 2.1a.

[†] Data Item No. 90002 "Structs: strength under flexural, local and inter-rivet buckling. (Computer program)".

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Figures 1 to 5d inclusive may be used to obtain a conservative estimate of P for cases where the strut is bowed. In these cases e is defined as shown in Sketch 2.1b. Similarly, Figures 1 to 5d inclusive may also be used to give a conservative estimate where the load is offset and the strut is bowed and these two effects are known to act together to increase the bending moment at the centre of the strut. In these cases e is defined as shown in Sketch 2.1c. For cases where the bow and offset loading are in opposition, or where a more accurate solution for bow alone is required, the program of Data Item No. 90002 should be used. If only bow exists, or the bow and offset loading act together as shown in Sketch 2.1c, an allowance may be made for the effect of manufacturing tolerances by increasing the bow. Where the manufacturing tolerance is not known it can be estimated from the greater of $(0.05k^2)/(y_p y_n)^{1/2}$ or $0.001l$.

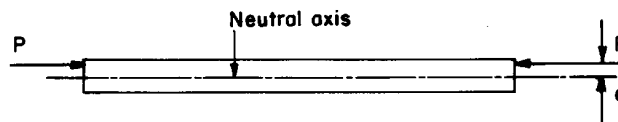


Figure 1a. Offset load

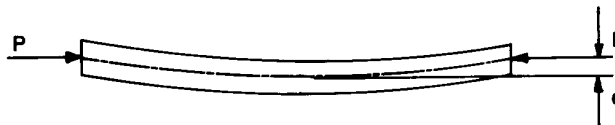


Figure 1b. Bow

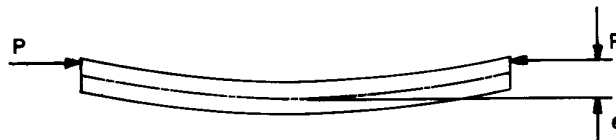


Figure 1c. Offset load and bow acting together

Sketch 2.1 Showing effective values of e for offset, bow and combinations of offset and bow

The stress-strain curve of the material is taken to be represented by the expressions given in Data Item No. 76016. The curved portion of the line for $m = \infty$ in Figure 1 is appropriate to a strut that buckles in the elastic range.

The equivalent lengths of a strut for simple conditions of restraint are given in the following table.

End conditions	Pinned	Pinned	Fixed	Fixed	Fixed
	Pinned	Fixed	Fixed	Free	Fixed in direction, free in translation
Equivalent length, l'	$1.0 \times l$	$0.7 \times l$	$0.5 \times l$	$2.0 \times l$	$1.0 \times l$

It should be noted that l' may have different values in different planes, and the minimum value of k/l' should be used. When the member is attached to a sheet the appropriate value of k is that about the neutral axis parallel to the plane of the sheet. (The effects of interaction between the stringer and the sheet are dealt with in Data Item No. 72012.)

The maximum allowable extreme fibre stress f_{all} normally lies between the 0.2 per cent proof stress in compression and a compressive stress of modulus equal to that of the ultimate tensile stress depending primarily on the shape of the cross section. For sections with a high form factor (see Item Nos Struct. 01.06.00 to 03) use of the ultimate tensile stress is more realistic. For the majority of practical struts with small e/c the failing stress is not very sensitive to changes in f_{all} within these limits.

The curves refer to flexural failure and the liability of the strut to local or torsional instability must therefore be checked.

3. DERIVATION

1. TIMOSHENKO, S.P. *Theory of elastic stability*. Second edition, Chapters 1, 3 and 4.
GERE, J.M. McGraw-Hill, 1961

The formula
$$f_{all} = f \left\{ 1 + \frac{e}{c} \sec \frac{l'}{2k} \left(\frac{f}{E_t} \right)^{1/2} \right\}$$

4. EXAMPLE

It is required to find the failing load of an I-section strut with fixed ends that has an initial bow estimated at $e/c = 0.15$ at the centre. The material of the strut is 2L 65 aluminium alloy, $l = 56.0$ in, minimum $k = 0.68$ in (about the centre-line of the web) and $A = 0.92$ in².

From Item No. 76016 for 2L 65 material not greater than 3/8 in thick,

$$f_t = 62\,700 \text{ lbf/in}^2, c_2 = 57\,500 \text{ lbf/in}^2, f_n = 52\,200 \text{ lbf/in}^2,$$

$$E = 11.4 \times 10^6 \text{ lbf/in}^2 \text{ and } m = 24.$$

For a strut with fixed ends, from the table, $l' = 0.5l$, so that

$$\frac{l'}{k} \left\{ \frac{f_n}{E} \right\}^{1/2} = 2.79.$$

An I-section strut bending about the centre line of the web may be idealised as a solid rectangular section which, from Item No. Struct. 01.06.02, is seen to have a high form factor.

Therefore, taking $f_{all} = |f_t|$, $f_{all}/f_n = 1.20$ and, interpolating from the figures,

for $e/c = 0.1$ (Figure 4c), $\frac{f}{f_n} = 0.85$

and for $e/c = 0.2$ (Figure 5c), $\frac{f}{f_n} = 0.76$

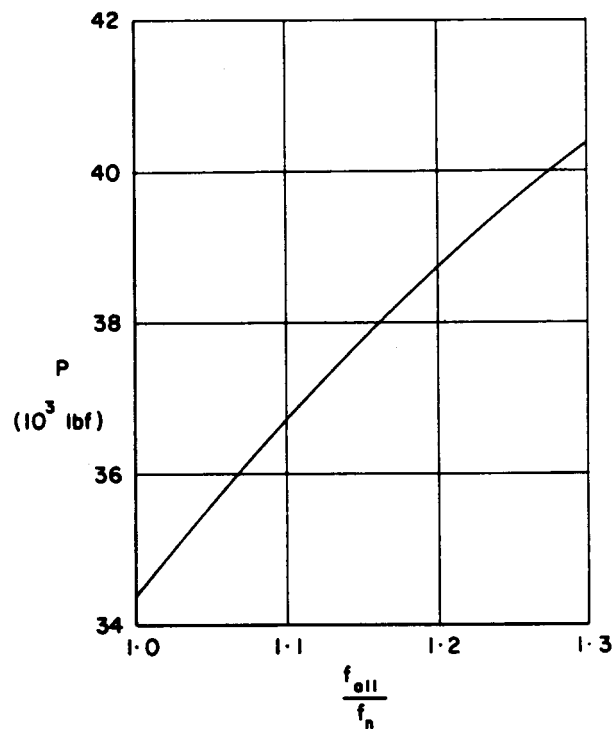
Hence, for $e/c = 0.15$, assuming linear interpolation,

$$\frac{f}{f_n} = 0.805$$

and the failing load

$$P = 0.805 \times 52\,200 \times 0.92 = 38\,700 \text{ lbf.}$$

Sketch 4.1 shows how the estimated failing load would vary if other values for f_{all} had been chosen.



Sketch 4.1

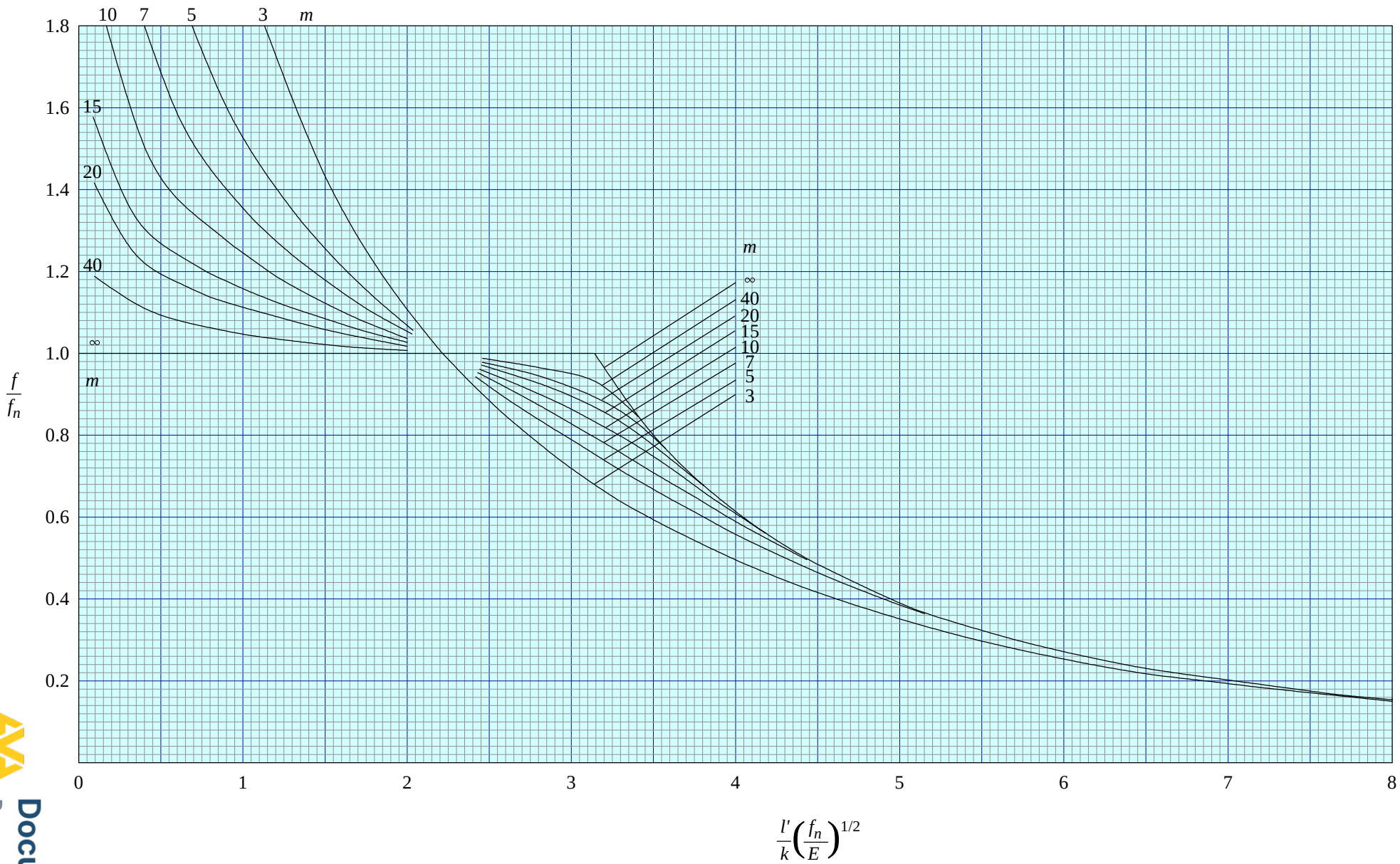


FIGURE 1 $\frac{e}{c} = 0$. ALL VALUES OF $\frac{f_{all}}{f_n}$

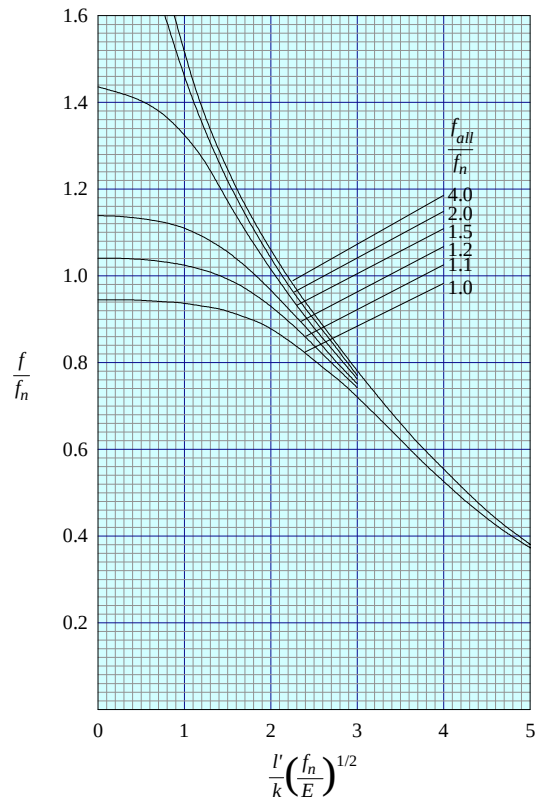


FIGURE 2a $\frac{e}{c} = 0.05, m = 5$

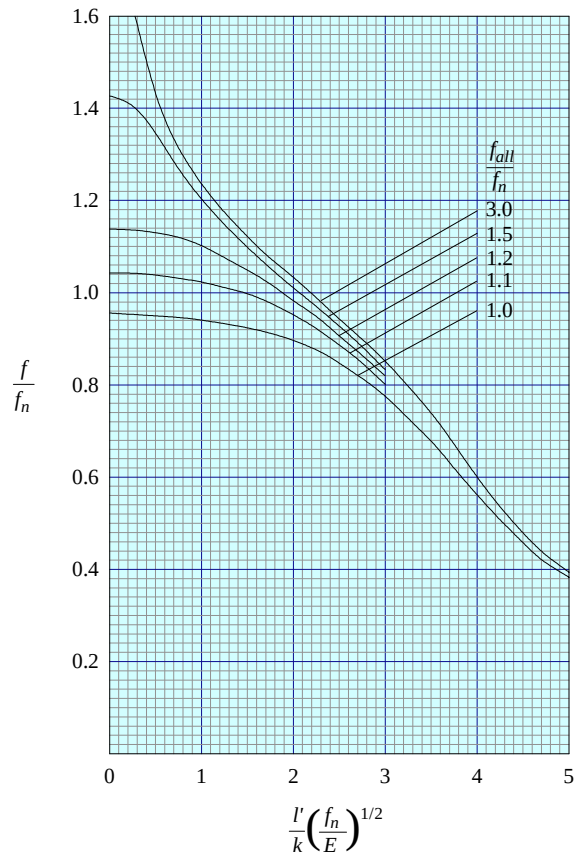


FIGURE 2b $\frac{e}{c} = 0.05, m = 10$