

TORSIONAL INSTABILITY OF STRINGERS AND STRUTS OF ANGLE SECTION

1. NOTATION

E_t	tangent modulus of elasticity at stress f_T (see Item No. 76016)	N/m ²	lbf/in ²
f_T	average compressive stress at which torsional instability begins	N/m ²	lbf/in ²
G	shear modulus of material	N/m ²	lbf/in ²
I_p	polar moment of inertia of section about centre of rotation	m ⁴	in ⁴
J	torsion constant of section as defined in Item No. Struct 00.07.00	m ⁴	in ⁴
k	radius of gyration of section about axis through centroid of section parallel to its base	m	in
l	effective length of stringer for torsional instability	m	in
t	thickness of section	m	in
Γ	total warping constant of section as defined in Item No. Struct 00.07.00	m ⁶	in ⁶
Γ_1	primary warping constant of section as defined in Item No. Struct 00.07.00	m ⁶	in ⁶
Γ_2	secondary warping constant of section as defined in Item No. Struct 00.07.00	m ⁶	in ⁶

Both SI and British Units are quoted but any coherent system of units may be used.

2. NOTES

This Item gives a method of calculating the critical compressive stress f_T at which torsional instability occurs for thin sections in which the primary warping constant, Γ_1 , is zero. Such sections are angles, tees, or any section that consists of straight parts meeting at a single point. This point is the shear centre, as defined in Item No. Struct 00.06.00, and in the sections considered approximates to the centre of rotation for torsional instability. Bulb angle stringers do not quite satisfy the conditions on which the theory is based, but provided the bulb diameter is not more than three times the mean thickness, experimental evidence shows that the results are fairly accurate. The data may be used for the torsional instability of stringers attached to a thin sheet, because the effects of the latter on torsional deformation of the stringer will be small, and the centre of rotation will alter appreciably.

The critical compressive stress is given by

$$f_T = \frac{GJ}{I_p} + \frac{\pi^2 E_t \Gamma_2}{I_p l^2}.$$

The first term on the right hand side of the above equation is the critical stress for a strut of which the ends are entirely free to rotate and warp, the instability mode being that of uniform twist. When end constraints entail a non-linear rate of twist the critical stress is increased by the second term on the right hand side of the above equation. The effective length l in the above equation depends on the nature of the end constraints and can best be described as the shortest distance between unwarped sections. The positions of the unwarped sections may generally be estimated from the symmetry of the structure or loading.

For a strut with ends fixed against rotation but free to warp, the effective length is equal to the actual length of the strut. If, however, the ends are also prevented from warping the effective length will be only half the actual length.

In a stringer fixed against rotation by uniformly spaced ribs, the unwarped sections would be at points mid-way between the ribs, and the effective length would be the rib pitch; in a panel test the ends as well as the middle sections are effectively constrained against warping and the effective length is equal to half the length of the panel, provided that there are no intermediate ribs.

The data may also be used to obtain the torsional instability stress for any open section strut of doubly symmetrical or point symmetrical section by using Γ instead of Γ_2 in the above formula.

3. DERIVATION

1. KAPPUS, R. Twisting failure of centrally loaded open-section columns in the elastic range. NACA tech. Memor. 851, 1938.
2. DUNN, L.G. An investigation of sheet-stiffener panels subjected to compression loads with particular reference to torsionally weak stiffeners. NACA tech. Note 752, 1940.
3. TIMOSHENKO, S. *Strength of materials*, Part II, second edition, Macmillan, 1941.
4. HOFF, N.J. Stresses in space-curved rings reinforcing the edges of cut-outs in monocoque fuselages. *Jl. R. aeronaut. Soc.*, Vol. 47, pp. 35-83, February 1943.

4. EXAMPLES

To find the torsional instability stress of the bulb angle section in Figure 1 when $l = 5$ in. The material properties are $m = 25$, $f_n = 50\,000$ lbf/in², $E = 10 \times 10^6$ lbf/in² and $G = 3.85 \times 10^6$ lbf/in².

For this section the value of J and Γ_2 may be calculated using the data given in Item Nos Struct 00.07.01 and 00.07.03, respectively. They are

$$J = 0.000233 \text{ in}^4 \text{ and } \Gamma_2 = 0.000049 \text{ in}^6.$$