

THE BUCKLING OF A COMPOSITE STRUT

1. NOTATION

E	Young's modulus	N/m^2	lbf/in^2
I	minimum moment of inertia of cross section of strut	m^4	in^4
l	length of constant section	m	in
P	buckling end load of composite strut	N	lbf

Suffixes

1	refers to length AB (see Figures 1 to 6)
2	refers to length BC (see Figures 1 to 6)

2. NOTES

The ratio $P(l_1 + l_2)^2 / (E_1 I_1)$ is plotted against the ratio $l_1 / (l_1 + l_2)$ for various values of the ratio $E_2 I_2 / (E_1 I_1)$ and for six conditions of end support. These conditions of end support are the various combinations of ends clamped, simply-supported or free.

The two parts of the strut are each of constant flexural rigidity and are rigidly attached end to end. The centroids of all sections lie on the axis of loading. The axes of minimum inertia of the two parts are co-planar. When this is not so the curves give conservative results.

The curves are drawn for $E_2 I_2$ greater than $E_1 I_1$.

It can be seen by inspection that combinations of curves may be used for symmetrical three-bay composite struts.

3. DERIVATION

1. CAPEY, E.C. Unpublished work, Royal Aircraft Establishment, Farnborough, June 1960.

For given values of l , I and E , buckling occurs at the lowest value of $P(l_1 + l_2)^2 / (E_1 I_1)$ that satisfies the appropriate transcendental equations in α , β and λ , where

$$\alpha^2 = Pl_1^2 / (E_1 I_1),$$

$$\beta^2 = Pl_2^2 / (E_2 I_2),$$

$$\lambda = l_2 / l_1.$$

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Both ends simply-supported

$$\lambda \alpha \cos \alpha \sin \beta + \beta \cos \beta \sin \alpha = 0.$$

One end simply-supported, other end clamped

$$\sin \alpha \cos \beta + \frac{\alpha \lambda}{\beta} \cos \alpha \sin \beta + \beta \left(1 + \frac{1}{\lambda}\right) \sin \alpha \sin \beta - \alpha(1 + \lambda) \cos \alpha \cos \beta = 0.$$

One end clamped, other end simply-supported

$$\cos \alpha \sin \beta + \frac{\beta}{\alpha \lambda} \sin \alpha \cos \beta + \alpha(1 + \lambda) \sin \alpha \sin \beta - \beta \left(1 + \frac{1}{\lambda}\right) \cos \alpha \cos \beta = 0.$$

Both ends clamped

$$2 + \left(\frac{\alpha \lambda}{\beta} + \frac{\beta}{\alpha \lambda}\right) \sin \alpha \sin \beta - 2 \cos \alpha \cos \beta - \alpha(1 + \lambda) \sin \alpha \cos \beta - \beta \left(1 + \frac{1}{\lambda}\right) \cos \alpha \sin \beta = 0.$$

One end clamped, other end free

$$\beta \cos \alpha \cos \beta - \alpha \lambda \sin \alpha \sin \beta = 0.$$

One end free, other end clamped

$$\alpha \lambda \cos \alpha \cos \beta - \beta \sin \alpha \sin \beta = 0.$$

4. EXAMPLE

To determine the load required to buckle a composite strut composed of two rigidly attached end-to-end struts when the ends are simply-supported and

$$E_1 I_1 = 0.25 \times 10^6 \text{ lbf in}^2,$$

$$E_2 I_2 = 0.5 \times 10^6 \text{ lbf in}^2,$$

$$l_1 = 20 \text{ in},$$

$$l_2 = 40 \text{ in},$$

$$l_1/(l_1 + l_2) = 0.333,$$

$$E_2 I_2/(E_1 I_1) = 2,$$

From Figure 1

$$P(l_1 + l_2)^2/(E_1 I_1) = 16$$

so that

$$P = 1110 \text{ lbf}.$$

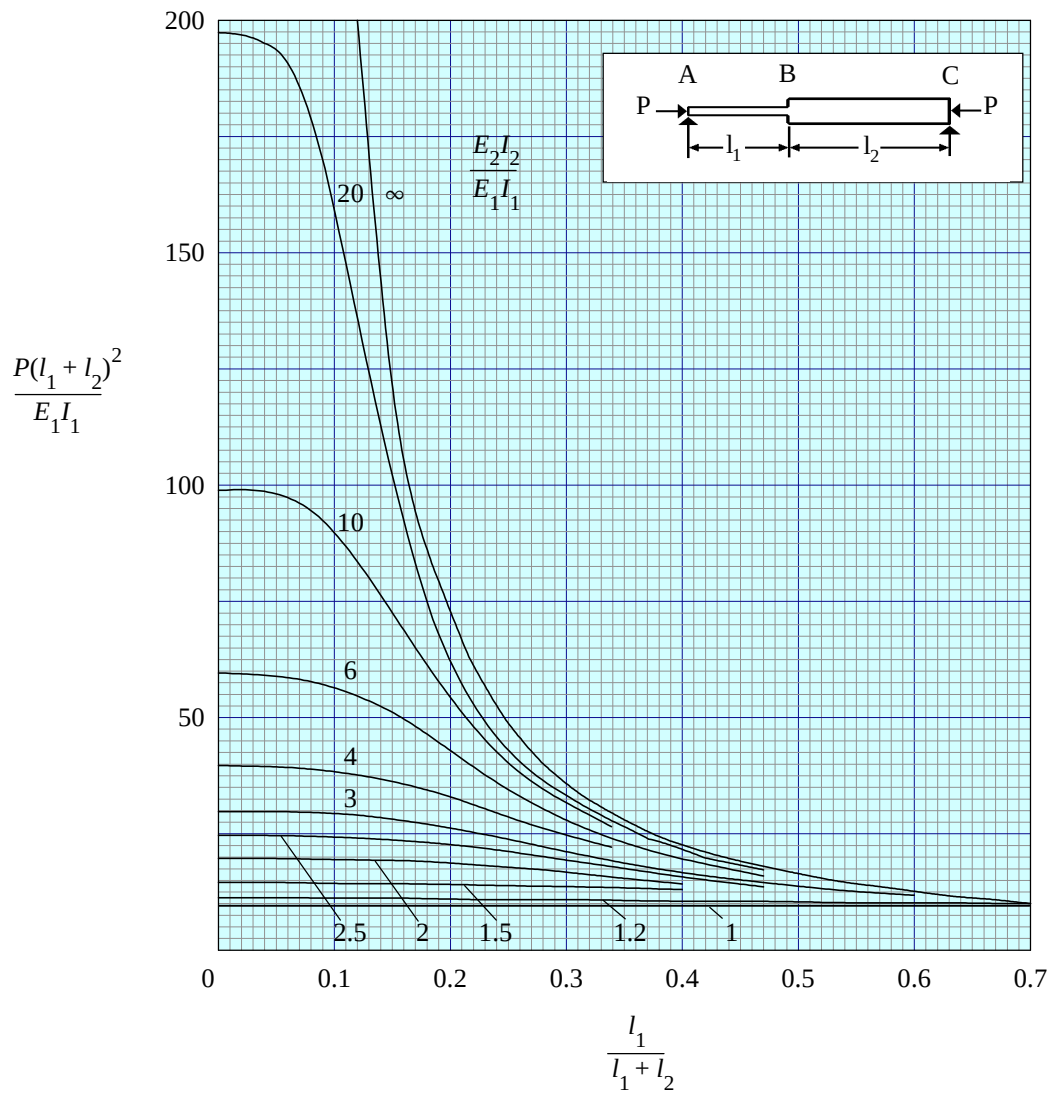


FIGURE 1 ENDS SIMPLY-SUPPORTED