

THE COMPRESSIVE INSTABILITY OF HYDRAULIC JACKS

1. NOTATION

a	distance between end of cylinder and piston	m	in
b	distance between end of piston rod and cylinder gland	m	in
E_c	Young's modulus of elasticity of cylinder material	N/m ²	lbf/in ²
E_R	Young's modulus of elasticity of piston rod material	N/m ²	lbf/in ²
I_c	moment of inertia of cross section of cylinder about axis perpendicular to plane of bending	m ⁴	in ⁴
I_R	moment of inertia of cross section of piston rod about axis perpendicular to plane of bending	m ⁴	in ⁴
L	overall length of jack	m	in
P_{cr}	critical compressive load	N	lbf
β	stiffness parameter = $\left(\frac{E_R I_R}{E_c I_c} \right)^{1/2}$		

Both SI and British units are quoted but any coherent system of units may be used.

2. NOTES

The charts present the variations of the critical compressive load with the values of jack length ratios a/L and b/L for eight values of the stiffness parameter β .

The charts have been derived from a theory using the concept of overlapping stiffness. It is assumed that the cylinder and piston rod have uniform section and material properties over their entire lengths and that their section centroids are initially perfectly aligned. It is assumed that the piston rod is simply-supported at the piston and the gland. The jack assembly is pin-jointed at both ends. For intermediate values of stiffness parameter β , the value of $P_{cr} L^2 / (\pi^2 E_R I_R)$ should be obtained by interpolation.

3. DERIVATION

1. KOSKO, E. The compressive instability of a hydraulic jack. Institute of Aerophysics, University of Toronto, Report No. 55, August 1958.
2. TORKINGTON, C. An investigation into the buckling instability of hydraulic jacks. College of Aeronautics Thesis, June 1960. (Unpublished experimental work).

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4. EXAMPLE

It is required to find the critical compressive load of a hydraulic jack, when

$$I_c = 0.144 \text{ in}^4, I_R = 0.016 \text{ in}^4,$$

$$E_c = E_R = 30 \times 10^6 \text{ lbf/in}^2,$$

$$L = 50 \text{ in}, a = 25 \text{ in and } b = 20.55 \text{ in}.$$

Therefore, $\beta = \left(\frac{0.016}{0.144} \right)^{1/2} = 0.333,$

$$\frac{a}{L} = 0.5 \text{ and } \frac{b}{L} = 0.411.$$

Then, since $\beta = 0.333$, from Figure 2, $P_{cr}L^2/(\pi^2 E_R I_R) = 1.87$

giving
$$P_{cr} = \frac{1.87 \times \pi^2 \times 30 \times 10^6 \times 0.016}{50^2} = 3540 \text{ lbf}.$$

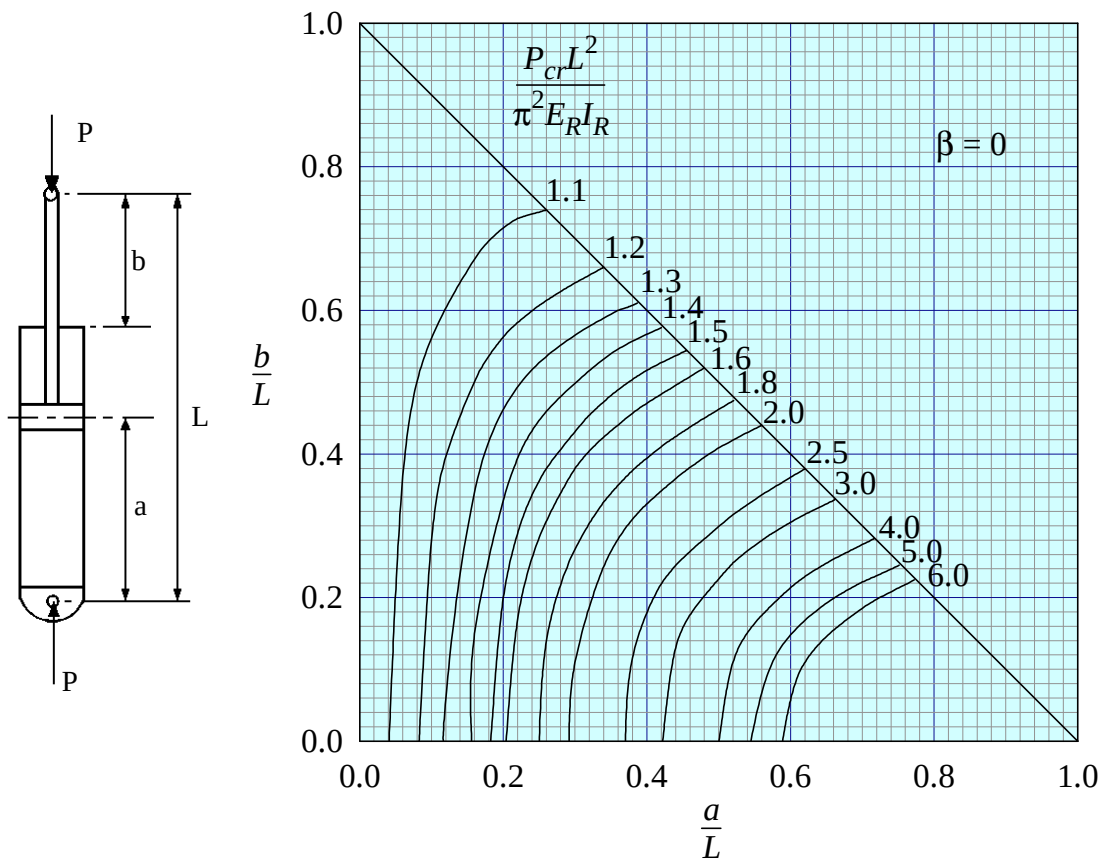


FIGURE 1 $\beta = 0$