

FORM FACTORS FOR CIRCULAR SECTIONS UNDER COMBINED BENDING AND AXIAL LOAD

1. NOTATION

D	outside diameter of tube	in	m
f_p	0.2 per cent proof stress of tube material	lbf/in ²	N/m ²
K	form factor, defined by $M_{max} = Kf_pZ$		
M	bending moment in tube	lbf in	N m
M_{max}	maximum permissible bending moment	lbf in	N m
P	axial load at centroid of tube	lbf	N
t	thickness of wall of tube	in	m
Z	section modulus $\frac{\pi}{32}D^3\left[1 - \left(1 - \frac{2t}{D}\right)^4\right]$	in ³	m ³

Both British and SI Units are quoted but any coherent system of units may be used.

2. NOTES

The form factor K is plotted against t/D in Figure 1 for $M/PD = \infty$, i.e. pure bending. In Figure 2 K is plotted against M/PD for various values of t/D , the curve for $t/D = 0$ being included for purposes of interpolation only. For convenience the form factor has been related to the 0.2 per cent proof stress of the material. Values of K taken from the curves give directly the values of the maximum permissible bending moment appropriate to a given constant ratio of bending moment and axial load as represented by M/PD . For other cases a process of successive approximation is necessary.

In the case of thin-walled tubes it is also necessary to check the possibility of failure due to instability, particularly when compressive end load is applied. The dotted portion of the curve in Figure 1 indicates the probable trend of K for thin tubes under pure bending when failure is influenced by instability.

3. DERIVATION

1. BARRETT, A.J. Note on the bending of some common beam sections into the plastic range. *Jl R. Aeronaut. Soc.*, Vol. 57, pp. 110-115, February 1953.
2. BARRETT, A.J. Unsymmetrical bending and bending combined with axial loading of a beam of rectangular cross-section into the plastic range. *Jl R. Aeronaut. Soc.*, Vol. 57, pp. 427-480, August 1953.

4. EXAMPLE

1. It is required to find the maximum permissible bending moment and tensile end load that, when combined in the ratio $M/PD = 2.5$, a circular section tube may be expected to carry when

$$D = 2.0 \text{ in}, \quad t = 0.4 \text{ in}, \quad \text{and} \quad f_p = 50\,000 \text{ lbf/in}^2.$$

Substituting in the expression given in the notation for Z gives $Z = 0.684 \text{ in}^3$ and from Figure 2, at $t/D = 0.2$, $K = 1.41$.

Hence,

$$M_{\max} = 1.41 \times 0.684 \times 50\,000 = 48\,000 \text{ lbf in}$$

and

$$P = \frac{M}{2.5D} = \frac{48\,000}{5.0} = 9600 \text{ lbf}.$$

2. It is required to find the maximum permissible bending moment for the tube of the first example when an end load of 14 400 lbf is applied.

With a fixed end load of 14 400 lbf the maximum permissible bending moment is found by successive approximation. After guessing an original value for M/PD , K is read from the curves or obtained by interpolation. This value of K gives a value for $M_{\max}$ and hence a new value for M/PD . This procedure is repeated until the value for $M_{\max}$ is sufficiently in agreement with its preceding value. This procedure is set out below commencing with an assumed value of M/PD of 1.5.

$\frac{M}{PD}$	K	$M_{\max}$ (lbf in)
1.50	1.36	46 500
1.61	1.37	46 900
1.63	1.375	47 000

$$\text{Hence } M_{\max} = 47\,000 \text{ lbf in.}$$