

CRITERIA OF YIELDING AND FAILURE

1. NOTATION

f_{ey}	equivalent direct yield stress	N/m ²	lbf/in ²
f_{et}	equivalent direct failing stress	N/m ²	lbf/in ²
f_x	direct stress acting in direction of y -axis	N/m ²	lbf/in ²
f_y	direct stress acting in direction of x -axis	N/m ²	lbf/in ²
f_1, f_2, f_3	principal stresses	N/m ²	lbf/in ²
q	shear stress	N/m ²	lbf/in ²
ν	Poisson's ratio		

Both SI and British units are quoted but any coherent system of units may be used.

2. NOTES

2.1 General

Most mechanical tests on materials are performed under a uniaxial tensile load. In order to predict the behaviour of materials under states of stress that are more complex than uniaxial tension it is necessary to be able to identify the magnitude of combined stresses which correspond to certain limiting states in the tension test.

For design purposes *yielding* and *failure* are defined in terms of *proof* and *ultimate* stress.

It is necessary to distinguish between the characteristic behaviour of two separate classes of materials, namely "brittle" and "ductile".

2.2 Brittle Materials

2.2.1 Principal Stress Criterion

Brittle materials behave sensibly elastically until failure and therefore only a true failure criterion need be considered. For such materials, experiment indicates that the stress that corresponds to the uniaxial tensile failing stress is the tensile principal stress of greater magnitude. This principal stress criterion is associated with the name of Rankine.

$$f_{et} = (f_1, f_2)_{max} = \left[\frac{f_x + f_y}{2} \pm \left\{ \left(\frac{f_x - f_y}{2} \right)^2 + q^2 \right\}^{1/2} \right]_{max}$$

2.3 Ductile Materials

2.3.1 Maximum Principal Strain Criterion

For ductile materials, most theoretical and experimental work has been concerned with the problem of yielding. Early attempts to define a yield criterion used the Rankine formula. However, it was found that with principal stresses of differing signs, yielding occurred at lower values than when all the stresses were of the same sign. Thus the maximum principal strain criterion of St. Venant was postulated, where Poisson's effects make some allowance for this.

$$f_{ey} = \text{maximum of } (f_1 - \nu f_2), f_2 - \nu f_1.$$

2.3.2 Maximum Shear Stress Criterion

Tresca and Guest formulated the criterion which states that yielding occurred when the maximum shear stress reached the value appropriate to the tension case, that is, half the tensile yield stress. The maximum shear stress is equal to half the difference between the principal stresses of greatest separation.

$$f_{ey} = \text{maximum of } |f_1|, |f_2|, |f_1 - f_2|.$$

2.3.3 Maximum Strain Energy Criterion

In an attempt to define a more general criterion, Beltrami, Haigh and Huber stated that yielding occurred when the strain energy density reached the value corresponding to that appropriate to the uniaxial tensile yield stress. This criterion again gives lower yield values for principal stresses of differing signs, and has the advantage of a single simple formula.

$$f_{ey}^2 = f_1^2 - 2\nu f_1 f_2 + f_2^2.$$

Experimental study has, however, shown that the variation of yield stress is due to a more fundamental fact, namely that for such combinations of stresses as are typical in structural work, the yield phenomenon is effectively unaffected by the values of the mean hydrostatic stress (the average of any three perpendicular direct stresses). This means that yielding is fundamentally a shearing or deformation phenomenon.

2.3.4 Von Mises and Hencky Criterion

For most aircraft metallic materials the von Mises-Hencky criterion is very widely used. This criterion associated yielding with the value of the strain energy density calculated after the mean hydrostatic stress has been subtracted from the direct stress components.

$$f_{ey}^2 = f_1^2 - f_1 f_2 + f_2^2,$$

which is an expression of the deformation energy associated with the stress system.

2.4 General Comments

The Maximum Shear Stress and the von Mises-Hencky criteria correspond most closely to material behaviour in the general case. The Maximum Shear Stress criterion is conservative for all stress combinations. The von Mises-Hencky criterion, while following more closely the trend of the experimental values, overestimates the stresses at yield for some combinations.