

AVERAGE AND EDGE STRESSES IN LONG FLAT RECTANGULAR PLATES IN COMPRESSION UNDER VARYING EDGE CONDITIONS

1. NOTATION

A_r	cross-sectional area of transverse member (see Section 2)	m ²	in ²
a	plate length	m	in
b	plate width	m	in
E	modulus of elasticity of plate material	N/m ²	lbf/in ²
f_a	average stress in plate = $\frac{\text{total load on plate}}{t \times b}$	N/m ²	lbf/in ²
f_b	stress at which plate first buckles	N/m ²	lbf/in ²
f_e	stress at edges of plate, parallel to direction of applied load	N/m ²	lbf/in ²
N	elastic stiffness per unit length against lateral movement of edges a	(N/m)/m	(lbf/in)/in
t	plate thickness	m	in
η	stiffness ratio after buckling		
λ	half wavelength of buckles in plate	m	in
μ_L	elastic edge restraint coefficient defined by $\mu_L = \frac{Nb}{Et}$		

Both SI and British units are quoted but any coherent system of units may be used.

2. NOTES

The ratio f_a/f_e is plotted against f_e/f_b for various values of the coefficient μ_L .

The plate is subjected to uniform compressive strain parallel to its edges a . It is assumed that the edges of the plate are all free to rotate and are constrained to remain in the plane of the plate. Figure 1 deals with the case in which, owing to the absence of edge members, the edges a are free to wave in the plane of the plate. The presence of edge members may tend to constrain the edges a to remain straight, and this condition, which may also arise from symmetry of loading and of the structure, is dealt with in Figure 2. It is assumed that the plate material is isotropic and that all stresses are within the elastic range.

In large panels typical of wing structures, having numerous stringers and similar uniformly and closely spaced traverse members, the restraint against lateral expansion of the plate will be mainly due to rib booms or frames attached to the plate along the edges b . The resistance offered by such a member, of length b and area A_r , averaged over the length a of the plate, is $EA_r/(ab)$, and with this value for N the coefficient μ_L equals $A_r/(at)$. The lateral constraint due to a heavy single transverse member will be distributed

along the panel by the stiffness of any adjoining structure (spar flanges, leading edge). On the other hand, bending in the plane of the plate of the edge members along the edges a will reduce the lateral restraint.

The ratio f_a/f_e depends largely on the wavelength of the buckles, which in turn is influenced by the ratio a/b . In this Item it has been assumed that λ/b has values appropriate to long plates, an assumption that in practice will be sufficiently accurate for plates with $a/b > 3$. The stiffness ratio after buckling is given by Item No. Struct. 02.01.24. If λ is known, an approximate value of η can be obtained from the figure of Item No. Struct. 02.01.24; if μ_L is known, an approximate value of η can be obtained by interpolation from the table on Item No. Struct. 02.01.24. Assuming η to remain constant, which will be sufficiently accurate up to $f_e/f_b < 4$, f_a is given by the equation

$$f_a/f_e = \eta + (f_b/f_e)(1 - \eta). \quad (2.1)$$

The effect of clamping the edges has been determined only for the case of $\mu_L = 0$. For this case the value of η is the same as that when the edges are simply-supported but with $\mu_L = 0.5$.

Values of f_b may be obtained from Item No. 72019. For plates with values of $f_e/f_b > 4$, see Item No. Struct. 02.01.03.

3. DERIVATION

This section lists selected sources that have assisted in the preparation of this Item.

1. COX, H.L. The buckling of a flat rectangular plate under axial compression and its behaviour after buckling. Aeronautical Research Council, UK, R and M No. 2041, 1946.
2. HEMP, W.S. Theory of flat panels buckling in compression. Aeronautical Research Council, UK, Report 8764, 1945.

4. EXAMPLE

It is required to determine the average stress in a long rectangular plate with edges simply-supported and restrained to remain straight. The plate dimensions are $t = 1.6$ mm and $b = 80$ mm, the value of μ_L is 0.75 and the material properties are $E = 68\,000$ MN/m² and Poisson's ratio is 0.3. The plate edge stress is 150 MN/m².

The plate buckling stress is determined from the lower set of curves of Figure 1 of Item No. 72019. In the notation of that Item, at $\mu_L = 0.75$, f_{xe}/f_0 for a long plate is 0.875. Thus,

$$f_b = 0.875 \times \frac{\pi^2 \times 68\,000 \times 10^6}{3 \times (1 - 0.3^2)} \left\{ \frac{1.6 \times 10^{-3}}{80 \times 10^{-3}} \right\}^2 = 86 \text{ MN/m}^2.$$

Therefore, $f_e/f_b = 150/86 = 1.74$ and from Figure 2, at $\mu_L = 0.75$, $f_a/f_e = 0.83$. Hence,

$$f_a = 0.83 \times 150 = 125 \text{ MN/m}^2. \quad (4.1)$$

A further example of the application of this Data Item is given in Section 7.2 of Item No. 72012.