

INITIAL BUCKLING OF SLIGHTLY CURVED PLATES UNDER COMBINED LONGITUDINAL AND CIRCUMFERENTIAL DIRECT STRESS (ALL EDGES SIMPLY-SUPPORTED)

1. NOTATION

a	axial length of plate	m	in
b	circumferential width of plate	m	in
E	Young's modulus of plate material	N/m ²	lbf/in ²
f_0	$\frac{\pi^2 E}{3(1 - \nu^2)} \left(\frac{t}{b} \right)^2$	N/m ²	lbf/in ²
f_x	axial compressive stress at buckling	N/m ²	lbf/in ²
f_θ	circumferential compressive stress at buckling	N/m ²	lbf/in ²
m	number of half waves in axial direction into which plate buckles		
p	pressure acting on complete cylinder; external is positive and internal is negative	N/m ²	lbf/in ²
R	radius of curvature of plate	m	in
t	plate thickness	m	in
ν	Poisson's ratio		

Both SI and British units are quoted but any coherent system of units may be used.

2. NOTES

The ratio f_x/f_0 is plotted against a/b for various values of f_θ/f_0 for the range of curvature covered by $b^2/Rt = 5, 10, 15, 20$ and 25 in Figure 1 to 5, respectively. For $b^2/Rt = 0$ (flat plate), see Item No. Struct 02.01.38.

The plate is loaded by uniform axial and circumferential stresses. The circumferential stresses are assumed to be due to internal or external pressure (that is, $f_\theta = pR/t$).

Prior to buckling it is assumed that the plate retains a circular cylindrical shape. At buckling all edges are free to rotate and are constrained against additional radial deformation. The longitudinal edges are constrained to remain straight. The plate buckles into one half wave across the width. The number of half waves along the length is given by the value of m appropriate to the regions bounded by the broken lines.

Issued December 1964

The curves apply only to the elastic range and to an initially perfect plate. There will be some reduction in the buckling stress due to initial irregularities and this may be appreciable when $b^2/Rt > 20$.

3. DERIVATION

1. SAYERS, K.H. The initial buckling of slightly curved panels under combined longitudinal and circumferential direct stresses. *Jl R. aeronaut. Soc.*, Vol. 68, No. 640, pp. 271-274, April 1964.

The formula

$$\frac{f_x}{f_0} = \frac{3(1-\nu^2)}{\pi^4} \left(\frac{ab}{Rt} \right)^2 \frac{m^2}{\left(\frac{a^2}{b^2} + m^2 \right)^2} + \frac{b^2 \left(\frac{a^2}{b^2} + m^2 \right)^2}{4m^2 a^2} - \frac{a^2}{m^2 b^2} \frac{f_\theta}{f_0}.$$

4. EXAMPLE

It is required to find the longitudinal stresses at which a slightly curved plate, with all edges simply-supported, first buckles when $a = 19$ in, $b = 6$ in, $t = 0.050$ in, $R = 68$ in, $E = 10 \times 10^6$ lbf/in² and an internal pressure of 2.94 lbf/in² is acting on it.

Then $f_\theta = pR/t = -4000$ lbf/in², $b^2/Rt = 10.6$, $a/b = 3.17$ and

$$f_0 = \frac{\pi^2 \times 10 \times 10^6}{3(1-0.3^2)} \left(\frac{0.050}{6.0} \right)^2 = 2510 \text{ lbf/in}^2.$$

Hence, $\frac{f_\theta}{f_0} = -\frac{4000}{2510} = -1.59$ and, by interpolation from Figure 1 to 5, $\frac{f_x}{f_0} = 2.42$.

Thus the initial axial buckling stress $f_x = 2.42 \times 2510 = 6070$ lbf/in².

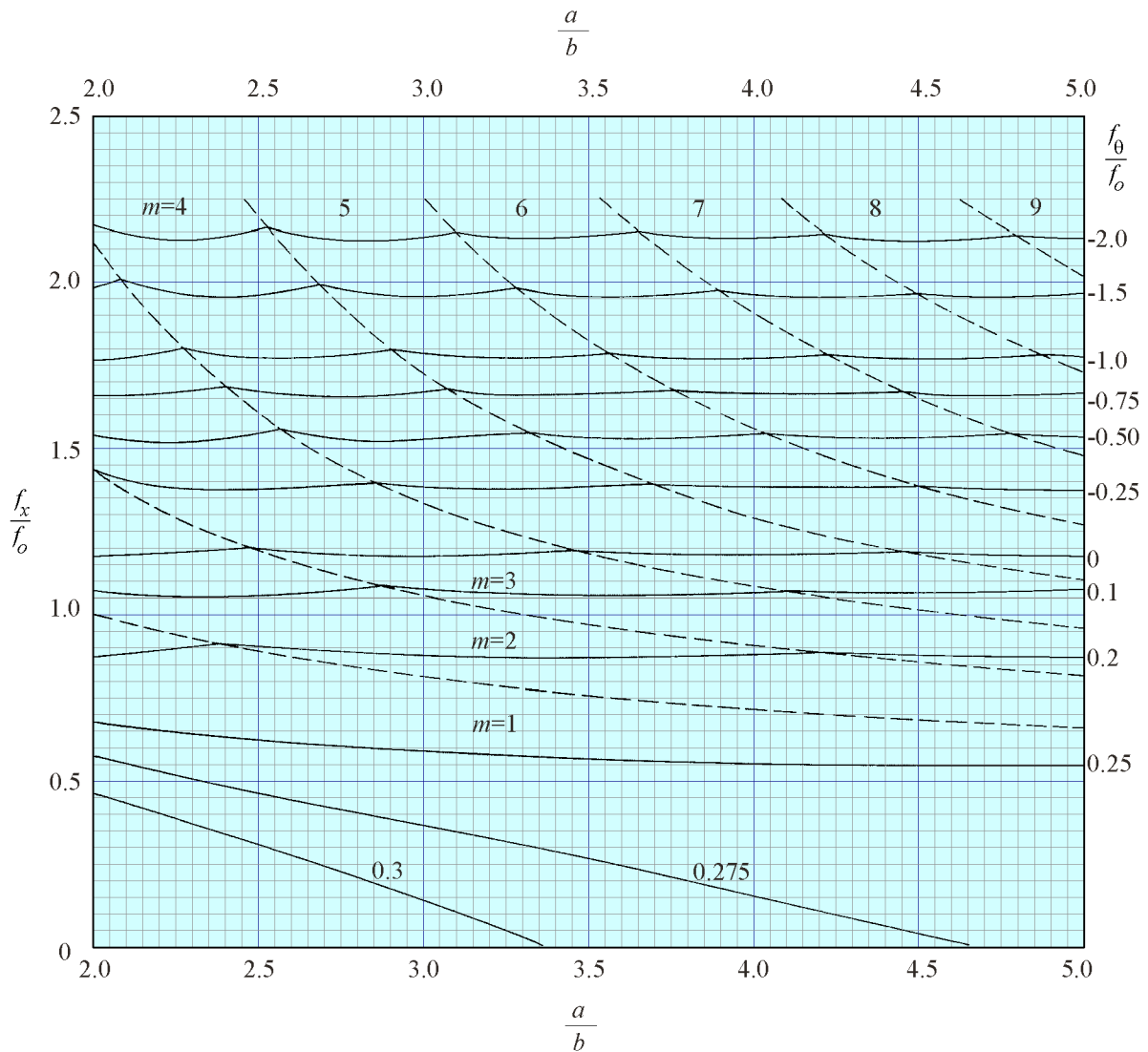
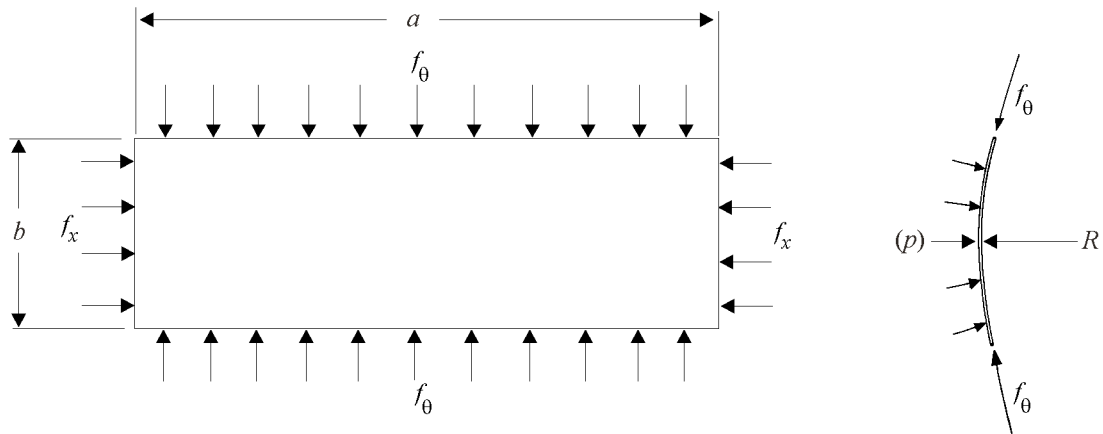


FIGURE 1 $\frac{b^2}{Rt} = 5$