

FLAT PANELS IN SHEAR. BUCKLING OF LONG PANELS WITH TRANSVERSE STIFFENERS

1. NOTATION

a	plate depth	m	in
b	effective stiffener spacing (see Section 2.1)	m	in
E_p	modulus of elasticity of plate material	N/m ²	lbf/in ²
E_s	modulus of elasticity of stiffener material	N/m ²	lbf/in ²
G	shear modulus of stiffener material	N/m ²	lbf/in ²
GJ	effective torsional stiffness of stiffener (see Section 2.1)	N m ²	lbf in ²
I	effective second moment of cross-sectional area of stiffener (see Section 2.1)	m ⁴	in ⁴
K	buckling stress coefficient defined by $q_b = KE_p(t/b)^2$		
q_b	shear stress at which plate first buckles	N/m ²	lbf/in ²
t	plate thickness	m	in
μ	$(E_s I_b / E_p a^2 t^3)^{1/2}$		
μ_c	least value of μ for which the stiffeners are effectively providing the condition of simple support		
ν	Poisson's ratio (taken as 0.3)		

Both SI and British units are quoted but any coherent system of units may be used.

2. NOTES

A long flat plate under shear first buckles at a stress as given by Item No. 71005^{*}. The addition of stiffeners increases the buckling stress and this Item deals with the case of identical stiffeners equally spaced. With relatively flexible stiffeners the plate and stiffeners buckle together, whereas with relatively rigid stiffeners, the plate first buckles between stiffeners. As the stiffness parameter μ increases the mode of buckling changes and this is indicated by the discontinuities of slope in the curves. When the stiffeners are completely rigid in bending, the limiting conditions of zero and infinite torsional stiffness are again covered by Item No. 71005.

The curves agree with results of tests in which the thickness of the attached leg of the stiffener was equal to or greater than the thickness of the plate. Thinner stiffeners may be forced by the sheet to buckle in a local mode. At present there is insufficient data to cover this effect in this Item.

^{*} Data Item No. 71005 "Buckling of flat plates in shear".

Figures 1 and 2 - K is plotted against μ for $a/b = 1.0, 1.5, 2.0, 3.0$ and 5.0 when the edges of the panel are clamped and simply-supported respectively and the stiffeners have no torsional stiffness.

The limiting value of K , as μ tends towards infinity, corresponds to the case when the stiffeners provide a simply-supported condition along the two edges of the plate of length a . When μ exceeds μ_c the value of K varies little, for the stiffeners are then effectively providing the condition of simple support. The values of μ_c are shown by the dotted curves of Figures 1 and 2.

Figures 3, 4, 5 and 6 - K is plotted against μ for $a/b = 1.0, 2.0, 3.0$ and 5.0 respectively, when the edges of the panel are simply-supported and the stiffeners have various values of torsional stiffness.

Figures 7, 8, 9 and 10 - K is plotted against μ for $a/b = 1.0, 1.5, 2.0$ and 3.0 respectively, when the edges of the panel are clamped and the stiffeners have various values of torsional stiffness.

The most efficient design of a panel may require stiffeners having μ less than μ_c but if the initial buckling stress is then exceeded the plate and stiffeners buckle together and early failure of the stiffeners may ensue. For general considerations regarding the dimensions to be used for the plate depth, reference should be made to Item No. 77014*.

2.1 Assessment of b , J and I

The “effective” values of b , J and I are dependent upon the stiffener design and its method of attachment to the plate. For open section stiffeners b should relate to the stiffener pitch. For open section stiffeners having multiple rows of attachments and more than one upright member, such as channels, and for closed section stiffeners, such as top-hat sections, b should relate to the plate between attachment lines of adjacent stiffeners.

The torsional stiffness of thin-walled open section stiffeners is usually very small and, even allowing for differential bending, can often be neglected. However, if μ is large an estimate of J may be necessary and in such cases it should relate to J of the stiffener alone. For closed section stiffeners, such as top-hat sections, J should include the plate acting with the stiffener to resist torsion. The case of a thin-walled circular tube stiffener arranged with half the tube on each side of the plate gives $J/I = 2$ so that, for Poisson's ratio = 0.3, $GJ/E_s I = 0.77$. (The value of 0.77 can be exceeded by using closed section stiffeners with width greater than their height.)

The analysis assumes that the centroids of the stiffeners lie in the mid-plane of the plate. When this is not the case the effective I may be calculated as either I of the stiffener alone about the plate mid-plane or, as I of the stiffener plus a contribution from the plate associated with the stiffener about their combined centroid. In practice the differences between these two assumptions are unlikely to result in a significant variation in q ; see the example (Section 4).

* Data Item No. 77014 “Flat panels in shear. Post-buckling analysis”.

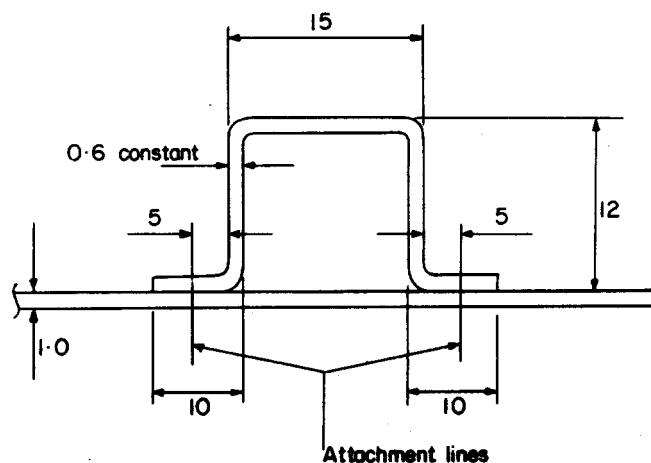
3. DERIVATION

This section lists selected sources that have assisted in the preparation of this Item.

1. STEIN, M.
FRALICH, R. Critical shear stress of infinitely long, simply-supported plate with transverse stiffeners. NACA tech. Note 1851, 1949.
2. COOK, I.T.
ROCKEY, K.C. Shear buckling of clamped and simply-supported infinitely long plates reinforced by transverse stiffeners. *Aeronaut. Q.*, Vol. XIII, part 1, pp. 41-70, February 1962.
3. COOK, I.T.
ROCKEY, K.C. Shear buckling of clamped and simply-supported infinitely long plates reinforced by closed-section transverse stiffeners. *Aeronaut. Q.*, Vol. XIII, part 3, pp. 212-222, August 1962.
4. ROCKEY, K.C.
COOK, I.T. Influence of torsional rigidity of transverse stiffeners upon the shear buckling of stiffened plates. *Aeronaut. Q.*, Vol. XV, part 2, pp. 88-131, May 1964.
5. ROCKEY, K.C.
COOK, I.T. Shear buckling of clamped infinitely long plates - influence of torsional rigidity of transverse stiffeners. *Aeronaut. Q.*, Vol. XVI, part 1, pp. 92-96, February 1965.

4. EXAMPLE

It is required to determine the buckling stress of a long simply-supported plate reinforced by top-hat section stiffeners. The stiffeners are attached to one side of the plate at a pitch of 85 mm. The plate dimensions are $a = 300$ mm, $t = 1.0$ mm and the material modulus $E_p = 72\,400$ MN/m². The stiffener dimensions are shown in Sketch 4.1 and its material properties are $E_s = 69\,600$ MN/m² and $G = 26\,800$ MN/m².



Sketch 4.1 Stiffener section (All dimensions in millimetres)

From the notes of Section 2.1 the value of I is calculated for the stiffener alone about the plate mid-plane as 2020 mm⁴. Also the value of J for the closed section is calculated as 1390 mm⁴. Since the section is

closed the distance between the attachment lines will be subtracted from the stiffener pitch to give b . Thus,

$$b = 85 - (2 \times 5 + 15) = 60 \text{ mm},$$

so that
$$\frac{a}{b} = \frac{300}{60} = 5.$$

Then, working in Newtons and millimetres,

$$\mu = \left(\frac{69\,600 \times 2020 \times 60}{2400 \times 300^2 \times 1.0^3} \right)^{1/2} = 1.14$$

and
$$\frac{GJ}{E_s I} = \frac{28\,600 \times 1390}{69\,600 \times 2020} = 0.265.$$

Therefore, from Figure 6 for $a/b = 5$ and simply-supported edges, $K = 7.8$ giving

$$q_b = 7.8 \times 72\,400 \times 10^6 \times \left(\frac{1.0}{60} \right)^2 = 157 \text{ MN/m}^2.$$

Note that calculating I for the stiffener and the plate beneath it about their joint centroid gives $I = 1410 \text{ mm}^4$ so that $\mu = 0.95$ and $GJ/E_s I = 0.380$. These give $K = 7.35$ and hence $q_b = 148 \text{ MN/m}^2$. This is less than 6 per cent different from the previous solution.

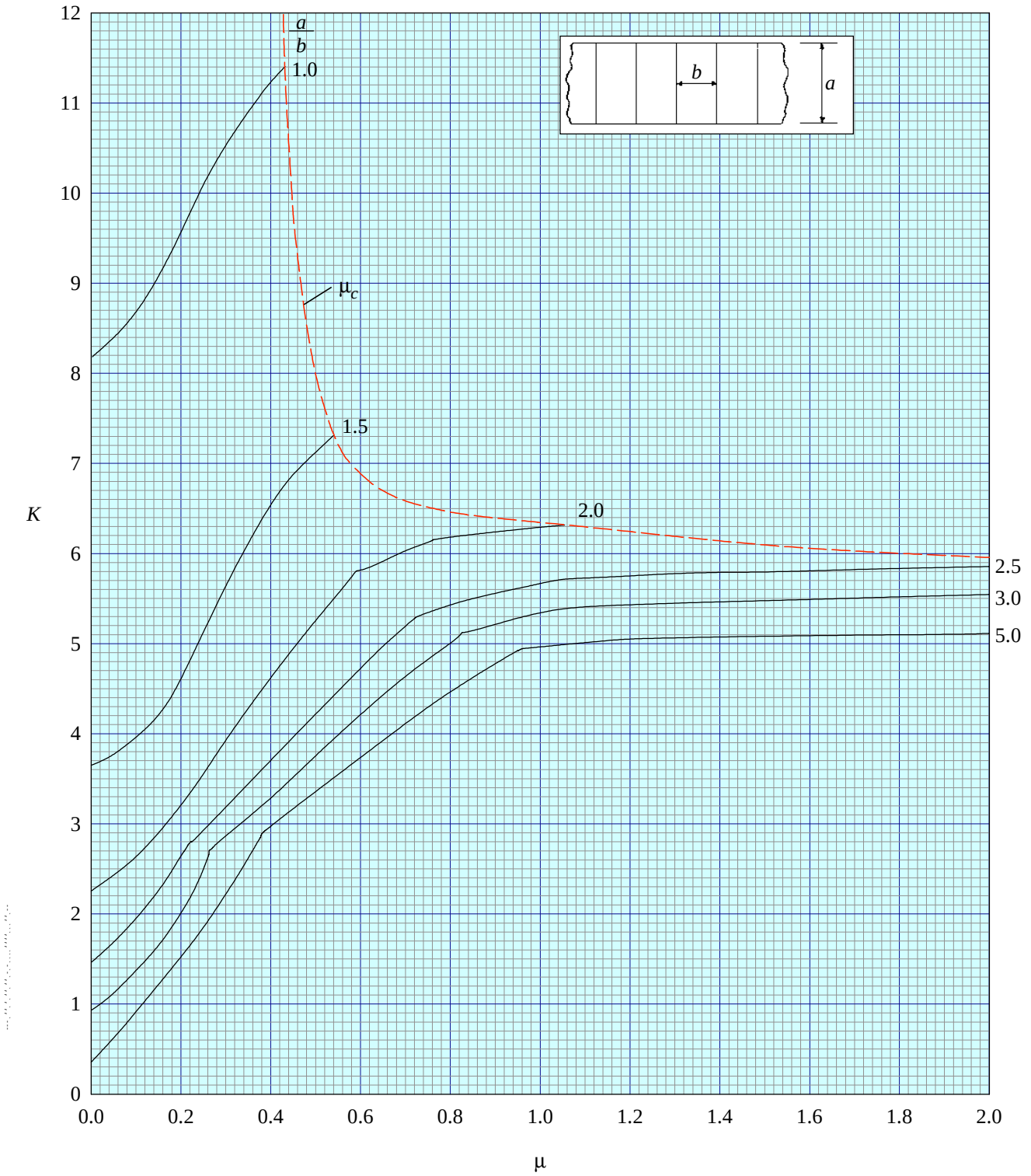


FIGURE 1 LONGITUDINAL EDGES CLAMPED, NO TORSIONAL STIFFNESS