

INITIAL BUCKLING OF FLAT PLATES UNDER COMPRESSION, BENDING AND SHEAR

1. NOTATION

a	length of plate	m	in
b	width of plate	m	in
E	Young's modulus of plate material	N/m ²	lbf/in ²
f_1	longitudinal compressive stress at edge AB	N/m ²	lbf/in ²
f_2	longitudinal stress at edge CD, positive if compression	N/m ²	lbf/in ²
f_{1b}, f_{2b}	values of stresses f_1 and f_2 respectively at which plate first buckles when no shear is present (see Item No. Struct. 02.04.04)	N/m ²	lbf/in ²
q	shear stress in plate	N/m ²	lbf/in ²
q_b	value of q at which plate first buckles in shear alone (see Item No. 71005)	N/m ²	lbf/in ²
t	thickness of plate	m	in

Both SI and British units are quoted but any coherent system of units may be used.

2. NOTES

Figure 1 gives, for various values of f_2/f_1 , the critical value of the ratio (f_1/f_{1b}) at which the plate first buckles under compression and bending combined with a shear stress, q . Alternatively, Figure 1 gives the critical value of the ratio q/q_b at which the plate buckles when already subjected to edge stresses f_1 and f_2 . Note that if any combination of f_1 , f_2 and q is represented by a point below the curve corresponding to the given f_2/f_1 there will be no buckling.

The plate is assumed to be isotropic, with a Poisson's ratio of 0.3, and simply-supported, but free from lateral restraint along all four edges. The curve corresponding to $f_2/f_1 = 1$, that is pure compression with shear, is the same as the upper curve in Item No. Struct 02.04.01 "Buckling stress ratios for plates under compression and shear". When the tensile stress exceeds the compressive stress the curve corresponding to $f_2/f_1 = -1$ may be used at least up to $f_2/f_1 = -1.3$.

The value of f_{1b} is obtained from Item No. Struct 02.04.04 "Initial buckling of flat plates under bending compression or tension", and that of q_b from the lowest curve on Figure 1 of Item No. 71005 "Buckling of flat plates in shear".

3. DERIVATION

1. – *Stahlbau kalendar* p. 392, 1939.
2. – *Stahlbau kalendar* p. 308, 1940.
3. TIMOSHENKO, S.P. *Theory of elastic stability*, Second edition, published by
GERE, J.M. McGraw-Hill, New York, 1961.

4. EXAMPLES

4.1 Example 1

It is required to find the edge stresses, f_1 and f_2 , at which the plate in the example given in Item No. Struct 02.04.04 first buckles when, in addition, a shear stress of 2150 lbf/in² is applied.

For the panel $b/a = 0.36$. Hence, using the lowest curve of Figure 1 of Item No. 71005, the shear buckling coefficient $K = 5.36$ so that $q_{be} = 5200$ lbf/in². This stress is sufficiently low for the effects of plasticity to be ignored so that $q_b = q_{be} = 5200$ lbf/in² giving $q/q_b = 0.41$. For $f_2/f_1 = -1.2$ the curve labelled $f_2/f_1 = -1.0$ on Figure 1 may be used to obtain a value of f_1/f_{1b} of $f_1/f_{1b} = 0.91$.

$$\text{Thus } f_1 = 0.91 \times 4170 = 3790 \text{ lbf/in}^2$$

$$\text{and } f_2 = -1.2 \times 3790 = -4550 \text{ lbf/in}^2.$$

4.2 Example 2

It is required to find the shear stress, q , at which a plate buckles when $a = 12$ in, $b = 3.2$ in, $t = 0.048$ in, $f_1 = 4000$ lbf/in², $f_2 = 2800$ lbf/in² and $E = 10 \times 10^6$ lbf/in².

$$\text{Hence, } \frac{a}{b} = 3.75, \frac{b}{a} = 0.267 \text{ and } \frac{f_2}{f_1} = 0.7.$$

From Item No. Struct 02.04.04,

$$K = K_{min} = 4.2 \text{ (since } a/b > 2.3 \text{)}.$$

$$\text{Hence, } f_{1b} = 4.2 \times 10 \times 10^6 \times (0.048/3.2)^2 = 9450 \text{ lbf/in}^2,$$

$$\text{giving } f_1/f_{1b} = 0.42.$$

From Figure 1, by interpolation for $f_2/f_1 = 0.7$,

$$q/q_b = 0.78.$$

From the lowest curve of Figure 1 of Item No. 71005, $K = 5.13$.

$$\text{Hence, } q_{be} = 5.13 \times 10 \times 10^6 \times (0.048/3.2)^2 = 11\,500 \text{ lbf/in}^2.$$