

MOMENTS IN CIRCULAR FRAME DUE TO CONCENTRATED LOADS AND COUPLES

1. NOTATION

M	bending moment in frame, positive if it produces tension on inside of frame	N m	lbf in
M_a	applied couple, positive if tending to rotate frame anti-clockwise	N m	lbf in
N_a	applied radial load, positive if acting outwards	N	lbf
R	radius of frame measured to centroid of section	m	in
T_a	applied tangential load, positive if tending to rotate frame anti-clockwise	N	lbf
θ	angular distance of point of application of load or couple from reference cross section, positive if anti-clockwise	degree	degree

Both SI and British units are quoted but any coherent system of units may be used.

2. NOTES

The ratios M/M_a , $M/(N_a R)$, $M/(T_a R)$ are plotted against the angle θ for loads and couples applied in the plane of the frame.

The load N_a is assumed to be balanced by tangential shear forces distributed around the circumference of the frame, in accordance with the simple theory of bending. The couple M_a is balanced by tangential shear forces distributed uniformly around the circumference of the frame. The distribution of shear forces which balance the load T_a is obtained by superimposing shear forces for a radial load and for a couple. These assumed distributions are indicated in the inset sketches. Departures from these distributions may occur and should be taken into account.

The frame is assumed to have constant flexural stiffness. No allowance is made for transverse shear deflection or circumferential extension of the frame centre-line, but these effects are not important provided that the depth of the section of the frame is less than one-tenth of R . Provided that the frame deflections are small, the bending moments due to a number of applied loads and couples may be found by superposition.

For direct and shear forces in the frame due to concentrated loads and couples see Data Items Nos Struct. 03.06.02 and 03.06.03.

3. DERIVATION

1. FAIRTHORNE, R.A. The stressing of thin rings. *Aircraft Engineering*, pp. 228-230, June 1939.
2. WISE, J.A. Analysis of circular rings in monocoque fuselages. *J. aeronaut. Sci.*, Vol. 6, No. 11, pp. 460-463, 1939.
3. FAHLBUSCH, H. WEGNER, W. Stress analysis of circular frames. NACA tech. Memorandum 999, 1941.

4. EXAMPLES

4.1 Example 1

To find the bending moment at the reference cross section in a circular frame of 36 in radius due to the following system of loads.

$$\begin{aligned}
 N_a &= +1000 \text{ lbf}, & \text{applied at } \theta &= 45^\circ, \\
 T_a &= -250 \text{ lbf}, & \text{applied at } \theta &= 225^\circ \\
 \text{and } M_a &= +400 \text{ lbf in}, & \text{applied at } \theta &= 240^\circ.
 \end{aligned}$$

From Figure 1 of this Data Item

$$\frac{M}{N_a R} = +0.05, \quad \frac{M}{T_a R} = +0.045 \text{ and } \frac{M}{M_a} = -0.109.$$

Hence,

$$\begin{aligned}
 M &= 0.05 \times 1000 \times 36 - 0.045 \times 250 \times 36 - 0.109 \times 400 \\
 &= 1351 \text{ lbf in.}
 \end{aligned}$$

4.2 Example 2

To find the bending moment at a cross section 90° anti-clockwise from the section of Example 1 under the same system of loads.

As we are counting the angle θ from the cross section anti-clockwise, the positions of the loads now are

$$N_a \text{ at } 315^\circ, \quad T_a \text{ at } 135^\circ \text{ and } M_a \text{ at } 150^\circ.$$

From Figure 1 of this Data Item

$$\frac{M}{N_a R} = +0.05, \quad \frac{M}{T_a R} = -0.045 \text{ and } \frac{M}{M_a} = +0.076.$$

Therefore,

$$\begin{aligned}
 M &= 0.05 \times 1000 \times 36 + 0.045 \times 250 \times 36 + 0.076 \times 400 \\
 &= 2235 \text{ lbf in.}
 \end{aligned}$$

For a more complex example see Data Item No. Struct. 03.06.06.