

RADIAL DEFLECTIONS OF CIRCULAR FRAME DUE TO CONCENTRATED LOADS AND COUPLES

1. NOTATION

E	modulus of elasticity of material of frame	N/m ²	lbf/in ²
I	moment of inertia of cross section of frame about section centroid	m ⁴	in ⁴
M_a	applied bending moment, positive if tending to rotate frame anti-clockwise	N m	lbf in
N_a	applied radial load, positive if acting outwards	N	lbf
R	radius of frame measured to centroid of section	m	in
T_a	applied tangential load, positive if tending to rotate frame anti-clockwise	N	lbf
$\delta_N, \delta_M, \delta_T$	radial deflections due to N_a, M_a and T_a , positive if outwards	m	in
θ	angular distance of point of application of load or couple from reference cross section, positive if anti-clockwise	degree	degree

Both SI and British units are quoted but any coherent system of units may be used.

2. NOTES

The ratios $\delta_N EI / N_a R^3$, $\delta_M EI / M_a R^2$ and $\delta_T EI / T_a R^3$ are plotted against the angle θ .

The conditions assumed are as described in Data Items Nos Struct. 03.06.01, 03.06.02 and 03.06.03.

3. DERIVATION

1. FAIRTHORNE, R.A. The stressing of thin rings. *Aircraft Engineering*, pp. 228-230, June 1939.

4. EXAMPLE

To find the radial deflection at a cross section of a frame for which $R = 0.9 \text{ m}$, $I = 0.08 \times 10^{-6} \text{ m}^4$, and $E = 69\,000 \text{ MN/m}^2$, under a system of loads such that

$$\begin{aligned} N_a &= -4500 \text{ N, applied at } \theta = 90^\circ, \\ M_a &= 45 \text{ N m, applied at } \theta = 315^\circ \\ \text{and } T_a &= 1100 \text{ N, applied at } \theta = 315^\circ. \end{aligned}$$

From Figure 1 of this Data Item

$$\frac{\delta_N EI}{N_a R^3} = -0.034,$$

$$\begin{aligned} \text{hence, } \delta_N &= \frac{-0.034 \times (-4500) \times 0.9^3}{69\,000 \times 10^6 \times 0.08 \times 10^{-6}} \\ &= 20.2 \text{ mm.} \end{aligned}$$

From Figure 1 also

$$\frac{\delta_M EI}{M_a R^2} = -0.060,$$

$$\begin{aligned} \text{hence, } \delta_M &= \frac{-0.060 \times 45 \times 0.9^2}{69\,000 \times 10^6 \times 0.08 \times 10^{-6}} \\ &= -0.4 \text{ mm.} \end{aligned}$$

Finally from Figure 1

$$\frac{\delta_T EI}{T_a R^3} = +0.019,$$

$$\begin{aligned} \text{hence, } \delta_T &= \frac{0.019 \times 1100 \times 0.9^2}{69\,000 \times 10^6 \times 0.08 \times 10^{-6}} \\ &= 2.76 \text{ mm.} \end{aligned}$$

Therefore, the total radial deflection at the reference cross section,

$$\begin{aligned} \delta &= 20.2 - 0.4 + 2.76 \\ &= 22.56 \text{ mm.} \end{aligned}$$