

BENDING MOMENT IN ELLIPTICAL FRAMES DUE TO INTERNAL PRESSURE

1. NOTATION

$2a$	length of major axis of elliptical frame	m	in
$2b$	length of minor axis of elliptical frame	m	in
I_s	moment of inertia of cross section of frame at point s	m^4	in^4
I_0	moment of inertia of cross section of frame at minor axis	m^4	in^4
M_s	bending moment at point s of frame with constant moment of inertia, positive if it produces tension on the inside of the frame	N m	lbf in
M_c	constant correction to bending moment in frame with variable moment of inertia	N m	lbf in
M'_s	$M_s + M_c$ = bending moment at point s of frame with variable moment of inertia	N m	lbf in
p	uniform normal internal loading on frame	N/m	lbf/in
s	distance along perimeter from minor axis	m	in
S	quarter of perimeter	m	in
y	distance of point on perimeter from major axis	m	in

Both SI and British units are quoted but any coherent system of units may be used.

2. NOTES

In Figure 1 the ratios $M_s/(pa^2)$ and y/b are plotted against the ratio s/S for various values of b/a .

The moments M_s refer to bending moments in an elliptical frame with constant moment of inertia. The curves can also be used to obtain the bending moment in any elliptical frame of varying moment of inertia provided that the major and minor axes are axes of symmetry. The effect of this variation of moment of inertia is simply to add a constant bending moment M_c , where

$$M_c = - \frac{\int_0^S (M_s/I_s) ds}{\int_0^S (1/I_s) ds} . \quad (2.1)$$

No allowance is made for transverse shear deflection or circumferential extension of the frame centre-line, but these effects are not important provided that the depth of the section of the frame is less than one-tenth of the radius of curvature.

The contribution of the skin to the stiffness of the frame must be taken into account.

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3. DERIVATION

1. TIMOSHENKO, S.P. *Strength of Materials*, Part II (second edition), McGraw-Hill, p. 90, 1941.

4. EXAMPLES

4.1 Example 1

To find the maximum bending moment in the frame of a fuselage of elliptical cross section with frames equally spaced at 0.38 m pitch when the internal pressure is 69 000 N/m², $a = 1.4$ m and $b = 1.1$ m.

Then $p = 69\,000 \times 0.38 = 26\,200$ N/m

and $\frac{b}{a} = 0.786$.

From Figure 1, the maximum bending moment occurs at $s/S = 1$ and by interpolation,

$$\frac{M_s}{pa^2} = 0.103,$$

hence, $M_s = 0.103 \times 26\,200 \times 1.4^2 = 5290$ N m.

4.2 Example 2

To find the bending moment correction and the bending moments at the major and minor axes of an elliptical frame when $a = 1.6$ m, $b = 0.8$ m, $p = 21\,000$ N/m and the moment of inertia varies as in the following table.

s/S	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
I_s/I_0	1.0	1.01	1.04	1.09	1.16	1.25	1.36	1.49	1.64	1.81	2.0

Then $\frac{b}{a} = 0.5$.

From Figure 1, for $b/a = 0.5$ and from the above values of I_s/I_0 the following table is obtained.

s/S	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$M_s/(pa^2)$	-0.156	-0.151	-0.136	-0.110	-0.073	-0.024	0.030	0.090	0.150	0.197	0.218
$1/(I_s/I_0)$	1	0.990	0.962	0.917	0.862	0.800	0.735	0.671	0.610	0.552	0.5
$M_s/(pa^2)/(I_s/I_0)$	-0.156	-0.150	-0.131	-0.101	-0.063	-0.019	0.022	0.060	0.091	0.109	0.109