

FLEXIBLE CIRCULAR FRAMES SUPPORTING A SHELL. DISPLACEMENTS DUE TO CONCENTRATED LOADS AND COUPLES

1. NOTATION

E	Young's modulus for the material of the frame	N/m ²	lbf/in ²
G	shear modulus for the material of the shell	N/m ²	lbf/in ²
I	effective moment of inertia of cross section of frame including an allowance for the effect of the attached skin	m ⁴	in ⁴
L	$2L_1L_2/(L_1 + L_2)$ where L_1 and L_2 are the distances from the loaded frame to the adjacent frames	m	in
L'	effective frame spacing (see Data Item No. Struct. 03.06.17)	m	in
M_a	applied couple, positive if tending to rotate the frame anti-clockwise	N m	lbf in
N_a	applied radial load, positive if acting outward	N	lbf
R	radius measured to centroid of frame section	m	in
t	thickness of skin of shell effective in shear	m	in
t'	equivalent thickness of skin of shell effective in carrying longitudinal stresses	m	in
T_a	applied tangential load, positive if tending to rotate the frame anti-clockwise	N	lbf
δ	radial displacement, positive if measured outward	m	in
η	tangential displacement, positive if measured anti-clockwise	m	in
θ	angle between the section at which the load or couple is applied and the section under consideration, positive if measured clockwise	degree	degree
ψ	angular displacement, positive if measured anti-clockwise	rad	rad

Both SI and British units are quoted but any coherent system of units may be used.

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2. NOTES

Nine sets of curves are given showing the radial, angular and tangential displacements of flexible circular frames supporting shells due to concentrated couples, concentrated tangential loads and concentrated radial loads.

Figure 1 – The ratio $\delta/(LM_a/GtR^2)$ is plotted against the angle θ for various values of the parameter GtR^4/EIL' and the curves are anti-symmetrical about the $\theta = 180^\circ$ ordinate.

Figure 2 – The ratio $\delta/(LT_a/GtR)$ is plotted against the angle θ for various values of the parameter GtR^4/EIL' and the curves are anti-symmetrical about the $\theta = 180^\circ$ ordinate.

Figure 3 – The ratio $\delta/(LN_a/GtR)$ is plotted against the angle θ for various values of the parameter GtR^4/EIL' and the curves are symmetrical about the $\theta = 180^\circ$ ordinate.

Figure 4 – The ratio $\psi/(LM_a/GtR^3)$ is plotted against the angle θ for various values of the parameter GtR^4/EIL' and the curves are symmetrical about the $\theta = 180^\circ$ ordinate.

Figure 5 – The ratio $\psi/(LT_a/GtR^2)$ is plotted against the angle θ for various values of the parameter GtR^4/EIL' and the curves are symmetrical about the $\theta = 180^\circ$ ordinate.

Figure 6 – The ratio $\psi/(LN_a/GtR^2)$ is plotted against the angle θ for various values of the parameter GtR^4/EIL' and the curves are anti-symmetrical about the $\theta = 180^\circ$ ordinate.

Figure 7 – The ratio $\eta/(LM_a/GtR^2)$ is plotted against the angle θ for various values of the parameter GtR^4/EIL' and the curves are symmetrical about the $\theta = 180^\circ$ ordinate.

Figure 8 – The ratio $\eta/(LT_a/GtR)$ is plotted against the angle θ for various values of the parameter GtR^4/EIL' and the curves are symmetrical about the $\theta = 180^\circ$ ordinate.

Figure 9 – The ratio $\eta/(LN_a/GtR)$ is plotted against the angle θ for various values of the parameter GtR^4/EIL' and the curves are anti-symmetrical about the $\theta = 180^\circ$ ordinate.

The couple M_a and the loads T_a and N_a are assumed to act in the plane of the frame and to be balanced entirely by tangential shear forces in the shell. These forces arise from the displacement of the frame as a whole in its own plane and from deformation of the frame from its original circular shape. In all cases the shear deflection in the skin between adjacent frames is taken into account and the displacements given are the total displacements of the frame relative to the line joining the centres of the nearest cross sections on either side that are not distorted. Rigid connection of frames to skins is assumed. The effective value of GtR^4/EIL' is reduced when flexible connection exists between the shell and frame.

The curves marked $GtR^4/EIL' = 0$ are for infinitely rigid frames which are displaced as a whole with no deformation. The radial displacements of a circular frame where the complete loading system is independent of the relative flexibility of the skin and frame are given by Data Item No. Struct. 03.06.04.

3. DERIVATION

1. WIGNOT, J.E. Analysis of circular shell-supported frames. NACA tech. Note 929,
COMBS, H. December 1943.
ENSRUD, A.F.

4. EXAMPLE

To find the radial displacements at $\theta = 0^\circ$ and $\theta = 60^\circ$ for a circular frame supporting a shell under equal and opposite loads N_a applied at $\theta = 0^\circ$ and $\theta = 180^\circ$, when

$$R = 40 \text{ in}, \quad t = 0.048 \text{ in}, \quad t' = 0.090 \text{ in}, \quad I = 1.24 \text{ in}^4,$$

$$E = 10 \times 10^6 \text{ lbf/in}^2, \quad G = 3.8 \times 10^6 \text{ lbf/in}^2, \quad L_1 = 60 \text{ in}, \quad L_2 = 95 \text{ in}$$

and $N_a = 5000 \text{ lbf}.$

Then
$$L = \frac{2 \times 60 \times 95}{60 + 95} = 73.5 \text{ in}.$$

Using Data Item No. Struct. 03.06.17,

$$\frac{GtR^4}{EIL} = 512 \quad \text{and} \quad \frac{R^6 t'}{IL^3} = 750,$$

hence
$$\frac{GtR^4}{EIL'} = 160.$$

Now
$$\frac{LN_a}{GtR} = 0.0504 \text{ in}$$

and from Figure 3 of this Data Item, for the radial displacement at $\theta = 0^\circ$, adding the deflections for each of the loads N_a ,

$$\delta / \frac{LN_a}{GtR} = 2.3 + 0.25,$$

so that
$$\delta = 2.55 \times 0.0504 = 0.129 \text{ in}.$$

For the radial displacement at $\theta = 60^\circ$,

$$\delta / \frac{LN_a}{GtR} = -0.9 - 0.2,$$

so that
$$\delta = -1.1 \times 0.0504 = -0.055 \text{ in}.$$

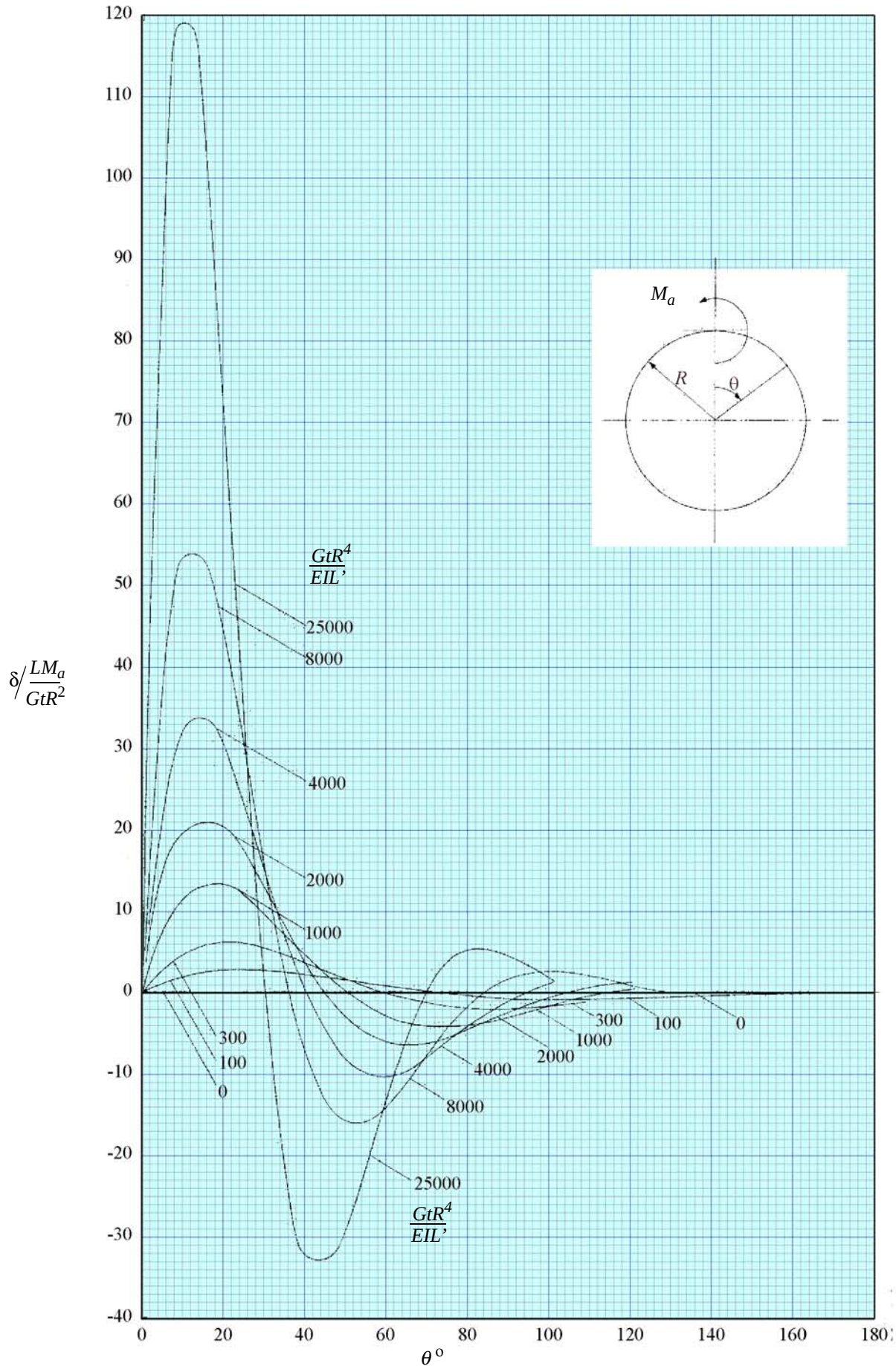


FIGURE 1 RADIAL DISPLACEMENTS DUE TO CONCENTRATED COUPLES