

# GRAPHICAL METHOD FOR ESTIMATING THE SPANWISE DISTRIBUTION OF AERODYNAMIC CENTRE ON WINGS IN SUBSONIC FLOW

## 1. INTRODUCTION

A graphical method is presented for the rapid estimation of the spanwise distribution of aerodynamic centre over uncambered and untwisted wings of zero thickness in unseparated, sub-critical flow. The basis of the method has been derived by Garner<sup>1</sup> from numerous calculations by Multhopp's<sup>2</sup> subsonic lifting-surface theory. In the 'tangent approximation' this method has been modified to take account of experimental evidence which shows that the aerodynamic centre near the tip lies further aft than theory predicts. The method has been extended to allow for the effect of wing thickness. Comments on the accuracy of Multhopp's subsonic lifting-surface theory will be found in Addendum A.

When coupled with a rapid estimation of spanwise loading presented in T.D.Memo.6403 (Ref. 3), the method determines approximately the distribution of pitching moment due to a change in incidence. The two procedures may be regarded as initial steps in the estimation of the chordwise pressure distribution<sup>4</sup>.

## 2. NOTATION

|                     |                                                                                                      |
|---------------------|------------------------------------------------------------------------------------------------------|
| $A$                 | aspect ratio of net wing ( $= 2s/\bar{c}$ )                                                          |
| $c$                 | local chord of wing                                                                                  |
| $\bar{c}$           | geometric mean chord $\left( = \int_0^1 c \, d\eta \right)$                                          |
| $c_r$               | root chord                                                                                           |
| $c_t$               | effective tip chord defined in Section 3.1 step (v)                                                  |
| $J$                 | see Figure 3                                                                                         |
| $m$                 | number of spanwise terms in theoretical solution                                                     |
| $M$                 | free-stream Mach number                                                                              |
| $s$                 | semi-span of net wing                                                                                |
| $t$                 | maximum thickness of aerofoil section                                                                |
| $t_{1/4}$           | thickness of aerofoil section at quarter-chord position                                              |
| $t_{3/4}$           | thickness of aerofoil section at three-quarter-chord position                                        |
| $X_{ac}$            | position of local aerodynamic centre measured as a fraction of the local chord from the leading edge |
| $\Delta X_{ac}$     | incremental thickness correction to $X_{ac}$                                                         |
| $(X_{ac})_{\eta r}$ | value of $X_{ac}$ in root region (see Section 4.1)                                                   |

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|                     |                                                                                                                         |
|---------------------|-------------------------------------------------------------------------------------------------------------------------|
| $(X_{ac})_{\eta t}$ | value of $X_{ac}$ in tip region (see Section 4.2)                                                                       |
| $X_0$               | value of $X_{ac}$ at $\eta = 0$ (see Figures 1 and 2)                                                                   |
| $X_{00}$            | value of $X_0$ for an unswept wing of the same aspect ratio (i.e. $\beta^{-1}\tan\Lambda_{1/4} = 0$ in Figures 1 and 2) |
| $\beta$             | $\sqrt{1 - M^2}$                                                                                                        |
| $\eta$              | distance from body side as a fraction of net semi-span                                                                  |
| $\lambda$           | Küchemann's interpolation function (see Section 4.1)                                                                    |
| $\Lambda_{1/4}$     | quarter-chord sweepback                                                                                                 |
| $\mu$               | see Equation (4.3)                                                                                                      |
| $\sigma$            | shape parameter for thickness correction (see Section 3.3)                                                              |

### 3. METHOD

The basic graphical method, discussed in Section 4, is derived from numerous calculations by Multhopp's<sup>2</sup> subsonic lifting-surface theory; the steps in the calculation are described in Section 3.1. In Section 3.2 a modification is suggested in the light of experimental evidence, and in Section 3.3 an additive term is proposed which allows for the effect of wing thickness. A worked example is given in Section 6 and is illustrated in Figure 6.

#### 3.1 The Basic Method

The application of the graphical method requires the knowledge of the semi-span, aspect ratio, quarter-chord sweepback, root chord and tip chord of the wing and the Mach number. The procedure is as follows:

- Step (i) Evaluate  $\beta A$  and  $\beta^{-1}\tan\Lambda_{1/4}$  at  $\eta = 0.25$ .
- Step (ii) Knowing  $\beta A$  and  $\beta^{-1}\tan\Lambda_{1/4}$ , obtain  $X_{00}$  and  $X_0$  from Figure 1 or 2.
- Step (iii) Evaluate  $\mu\eta\beta s/c_r = \{\eta\beta s/c_r\}(1.10 + 0.267\beta^{-1}\tan\Lambda_{1/4})$  for  $0 \leq \eta \leq 0.6$ , and hence determine related values of  $J$  from the curve in Figure 3. (3.1)
- Step (iv) Determine  $(X_{ac})_{\eta r} = X_{00} + (X_0 - X_{00})J$ . (3.2)
- Step (v) Estimate the effective tip chord.  $c_t$  is defined as the exact tip chord or 0.4 times the wing chord at  $\eta = 0.75$ , whichever is the larger.
- Step (vi) Evaluate  $(1 - \eta)\beta s/c_t$  for  $0.4 \leq \eta \leq 1.0$  and  $\beta^{-1}\tan\Lambda_{1/4}$  at  $\eta = 0.75$ , and hence determine  $(X_{ac})_{\eta t}$  from Figure 4.
- Step (vii) Plot the values of  $(X_{ac})_{\eta r}$  and  $(X_{ac})_{\eta t}$  against  $\eta$ . Joint the two curves by the common tangent: if this is not possible, the two curves are to be faired together (see Section 3.2).

The procedure is illustrated in Figure 6 and a worked example given in Section 6.

### 3.2 Tangent Approximation

Experimental evidence shows that the local aerodynamic centre near the tip lies further from the leading edge than theory predicts. It is therefore recommended that the following empirical modification to the basic method be used. The curve of  $(X_{ac})_{\eta t}$  should be replaced by the common tangent extended to  $\eta = 1$  if this exists; otherwise the tip region is represented by the tangent at the point where the faired portion joins the curve of  $(X_{ac})_{\eta t}$ .

### 3.3 Thickness Correction

An additive correction is suggested which allows for the effect of aerofoil thickness according to approximate two-dimensional theory (Ref. 5, Fig. 1b) with modifications for sweepback and Mach number. The correction is dependent on a thickness parameter

$$\frac{t}{c} \sec \Lambda_{1/4},$$

and also on a shape parameter

$$\sigma = \frac{(t_{1/4} - t_{3/4})}{t},$$

where  $t_{1/4}$  and  $t_{3/4}$  and the maximum thickness  $t$  are defined as those of the streamwise section.

From a knowledge of these two parameters, the correction is

$$\Delta X_{ac} = \frac{1}{\sqrt{1 - M^2 \cos^2 \Lambda_{1/4}}} f\left(\frac{t}{c} \sec \Lambda_{1/4}, \sigma\right), \quad (3.3)$$

where  $f\left(\frac{t}{c} \sec \Lambda_{1/4}, \sigma\right)$  is given in Figure 5. For wings with curved tips the local quarter-chord sweepback,  $\Lambda_{1/4}$ , is used in calculating the thickness correction.

#### 4. FOUNDATIONS OF BASIC METHOD

A simple graphical method has been derived for estimating the spanwise distribution of local aerodynamic centre in satisfactory agreement with lifting-surface theory. From Reference 2 theoretical values of  $X_{ac}$  can be calculated at the spanwise stations  $\eta = \eta_n = \sin\{n\pi/(m+1)\}$ , where  $n = 0, 1, \dots, \frac{1}{2}(m-1)$ . The basic method<sup>1</sup> sets out to represent the results of about 40 such calculations with  $m = 11$  or 15.

##### 4.1 Root Region

First of all, a chart was prepared for  $X_0$ , the value of  $X_{ac}$  at  $\eta = 0$ . This was found to depend on  $\beta A$  and  $\beta^{-1} \tan \Lambda_{1/4}$  and was determined in reasonable accord with theory by the carpet in Figure 1. For small values of  $\eta$  a relationship

$$(X_{ac})_{\eta r} = X_{00} + (X_0 - X_{00})J \quad (4.1)$$

was sought as a generalisation of Küchemann's<sup>6</sup> formula

$$X = \frac{1}{4} + \frac{1}{2\pi} \Lambda_{1/4} \lambda\left(\frac{\eta s}{c}\right) \quad (4.2)$$

for an infinite swept wing in incompressible flow.  $X_{00}$  is defined as the value of  $X_0(\beta A)$  from Figure 1 or 2 appropriate to zero quarter-chord sweepback. The function  $J(\mu\eta\beta s/c_r)$  in Figure 3 is analogous to Küchemann's function  $\lambda(\eta s/c)$ , defined in Figure 23 of Reference 6, and  $\mu$  was chosen so that Equation (4.1) gives the best correlation with lifting-surface theory at the Multhopp<sup>2</sup> collocation station  $\eta = \sin\{\pi/(m+1)\}$  adjacent to the root section; this procedure led to

$$\mu = 1.10 + 0.267\beta^{-1} \tan \Lambda_{1/4}, \quad (4.3)$$

no significant and consistent effect of aspect ratio being found. Since  $J(1) = 0$ , the root effect can be assumed to have disappeared when  $\eta \geq c_r/(\mu\beta s)$ .

There are limited data from lifting-surface theory for large values of the aspect ratio, but it is feasible to use Equation (4.2) to assist an extrapolation of Figure 1 to cases in which  $\beta A > 4$ . This logical extension of the basic method is given in Figure 2, although it remains largely unsubstantiated.

##### 4.2 Tip Region

Küchemann's<sup>6</sup> theory suggests that  $(1-\eta)\beta s/c_t$  may be an important parameter in estimating values of  $X_{ac}$  near the wing tip. Good correlation was found when  $X_{ac}$  from lifting-surface theory for swept wings was plotted against  $(1-\eta)\beta s/c_t$ , except perhaps for the outermost collocation station at  $\eta = \cos\{\pi/(m+1)\}$  in which full confidence is lacking. The approximate curve of  $(X_{ac})_{\eta t}$ , labelled  $\beta^{-1} \tan \Lambda_{1/4} \geq 0.4$  in Figure 4, is independent of aspect ratio and sweepback. For unswept wings rather higher values of  $X_{ac}$  were found, which approximate to the curve  $\beta^{-1} \tan \Lambda_{1/4} = 0$ . Linear interpolation is suggested for  $0 < \beta^{-1} \tan \Lambda_{1/4} < 0.4$ , and a curve for  $\beta^{-1} \tan \Lambda_{1/4} = 0.2$  is included in Figure 4.

### 4.3 Intermediate Region

Figures 1 to 3 with Equations (4.1) and (4.3) for the root region and Figure 4 for the tip region define curves of  $X_{ac}$  against  $\eta$ . These curves either have a unique common tangent or occasionally must be faired together. The common tangent, or fairing, provides a satisfactory link in the region of mid-semi-span, as illustrated in Figure 6.

### 4.4 Curved and Highly Tapered Planforms

The graphical method can be applied with little modification to planforms with curved leading edges. When the curvature extends to the root, the value of  $\Lambda_{1/4}$  at  $\eta = 0.25$  should be used in Section 4.1. There is a limited amount of evidence to suggest that this is preferable to using either the root or the local sweepback.

The procedure in Section 4.2 presupposes that the tip chord is non-zero. For curved and pointed tips adequate correlation with theory in the region of the tip is found if  $c_t$  is taken as 0.4 times the wing chord at  $\eta = 0.75$ . This value of  $c_t$  is also used for highly tapered wings, if it exceeds the actual tip chord.

For the tip region the sweepback is only significant when  $\beta^{-1} \tan \Lambda_{1/4} < 0.4$ ; it is then suggested that the value of  $\Lambda_{1/4}$  at  $\eta = 0.75$  should be used in Section 4.2.

### 4.5 Range of Planform

The graphical method can be applied to a wide range of planform, but the present analysis does not include negative values of  $\Lambda_{1/4}$ . For straight tapered wings of equivalent aspect ratios ( $\beta A$ ) greater than 4 an extrapolation of Figure 1 is given in Figure 2. For slender wings the method may have to be extrapolated below  $\beta A = 1$ , but the sub-division of the span into three regions is hard to justify. In cases of attached flow on such wings, present information suggests that a more representative solution may be obtained by extending the root curve,  $(X_{ac})_{\eta r}$ , over the whole semi-span; this recommendation is based on limited experimental evidence and slender wing theory for curved-tipped wings. In the presence of leading-edge vortices the graphical method is inapplicable.

The modification for curved and highly tapered planforms suggested in Section 4.4 is based on only limited theoretical data, but appears to be satisfactory. The procedure of Section 3.1 has been applied to wings with cranked leading or trailing edges, but such results should be used with caution, particularly in the region of the crank.

## 5. COMPARISONS

### 5.1 Comparison with Theory

The basic graphical method has been applied to a number of combinations of planform and Mach number for which spanwise distributions of aerodynamic centre are available by Multhopp's<sup>2</sup> subsonic lifting-surface theory. Comparisons between the basic method and theory for a range of planforms are shown in Figure 7.

Figure 7(a) gives results for wings of moderate aspect ratio. The effect of taper is seen to increase the gradient of  $X_{ac}$  near the tip, thus tending to localise the tip effect. The magnitude of the root effect increases with sweepback. The latter tendency is observed in Figure 7(b) which covers a range of sweepback for low aspect ratio planforms with medium taper. Three slender wings of special interest are shown in Figure 7(c). The gothic and ogee planforms are particular instances where leading-edge curvature extends to the root; the effective sweepback near the root can be considered respectively greater and less than the actual sweepback (see Section 4.4). All three slender wings illustrate the use of the effective tip chord for highly tapered and curved tips.

The discrepancies in  $X_{ac}$  between the basic method and lifting-surface theory are of the order 0.01 and rarely exceed 0.02. The greatest uncertainty is found close to the tip, but this is absorbed into the empirical modification to account for the larger experimental values in this region.

### 5.2 Comparison with Experiment

The tangent approximation to the graphical method (see Section 3.2) is used throughout the comparisons with experimental data on wings (Refs 7 to 11) in Figures 8 and 9. These also show the increments to the estimated  $X_{ac}$  due to thickness (see Section 3.3), details of which are found in Table 5.1.

Figures 8(a) and 8(b) give experimental data from low-speed tunnels, showing in Figure 8(a) a range of aspect ratio for untapered wings of 45° sweepback and in Figure 8(b) the effect of thickness-chord ratio for a wing of 60° sweepback with curved tips. Although in Figure 8(a) the large measured values of  $X_{ac}$  nearest to the tip are thought to arise from flow separation, this is not primarily so in Figure 8(b). In the latter case the thickness correction of Equation (3.3) goes some way towards eliminating the discrepancies for both the 5% and 9% streamwise sections. Figure 8(b) illustrates the practical need for the empirical tangent approximation instead of the basic graphical method in which  $X_{ac}$  would fall to 0.08 at the extreme tip. Figure 8(c) shows how well the tangent approximation with thickness correction can work for a planform with a cranked trailing-edge and varying thickness-chord ratio across the span.

In Figure 9 the variation with Mach number for various planforms is shown. The cropped delta and swept tapered planforms in Figures 9(a) and 9(b) show good agreement between experiment and the fully modified graphical method. In Figure 9(c), near the root of the curved-tipped planform, the graphical method tends to over-estimate  $X_{ac}$ . Towards the tip, however, reasonable agreement is shown. There are general indications that the thickness correction should increase with Mach number. On straight-tapered wings it sometimes appears that the slope of the experimental curve of  $X_{ac}$  against  $\eta$  is more negative than that of the tangent near  $\eta = 0.8$ , while the reverse is often true of curved-tipped wings.

The discrepancies in  $X_{ac}$  between the tangent approximation with thickness correction and experiment are of the order 0.01 in low-speed flow and of order 0.02 in subcritical compressible flow; they rarely exceed 0.03, unless flow separation occurs.

TABLE 5.1

| Figure Number | Section    | $\eta$               | $\sigma$ | $\frac{t}{c} \sec \Lambda_{1/4}$ | $M$ | $\Delta X_{ac}$          |
|---------------|------------|----------------------|----------|----------------------------------|-----|--------------------------|
| 8(a)          | RAE 101    | all                  | 0.530    | 0.170                            | 0   | 0.016                    |
| 8(b)          | 5% RAE 101 | < 0.6<br>0.8<br>0.95 | 0.530    | 0.100<br>0.115<br>0.187          | 0   | 0.0085<br>0.010<br>0.018 |
|               | 9% RAE 101 | < 0.6<br>0.8<br>0.95 | 0.530    | 0.180<br>0.207<br>0.337          | 0   | 0.175<br>0.020<br>0.070  |
| 8(c)          | —          | < 0.59               | 0.202    | 0.100                            | 0   | 0.017                    |
|               |            | > 0.59               | 0.226    | 0.095                            | 0   | 0.016                    |
| 9(a)          | RAE 102    | all                  | 0.474    | 0.0855                           | 0.8 | 0.011                    |
|               |            |                      |          |                                  | 0.9 | 0.012                    |
| 9(b)          | RAE 102    | all                  | 0.474    | 0.093                            | 0.6 | 0.011                    |
|               |            |                      |          |                                  | 0.8 | 0.011                    |
| 9(c)          | RAE 103    | < 0.5<br>0.8<br>0.9  | 0.407    | 0.120<br>0.155<br>0.223          | 0.6 | 0.016<br>0.021<br>0.034  |
|               |            | < 0.5<br>0.8<br>0.9  | 0.407    | 0.120<br>0.155<br>0.223          | 0.8 | 0.017<br>0.022<br>0.035  |
|               |            | < 0.5<br>0.8<br>0.9  | 0.407    | 0.120<br>0.155<br>0.223          | 0.9 | 0.018<br>0.022<br>0.023  |
|               |            | < 0.5<br>0.8<br>0.9  | 0.407    | 0.120<br>0.155<br>0.223          | 0.9 | 0.018<br>0.022<br>0.023  |
|               |            | < 0.5<br>0.8<br>0.9  | 0.407    | 0.120<br>0.155<br>0.223          | 0.9 | 0.018<br>0.022<br>0.023  |
|               |            | < 0.5<br>0.8<br>0.9  | 0.407    | 0.120<br>0.155<br>0.223          | 0.9 | 0.018<br>0.022<br>0.023  |

### 5.3 Comparison with Experiment with a Body Present

The net wing is used in all calculations when a body is present. Figure 8c shows low-speed experimental data for a cranked wing with and without a body. Very good agreement with the graphical method is found in both cases. Figure 10a shows a straight tapered wing and Figure 10b a curved-tipped warped wing each with a body present. These two examples<sup>12</sup> provide a rather more crucial test of whether the net wing concept is satisfactory in the case of a wing-body combination because, as illustrated in the Figures, the wing-body settings and other features of the wing-body junction are far from symmetrical. The order of agreement achieved between predicted and measured values is therefore encouraging.