

METHOD FOR THE RAPID ESTIMATION OF THEORETICAL SPANWISE LOADING DUE TO A CHANGE OF INCIDENCE

SUMMARY

This document supersedes Royal Aeronautical Society Transonic Data Memorandum 6208 the subject of which was identical to that of this Memorandum but had an upper limit of application of $\beta A = 4$ set by the lack of theoretical data for wings of high aspect ratio.

This lack of data was originally remedied by calculations made by the British Aircraft Industry on behalf of the Society's Transonic Aerodynamics Committee and led to T.D. Memo. 6403 being issued in March 1964.

In March 1971 the detailed procedure in Section 3 was further revised to make use of charts of $\bar{\eta}$ calculated¹³ by the further development of Multhopp's lifting-surface theory contained in Reference 14. The charts are presented in Figures 1 to 4. Their use in conjunction with Equation (3.2) yields in certain cases, principally highly swept wings, estimates of $cC_L/\bar{c}\bar{C}_L$ in closer agreement with lifting-surface theory calculations than did the original procedure in Section 3.

Complete revision of Figures 8a and 8b and the illustrative examples in Section 5 is considered to be unwarranted at the present time. The original examples in Figures 9 to 15 remain typical of the revised method.

A program Listing of the method written in FORTRAN IV is in Appendix A.

1. INTRODUCTION

A method is presented for the rapid estimation of the spanwise load distribution over uncambered and untwisted wings in unseparated subcritical flow. It covers a wide range of planforms having

- (a) arbitrary leading edges and unswept trailing edges,
- (b) straight or cranked leading edges and arbitrary trailing edges.

In addition a special modification allows for sweptback planforms having curved leading edges with continuous slope and streamwise tips of zero chord.

The first step in the method evaluates the spanwise centre of pressure and may be useful in its own right. When coupled with a corresponding method for local aerodynamic centre¹, the procedure may often be an adequate substitute for calculations by lifting-surface theory to obtain the incremental loading due to a change of lift coefficient. Therefore it may also be useful as a preliminary step in the RAE Standard Method² for calculating the chordwise pressure distribution on thick swept wings in subcritical flow.

The method, derived by Garner³, is based on numerous calculations⁴ by Multhopp's subsonic lifting-surface theory⁵. Comparisons are made with experimental data as well as with the basic calculations.

Attention is drawn to the further development of Multhopp's lifting-surface theory contained in Reference 14 and used in the March 1971 revision (see Summary). This development involves an increase in the number of spanwise integration points relative to the number of collocation sections and improves the accuracy of calculating chordwise loading. Appendix A provides a computer program for the enumeration of the method.

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2. NOTATION

A	aspect ratio ($= 2s/\bar{c}$)
A^*	see Equations (3.3) and (3.4)
c	local chord of wing
$\bar{c}$	geometric mean chord ($= \int_0^1 c \, d\eta$)
c_r	root chord
c_r^*	see Equation (3.5)
c_t	tip chord
C_L	local lift coefficient
$\bar{C}_L$	overall lift coefficient ($= \int_0^1 \frac{c C_L}{\bar{c}} \, d\eta$)
$F(\eta, \bar{\eta})$	see Equation (4.3) and Figure 5
$G(\eta)$	see Equation (4.4) and Figure 6
$H(\eta)$	see Equation (4.5) and Figure 7
m	number of spanwise terms in theoretical solution
M	Mach number
N	number of chordwise terms in theoretical solution (Figure 8 only)
R	Reynolds number based on geometric mean chord
s	semi-span of wing
x_T	distance of trailing edge, measured in free-stream direction from apex (<i>i.e.</i> wing root leading edge)
α	incidence
β	$\sqrt{1 - M^2}$
η	spanwise distance from root as a fraction of semi-span
$\bar{\eta}$	spanwise centre of pressure ($= \int_0^1 \frac{c C_L}{\bar{c} \bar{C}_L} \eta \, d\eta$)
$\bar{\eta}^*$	value of $\bar{\eta}$ for wing with curved leading edge

$\bar{\eta}_\Delta$	value of $\bar{\eta}$ for a planform having the required chord distribution and span, but an unswept trailing edge
κ	taper parameter $\left(= \int_0^1 \frac{c\eta}{c} d\eta \right)$
Λ_0	sweepback of leading edge at $\eta = 0$
$\Lambda_{1/2}$	sweepback of mid-chord line at $\eta = 0$
Λ_1	sweepback of trailing edge at $\eta = 0$
Λ_1^*	see Equation (3.1)

In subsonic compressible flow the method embodies an analogous planform in which the dimensions normal to the free-stream direction are shrunk by a factor β . The relevant listed symbols thus relate to the true wing planform and not to the analogous one.

3. METHOD

Charts used in the rapid estimation of the theoretical spanwise loading are presented in Figures 1 to 7. The spanwise centre of pressure ($\bar{\eta}$) is derived from a knowledge of aspect ratio, mid-chord sweepback ($\Lambda_{1/2}$), a taper parameter (κ) and Mach number. The taper parameter caters for arbitrary leading edges and its numerical range $0.333 \leq \kappa \leq 0.5$ excludes only wings with inverse taper. The method is restricted to a general upper limit of reduced aspect ratio $\beta A = 8$ and to a range of sweepback defined by $0 \leq A \tan \Lambda_{1/2} \leq 6$. The lower limit of reduced aspect ratio is 1.5, except that for wings with straight leading and trailing edges and $A \tan \Lambda_{1/2} \leq 2$ it is known that $\bar{\eta}$ converges rapidly to the value corresponding to elliptic loading ($\bar{\eta} = 0.4244$) as $\beta A \rightarrow 0$. This condition may sometimes provide a useful fifth point on the plot required in operation (iii) of the rapid calculation procedure.

The method should be applied with caution to planforms differing greatly from those shown in Figures 8a and 8b; in particular, it is recommended that for cranked leading or trailing edges, its use should be restricted to discontinuities of less than about 40° in sweep angle.

The current procedure (revised March 1971) for wings with *straight or cranked leading edges or unswept trailing edges* is as follows:

- (i) Evaluate $\kappa = \int_0^1 \frac{c\eta}{c} d\eta$

$$= \frac{c_r + 2c_t}{3(c_r + c_t)}$$
 for a straight tapered wing.
- (ii) Knowing κ and $A \tan \Lambda_{1/2}$, where $\Lambda_{1/2}$ is the sweepback of the mid-chord line at $\eta = 0$, obtain $\bar{\eta}$ from Figures 1 to 4 for $\beta A = 1.5, 3, 5$ and 8 .
- (iii) Plot $\bar{\eta}$ against β to obtain $\bar{\eta}$ for required values of M .

(iv) Evaluate $\tan \Lambda_1^* = \int_0^{x_T/s} (12\eta - 6) d\eta$ (3.1)

$$= \tan \Lambda_0 - \frac{12}{A}(1 - 2\kappa) \text{ for a straight leading edge.}$$

Note that $\Lambda_1^* = \Lambda_1$ for wings having straight trailing edges.

(v) Obtain $F(\eta, \bar{\eta})$, $G(\eta)$ and $H(\eta)$ from Figures 5, 6 and 7.

(vi) Evaluate the local spanwise loading from the equation

$$\frac{cC_L}{\bar{c}\bar{C}_L} = F(\eta, \bar{\eta}) + (A \tan \Lambda_1) G(\eta) + (\beta A - 4)(1 + 3.5\beta^{-1} \tan \Lambda_1^*) H(\eta), \quad (3.2)$$

where the last term is omitted if $\beta A < 4$.

The following procedures, depending on the type of swept trailing edge, are suggested for sweptback wings with *curved leading edges*. Here 'curved leading edges' are defined as having continuous slope (apart from the root) resulting in streamwise wing tips of zero chord. The procedures are only applicable when the sweepback of the leading and trailing edges at the wing root are approximately equal.

(a) Operation (i) is completed as before.

(b) An empirical parameter (A^*) is now introduced.

For straight trailing edges

$$A^* = -1.2A + 8.8 \left(\frac{2c_r}{s} + \tan \Lambda_1 - \tan \Lambda_0 \right)^{-1}. \quad (3.3)$$

For curved or cranked trailing edges

$$A^* = -1.2A + 8.8 \left(\frac{2c_r^*}{s} + \tan \Lambda_1^* - \tan \Lambda_0 \right)^{-1}, \quad (3.4)$$

where $c_r^* = \int_0^{x_T} (4 - 6\eta) d\eta$ (3.5)

and $\tan \Lambda_1^*$ is defined by Equation (3.1).

(c) Operation (ii) now requires the evaluation of

$$\bar{\eta}^* = (\bar{\eta} - \bar{\eta}_\Delta) \frac{A^*}{A} + \bar{\eta}_\Delta,$$

where $\bar{\eta}^*$ is the value of $\bar{\eta}$ for a wing with curved leading edges and $\bar{\eta}_\Delta$ is obtained from Figures 1 to 4 with the true κ and βA but with $A \tan \Lambda_{1/2} = 6(1 - 2\kappa)$.

(d) Operations (iii) to (vi) follow as before but with $\bar{\eta}$ replaced by $\bar{\eta}^*$.

4. BASIS OF METHOD

The charts in Figures 1 to 4 are derived directly from theoretical data in Reference 13 based on the lifting-surface method of Reference 14. The taper parameter κ seems to be quite satisfactory for curved leading edges.

If the trailing edge is cranked or curved, $\bar{\eta}$ is evaluated for a trailing edge of constant sweepback Λ_1^* in Equation (3.1) such that κ , A , s and the leading edge remain unaltered. The formulae and charts hold for cranked leading edges provided the discontinuity in slope is not too severe. Since Λ_0 does not appear explicitly in Equation (3.2), there is no need to define an equivalent leading edge of constant sweepback to deal with cranked leading edges, in the way that the method introduces Λ_1^* to accommodate cranked trailing edges.

The theoretical data available for wings with curved leading edges are sparse. Nevertheless an attempt has been made to include them by introducing an empirical parameter A^* , given by Equation (3.3) or (3.4), which reduces to A when the leading edge is straight. This special modification affects only the equation for $\bar{\eta}^*$ and does not enter into Figures 1 to 4.

The original accuracy of the estimation of $\bar{\eta}$ illustrated in Figures 8a and 8b for wings at $M = 0$ is now improved significantly. The two diagrams illustrate respectively the range of planforms of aspect ratios $1 \leq A \leq 4$ and $4 \leq A \leq 7$ for which this method has been found satisfactory. There is additional evidence of the adequacy of the method in Figure 12 of Reference 13.

The spanwise loading can be represented adequately in the form

$$\frac{cC_L}{\bar{c}\bar{C}_L} = (P_0 + P_1|\eta| + P_2\eta^2 + P_4\eta^4)\sqrt{1-\eta^2}. \quad (4.1)$$

For wings with unswept trailing edges it is sufficient to take $P_1 = 0$; similarly $P_4 = 0$ if $\beta A \leq 4$. Then P_0 and P_2 may be determined from the conditions

$$\left. \begin{aligned} \int_0^1 \frac{cC_L}{\bar{c}\bar{C}_L} d\eta &= 1 \\ \int_0^1 \frac{cC_L}{\bar{c}\bar{C}_L} \eta d\eta &= \bar{\eta} \end{aligned} \right\}. \quad (4.2)$$

This leads to $cC_L/\bar{c}\bar{C}_L = F(\eta, \bar{\eta})$, where

$$F(\eta, \bar{\eta}) = (3.395 - 5\bar{\eta})\sqrt{1-\eta^2} + (20\bar{\eta} - 8.488)\eta^2\sqrt{1-\eta^2} \quad (4.3)$$

is given in Figure 5.

In the case of wings with swept trailing edges it is expedient to include $P_1|\eta|$ in Equation (4.1). The conditions of Equations (4.2) then determine $cC_L/\bar{c}\bar{C}_L$ in the form of Equation (3.2) for $\beta A \leq 4$ with

$$G(\eta) = -0.01484(1 - 6.666|\eta| + 7.316\eta^2)\sqrt{1-\eta^2}; \quad (4.4)$$

the numerical coefficient 0.01484 and the factor $A \tan \Lambda_1$ in Equation (3.2) have been chosen empirically. In cases where $\beta A > 4$ the term $P_4 \eta^4$ is needed in Equation (4.1). To maintain the conditions of Equations (4.2), the function $H(\eta)$ in the final term of Equation (3.2) takes the form

$$H(\eta) = 0.003(1 - 14.5\eta^2 + 21\eta^4)\sqrt{1 - \eta^2}; \quad (4.5)$$

the numerical coefficient 0.003 and the factor $(\beta A - 4)(1 + 3.5\beta^{-1} \tan \Lambda_1^*)$ in this small correction term have been chosen empirically from limited theoretical evidence. The functions $G(\eta)$ and $H(\eta)$ are given in Figures 6 and 7.

It should be noted that the empirical parameter A^* does not appear explicitly in Equation (3.2). The equivalent trailing-edge sweepback Λ_1^* appears only in the last term which is omitted unless $\beta A > 4$. It is emphasised that the method is based on potential flow and no allowance is made for viscous effects and in particular for the effects of separation.

5. COMPARISONS WITH THEORY AND EXPERIMENT

In Figures 9 to 13 comparisons with Multhopp's theory⁵ are made. It should be noted that the term $G(\eta)$ in Equation (3.2) imposes a positive slope of $cC_L/\bar{c}\bar{C}_L$ from $\eta = 0$ when the trailing edge is swept back and a negative slope when the trailing edge is swept forward. These effects are well illustrated in Figure 9.

The adequacy of the present method is illustrated in Figure 9 for wings of small or moderate aspect ratio, in Figure 10 for constant chord wings of moderate or high aspect ratio and in Figure 11 for high-aspect-ratio tapered wings.

Comparisons with the experiments of References 6 to 12 are included in Figures 12 to 15. The present rapid method for estimating spanwise loading in Figure 12 is seen to be satisfactory at moderate and high subsonic Mach numbers for a wing with curved leading edge. Satisfactory agreement with lifting-surface theory and experiment is also to be found in Figure 13 for a cropped delta wing of small effective aspect ratio ($\beta A = 1.18$), a tapered swept wing at $M = 0.6$ and a 5% thick swept wing with curved tip at $M = 0$. Figure 13 also includes an example with a cranked trailing edge for which satisfactory agreement with experiment is obtained.

Figures 14 and 15 compare the rapid method with the experiments of References 10 to 12. In these cases solutions by Multhopp's lifting-surface theory are not available. Figure 14 shows results for constant-chord sweptback wings of moderate aspect ratio. In producing this comparison it was necessary to amend slightly the values of $\bar{C}_L$ from Reference 10 so that the experimental results would give an average value of $cC_L/\bar{c}\bar{C}_L$ of about unity. It seems likely that there is some scatter in the experimental results and under this circumstance the comparison is as good as could be expected. Figure 15 shows results for tapered sweptback wings of high aspect ratio. Again in presenting the results it was sometimes necessary to apply a correction factor to ensure that the experimental distribution was consistent with the first of Equations (4.2).

By Equations (3.1) and (4.4) the slope of the $cC_L/\bar{c}\bar{C}_L$ curve against η at $\eta = 0$ is equal to $0.0989 A \tan \Lambda_1$; although for swept trailing edges this represents a departure from theoretical principles, it is well justified by numerical data both from lifting-surface theory in Figures 10 to 12 and from experiment in Figures 14 and 15. There appear to be systematic disparities between estimated and measured spanwise loadings on sweptback wings of high aspect ratio; it will be seen that for both the examples in Figure 15 and the $A = 5$ wing in Figure 14, the calculated curves tend to lie below the experimental points for $\eta < 0.2$, above those for $0.4 < \eta < 0.7$ and below those for $\eta > 0.9$. These differences are attributed to viscous effects at model scale and would not be expected at flight Reynolds numbers.

6. REFERENCES

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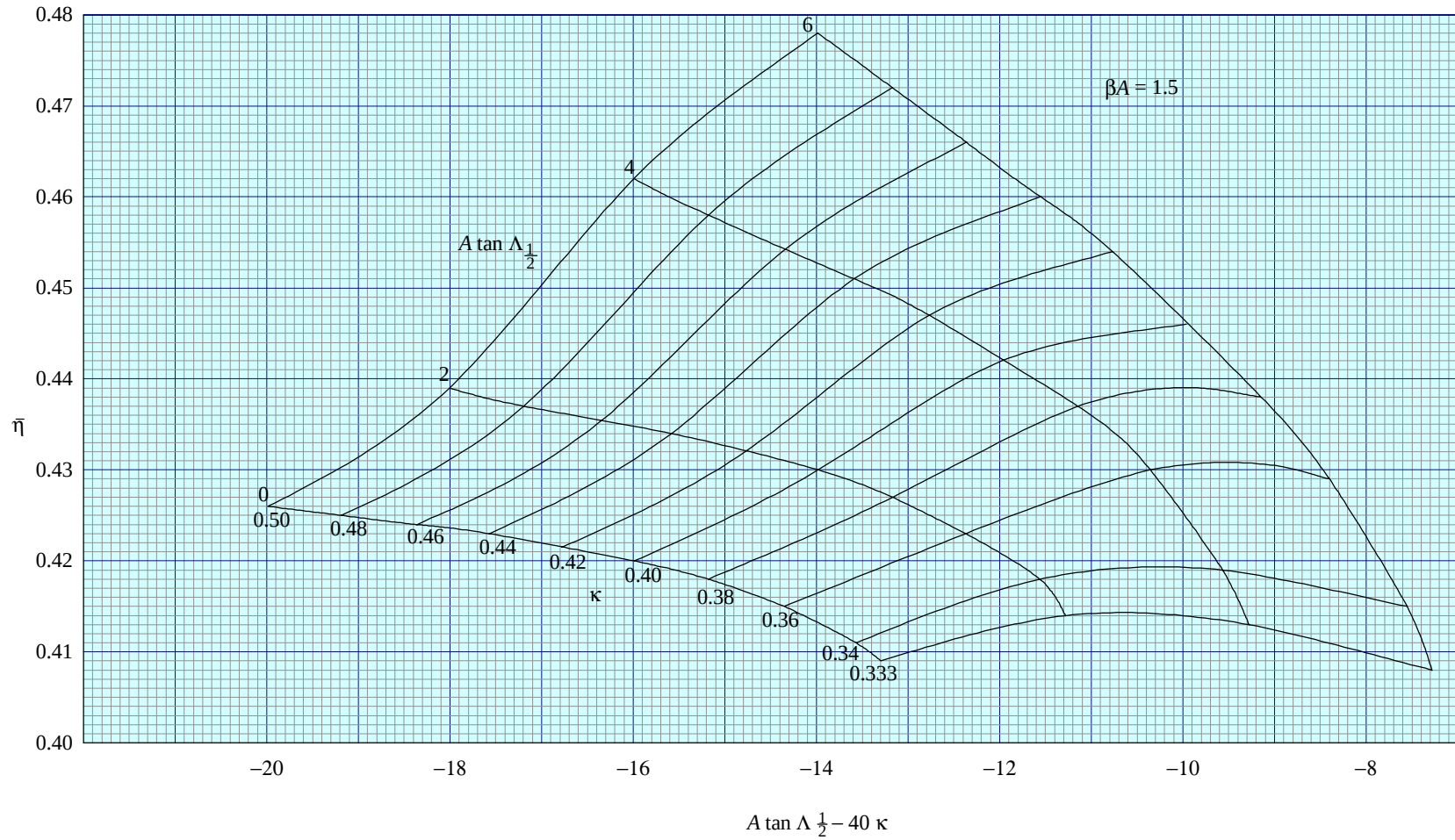


FIGURE 1 THEORETICAL SPANWISE CENTRE OF PRESSURE

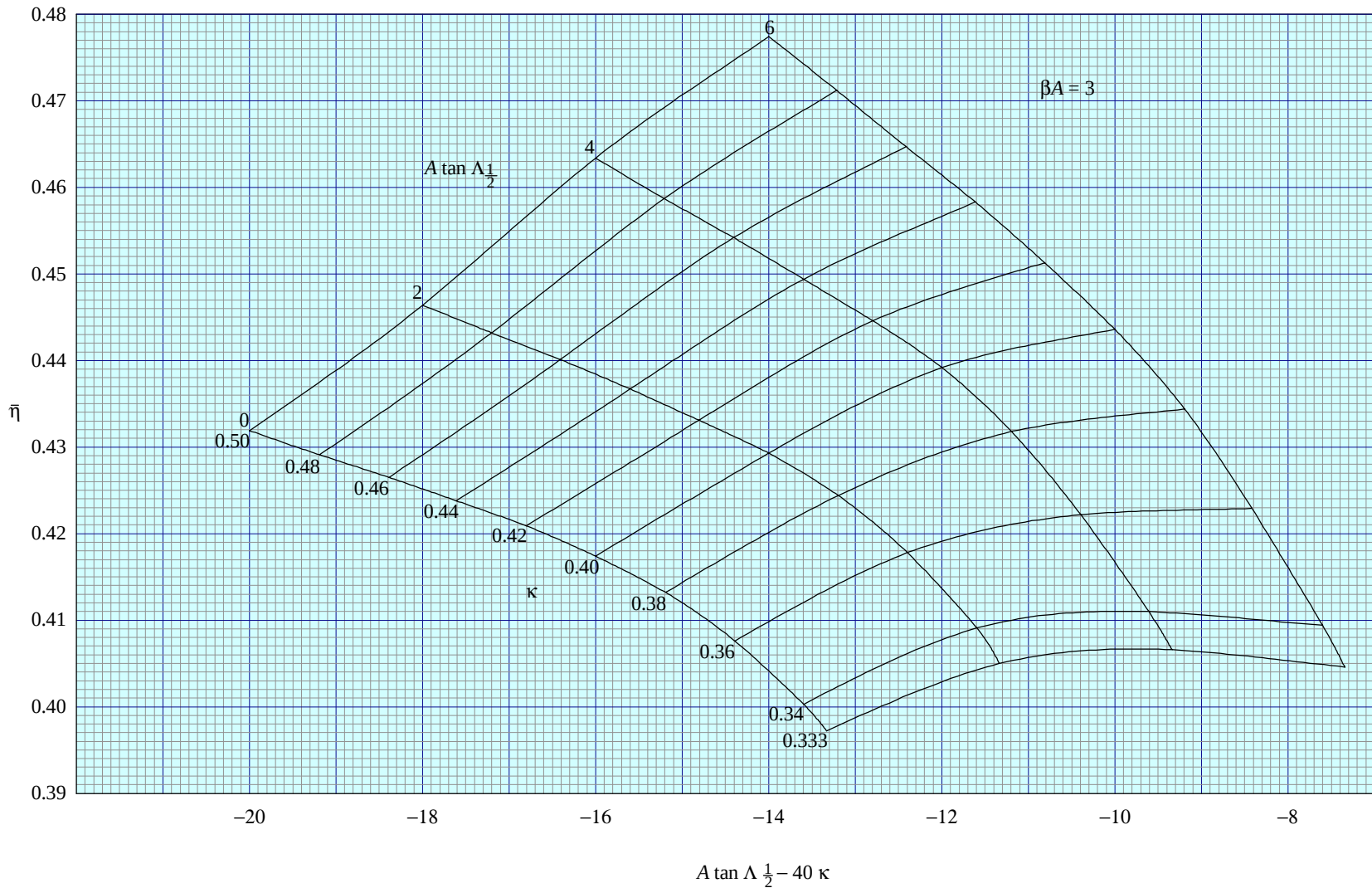


FIGURE 2 THEORETICAL SPANWISE CENTRE OF PRESSURE

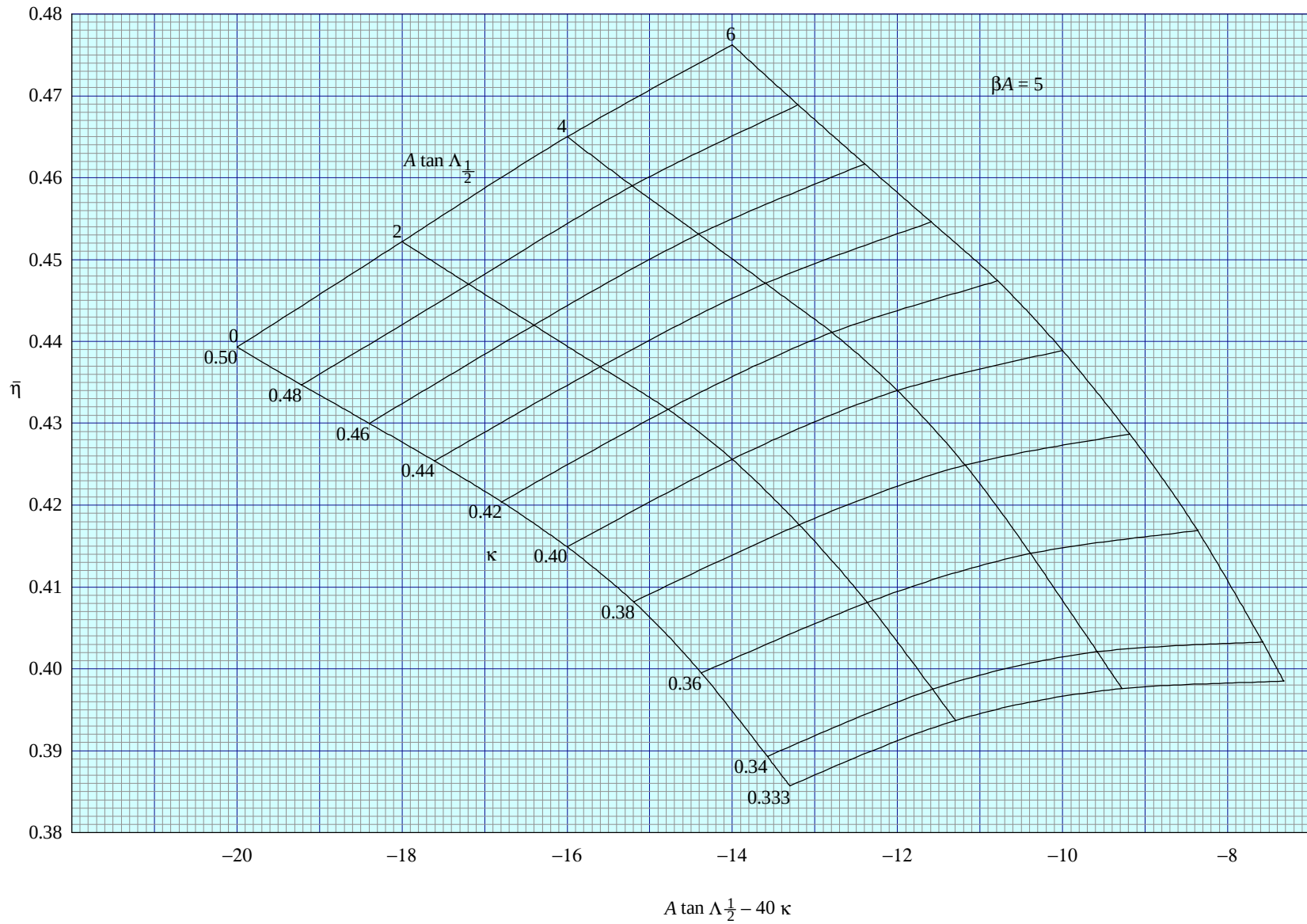


FIGURE 3 THEORETICAL SPANWISE CENTRE OF PRESSURE

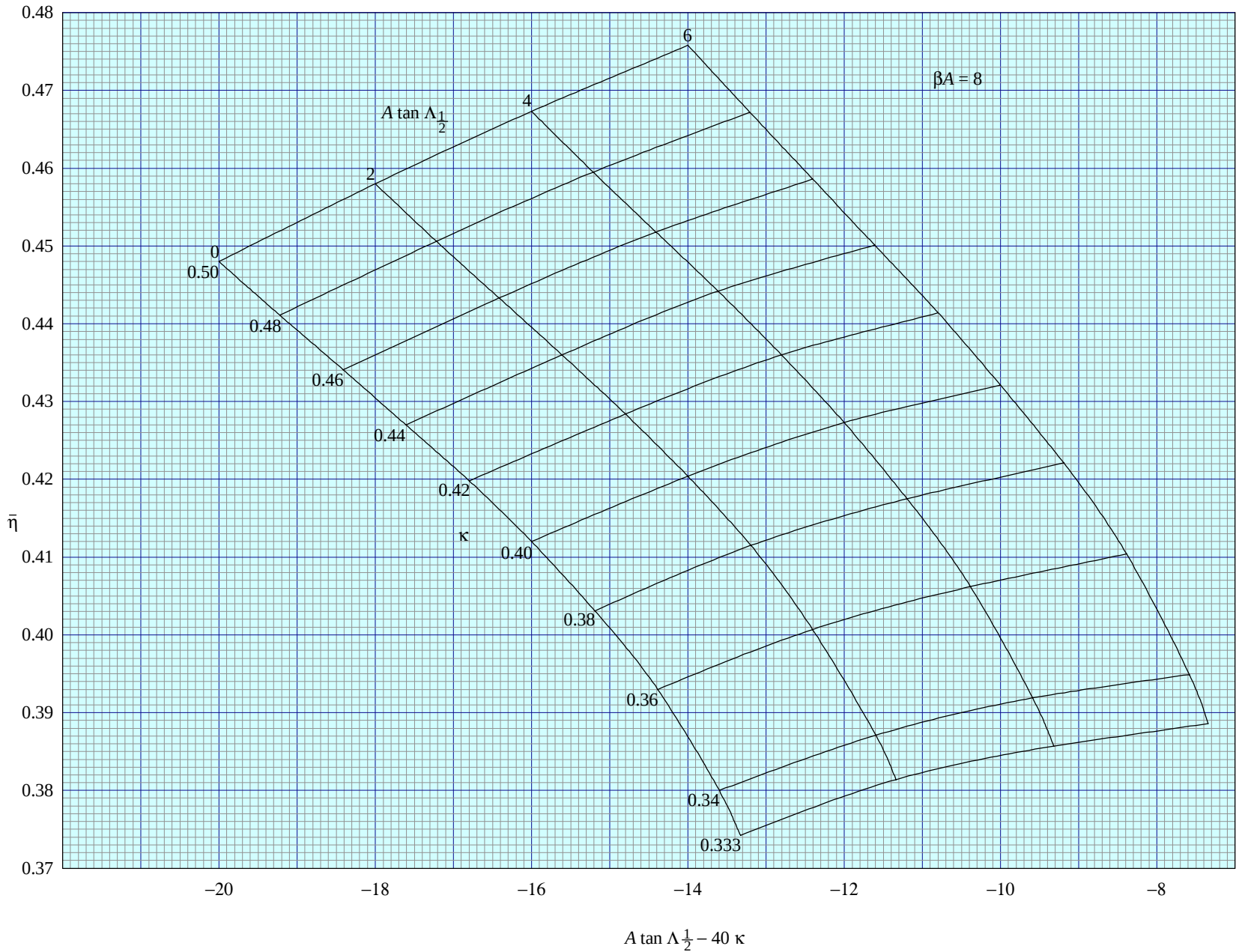


FIGURE 4 THEORETICAL SPANWISE CENTRE OF PRESSURE

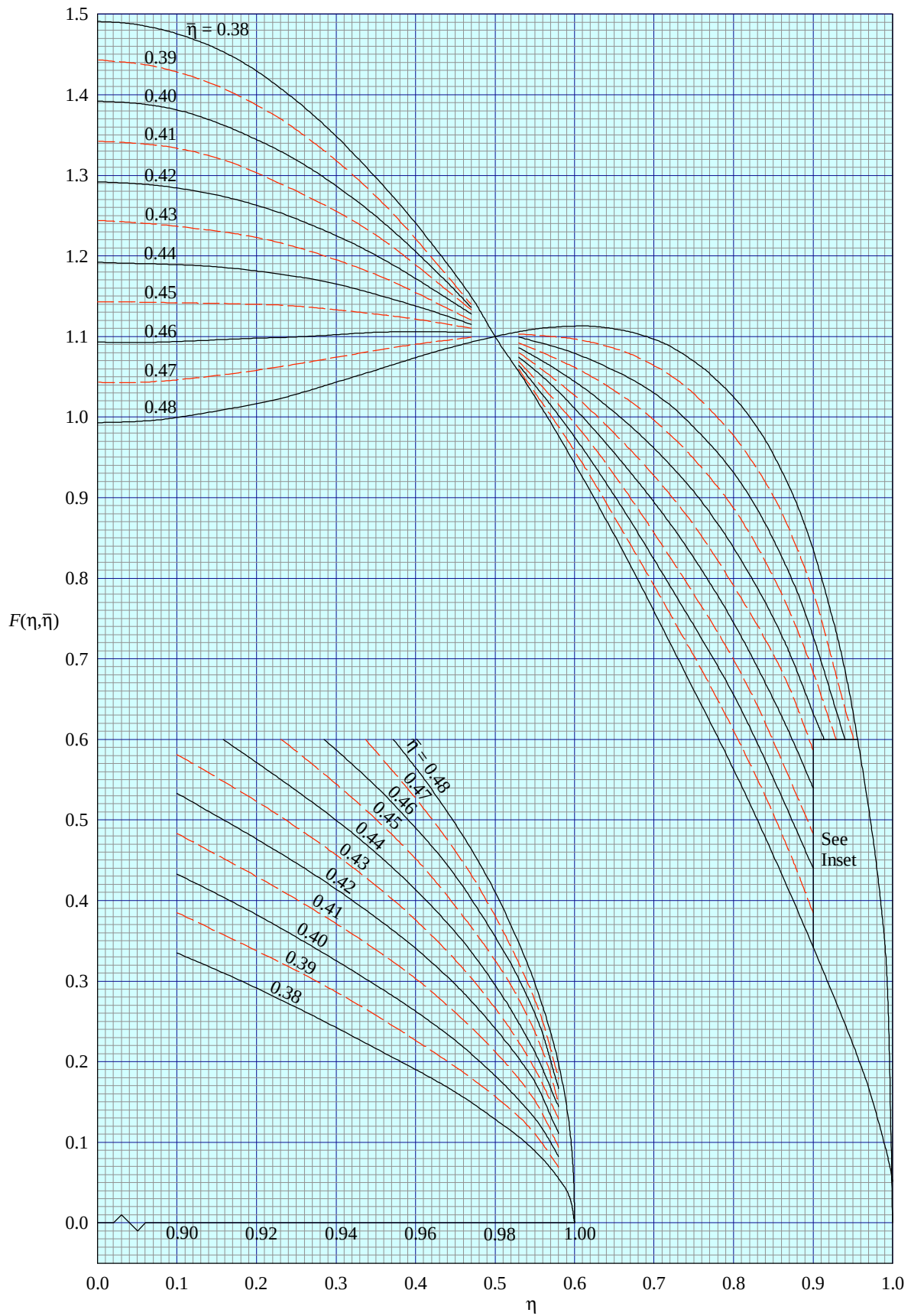


FIGURE 5 THE FUNCTION $F(\eta, \bar{\eta})$

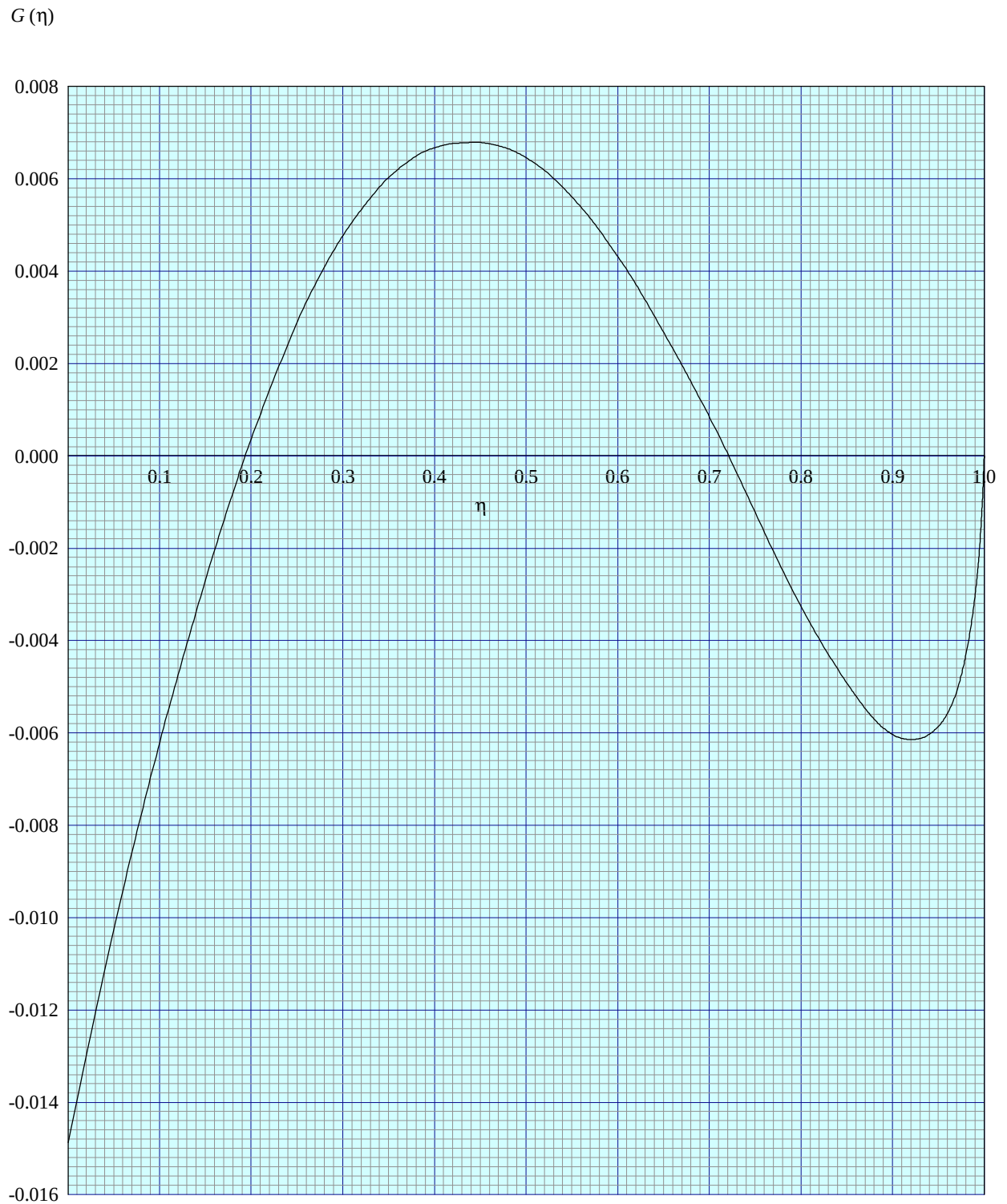


FIGURE 6 THE FUNCTION $G(\eta)$