

A METHOD OF ESTIMATING DRAG-RISE MACH NUMBER FOR TWO-DIMENSIONAL AEROFOIL SECTIONS

SUMMARY

A method is given of estimating drag-rise Mach number for any two-dimensional aerofoil. This Mach number is defined as that beyond which any further increase would lead to a rapid increase of drag coefficient due to the development of shock-wave drag. It is evaluated as a function of lift coefficient.

The method is based on the observation that, in general, the shock-wave drag first develops rapidly as the shock wave moves downstream away from the aerofoil crest, and therefore that the drag-rise Mach number is that at which the shock coincides with the crest. The problem of predicting the drag-rise Mach number is thus reduced to that of predicting the Mach number for shock-at-crest, which is achieved by applying a simple criterion to the subsonic pressure distribution.

The validity of the method has been demonstrated by comparison with experimental data for a large number of aerofoils; the aerofoils and the different sources of the data are listed in Appendices. On the basis of this comparison it is concluded that the present method should predict the Mach number for the rapid drag-rise to an accuracy of ± 0.015 for the majority of cases. For a few particular cases of aerofoil shape and pressure distribution a slow but significant growth of shock-wave drag may precede the rapid increase predicted by the present method; a procedure is included for recognising these cases.

1. NOTATION

(a_1)	slope of lift-coefficient curve with incidence for the two-dimensional aerofoil section
c	aerofoil chord
C_D	drag coefficient
C_L	lift coefficient (see also Figure 4)
C_{LD}	lift coefficient when $M = M_D$
C_p	pressure coefficient
C_{pcrest}	pressure coefficient at the aerofoil crest; the crest being defined as the point at which the aerofoil surface is tangential to the undisturbed free-stream direction
F	the Kármán-Tsien compressibility correction factor
H	stagnation pressure in undisturbed stream
M	Mach number
M_{crit}	Mach number at which sonic flow first occurs on upper surface
M_D	drag-rise Mach number (see also Figure 4)

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p	local static pressure
R_e	Reynolds number based on chord length
t_{90}	thickness of aerofoil at 90 per cent chord
t_{99}	thickness of aerofoil at 99 per cent chord
x	chordwise distance measured from the leading edge
z	aerofoil ordinate measured normal to the chord
z'	aerofoil ordinate measured normal to the free-stream direction from an arbitrary datum
α	angle of incidence (see also Figure 4)
α_0	angle of zero lift
β_D	$(1 - M_D^2)^{1/2}$; the Prandtl-Glauert compressibility correction factor evaluated at the drag-rise Mach number
γ	ratio of the specific heats of air, here taken as 1.40
Δ	indicates a small increment
θ	aerofoil surface-slope relative to the chord line
τ_a	trailing-edge angle given by $\tan (\tau_a/2) = (t_{90} - t_{99})/(0.18c)$

Suffixes

I	indicates incompressible viscous flow
IT	indicates theoretical incompressible inviscid flow

2. INTRODUCTION

In the preliminary stages of an aircraft design estimates are often needed of the drag-rise Mach number of two-dimensional aerofoils equivalent to the section shapes of the wing, to help either in choosing the section shapes or in predicting the drag-rise Mach number of the aircraft itself. The method now presented provides the best available theoretical estimates and should be sufficiently straightforward to use at any stage, either directly for individual aerofoils, or indirectly via generalised charts for families of related aerofoils. The basis of the method was first described by Sinnott in Reference 1 but the procedure presented here includes certain refinements which lead to greater accuracy.

The drag rise in question develops with increasing free-stream Mach number when a shock wave forms to terminate the region of local supersonic flow adjacent to the surface of the aerofoil, and subsequently grows in strength and in height above the surface. The source of this wave drag is the loss of momentum at the shock, but the drag itself is transmitted to the aerofoil by readjustments in the pressure distribution over the surface. A full prediction of the wave drag development would thus require a detailed prediction of the shock growth or, alternatively, of the related development in the mixed subsonic/supersonic velocity distribution on the surface. Fortunately, however, it is possible to define a stage in the development of the local supersonic flow, up to which any shock-wave drag is generally trivial but beyond which the drag can be expected to rise rapidly. This stage is that at which the shock, either on first being formed or on moving from an upstream station, first appears immediately to the rear of the crest of the aerofoil; the crest being defined as the point at which the aerofoil surface is tangential to the undisturbed free-stream direction. Furthermore, this stage, which from now on will be referred to as the shock-at-crest condition, can be predicted directly by means of a simple criterion from the pressure at the crest position at low speeds. The use of the shock-at-crest condition thus leads to a definition of drag-rise Mach number that not only has considerable relevance in practice but also lends itself readily to prediction.

The method, discussed in Section 4, includes an itemised procedure that leads to the prediction of a drag-rise locus, or boundary, with lift coefficient and Mach number as parameters. The final step of the procedure identifies a certain class of pressure distribution for which the slow development of shock-wave drag at free-stream Mach numbers below that corresponding to the shock-at-crest condition may not be trivial.

3. THE CRITERION FOR DRAG-RISE MACH NUMBER

The criterion is

M_D is the value of free-stream Mach number for which the crest pressure, calculated from the low-speed value by application of the Kármán-Tsien compressibility factor, equals $0.515H$.

This simple criterion is derived from the definition that the drag-rise Mach number coincides with the Mach number for shock-at-crest and from the prediction of the shock-at-crest condition in Sinnott's transonic aerofoil theory². The 'equivalent shock downstream pressure' is taken as the value that would exist at the position of the shock in a pressure distribution derived by applying the appropriate compressibility factor to the low-speed distribution. For the weak shocks appropriate to the present context, this equivalent downstream pressure can be taken as constant and equal to $0.515H$. The shock will thus coincide with the crest at the Mach number for which the crest pressure, factored from its low-speed value, equals $0.515H$ and this leads to the criterion quoted above.

4. THE METHOD

The method, embodied in a procedure detailed in Section 4.1, is illustrated by the five sketches in Figure 1a. It is necessary to specify only the aerofoil geometry (Sketch 1) and to prescribe values of incidence (α_{IT}) for which the incompressible-flow pressure distributions (Sketch 2) are to be calculated. The incompressible-flow pressure distribution constitutes the kernel of the process. Its integral is the lift coefficient (Sketch 3) which, when account is taken of viscous effects on lift-curve slope, yields the true incidence (α_I). This, coupled to a knowledge of the aerofoil surface-slope (Sketch 4), identifies the crest position. Recourse to the incompressible-flow pressure distribution gives the low-speed value of the crest pressure which is related to M_D by a unique law (Sketch 5) derived from the drag-rise criterion and the appropriate compressibility factor. Finally the lift coefficient, C_{LI} , is factored for compressibility to yield C_{LD} .

The drag-rise boundary is formed from the locus of the points (M_D , C_{LD}) and will normally be associated with the incremental velocities on the upper surface of the aerofoil. Usually, therefore, the procedure need only be applied to the upper surface. However, for cambered aerofoils at small positive or at negative values of lift coefficient, the boundary may be associated with the velocities on the lower surface. Under these circumstances, the steps from (vi) onwards should be repeated for the lower surface and the smaller value of M_D (and its associated value of C_{LD}) used to form the drag-rise boundary.

4.1 Procedure

The procedure detailed below is based on a theoretical incompressible inviscid pressure distribution derived for an explicit aerofoil geometry. The method may be applied to a design pressure distribution for which the equivalent aerofoil section has not been accurately established, but the uncertainty regarding the position of the crest must then be expected to detract from the accuracy. The method can be applied without loss of accuracy to known low-speed experimental data, in which case steps (i) to (v) would be unnecessary.

The individual steps in the procedure follow:-

- (i) **Determine the chordwise incompressible inviscid pressure distribution for a chosen set of incidence values (α_{IT}) and integrate each pressure distribution to obtain the lift coefficient (C_{LIT}).**

It is recommended that the Weber formula^{3, 4} be used to obtain the pressure distribution (see Section 5.1)

- (ii) **Assume that for any given value of lift coefficient the pressure distributions in incompressible inviscid flow and in incompressible viscous flow are identical in the vicinity of the crest, but are associated with different angles of incidence, namely α_{IT} and α_I . The latter must be determined (step (v)).**

The analysis now proceeds on the basis of the set of pressure distributions derived in (i) with the corresponding set of lift coefficients (C_{LIT}) equated to C_{LI} .

- (iii) **From the data obtained above, calculate the lift-curve slope in incompressible inviscid flow (a_1)_{IT} at zero lift.**
- (iv) **Determine the lift-curve slope in incompressible viscous flow (a_1)_I from Figure 2 using the value of (a_1)_{IT} obtained in (iii).**

Figure 2 assumes transition to occur at the leading edge but requires a knowledge of the Reynolds number (R_e). The latter will usually relate to specific flight conditions which implies a pre-knowledge of the drag-rise Much number. However, the procedure is not greatly sensitive to Reynolds number and an initial assumption that $M_D = 0.7$ – made purely for the purpose of evaluating R_e – will almost certainly be adequate.

- (v) **Determine the incidence in incompressible viscous flow (α_I) corresponding to the values of C_{LI} obtained by the assumption of (ii) above, from**

$$\alpha_I = C_{LI}/(a_1)_I + (\alpha_0)_{IT}.$$

It should be noted that this equation assumes that the zero lift angle in viscous flow is unchanged from that in inviscid flow. This assumption is adequate to the purpose of this paper but may not be valid in other contexts.

The procedure so far has led to a set of pressure distributions each associated with a value of C_{LI} and α_I .

- (vi) **Determine the aerofoil upper-surface slope (θ) for a series of chordwise stations such that the numerical range of θ slightly exceeds the numerical range of α_I , taking into account both positive and negative values of α_I .**
- (vii) **Determine:-**
 - (a) **the chordwise position of the crest for each value of α_I – the crest being defined as the point at which the aerofoil surface is tangential to the undisturbed free-stream direction, i.e. $\theta = \alpha_I$,**
 - (b) **the incompressible pressure coefficient at the crest (C_{pcrest}) for each value of α_I .**
- (viii) **Use C_{pcrest} to determine M_D from Figure 3.**

This figure presents the relationship that

$$C_{pcrest} = \frac{0.515(1 + 0.2M_D^2)^{3.5} - 1}{0.7FM_D^2}$$

derived from the more general expression

$$C_{pI} = \frac{(p/H)\{1 + \frac{1}{2}(\gamma-1)M_D^2\}^{\gamma/(\gamma-1)} - 1}{\frac{1}{2}\gamma FM_D^2}.$$

F is the Kármán-Tsien compressibility factor and in the former expression is given by

$$F = [\beta_D + \frac{1}{2}(1 - \beta_D)C_{pcrest}]^{-1}.$$

- (ix) **Use the Prandtl-Glauert compressibility correction factor (β_D) evaluated at the drag-rise Mach number (M_D) to obtain the lift coefficient C_{LD} from**

$$C_{LD} = C_{LI}/\beta_D.$$

- (x) **Plot the drag-rise boundary as the locus of the points (C_{LD} , M_D).**
- (xi) **Examine the pressure distribution in relation to the criterion introduced in Section 4.2.1. to determine the incidence range for which a premature slow growth of wave drag with Mach number may be anticipated.**

The reason for using different compressibility correction factors in steps (viii) and (ix) is given in Section 5.2. Other differences between the above procedure and that of Reference 1 are discussed in Sections 5.3 and 5.4.

With the possible exception of obtaining the incompressible pressure distribution, the process itemised above may be readily evaluated by hand. Nevertheless, if the method is to be used regularly a digital computer programme should be of value. A flow diagram is given in Figure 1b.

4.2 Applicability

An assessment of experimental data (Section 6 and Figures 4 to 7) has shown that the method has a wide range of applicability. This stems from the general validity of the proposition that the shock-at-crest condition corresponds to the onset of the rapid increase in drag coefficient, and hence by definition to the drag-rise Mach number, and from the accuracy with which this condition can now be predicted. The applicability is conditional upon the flow over the section being everywhere attached, but other limitations are mostly ones of degree only, arising from certain special cases in which the proposition that the shock-at-crest condition corresponds to the rapid increase in drag coefficient needs some qualification.

4.2.1 The shock-at-crest condition

The correspondence between the rapid drag rise and the shock-at-crest condition on which the method is based is valid for the majority of cases. Sinnott⁶, however, demonstrated the failure of the method for one case and Reference 7 suggests that two classes of special cases exist for which the significance of the shock-at-crest condition needs further clarification. These classes represent two forms of behaviour which can occur when a suction peak exists ahead of the crest and arise when the development of the local supersonic flow from the peak is either:-

- (a) so conditioned that the shock grows significantly in strength even before reaching the crest, or
- (b) so contrived that the shock can be kept weak and short even after passing over the crest.

Features of the pressure distributions for cases (a) and (b) have led to their being referred to respectively as of 'triangular' or 'peaky' type. These descriptions derive principally from the shape of the pressure distribution when presented in the form of a suction loop generated by plotting pressure against a skewed z ordinate (designated z') as illustrated in Figure 8. The same identifying features may also be recognised in the conventional plot of pressure against chordwise distance, but the former plot is the more pertinent because it combines the relevant features of both the chordwise pressure distribution and the aerofoil shape.

For the triangular case the characteristic feature of the incompressible-flow suction loop lies in its full, triangular form (Figure 8). This figure also illustrates the reduction in the size of the suction loop effected by the first significant-drag-producing shock wave. A second feature of the triangular case is the failure of the flow over the foremost part of the chord to achieve the expansion necessary to maintain, with increasing free-stream Mach number, an overall balance representing zero wave drag in the face of pressure changes over other parts of the chord. This feature is apparent when the suction loop formed by the actual distribution is compared with the 'equivalent shock-free distribution' derived by applying the appropriate compressibility factor to the low-speed pressure distribution.

A precise criterion for recognising the triangular case is given later.

The characteristic feature of the peaky case is the pronounced hollow downstream of the peak suction. This hollow can be identified in the incompressible chordwise pressure distribution and, more readily, in the incompressible-flow suction loop as indicated in Figure 9. When, with increasing free-stream Mach number, the local flow becomes supersonic a suction peak is retained near the leading edge and this 'leading-edge' suction, integrated over the forward facing areas, remains sufficiently high to balance some development of low pressure supersonic flow to the rear of the crest.

The following additional remarks relate to the significance of the shock-at-crest condition and have a bearing on the applicability of the method.

(a) The case having a suction loop of triangular form

In this case, the development of the local supersonic flow leads to significant shock strengths and wave drag even while the shock is upstream of the crest. However for small or moderate angles of incidence the *rapid* increase in shock strength is usually deferred until the shock reaches the crest whereupon the *rapid* increase in drag is still encountered. An analysis of experimental results (Section 6) has shown that for this case the shock-at-crest condition and the drag-rise Mach number, M_D , predicted by the present method continue to give a good indication of the Mach number for the rapid drag-rise, but, below M_D , at the free-stream Mach numbers for which a region of local supersonic flow exists, a significant supercritical drag-creep occurs. This can result in increases in C_D varying from perhaps 0.002 near zero lift to 0.01 (or more in extreme cases) at moderate values of C_L (say, 0.6) at $M = M_D$, prior to the start of the steep drag-rise. The typical variation of drag coefficient at constant incidence in such a case is illustrated in several of the examples in Figure 6. The NPL 491 aerofoil (Figure 6a) provides a relatively extreme example; the first occurrence of local sonic flow (M_{crit}) on the upper surface, derived from two-dimensional wind-tunnel data, being indicated for $\alpha = 2^\circ$ and 3° to illustrate that the drag-creep in question is a supercritical phenomenon.

The strength of the early shocks upstream of the crest generally becomes progressively stronger as incidence is increased on any given aerofoil and the distinction between the initial slower rise and the later more rapid rise becomes less marked as the supercritical drag creep becomes more pronounced. Eventually shock-induced separation tends to occur – at some point before the predicted M_D (see Section 4.2.2). The drag variation is then of the form indicated in the top curve of Figure 5e.

An empirical criterion for identifying the features of the aerofoil and its pressure distribution which contribute to the full triangular form of the suction loop has been established. It is implicit in the development of the local supersonic flow associated with such a suction loop that this criterion will identify cases which exhibit a supercritical drag-creep. Thus the criterion may be stated in the following terms:-

A premature slow growth of wave drag should be anticipated if the negative increment in C_p in the incompressible-flow pressure distribution exceeds in magnitude 0.07 for an increment of 0.25 in $\sum_{crest}^x \theta \delta(x/c)$, or, for a decrement of 0.0044 in z/c , in moving upstream from the crest at the appropriate incidence, θ being measured in degrees.

The summation is an approximation to the change in ordinate of the aerofoil but in the context of the procedure (Section 4.1) may be the more convenient to use.

The derivation of this criterion is illustrated in Figures 10 and 11 and is described in Appendix A. In certain cases (the NACA 0015-63 aerofoil in Figure 11 is an example) the criterion will predict a supercritical drag-creep at all values of incidence and it is implicit in the criterion that as the incidence is increased, all aerofoils will eventually develop a pressure distribution of the form that leads to a supercritical drag-creep. The incidence above which this happens for any particular aerofoil will depend on the rate at which adverse pressure gradients develop with increasing incidence, and the rate at which the crest moves forward into them. This topic is the subject of further discussion in Appendix A.

(b) The case having a suction loop of peaky form

In this case the onset of the rapid drag-rise may be delayed until the shock is substantially downstream of the crest. The predicted value of M_D may therefore be conservative (by as much as 0.02 or possibly more).

The attainment of the optimum effect at a specified design lift coefficient, however, is critically dependent on the precise features of the surface slope and pressure distribution (see Reference 7). It requires a specialised design procedure which is outside the scope of this document.

A degree of the favourable effect is sometimes obtained for a limited range of incidence on conventional aerofoils – usually, if at all, just above the incidence corresponding to the condition of a roof-top pressure distribution in compressible flow ($M = M_D$). The results for the 12% RAE 103 aerofoil in Figure 5e provide an illustration of this.

It is emphasised that the term peaky or triangular is strictly applicable to the form of the suction loop and may be relevant to an aerofoil over only a limited range of incidence. The majority of cases as demonstrated by Figures 7a and 11 conform to neither of these patterns. For these the correspondence relating the shock-at-crest condition to the rapid drag-rise is undoubtedly valid and the method presented here provides an excellent prediction of the Mach number corresponding to the development and rapid growth of wave drag.

4.2.2 The attached-flow condition

The method should only be applied when the flow is attached. This implies that the derived (C_{LD} , M_D) locus will cease to be valid when either subsonic-type separation under strong adverse pressure gradients or shock-induced separation occurs at Mach numbers below M_D .

The limit set by subsonic-type separation can, if necessary, be estimated by the available methods⁸ for predicting the separation of either laminar or turbulent boundary layers in subsonic flow.

Shock-induced separation should present no limitation to the applicability for cases in which a supercritical drag-creep is not encountered, at least for boundary layers that are turbulent at the shock, because the separation will then almost certainly be delayed until the strength of the shock is considerably greater than that which provokes the rapid drag increase appropriate to M_D .

The development of relatively strong shock waves upstream of the crest, however, can restrict the applicability of the method with increasing severity as incidence is increased progressively beyond that at which a premature slow growth of wave drag is first encountered. This arises when the shock becomes strong enough to cause shock-induced boundary-layer separation while it is still upstream of the crest, and hence before the predicted value of M_D . The premature drag-rise then becomes even more severe and in some cases quite rapid. Under this circumstance the method is no longer applicable and would be expected to fail completely. An example of such a case is shown in Figure 6c for $\alpha = 6^\circ$.

4.3 Accuracy

Comparisons with experimental data (Section 6) have shown that the method predicts the onset of the rapid drag-rise to within ± 0.015 in Mach number for the majority of cases. Alternatively, the accuracy of the method may be stated as being equivalent to an R.M.S. error of 0.01 in Mach number between prediction and experiment. No significant shock-wave drag should be present at the predicted values of M_D , except for the cases identified by the criterion of Section 4.2.1. For these a slow but significant growth of shock-wave drag would be expected at lower Mach numbers.

For those aerofoils designed using specialised techniques to have a 'peaky' pressure distribution, or for those which fortuitously develop a degree of this favourable effect, the method should give an estimate of M_D which is conservative by 0.02 or possibly more.

5. THE DETAILED DEVELOPMENT OF THE METHOD

The steps in the development of the method that will be of most interest to the user are those that have contributed and are essential to the level of accuracy and generality achieved in predicting the shock-at-crest condition from a knowledge only of the aerofoil geometry. This development – from the stage at which Nitzberg and Crandall⁹ used as the criterion sonic velocity at the crest – has been based on the more realistic criterion of Section 3, the framework provided by Sinnott¹, and on further studies of the viscous effects and compressibility correction factors.

5.1 Incompressible Pressure Distributions

The method in its present form has been developed around the use of the Weber formula for calculating the incompressible inviscid pressure distribution because, although this formula is based on linearised theory, it takes into account many important non-linear terms. Furthermore, its use is consistent with the adoption of the Weber formula throughout a large section of the British Aircraft Industry as a ready means of determining the pressure distribution for the more general case of finite, swept wings of non-zero thickness¹⁰.

The procedure of Section 4.1. is not restricted, however, to the Weber theory and any equally valid alternative method may be substituted.

5.2 The Compressibility Correction Factors

The values of shock-downstream pressure used in Sinnott's transonic aerofoil theory², and in particular the value of $0.515H$, which defines the location of a weak shock wave (Section 3), were established empirically. In deriving his empirical relationships, Sinnott² made some use of a compressibility correction factor but he was careful to avoid uncertainties due to tunnel interference in particular experiments by considering changes for small increments only in Mach number. This also avoided the dependence on any one compressibility factor and hence any further uncertainties on this score. He formulated his theory in such a way that it could be used with the factor considered most appropriate for a particular purpose and could be adapted in the light of subsequent progress in the derivation and justification of improved rules.

It is now clear that the most generally appropriate compressibility correction factor will be a function of chordwise position (or surface slope) as well as of the value of the incompressible velocity or pressure coefficient at any position. The precise form of this function has not yet been determined. In the meantime, an analysis of representative experimental data¹¹ has indicated that the variation with Mach number of the pressure coefficient near the crest position is well represented by the Kármán-Tsien rule and that this rule can be used to predict M_D from the incompressible-flow value of the crest pressure coefficient. The variation of overall lift coefficient is, on the other hand, best represented by the Prandtl-Glauert rule. These different rules are thus used respectively in steps (viii) and (ix) of the procedure and are strongly recommended for the present. It will be clear, however, that the method is still formulated in such a way as to be readily adapted if and when a more precise method of dealing with the compressibility corrections should become available.

5.3 The Correction for Viscous Flow

The pressure distribution *near the crest* obtained for a given lift coefficient and Mach number is relatively insensitive to viscous effects provided, of course, attention is confined to attached flow. As indicated by Sinnott¹, therefore, the major influence of viscosity can be eliminated by deriving a drag-rise boundary directly in terms of Mach number and lift coefficient. However, it has now been shown¹¹ that in selecting the crest pressure from the theoretical inviscid pressure distribution for the purpose of estimating M_D for a given value of C_{LI} (and hence C_{LD}) it is frequently important to know the crest position more accurately than would be given by taking the inviscid flow lift-curve slope, $(a_1)_{IT}$, and angle of incidence α_{IT} . The procedure for estimating α_I was thus evolved as a further development from Reference 1 and was instrumental in widening the range of aerofoils for which the predictions agree well with experiment. As might be expected this is particularly relevant to those cases in which a substantial pressure gradient exists in the neighbourhood of the crest. On the other hand the sensitivity of M_D to this correction is not such as to merit a more sophisticated estimation of the lift-curve slope $(a_1)_I$ than that indicated in Section 4.1.

5.4 The Restriction to a Single Criterion for Predicting Drag-Rise

In Reference 1 an alternative definition of drag-rise was suggested, to be invoked in certain special cases in preference to the definition in Section 3. This alternative definition was chosen to give the higher values of M_D appropriate to a particular 'roof-top' pressure distribution which led to a delayed formation of the shock wave. However, the alternative definition was not found to be generally applicable in the manner expressed in Reference 1 to the wider range of experimental results now considered. The overall correlation with experiment was in fact slightly better using the general definition only. This also has the merit of giving the more conservative prediction in the cases of doubt. The use of this particular alternative definition is therefore not recommended, but certain results in Figures 7a and 11 clearly demonstrate the need for further analysis leading to a more appropriate definition of the régime in which the formation of the shock is delayed, and a criterion by which these cases can be recognised.

6. EXPERIMENTAL VERIFICATION

A preliminary step in the verification of the method was to collect a brief bibliography of experimental drag data on two-dimensional aerofoils at high subsonic speeds. This was restricted to data obtained at Reynolds numbers (at $M = 0.7$) greater than 10^6 , to data believed to be free from condensation effects and data either free from tunnel interference effects or corrected adequately for them. This bibliography although not necessarily exhaustive may be of wider use in its own right and so it is included here as Appendix B. In particular, the aerofoils for which data have been collected, nearly 100 in number, are listed in Tables B1.1 and B1.2 of this Appendix.

Initially the drag-rise boundary was calculated for all but a few of these aerofoils using a method closely resembling that outlined in Reference 1. The results obtained highlighted the need to take account of the effect of viscosity on lift-curve slope (see Section 5.3) in determining the crest position for a given lift coefficient. The inclusion of this step proved particularly significant for the cases having suction loops exhibiting or approaching full triangular form since for them the crest is in a region of marked pressure gradient so that small changes in crest position noticeably affect C_{pcrest} and hence M_D . This first study also prompted an examination of available compressibility laws (see Section 5.2) to establish which most nearly represented the variation of C_{pcrest} and C_L with Mach number.

The method in its developed form (Section 4.1) was applied to some 30 of the aerofoils, selected to represent the whole range of aerofoil design parameters but also to include, relative to the initial selection, an unrepresentatively large proportion of those showing a supercritical drag creep. These aerofoils and the values of incidence for which specific comparisons have been made between prediction and experiment are listed in Appendix C. The incidence range for each aerofoil was limited either by the availability of data or by the requirement that for strict applicability of the method neither the C_L - v - α nor the C_D - v - M curve should be influenced by boundary-layer separation prior to M_D . However, the latter requirement was waived in a few cases to illustrate (for example, the 12% RAE 103 aerofoil on Figure 11) that when shock-induced separation occurs before the predicted value of M_D , it is a natural sequel to the type of supersonic flow development for which a supercritical drag creep occurs.

The predicted (M_D , C_L) locus (Figure 4) was obtained for each aerofoil from a set of arbitrarily chosen incidence values, α_{IT} , with Reynolds number assumed to be that corresponding to $M = 0.7$ in the appropriate wind tunnel tests and with transition assumed to be at the leading edge.

The experimental values of drag-rise Mach number (M_{De}) were assessed, in general, from measurements of drag coefficient made at constant values of incidence, α_e , for which C_L varied with Mach number.

Figure 4 illustrates that in attempting to correlate prediction with experiment, there are, for a given assessment of M_{De} , two possible interpretations of the predicted drag-rise Mach number, although the predicted boundary itself is quite unique. These possibilities are M_{Dp} (constant incidence) and M_{Dpe} (constant lift coefficient, namely, the value of C_L measured at the experimental drag-rise Mach number). The latter interpretation has been used throughout this memorandum because it is the more appropriate to aircraft design and also leads to the more conservative assessment of the accuracy of the method, although it has the disadvantage of being dependent on M_{De} .

The definition of M_{De} is itself open to some debate and for the primary analysis in this memorandum the experimental drag-rise Mach number has been taken as that Mach number for which C_D first noticeably departs from an extrapolation of the subcritical curve, assuming no wave drag, as illustrated in Figure 5a. The measured subcritical C_D - v - M curves will have included variations of profile drag coefficients that are due to compressibility effects which arise in the absence of shock waves, to changing Reynolds number (which increased with increasing Mach number) and, possibly, to changing transition position (where this was not deliberately fixed near the leading edge). The extrapolated subcritical curve will, by implication, also include these effects, varying in a consistent manner with Mach number. Thus it was thought that this definition of M_{De} obviated any need to correct the experimental data to constant Reynolds number and transition position.

For aerofoil sections showing a supercritical drag-creep, M_{De} , by virtue of the above definition, lies at the beginning of the drag-creep rather than at the beginning of the rapid rise. In order to assess the accuracy with which the present method continues to predict the beginning of the rapid drag rise, a definition of the experimental *rapid* drag-rise Mach number, M_{Der} , has been invoked. This is that M_{Der} is the free-stream Mach number for which $dC_D/dM = 0.05$. This slope is far less than the maximum slope of the C_D - v - M curve but greater than that of the supercritical drag-creep. In the absence of a supercritical drag-creep, M_{Der} would therefore not differ noticeably from M_{De} . As is the case for M_{De} , two interpretations of the predicted drag-rise Mach number corresponding to M_{Der} are possible, namely, M_{Dper} and M_{Dp} (see Figure 4).

In Figures 5 and 6 the predicted values of drag-rise Mach number, M_{Dpe} , are indicated by arrows for direct comparison with typical experimental results.

Figure 5 presents a selection which excludes aerofoils designed specifically to exploit peaky characteristics and those falling within the criterion defined in Section 4.2.1 at very low values of incidence. The agreement between prediction and experiment is seen to be extremely good. Attention is drawn to the data for the 12% thick RAE 103 aerofoil in Figure 5e which shows results that are optimistic compared with prediction for small finite values of incidence. This feature has been observed on other similar aerofoils and is an example where the local supersonic flow development is approaching that achieved with peaky velocity distributions.

Figure 6 presents a selection of results for aerofoils for which a drag creep would be expected, by virtue of the criterion of Section 4.2.1 for incidences above a certain small value in each case and for these incidences M_{Der} and M_{Dper} are also indicated. It will be seen that, in general, the characteristic creep preceding M_{Der} becomes more pronounced as incidence is increased for a given aerofoil.

The complete set of results is presented in Figure 7a as a correlation between prediction and the initial drag-rise determined from experimental data. Some of the scatter in Figure 7a can be attributed to the uncertainty in this interpretation of M_{De} but it was chosen after much consideration as the most consistent and the most apt of several possible interpretations. This uncertainty in interpretation is clearly increased by the supercritical drag-creep when present, but would in no way account for the consistent lack of agreement of the initial drag-rise with the predicted value for the examples identified by the criterion of Section 4.2.1 – shown in Figure 7a by solid points. All but one of these points are displaced from the line of perfect correlation by more than -0.015 in M_{De} . The extent of the displacement is a measure of the Mach number range over which the slow supercritical drag-creep can be expected. Some of the points on Figure 7a which show M_{De} to exceed M_{Dpe} by more than 0.015 are attributable to cases exhibiting varying degrees of the favourable effect associated with peaky velocity distributions.

The cases represented by solid points in Figure 7a are re-presented in Figure 7b in the form of a correlation between the experimental rapid drag-rise (M_{Der}) and the corresponding prediction (M_{Dper}). Good correlation is now obtained, indicating that the shock-at-crest condition retains its importance in marking the onset of a rapid increase in wave drag even for those cases subject to a significant supercritical drag-creep. Figures 7a and 7b, taken together, justify a high degree of confidence being placed in the method.

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Although the references listed above can generally be obtained on application to the appropriate quarter, no guarantee is given that they can always be obtained.