

A METHOD FOR ESTIMATING THE PRESSURE DISTRIBUTION BETWEEN THE CREST AND THE TRAILING EDGE ON THE SURFACE OF AN AEROFOIL SECTION IN A SONIC STREAM

1. INTRODUCTION

The transonic flow past the surface of an aerofoil, at stream speeds below sonic, is characterised by the presence of a region of local supersonic flow enclosed between a sonic line, the aerofoil surface, and, at the downstream end, a terminating shock wave. Transition from this phase to one which is characterised by the presence of a small region of subsonic flow at the leading edge, embedded in an otherwise wholly supersonic flow, occurs as the free-stream speed approaches sonic. This transition is achieved by a growth of the local supersonic region outwards and rearwards, the rearwards growth being controlled by the movement of the terminating shock (Figure 1a) which approaches the trailing edge as the stream speed nears sonic. The corresponding development of the surface pressures is illustrated by an example in Figure 1b.

The pressure distribution that is established when the free stream is sonic ($M_0 = 1$) is significant for two reasons. Firstly, it is essentially invariant for a small range of stream speeds on either side of sonic speed and is often referred to as the 'sonic-range' pressure distribution; the corresponding near-stationary point in the variation of surface pressure with M_0 at any given fixed point upstream of the shock wave is often referred to as the 'sonic-freeze'. Secondly, the form of that part of the pressure distribution between the crest (defined as the point at which the aerofoil surface is tangential to the undisturbed free-stream direction, as shown in Figure 2) and the terminating shock wave remains similar for an even wider range of speeds below ($M_0 = 1$), Sinnott¹ shows how this part of the pressure distribution may be derived from the sonic-range pressure distribution as a stage in his method for predicting mixed supersonic/subsonic pressure distributions which, in turn, form the basis for predicting the onset of the effects of shock-induced separation, such as buffeting. It is in this context that the need to estimate the sonic-range pressure distribution is most likely to arise.

The method presented here is similar in principle to that suggested by Sinnott² but differs appreciably in detail, and is founded on data that are far more extensive and general than those available to Sinnott. Firstly, the flow conditions at the crest and the trailing edge are established in terms of the Prandtl-Meyer flow deflection angle ω (see Section 2.2). The Prandtl-Meyer angle at the crest is obtained by a method based on the local linearisation theory developed by Spreiter and Alksne³ for sharp-nosed non-lifting aerofoils, but here extended empirically to both round-nosed and lifting aerofoils. The Prandtl-Meyer angle at the trailing edge is predicted from an empirical relation similar to that suggested by Holder⁴. Then, between the crest and the trailing edge, the distribution of the expansion wave that is generated directly by the curved surface of the aerofoil (Figure 2) and the distribution of the interacting compression wave that is reflected from the sonic line are based on theoretical relationships for sonic flow.

To apply the procedure, detailed in Section 3, it is necessary to specify only the aerofoil geometry and the geometric angle of incidence for which the calculation is to be made.

2. NOTATION

c	aerofoil chord
C_p	pressure coefficient
H	stagnation pressure in undisturbed stream
M	local Mach number
M_0	free-stream Mach Number
p	local static pressure
R	a parameter involving the leading-edge shape (see Section 3.1)
T	a parameter involving the thickness-chord ratio and the change in surface slope near the crest (see Section 3.1)
t	aerofoil maximum thickness
X	a parameter involving the chordwise distance (see Section 3.3)
x	chordwise distance measured from the leading edge
Δx	chordwise distance between points on the aerofoil surface where $(\theta - \alpha) = 4^\circ$ and 2° (see Figure 2)
α	angle of incidence
γ	ratio of specific heats of air, here taken as 1.4
δ_{te}	geometric flow-deflection at the trailing edge assuming zero downwash
θ	aerofoil surface slope relative to the chord line
ρ	aerofoil nose radius
ω	Prandtl-Meyer angle corresponding to local Mach number on the aerofoil surface
$\tilde{\omega}$	effective compression angle
$\Delta\omega_{cr}$	correction term dependent on R
Suffixes	
cr	pertaining to the crest position
te	pertaining to the trailing edge

2.1 Units

Throughout this document all angles are given in degrees.

2.2 Definition of ω

The Prandtl-Meyer angle, ω , is defined as the angle through which a streamline would have to be deflected in a simple-wave (or Prandtl-Meyer) expansion in order to accelerate the flow from an initial local Mach number of unity to the value in question.

3. METHOD[†]

The method is based on theoretical relationships for sonic flow, modified and extended in applicability by empirical factors derived from an analysis of numerous experimental results. It has been found to give reliable results for a wide range of aerofoil section shapes but for aerofoils having zero or very small leading-edge radius the prediction of ω_{cr} may be in error for angles of incidence greater than 2° (see Sections 4.1 and 6.1).

The local flow characteristics on the surface of the aerofoil are determined in three stages, being expressed in each case in terms of the Prandtl-Meyer flow-deflection angle ω . The three stages comprise the evaluation of ω at the crest, the evaluation of ω at the trailing edge and the distribution of ω between the crest and trailing edge. In a fourth and final stage ω is converted to a pressure ratio, a pressure coefficient or a local Mach number as indicated in Section 3.4.

The procedures in Section 3.1 to 3.3 detail the application of the method to the upper surface of an aerofoil for a given incidence, and with the geometry specified in terms of co-ordinates and surface slopes. However, the procedure may readily be applied to the lower surface by inverting the aerofoil and changing the sign of the incidence.

3.1 The Evaluation of ω at the Crest

- (i) Determine the chordwise stations corresponding to

- (a) $\theta = \alpha$,
- (b) $\theta = \alpha + 2^\circ$,
- (c) $\theta = \alpha + 4^\circ$,

and hence evaluate
$$\frac{\Delta x}{c} = \left(\frac{x}{c}\right)_{\theta = \alpha + 2^\circ} - \left(\frac{x}{c}\right)_{\theta = \alpha + 4^\circ}.$$

- (ii) Evaluate the thickness parameter T and the leading-edge parameter R , where

$$T = \frac{(\frac{1}{2}t/c)^{2/3}}{\Delta x/c},$$

$$\text{and } R = \frac{\left(\frac{\Delta x}{c}\right)\left(\frac{\rho}{c}\right)}{\left(\frac{x}{c}\right)_{cr}\left(\frac{t}{c}\right)}.$$

[†] Attention is drawn to the alternative and more comprehensive method for predicting the complete sonic-range pressure distribution on an aerofoil in T.D. Memo 69013. The results of applying the methods of T.D. Memos 6511 and 69013 are in substantial agreement over that part of the chord where comparison is possible, *i.e.* between the crest and the trailing edge. T.D. Memo 6511 is, therefore, retained in the Series since it provides the more rapid estimate of the sonic-range pressure distribution where this is required downstream of the crest only.

- (iii) Given T , determine $(\omega_{cr} - \Delta\omega_{cr})$ from Figure 3.
- (iv) Given R , determine $\Delta\omega_{cr}$ from Figure 4.
- (v) Evaluate ω_{cr} , where

$$\omega_{cr} = (\omega_{cr} - \Delta\omega_{cr}) + \Delta\omega_{cr}.$$

3.2 The Evaluation of ω at the Trailing Edge

- (i) Determine δ_{te} , where

$$\delta_{te} = \alpha - \theta_{te} \quad (\text{see Figure 2}).$$

(Note that θ_{te} is a negative quantity and that this relationship assumes zero downwash.)
- (ii) Determine ω_{te} from the relation

$$\omega_{te} = \delta_{te} + 1^\circ.$$

3.3 The Evaluation of ω at Points Between the Crest and Trailing Edge

- (i) Evaluate $\tilde{\omega}_{te}$, where

$$\tilde{\omega}_{te} = \omega_{cr} - 1^\circ.$$

The remaining calculations must be performed for sufficient values of x , between the crest and trailing edge, to establish a well-defined pressure distribution.

- (ii) Evaluate the chordwise parameter X , where

$$X = \frac{x - x_{cr}}{c - x_{cr}}.$$

- (iii) Given X , determine $\tilde{\omega}/\tilde{\omega}_{te}$ from Figure 5 and hence $\tilde{\omega}$ at each chordwise station x .
- (iv) Determine the local Prandtl-Meyer flow-deflection angle

$$\omega = \omega_{cr} + (\alpha - \theta) - \tilde{\omega}.$$

3.4 The Conversion of ω to Other Quantities

Use ω to determine either

- (a) the static pressure ratio (p/H) from Figure 6,
- (b) the pressure coefficient (C_p) from Figure 7[†],
- or (c) the local Mach number (M) from Figure 8.

[†] It should be noted that although Figures 6 and 8 can be used generally if desired, Figure 7 applies only for $M_0 = 1$.

Figures 6, 7 and 8 are derived from the following equations, taking $\gamma = 1.4$:

$$\frac{\pi}{180}(\omega + 90) = \operatorname{cosec}^{-1} \left[\frac{2}{\gamma - 1} \left\{ \left(\frac{p}{H} \right)^{-(\gamma - 1)/\gamma} - 1 \right\} \right]^{1/2} + \left(\frac{\gamma + 1}{\gamma - 1} \right)^{1/2} \tan^{-1} \left\{ \frac{2}{\gamma + 1} \left(\frac{p}{H} \right)^{-(\gamma - 1)/\gamma} - 1 \right\}^{1/2},$$

$$C_p = \frac{\left(\frac{p}{H} \right) \left[1 + \frac{1}{2}(\gamma - 1)M_0^2 \right]^{\gamma/(\gamma - 1)} - 1}{\frac{1}{2}\gamma M_0^2},$$

$$M = \left(\frac{2}{\gamma - 1} \right)^{1/2} \left\{ \left(\frac{p}{H} \right)^{-(\gamma - 1)/\gamma} - 1 \right\}^{1/2}.$$

4. BASIS OF THE METHOD

The method has been built upon a number of empirical correlations, some of which are theoretically based, and all of which have been substantiated by experimental results on a wide variety of symmetrical and cambered aerofoils.

4.1 Prediction of ω_{cr}

It has been demonstrated^{3, 5} that the value of the sonic-range pressure coefficient at the crest, calculated by the local linearisation theory of Spreiter and Alksne³ for a family of sharp-nosed non-lifting aerofoils, compares favourably with experiment at free-stream Mach numbers close to unity. For affinely related aerofoils at zero incidence the theory shows that $C_{p_{cr}}$ is directly proportional to $(\frac{1}{2}t/c)^{2/3}$ and Thompson⁶ noted that by introducing the parameter $\Delta x/c$, first used by Sinnott², the theoretical results of Spreiter and Alksne for a family of parabolic-arc aerofoils produced a unique curve of ω_{cr} versus T . He then found that all values of ω_{cr} calculated by Spreiter for a range of sharp-nosed aerofoils collapsed onto this same unique curve. Furthermore, it has now been shown that experimentally determined crest pressures for a large number of round-nosed aerofoils of arbitrary shape, over a range of incidence, also collapse onto the same curve, as illustrated in Figure 9 – the points represented by solid symbols having a special significance discussed in the next paragraph.

The collapse is not complete for all the aerofoils shown in the correlation. In particular, some round-nosed aerofoils (and most sharp-nosed sections at incidence) were found to exhibit an over-expansion peak near the leading edge under conditions for which the sonic point was so situated on the leading edge as to allow a large flow deflection in the initial rapid expansion. In some cases this phenomenon, illustrated in Figure 10, caused the crest pressure to deviate considerably from the value predicted by the curve in Figure 9. Thus, to improve the collapse of Figure 9 even further, a correction term $\Delta\omega_{cr}$ was introduced to account for this deviation, and is presented in Figure 11 as a function of the leading-edge correction parameter R .

The modified correlation of experimental data ($\omega_{cr} - \Delta\omega_{cr}$), which results when the correction term given by the mean line of Figure 11 is taken into account, is shown by Figure 12 where (as on Figure 9) the points for which the correction term $\Delta\omega_{cr}$ is significant (*i.e.* $R > 0.04$) are shown by solid symbols. The overall accuracy in the estimation of ω_{cr} is now implicit in Figure 12. For most cases $\Delta\omega_{cr}$ is zero and the curve for the parabolic-arc family of aerofoils, derived from the theory of Spreiter and Alksne, remains unchanged between Figures 9 and 12 since ρ/c , and hence $\Delta\omega_{cr}$, is zero. For the same reason Figure 11 cannot be used to estimate the effect of the leading-edge over-expansion on the crest pressure for sharp-nosed sections at incidence. However, the deviation of ω_{cr} from the predicted value given by the curve in Figure 9, for incidences below about 2° , is relatively small for the majority of sharp-nosed sections.

Data for 26 round-nosed sections at varying incidence were used in the compilation of these correlations, and the maximum deviation from the mean lines is about $\pm 1^\circ$. Of the data analysed only the NPL 491 aerofoil (for which the position of maximum thickness is at $x/c = 0.2$) at incidences greater than 2° , shows deviations considerably in excess of this. Above this incidence, the crest of this aerofoil is forward of about $x/c = 0.12$, which draws attention to a possible need for caution when the crest position is very near the leading edge.

4.2 Prediction of ω_{te}

A correlation of ω_{te} and δ_{te} was first suggested by Holder⁴. In the present work a similar correlation has been established but extended to very much higher values of δ_{te} , as shown in Figure 13. The analysis shows that for values of δ_{te} from about 1.0° to 18.5° , the mean variation of ω_{te} with δ_{te} can be expressed empirically as

$$\omega_{te} = \delta_{te} + 1^\circ.$$

Some comment is appropriate on the method adopted in analysing the experimental results to derive the value of ω_{te} plotted in Figure 13. In most instances the measured data indicate that the influence of the trailing-edge shock-wave spreads upstream of the trailing edge because of the interaction of the shock and the boundary layer. In the method herein presented, no attempt is made to account for these effects and the value of ω_{te} is appropriate to flow in which the influence of the trailing-edge shock does not spread forward onto the aerofoil. For the purpose of the correlation in Figure 13 the experimental points were extrapolated from a point upstream, free from the influence of the shock, to give an equivalent value of ω_{te} (see Figure 10). The extrapolation was performed using a procedure similar to that in Section 3.3 and therefore consistent with the application of the method.

4.3 Distribution of ω Between Crest and Trailing Edge

The variation of ω between crest and trailing edge can be determined from (i) the strength of the simple wave that would be generated by the change of slope at the surface if the supersonic flow approaching the crest were uniform, and (ii) from the strength of the compressive wave that must be superimposed on this to account for the non-uniformity of the flow. The compressive wave can be considered as originating from the sonic line bounding the local supersonic flow. Once ω_{cr} and ω_{te} have been established the total magnitude of the compression between crest and trailing edge is known from the following expression:

$$\tilde{\omega}_{te} = \omega_{cr} + \delta_{te} - \omega_{te}.$$

From the relation given in Section 4.2, this expression simplifies to:

$$\tilde{\omega}_{te} = \omega_{cr} - 1^\circ.$$

Sinnott² showed that by representing the distribution of the compression in the non-dimensional form shown in Figure 5, the same distribution was obtained for all aerofoils, at least to the same order of accuracy as obtained in the empirical relations for ω_{te} and ω_{cr} . In his method for predicting sonic-range pressure distributions Sinnott² used the theoretical distribution for a double-wedge aerofoil but here, the solution of Spreiter and Alksne³ for a 10% thick parabolic-arc section is used (Figure 5) because it gives an equally good correlation and is more consistent with the derivation of ω_{cr} .

5. ACCURACY

The accuracy of prediction obtained with the present method, within the limits of applicability discussed in the next Section, would be expected to correspond to the accuracy with which ω_{cr} and ω_{te} can be predicted. Figures 11–14 suggest this to be within $\pm 1^\circ$ on ω (or to the corresponding limits on pressure or local Mach number) for the majority of cases. Some additional error may accrue from the adequacy with which Figure 5 represents the distribution of compression between crest and trailing edge. The comparisons with experiment shown in Figure 14 suggest that this latter effect does not significantly affect the accuracy because maximum deviations in the local pressure ratio, p/H , are about ± 0.015 (corresponding to about $\pm 1^\circ$ on ω for ω less than about 20°) except near the trailing edge, where the experimental results are influenced by the trailing-edge shock. The discrepancies near the trailing edge do not materially detract from the value of the method.

Although the overall accuracy of the method may be judged by the adequacy with which it represents experimental data, the question arises whether some allowance must also be made for inherent inaccuracies in these data. In this respect it is appropriate to add that the experimental data have all been obtained from tests at the National Physical Laboratory, where results⁵ have shown that errors due to wind tunnel wall interference lie within, and indeed partly account for, the scatter of the correlations. These results⁵ have also shown that if tunnel interference effects are eliminated, the agreement between experiment and the theory of Spreiter and Alksne is extremely good for those aerofoils for which comparison is possible.

6. APPLICABILITY OF THE METHOD

The method presented in this Memorandum may be applied to a wide range of round-nosed lifting aerofoils and also, in a more limited extent, to sharp-nosed aerofoils.

Because the method is partly founded on experimental data, viscous effects, other than boundary-layer/shock-wave interaction near the trailing edge, are included. Also, since the results used or derived in this method agree well with the inviscid theory of Spreiter and Alksne³ wherever comparison is possible (see also Reference 5), it follows that the method may be applied over a wide range of Reynolds number.

6.1 Application to Sharp-Nosed Aerofoils

Experimental evidence indicates that for sharp-nosed sections, or sections with a leading-edge radius less than about $0.001c$, the procedure for estimating ω_{cr} should be limited to an incidence less than about 2° .

6.2 Application to Round-Nosed Aerofoils

It is not possible to specify precise limits for which the method is applicable to round-nosed aerofoils. However, the values of the geometric parameters given below represent the range of available experimental data used to establish the validity of the method, but do not necessarily represent the ultimate limits of applicability.