



ESDU TECHNICAL MEMORANDUM

Derivation of lubricant entrainment velocity for disc cams with six different types of follower

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THE PREPARATION OF THIS TECHNICAL MEMORANDUM

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DERIVATION OF LUBRICANT ENTRAINMENT VELOCITY FOR DISC CAMS WITH SIX DIFFERENT TYPES OF FOLLOWER

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DERIVATION OF LUBRICANT ENTRAINMENT VELOCITY FOR DISC CAMS WITH SIX DIFFERENT TYPES OF FOLLOWER

1. INTRODUCTION

Expressions for the velocity of entrainment of a lubricant film between a cam and a follower are required in ESDU 94008 (Reference 2) and the associated computer programs (ESDUpac A9408 and VIEWpac W9408). The full derivation only for the case of a disc cam with a domed follower is included in that Data Item. The full derivations for the other follower types are presented in this memorandum.

2. NOTATION

The units used in this memorandum are predominantly SI.

B	offset of cranked follower arm (measured in positive sense above X_m -axis)	m
D	distance between cam rotational centre and follower pivot	m
d	distance along x-axis from cam centre to point of intersection of contact normal with X_m -axis	m
E	follower offset	m
$g(t)$	intermediate calculation parameter	–
L	distance between follower arm pivot and roller centre (or centre of follower face radius)	m
R_f	radius of follower	m
t	time	s
V_c	velocity of cam surface at contact point (relative to contact point)	m/s
$(V_c)_x, (V_c)_y$	components of velocity of cam surface at contact point (relative to contact point) in x- and y-directions, respectively	m/s
V_{ent}	lubricant entrainment velocity	m/s
$(V_{ent})_x, (V_{ent})_y$	components of lubricant entrainment velocity in x- and y-directions, respectively	m/s
V_f	velocity of follower surface at contact point (relative to contact point)	m/s
$(V_f)_x, (V_f)_y$	components of velocity of follower surface at contact point (relative to contact point) in x- and y-directions, respectively	m/s
X_m, Y_m	axes fixed in machine frame	

$\dot{x}_c, \dot{y}_c$	components of velocity of cam surface at contact point (relative to fixed machine axes) in x - and y -directions, respectively	m/s
$\dot{x}_f, \dot{y}_f$	components of velocity of follower surface at contact point (relative to fixed machine axes) in x - and y -directions, respectively	m/s
x_p, y_p	coordinates of contact point (relative to fixed machine axes) in x - and y -directions, respectively	m
$\dot{x}_p, \dot{y}_p$	components of velocity of contact point (relative to fixed machines axes) in x - and y -directions, respectively	m/s
y	translational displacement of follower	m
$\dot{y}$	translational velocity of follower	m/s
$\ddot{y}$	translational acceleration of follower	m/s ²
y_m	instantaneous height of roller centre (or centre of follower face radius or point of intersection of flat follower with Y_m -axis) above X_m -axis	m
$Z_1, Z_2, \dots, Z_n$	intermediate calculation parameters	—
α	pressure angle	degree
ψ	rotational displacement of follower	rad
$\dot{\psi}$	rotational velocity of follower	rad/s
$\ddot{\psi}$	rotational acceleration of follower	rad/s ²
ψ_m	angle between X_m -axis and follower arm	rad
ω_c	angular velocity of cam	rad/s

One or two dots above a variable indicates the first or second derivative, respectively, of that variable with respect to time.

3. GENERAL FORM OF EQUATIONS

The lubricant entrainment velocity is defined as

$$V_{ent} = 0.5 |V_c + V_f|. \quad (3.1)$$

It is therefore necessary to derive expressions for V_c and V_f . However the contact point moves as the cam rotates and so great care is required. It is most convenient to work with x-and y-components of velocities until the final evaluation of entrainment velocity.

The components of velocity of the cam surface at the contact point (relative to the contact point) are given by

$$(V_c)_x = \dot{x}_c - \dot{x}_p \quad (3.2)$$

and
$$(V_c)_y = \dot{y}_c - \dot{y}_p. \quad (3.3)$$

Similarly the components of velocity of the follower surface at the contact point (relative to the contact point) are given by

$$(V_f)_x = \dot{x}_f - \dot{x}_p \quad (3.4)$$

and
$$(V_f)_y = \dot{y}_f - \dot{y}_p. \quad (3.5)$$

With use of Equation (3.1), the components of the entrainment velocity are given by

$$(V_{ent})_x = 0.5 |(V_c)_x + (V_f)_x| \quad (3.6)$$

and
$$(V_{ent})_y = 0.5 |(V_c)_y + (V_f)_y|. \quad (3.7)$$

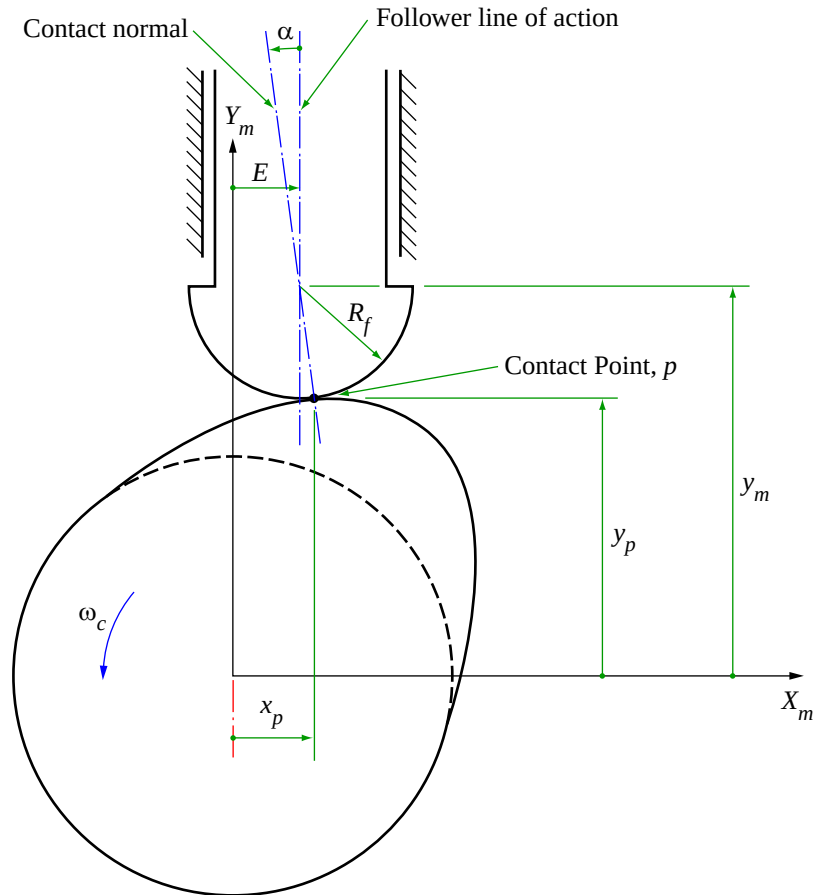
The magnitude of the entrainment velocity is given by

$$V_{ent} = \left([(V_{ent})_x]^2 + [(V_{ent})_y]^2 \right)^{1/2}. \quad (3.8)$$

Entrainment velocity is calculated in ESDUpac A9408 at intervals of one degree of cam angle during one full rotation of the cam. It remains to find expressions for $\dot{x}_p, \dot{y}_p, \dot{x}_c, \dot{y}_c, \dot{x}_f$ and $\dot{y}_f$ for each follower type.

4. DISC CAM WITH TRANSLATING DOMED FOLLOWER

Firstly, equations for the velocities of the surfaces of the cam and follower at the contact point relative to the fixed axes (X_m , Y_m) can be derived. Sketch 4.1 shows the cam and follower in contact at a point defined by coordinates x_p and y_p relative to the fixed axes.



Sketch 4.1 Disc cam with translating domed follower

By observation of Sketch 4.1 it can be seen that the instantaneous velocity of a particle on the cam surface at the contact point (relative to the fixed axes X_m , Y_m) in the x -direction is given by

$$\dot{x}_c = -\omega_c y_p \quad (4.1)$$

and similarly in the y -direction by

$$\dot{y}_c = \omega_c x_p. \quad (4.2)$$

The negative sign in Equation (4.1) arises because of the conventional direction of rotation of the cam and the fact that y_p is always positive.

The coordinates of the contact point are given by

$$x_p = E + R_f \sin \alpha \quad (4.3)$$

$$\text{and} \quad y_p = y_m - R_f \cos \alpha. \quad (4.4)$$

The instantaneous velocity of a particle on the follower surface at the contact point (relative to the fixed axes X_m, Y_m) is known. The x -component is given by

$$\dot{x}_f = 0 \quad (\text{due to constraint provided by follower guide}) \quad (4.5)$$

and the y -component is given by

$$\dot{y}_f = \dot{y}. \quad (4.6)$$

Note that $\dot{y} = \dot{y}_m$ because $y_m = y + \text{constant}$.

The next step is to find the x - and y -components of velocity of the contact point relative to the fixed machine axes.

It is necessary to differentiate the expressions given in Equations (4.3) and (4.4) with respect to time. Differentiating Equation (4.3) gives

$$\dot{x}_p = R_f \dot{\alpha} \cos \alpha. \quad (4.7)$$

The pressure angle is given in ESDU ME2 (Reference 1) by

$$\alpha = \tan^{-1} \left[\frac{\left(\frac{\dot{y}}{\omega_c} \right) - E}{y_m} \right]. \quad (4.8)$$

An expression for the rate of change of pressure angle with respect to time must also be found and by observation of Equation (4.8) the following substitution is required.

$$g(t) = \frac{\frac{\dot{y}}{\omega_c} - E}{y_m} \quad (4.9)$$

and so
$$\dot{\alpha} = \frac{d}{dt} \left(\tan^{-1} (g(t)) \right),$$

i.e.
$$\dot{\alpha} = \frac{1}{1 + g(t)^2} \dot{g}(t) \quad (4.10)$$

and by the quotient rule of differentiation

$$\dot{g}(t) = \frac{\frac{y_m \ddot{y}}{\omega_c} - \dot{y} \left(\frac{\dot{y}}{\omega_c} - E \right)}{y_m^2}. \quad (4.11)$$