



ESDU TECHNICAL MEMORANDUM

Derivation of contact force and radius of curvature of cam profile for disc cams with six different types of follower

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DERIVATION OF CONTACT FORCE AND RADIUS OF CURVATURE OF CAM PROFILE FOR DISC CAMS WITH SIX DIFFERENT TYPES OF FOLLOWER

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DERIVATION OF CONTACT FORCE AND RADIUS OF CURVATURE OF CAM PROFILE FOR DISC CAMS WITH SIX DIFFERENT TYPES OF FOLLOWER

1. INTRODUCTION

Expressions for the contact force between a cam and a follower and for the radius of curvature of the cam profile at the contact point are required in ESDU 93001^[3], ESDU 94008^[4] and ESDU 94011^[5], and the associated computer programs, ESDUpacs A9301, A9408 and A9411, respectively. Full derivations of these quantities are not always given in these Data Items for every follower type and so this memorandum provides a record of all equations used in these computer programs.

2. NOTATION

The units used in this Memorandum are predominantly SI.

B	follower arm crank length (see Sketch 9.1 for sign convention)	m
c	distance between follower cam pivot and contact point	m
D	distance between cam rotational centre and follower pivot (see Sketches 7.1, 8.1 or 9.1 for sign convention)	m
E	follower offset (see Sketches 4.1, 5.1 or 6.1 for sign convention)	m
F_E	external force acting on follower (positive to increase contact force)	N
F_{xi}, F_{yi}	components of resultant of external forces acting on part i in x - and y -directions, respectively	N
H	follower lift corresponding to one segment	m
I_f	mass-moment of inertia of follower system about follower pivot	kg m ²
k_{SC}	compression spring rate	N/m
k_{ST}	torsion spring rate	Nm/rad
L	distance between follower pivot and centre of follower radius	m
m_f	mass of follower system	kg
P_C	contact force normal to cam profile	N
P_S	compression spring force	N
P_{ij}	force exerted by part i on part j	N
P_{xij}, P_{yij}	components of joint force P_{ij} in x - and y -directions, respectively	N

P_{SP}	compression spring preload	N
R_b	reference circle radius of cam	m
R_f	radius of follower	m
R_r	radius of roller	m
T_E	external moment acting on moving follower (positive to increase contact force)	Nm
T_S	torsion spring moment	Nm
T_{SP}	torsion spring preload	Nm
T_{ij}	torque exerted by part i on part j	Nm
w	normalised follower displacement	—
w'	non-dimensional follower velocity	—
w''	non-dimensional follower acceleration	—
y	translational displacement of follower	m
$\dot{y}$	translational velocity of follower	m/s
$\ddot{y}$	translational acceleration of follower	m/s ²
y_b	height of centre of follower radius above X_m -axis at inner dwell	m
y_0	height of centre of follower radius above X_m -axis at beginning of segment	m
y_m	instantaneous height of centre of follower radius (or height of intersection of flat-faced follower with Y_m -axis) above X_m -axis	m
α	pressure angle	degree
ρ	radius of curvature of pitch curve	m
ρ_p	radius of curvature of cam profile	m
Φ	total angle of cam rotation corresponding to one segment	rad
ϕ	cam angle of rotation (from start of Segment 1)	degree
Ψ	follower swing corresponding to one segment	rad
ψ	rotational displacement of follower	rad
$\dot{\psi}$	rotational velocity of follower	rad/s

$\ddot{\psi}$	rotational acceleration of follower	rad/s ²
ψ_b	angle between X_m -axis and line joining follower pivot to centre of follower at inner dwell	rad
ψ_m	instantaneous angle between X_m -axis and line joining follower pivot to centre of follower radius	rad
ψ_0	angle between X_m -axis and line joining follower pivot to centre of follower radius at beginning of segment	rad
ω_c	angular velocity of cam	rad/s

3. CONVENTIONS

The convention adopted for forces and moments in this memorandum is such that P_{x23} is the component of force in the x -direction exerted by Part 2 on Part 3. Thus

$$P_{x23} = -P_{x32}$$

The magnitude of the force exerted by Part 2 on Part 3 is given by

$$P_{23} = \left(P_{x23}^2 + P_{y23}^2 \right)^{1/2}.$$

Similarly, T_{45} is the moment (positive in the anticlockwise sense) exerted by Part 4 on Part 5. Thus

$$T_{45} = -T_{54}.$$

The cam is assumed to rotate in an anticlockwise sense at constant velocity.

4. DISC CAM WITH TRANSLATING ROLLER FOLLOWER

Sketch 4.1(a) shows a typical sub-system composed of a disc cam with an offset translating roller follower. A compression spring is used to provide force closure. The sign conventions used are shown on Sketch 4.1(b) and diagrams showing the forces acting on each part are shown in Sketch 4.1(c).

4.1 Derivation of Contact Force

The force between the cam and the follower is due to the action of the spring, the inertia of the follower system and the external force acting on the follower. The follower is constrained to move only in the y -direction and it is assumed that the friction force, P_{y14} , between the follower and the guide is zero. However in practice friction can be included by considering it as part of the external force, F_{y4} . The sum of the components of force acting on the follower in the y -direction is equal to the product of the mass of the follower and the acceleration of its centre of mass in the y -direction. Thus for equilibrium of the follower

$$P_{y34} + P_{y54} + F_{y4} = m_f \ddot{y}, \quad (4.1)$$

where P_{y34} is the force exerted by the roller on the follower,

P_{y54} is the force exerted by the spring on the follower,

F_{y4} is the external force exerted on the follower

and m_f is the mass of the follower system.

The instantaneous motion (displacement, y , velocity, $\dot{y}$, and acceleration, $\ddot{y}$) of the roller centre during a rise or return segment is required. These quantities are given by

$$y = wH, \quad (4.2a)$$

$$y_m = y_0 + wH, \quad (4.2b)$$

$$\dot{y} = \frac{w' \omega_c H}{\Phi} \quad (4.3)$$

$$\text{and} \quad \ddot{y} = \frac{w'' \omega_c^2 H}{\Phi^2}. \quad (4.4)$$

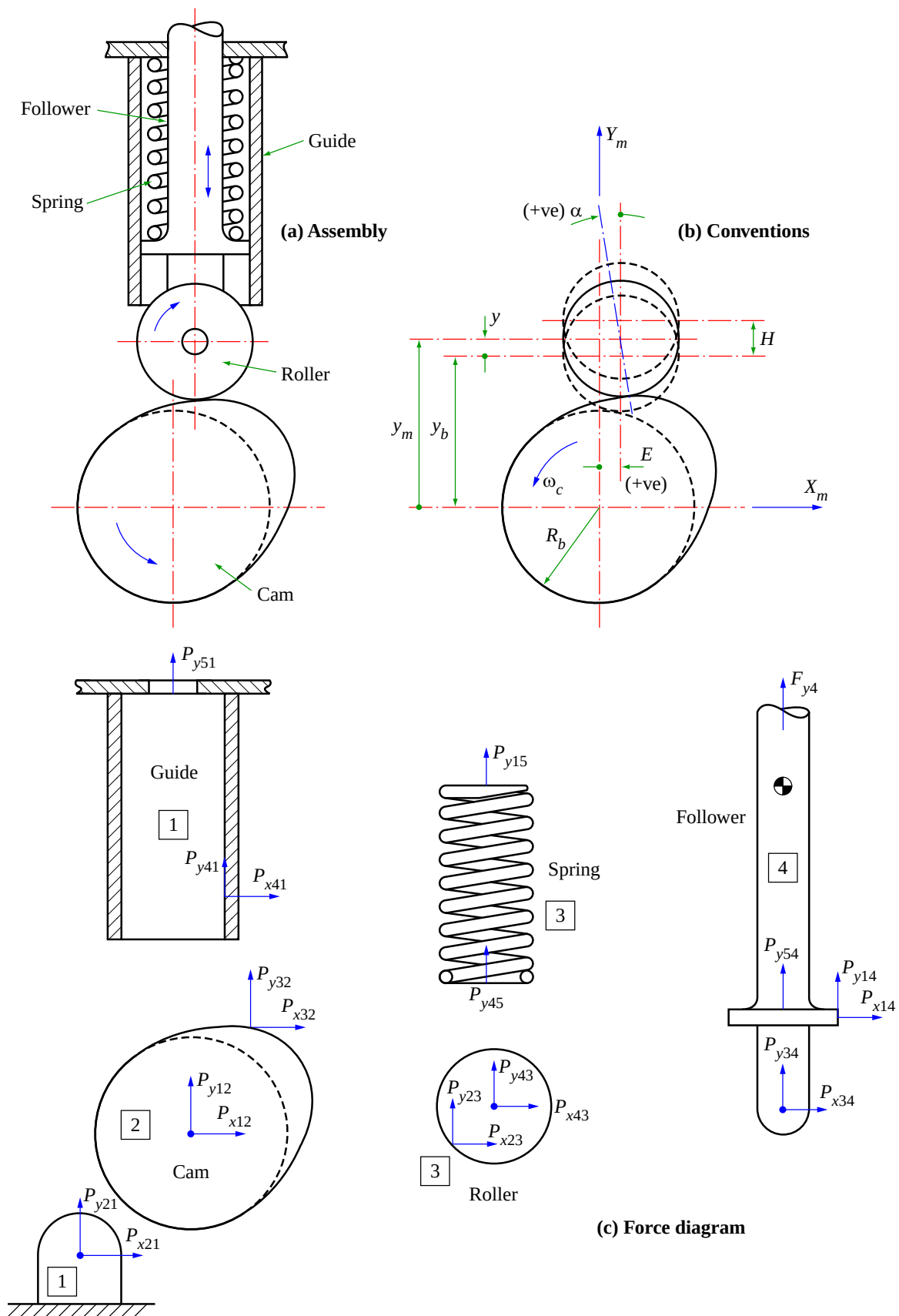
where y_0 is the height of the roller centre above the X_m -axis at the start of the motion segment (rise or return),

H is the total follower lift corresponding to one motion segment (in this memorandum it is taken to be a positive scalar quantity for both rise and return motion segments),

Φ is the total cam angle corresponding to one motion segment (note that units are radians),

ω_c is the angular velocity of the cam

and w, w' and w'' are the dimensionless follower displacement, velocity and acceleration, respectively (expressions for these parameters corresponding to commonly-used cam laws are given in ESDU 82006^[2]).



**Sketch 4.1 Cam and follower system
(translating roller follower)**

Note that:

- (a) w and w' are positive during a rise motion and negative during a return motion
- and (b) w'' changes sign during every motion segment (it is positive during the first part of a rise segment and negative during the first part of a return segment).

The spring force is composed of the preload, P_{SP} , with the product of the spring rate, k_{SC} , and the follower displacement from inner dwell ($y_m - y_b$), that is

$$P_S = P_{SP} + k_{SC}(y_m - y_b), \quad (4.5)$$

where

$$y_b = \left((R_b + R_r)^2 - E^2 \right)^{1/2}. \quad (4.6)$$

It is assumed that the mass-moment of inertia of the rotating parts of the roller is negligible, which implies that the torque to accelerate the outer race and the rolling elements is negligible, and so only two forces act on the roller. These forces must be of equal magnitude and opposite in direction. Therefore, for equilibrium of the roller

$$P_{23} + P_{43} = 0 \quad (4.7)$$

and the force exerted on the roller by the cam is given by

$$\begin{aligned} P_{23} &= -P_{43}, \\ P_{23} &= \frac{P_{y43}}{\cos \alpha}, \end{aligned} \quad (4.8)$$

where α is the pressure angle (see ESDU ME2^[1]) given by

$$\alpha = \tan^{-1} \left(\frac{\left(\frac{w'H}{\Phi} \right) - E}{y_m} \right). \quad (4.9)$$

Note that for a rise segment w' is positive and for a return segment w' is negative.

Substituting Equation (4.1) into Equation (4.8) gives

$$P_{23} = \frac{-P_{y54} - F_{y4} + m_f \ddot{y}}{\cos \alpha}. \quad (4.10)$$

The external force, F_E , and the spring force, P_S , are considered as positive (by convention) when they act to increase the contact force on the cam. So

$$F_E = -F_{y4} \quad (4.11)$$

and

$$P_S = -P_{y54}. \quad (4.12)$$

Thus the expression for contact force, as used in the programs, is obtained by substituting Equations (4.11), (4.12) and (4.5) in Equation (4.10) to give

$$P_C = P_{23} = \frac{(P_{SP} + k_{SC}(y_m - y_b)) + F_E + m_f \ddot{y}}{\cos \alpha} \quad (4.13)$$

Contact force is calculated in ESDUpacs A9301 and A9408 at intervals of one degree of cam angle during one full rotation of the cam.

4.2 Radius of Cam Profile Curvature

The radius of curvature of the cam profile is required for the calculation of contact stress and lubricant film thickness. The derivation of an expression for this quantity is given in ESDU ME2^[1] where the radius of curvature of the pitch curve for a cam driving a translating roller follower is given by

$$\rho = \frac{y_m^2}{\frac{H}{\Phi} \cos^2 \alpha \left(w' \sin \alpha - \frac{w''}{\Phi} \cos \alpha \right) + y_m \cos \alpha} \quad (4.14)$$

and the radius of curvature of the cam profile is given by

$$\rho_p = \rho - R_r \quad (4.15)$$

This quantity is determined in ESDUpacs A9301 and A9408 at intervals of one degree of cam angle during one full rotation of the cam.