



ESDU TECHNICAL MEMORANDUM

Calculation of planar area and moment properties – theoretical background

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ESDU TECHNICAL MEMORANDA

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THE PREPARATION OF THIS TECHNICAL MEMORANDUM

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CALCULATION OF PLANAR AREA AND MOMENT PROPERTIES – THEORETICAL BACKGROUND

CONTENTS

	Page
1. INTRODUCTION	1
2. THE SLICE APPROACH	1
3. DERIVATION OF PROCEDURES	4
3.1 Moments of Area	4
3.2 Product Moment of Area	7
3.3 Principal Axes and Moments	9
4. INERTIA PROPERTIES	10

CALCULATION OF PLANAR AREA AND MOMENT PROPERTIES – THEORETICAL BACKGROUND

1. INTRODUCTION

Stress analysis and rigid body motion are just two of the fields of mechanics in which area and area moment properties are frequently required. Such properties are given by an integral of the general form

$$\int (\text{distance})^n d(\text{area}) ,$$

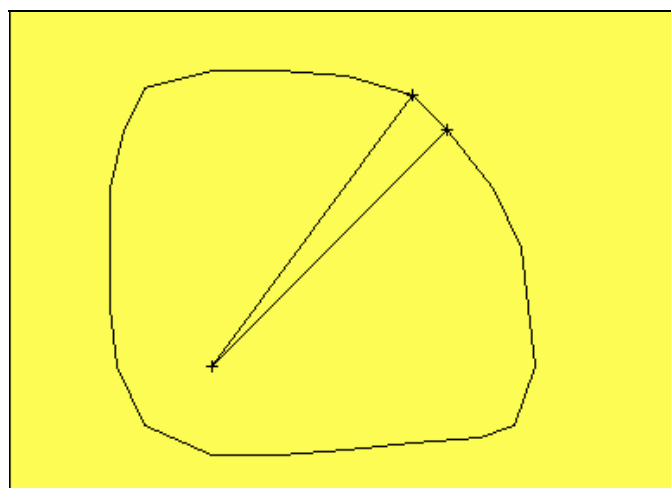
where distances are measured from some moment axis to the elemental area and the integration is applied over the whole of the area of interest (the zeroth moment when $n = 0$ gives the area).

In the numerous text books that derive the various moments, a cartesian approach is invariably taken whereby the shape of interest is decomposed into essentially rectangular elements in order for the integration to be applied. This approach is perfectly acceptable for the relatively simple shapes that are normally considered but becomes less so when the shape is complex. The approach becomes particularly messy when shapes with re-entrant boundaries are to be analysed.

In this note an alternative, pseudo polar, approach is developed in which a triangular “slice” element is used for the required integration. It is demonstrated that such an approach leads to easily implemented computational algorithms that are general and can cope with shapes of arbitrary complexity. Having obtained the area and moment properties, the mass-based properties can be easily calculated for a real shape, or plate, of finite thickness.

2. THE SLICE APPROACH

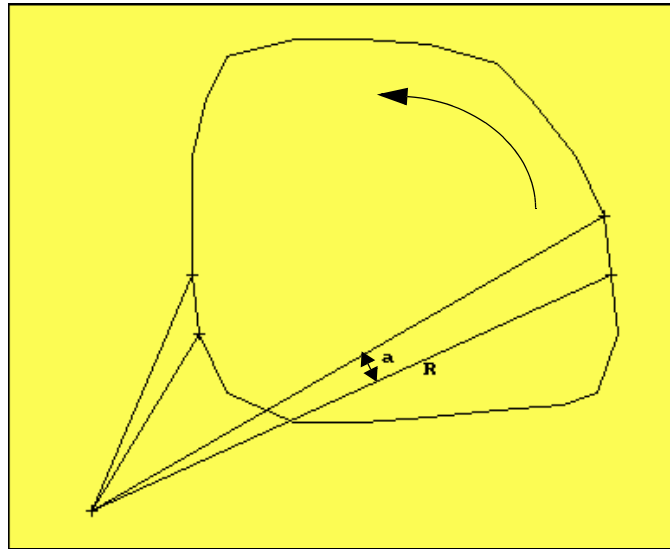
Consider any area with a closed boundary and take an origin inside the boundary. Draw radii from the origin to two adjacent points on the boundary of the shape so as to define a portion of the shape in the same way that one would slice a cake as shown in Sketch 2.1.



Sketch 2.1 Shape with origin inside boundary

Provided points are distributed sufficiently frequently around the boundary, any moment property of the complete shape can be obtained by summing the contributions to the moment of all such “slices”. In practice, one would generate the slices by tracing around the shape taking each pair of adjacent points in turn until the starting point was reached again.

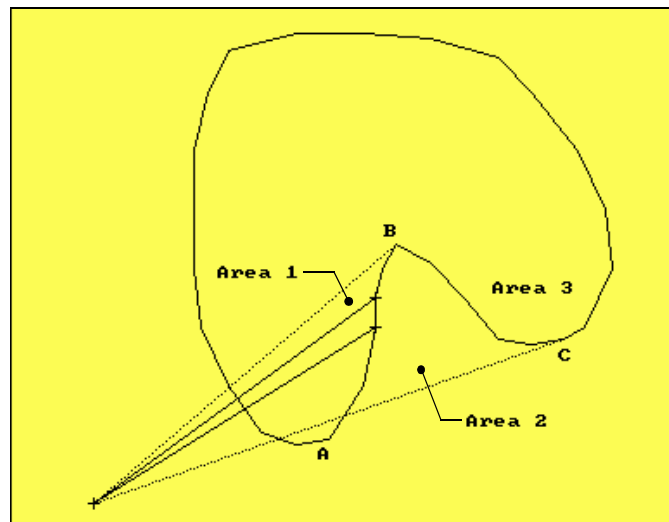
It is, perhaps, not so obvious that the procedure just outlined for generating slices of the shape will work just as well if the origin is outside of the boundary, provided a suitable sign convention is adopted for the tracing direction. As can be seen from Sketch 2.2, tracing around the shape will generate two types of slices. When tracing the points on the side of the shape furthest from the origin, each slice will only partly consist of shape area. When tracing the points on the side of the shape nearest to the origin, the slices will include no area from the shape. However, it will be observed that the area swept out by the slices to the side closer to the origin exactly matches the “extra” area swept out by the slices to the further side of the shape.



Sketch 2.2 Shape with origin outside boundary

The expression for the area of a slice is seen to be of the form $a \times R^2/2$ where R is the length of the radial generator of the slice and a is the angle swept out by this generator in going from one point on the perimeter of the shape to the next. It is seen that the direction of sweep of this generator when tracing points on the side of the shape further from the origin is opposite to the direction when tracing points on the nearer side. By adhering to the usual sign convention of angles, that an anticlockwise rotation is positive, all areas of the slices that are not part of the shape area can be made to “cancel” out during a full trace of the shape boundary.

This same cancelling out process operates equally when tracing “difficult” re-entrant shapes such as that shown in Sketch 2.3.



Sketch 2.3 Shape with reentrant boundary