



ESDU TECHNICAL MEMORANDUM

The implications of flow property profiles on determination and application of non-dimensional pressure loss coefficients for flow of incompressible fluids

B.C. Freeman

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The development and preparation of this Technical Memorandum were undertaken by

Mr B.C. Freeman

— Senior Engineer.

THE IMPLICATIONS OF FLOW PROPERTY PROFILES ON DETERMINATION AND APPLICATION OF NON-DIMENSIONAL PRESSURE LOSS COEFFICIENTS FOR FLOW OF INCOMPRESSIBLE FLUIDS

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THE IMPLICATIONS OF FLOW PROPERTY PROFILES ON DETERMINATION AND APPLICATION OF NON-DIMENSIONAL PRESSURE LOSS COEFFICIENTS FOR FLOW OF INCOMPRESSIBLE FLUIDS

1. NOTATION

		SI Unit
A	flow area	m^2
C_{pr}	static-pressure recovery coefficient, equal to $-K_s$	-
D	duct diameter or equivalent diameter	m
f	Fanning friction factor, $\Delta p / [\frac{1}{2} \rho V^2 4(L/D)]$	-
J_{KE}	kinetic energy profile factor, $\int \frac{1}{2} V^2 d\dot{m} / \frac{1}{2} \bar{V}_{CM}^2$	-
J_M	momentum profile factor, $\int V d\dot{m} / \bar{V}_{CM}$	-
K	pressure-loss coefficient	-
K_s	static-pressure loss coefficient	-
K_t	total-pressure loss coefficient	-
$\dot{m}$	mass flow rate	kg/s
p	absolute pressure	Pa
p_s	absolute static pressure	Pa
p_t	absolute total pressure	Pa
Q	volume flow rate	m^3/s
q	kinetic pressure (equal to dynamic pressure for incompressible flow)	Pa
Re	Reynolds number	-
V	flow velocity	m/s
$\bar{V}_{CM}$	continuity-mean (mass-derived or bulk mean) velocity (Equation 3.1)	m/s
ϵ_{Kt}	error in K_t defined by Equations (7.4) and (7.8)	-
ϵ_{pt}	error in p_t defined by Equations (7.5) and (7.9)	-
Λ	function defined by Equations (7.2), (7.3), (7.6) and (7.7)	-
ρ	fluid density	kg/m^3

Subscripts

aw	refers to area-weighted mean value
$/aw$	refers to loss coefficient based on area-weighted mean kinetic pressure
CM	refers to continuity mean value
$/CM$	refers to loss coefficient based on continuity mean kinetic pressure
d	refers to conditions at downstream measurement section
KE	refers to kinetic-energy-derived mean value
M	refers to momentum-derived mean value
mw	refers to mass-flow-weighted mean value
$/mw$	refers to loss coefficient based on mass-flow-weighted mean kinetic pressure
ref	refers to conditions at reference plane
$tilde$	refers to loss coefficient based on reference (mass-flow-weighted) mean total pressure
$/tilde$	refers to loss coefficient based on reference (mass-flow-weighted) mean kinetic pressure
u	refers to conditions at upstream measurement section
1	refers to value at upstream reference plane
2	refers to value at downstream reference plane
3	refers to value at intermediate reference plane

Superscripts

$-$	an over-bar denotes mean value of type defined by subscripts
$\sim$	an over-tilde denotes a property of the reference mean flow or derived property using reference mean flow properties (see also subscripts for alternative notation used on Figures)

1.1 Use of Subscripts

In this Note, because of the wide range of definitions of loss coefficient, loss coefficients are identified by extensive use of subscripts, *e.g.* $K_{t,aw/CM,1}$ identifies a total-pressure loss coefficient based on area-weighted mean total pressure and continuity-mean kinetic pressure at the upstream reference plane. Provided that the basis is clearly defined, this level of subscripting is not necessary in the general presentation of loss coefficient data.

2. INTRODUCTION

The fundamental definition of a pressure loss coefficient is

$$K = \frac{\Delta p}{q_{ref}}, \quad (2.1)$$

where Δp is the change in static or total pressure between two reference sections and q_{ref} is a reference kinetic pressure or dynamic pressure. This definition implicitly assumes that the flow at the reference sections is uniform or one-dimensional.

In contrast to the fundamental definition, the interpretation of measurements from experiment or computation uses measurements made in flow that will usually not be uniform at either the measurement sections or the reference section.

This Note explores the implications of this dichotomy which leads to the need to make choices in the derivation of a pressure-loss coefficient when deriving one from experiment or computation or a requirement to know what choices were made in the derivation when applying given loss coefficients. The essential requirement is to define appropriate mean values to represent the profiled flow in a one-dimensional manner.

Given a profile of a property across a section in a flow, mean values of that property can be defined in various ways^{3,4,6}. The magnitude of the differences between the different mean values depends on the severity of the profile (and for some definitions, on the severity of other profiles). It is not possible to define a set of mean property values that, on their own, fully represent the profiled flow, a minimum number of mean-set factors relating sectional properties to their mean-set values is always required as well.

In the present context the profiles are defined to be for fully-developed axial flow. For fully-developed laminar flow, differences between different definitions can be large but for full-developed turbulent flow smaller differences result. However, even in turbulent flow these differences can contribute significantly to the apparent spread between experimental results from different sources and to the uncertainty of predicted pressure losses.

For flow of an incompressible fluid, the value taken for q_{ref} is the kinetic pressure $\frac{1}{2}\rho V^2$. The kinetic pressure is conventionally assumed to be equal to the dynamic pressure ($p_t - p$) but this is only universally true in one-dimensional flow, for other cases equality will depend on compatible definition of mean p_t and mean V^2 .

This note focuses primarily on derivation or application of "standard" pressure loss coefficients for which the flow at the measurement sections is a fully-developed axial flow. The derivation applies for flow of incompressible fluids, the case of compressible fluids is more complex and will be considered elsewhere.

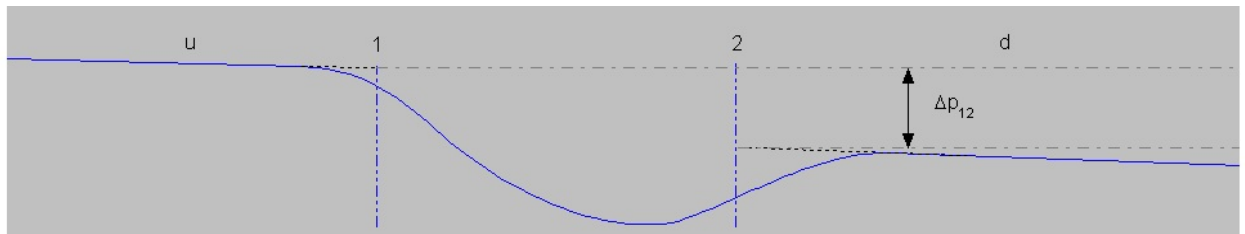
2.1 The Flow Model

Sketch 2.1 shows schematically a duct system with a component^{1*} other than straight duct inserted between Sections 1 and 2 and a notional pressure change along it. In the real flow (solid line) at Sections 1 and 2, and possibly also at Sections u and d, the flow property profiles are affected by the presence of the component, its influence extending upstream and downstream from the component itself.

The pressure loss between Sections u and d will depend on the nature of flow at the measurement sections and their distances from the component as well as on the nature of the component itself. The dependency on conditions at the measurement sections can be removed by requiring them to be in regions of fully-developed flow and the dependency on location by then applying a "correction" that is a function of the distance of the measurement sections from the reference Sections 1 and 2.

¹

This component could in reality comprise a number of closely-spaced components.



Sketch 2.1 Schematic duct system and notional pressure change along it

The pressure change^{2*} that is to be correlated can then be defined by

$$\bar{p}_1 - \bar{p}_2 = \bar{p}_u - \bar{p}_d - (\bar{p}_u - \bar{p}_1) - (\bar{p}_2 - \bar{p}_d) \quad (2.2)$$

$$\Delta p_{12} = \Delta p_{ud} - \Delta p_{u1} - \Delta p_{d2} \quad (2.3)$$

where Δp_{u1} and Δp_{d2} are the pressure losses that would arise from fully-developed flow in ducts of lengths $u1$ and $d2$.

This "correction" from the measured pressures to the reference pressures can be represented graphically by extrapolating the upstream and downstream pressure gradients from Sections u to 1 and d to 2 (dashed lines on Sketch 2.1). Is not essential to the definition process to infer an idealised extrapolation of the flow itself and caution is needed if this inference is to be followed as such an extrapolation can be interpreted in two ways;

- (i) the flow is extrapolated as a fully-developed flow between the relevant sections, implying fully-developed flow conditions in the idealised flow at 1 and 2 . Because the flow is fully-developed, only the magnitudes of the properties change and not their profiles.
- (ii) the flow at Sections u and d is represented by appropriate set of mean values and the mean properties are extrapolated to Sections 1 and 2 with changes in magnitude equal to those that would occur for a fully-developed flow over duct lengths $u1$ and $d2$. The appropriate profile factors at u and d are retained as part of the flow description at Sections 1 and 2 .

The distinction is subtle, option (i) implies an idealised flow that is fully-developed at Sections 1 and 2 and has yet to be represented one-dimensionally whereas option (ii) simply provides an idealised one-dimensional flow description at Sections 1 and 2 .

In the case considered where the real flow is required to be fully-developed at Sections u and d the distinction is irrelevant practically except in strict notational terms; for (i) the relevant profile factors are calculated at 1 and 2 whereas for (ii) they are calculated at u and d . Since the profiles are identical for fully-developed flow at each section, the two definitions give the same results.

In this Note, option (ii) is implied and this is recommended for Data Item application (if either is used) as it removes the need to imply a fully-developed idealised flow at 1 and 2 which conflicts with the real flow conditions. Furthermore option (ii) can be extended more readily to cases where the measurement Sections u and d are not in regions of fully-developed flow.

It must be emphasised that although the Sections 1 and 2 are located at the physical entry and exit of the component, the flow conditions at Sections 1 and 2 used in the calculation are those defined by a flow model and not those existing in the real duct flow.

3. MEAN PROPERTIES AND PROFILE FACTORS

Mean property values for a non-uniform (profiled) flow can be derived in a variety of ways. They may form part of a consistent mean-value-set that, with a minimum number of profile factors, completely represents the original flow or they may be defined on the basis of some convenient averaging process. Reference 6 discusses derivation of mean values for flow of incompressible fluids.

Reference 6 considers three main mean-value-sets, the reference or entropy-derived set, the mass-flow-derived set and the momentum-mixed set. The first represents the sectional entropy exactly and allows entropy changes through a system to be tracked correctly and hence true efficiency and energy transfer in the flow process can be defined. However, Reference 6 acknowledges that the overwhelming simplicity of the mass-derived or continuity mean velocity, requiring no knowledge of the flow profiles, makes it the prime candidate for mean velocity definition. Since it is not part of the reference or momentum-mixed mean-value-sets, profile factors are required to allow its use. However, because of the nature of flow of an incompressible fluid, it is possible to define all of the profile factors required in terms of two factors which form part of the continuity or mass-derived mean-value-set.

In the present context, for definition or application of pressure-loss coefficients, with fully-developed axial flow at the measurement sections, the properties of interest are velocity, total pressure and kinetic pressure, the static pressure and density being uniform. Furthermore, because the flow is required to be fully-developed, the values of the profile factors are predetermined and the relationships between the different mean value definitions can be established without measurement of velocity or total-pressure profiles.

3.1 Continuity or Mass-derived Mean Values

The continuity-mean or mass-derived mean velocity (sometimes called the bulk-mean velocity) is defined by

$$\bar{V}_{CM} = \frac{\dot{m}}{\rho A}. \quad (3.1)$$

The continuity-mean kinetic pressure and continuity-mean total pressure are then defined for consistency as

$$\bar{q}_{CM} = \frac{1}{2} \rho \bar{V}_{CM}^2, \quad (3.2)$$

$$\bar{p}_{t,CM} = p + \frac{1}{2} \rho \bar{V}_{CM}^2. \quad (3.3)$$

These values form part of the mass-derived or continuity mean-value-set. The continuity-mean velocity represents the sectional mass flow rate correctly but mean-set profile factors are required to derive the sectional momentum and sectional kinetic energy if $\bar{V}_{CM}$ is used. The factor for momentum is defined by

$$\int_m V d\dot{m} = J_M \dot{m} \bar{V}_{CM} \quad (3.4)$$

leading to

$$J_M = \frac{\int_m V d\dot{m}}{\dot{m} \bar{V}_{CM}} = \frac{\int_A V \rho V dA}{\rho \bar{V}_{CM}^2 A} = \int_0^1 \left(\frac{V}{\bar{V}_{CM}} \right)^2 \frac{dA}{A} \quad (3.5)$$

and the factor for kinetic energy by

$$\int_m \frac{1}{2} V^2 d\dot{m} = J_{KE} \frac{1}{2} \dot{m} \bar{V}_{CM}^2 \quad (3.6)$$

leading to

$$J_{KE} = \frac{\int_m \frac{1}{2} V^2 d\dot{m}}{\frac{1}{2} \dot{m} \bar{V}_{CM}^2} = \frac{\int_A \frac{1}{2} V^2 \rho V dA}{\frac{1}{2} \rho \bar{V}_{CM}^3 A} = \int_0^1 \left(\frac{V}{\bar{V}_{CM}} \right)^3 \frac{dA}{A}. \quad (3.7)$$

Figure 1 shows the values of J_M and J_{KE} for laminar (see Appendix A) and turbulent (from Reference 1), fully-developed flow in smooth circular ducts. The symbols β and α respectively are sometimes used for these quantities.

If the flow is laminar at both upstream and downstream measurement sections, $J_d = J_u$. However, if the flow is turbulent at both upstream and downstream measurement sections, this is only the case if the duct areas are equal.

For incompressible flow, the upstream and downstream continuity-mean velocities can be related through the requirement of mass flow continuity by

$$\frac{\bar{V}_{CM,1}}{\bar{V}_{CM,2}} = \frac{A_2}{A_1} \quad (3.8)$$

and the corresponding Reynolds numbers by

$$\frac{\bar{Re}_{CM,1}}{\bar{Re}_{CM,2}} = \sqrt{\frac{A_2}{A_1}} \quad (3.9)$$

3.2 Reference or Entropy-derived Mean Values

For the reference mean-value-set, Reference 6 shows that the appropriate mean kinetic and total pressures are the mass-flow weighted means. Then

$$\tilde{q} = \bar{q}_{mw} = \frac{1}{\dot{m}} \int_{\dot{m}} q \, d\dot{m} = \frac{1}{\rho A \bar{V}_{CM}} \int_A \frac{1}{2} \rho V^2 \rho V \, dA \quad (3.10)$$

$$= \frac{1}{2} \rho \bar{V}_{CM}^2 \int_0^1 \left(\frac{V}{\bar{V}_{CM}} \right)^3 \frac{dA}{A} = J_{KE} \frac{1}{2} \rho \bar{V}_{CM}^2 \quad (3.11)$$

$$\text{and} \quad \tilde{p}_t = \bar{p}_{t,mw} = \frac{1}{\dot{m}} \int_{\dot{m}} p_t \, d\dot{m} = p + \frac{1}{\dot{m}} \int_{\dot{m}} q \, d\dot{m} = p + J_{KE} \frac{1}{2} \rho \bar{V}_{CM}^2 \quad (3.12)$$

The corresponding reference-mean velocity can then be derived from

$$\tilde{q} = \frac{1}{2} \rho \tilde{V}^2 = J_{KE} \frac{1}{2} \rho \bar{V}_{CM}^2 \quad (3.13)$$

$$\text{from which} \quad \tilde{V} = \sqrt{J_{KE}} \bar{V}_{CM} \quad (3.14)$$

This is equal to the rms or root-mean-square velocity derived from the mass-flow weighted mean of local V^2 and is not the same as $\bar{V}_{mw}$.

3.3 Momentum-mixed Mean Values

For the momentum-mixed mean-value-set, the mean velocity is defined from the sectional absolute stream force such that

$$\dot{m} \bar{V}_{MM} + pA = \int (V \, d\dot{m} + p \, dA) \quad (3.15)$$

$$\text{leading to} \quad \bar{V}_{MM} = \frac{1}{\dot{m}} \int_{\dot{m}} V \, d\dot{m} = \int_0^1 \left(\frac{V}{\bar{V}_{CM}} \right)^2 \frac{dA}{A} = J_M \bar{V}_{CM} \quad (3.16)$$

The corresponding mean kinetic and total pressures are then

$$\bar{q}_{MM} = J_M^2 \frac{1}{2} \rho \bar{V}_{CM}^2 \quad (3.17)$$

and
$$\bar{p}_{t,MM} = p + J_M^2 \frac{1}{2} \rho \bar{V}_{CM}^2. \quad (3.18)$$

3.4 Mass-flow-weighted Mean Values

The mass-flow-weighted mean kinetic pressure and total pressure form part of the reference or entropy-derived mean-value-set and will, in this Note, be applied in that context. Note that the mass-flow-weighted mean velocity is not part of the reference mean-value-set.

3.5 Area-weighted Mean Values

Area-weighted mean kinetic pressure and mean total pressure can be defined as

$$\bar{q}_{aw} = \frac{1}{A} \int_A q \, dA = \frac{1}{2} \rho \bar{V}_{CM}^2 \int_0^1 \left(\frac{V}{\bar{V}_{CM}} \right)^2 \frac{dA}{A} = J_M \frac{1}{2} \rho \bar{V}_{CM}^2 \quad (3.19)$$

and
$$\bar{p}_{t,aw} = \frac{1}{A} \int_A p_t \, dA = p + \frac{1}{A} \int_A q \, dA = p + J_M \frac{1}{2} \rho \bar{V}_{CM}^2. \quad (3.20)$$

The area-weighted mean velocity does not form part of a consistent mean-value-set with the area-weighted mean kinetic and total pressures, the consistent value is given instead by $\sqrt{J_M} \bar{V}_{CM}$.

4. STATIC-PRESSURE LOSS COEFFICIENTS

The static-pressure loss coefficient for profiled flow is defined by

$$K_s = \frac{\bar{p}_{s,1} - \bar{p}_{s,2}}{\bar{q}_{ref}} \quad (4.1)$$

where subscript *ref* defines the plane at which the reference kinetic pressure is taken.

For fully-developed axial flow, the static pressure across a section is uniform and the choice in definition of the static-pressure loss coefficient reduces to the selection of an appropriate value for $\bar{q}_{ref}$ in the case of derivation of the loss coefficient or a requirement to know what value of $\bar{q}_{ref}$ was used in the derivation when applying given loss coefficients.

4.1 Choice of Reference Plane

If the inlet and outlet duct areas differ, it is usual to base the pressure loss coefficient on kinetic pressure in the smaller duct area.

Thus, usually, for flows in ducts with a contraction of area, the pressure loss coefficient will be based on the kinetic pressure at the downstream reference plane, Section 2, whilst for ducts with an expansion of area, the pressure loss coefficient will be based on the kinetic pressure at the upstream reference plane, Section 1.

For some duct components, such as orifices, the inlet and outlet areas are equal but the cross section at some well-defined intermediate section is smaller, it is then usual to base the pressure loss coefficient on the kinetic pressure at a notional intermediate reference plane, Section 3.

The kinetic pressure at either the upstream or the downstream reference plane can be defined from the idealised flow conditions described in Section 2. For use of conditions at Section 3, however, it is necessary to consider how those conditions are defined. Firstly, since the mass flow rate through the section must be the same as through the upstream and downstream reference planes, the continuity-mean velocity can be defined (adapting Equation (3.8)) as

$$\bar{V}_{CM,3} = \frac{\dot{m}}{\rho A_3} = \bar{V}_{CM,1} \left(\frac{A_1}{A_3} \right) = \bar{V}_{CM,2} \left(\frac{A_2}{A_3} \right) \quad (4.2)$$

and the corresponding kinetic pressure as

$$\bar{q}_{CM,3} = \frac{1}{2} \rho \bar{V}_{CM,3}^2 = \frac{1}{2} \rho \bar{V}_{CM,1}^2 \left(\frac{A_1}{A_3} \right)^2 = \frac{1}{2} \rho \bar{V}_{CM,2}^2 \left(\frac{A_2}{A_3} \right)^2 \quad (4.3)$$

In order to use any other mean kinetic pressure, some statement has to be made regarding either the assumed conditions at the notional Section 3 or the assumed flow process between Section 1 and Section 3. Given that the flow conditions at other reference planes are assumed to be those for fully-developed flow it is appropriate to impose this condition for Section 3. This is sufficient to allow definition of all other mean kinetic pressures. Alternatively, the flow could be defined as uniform, in which case all kinetic pressures are the same.

For the present purpose, no other properties are required but if they were to be required it would be necessary to define the process between Section 1 and Section 3.

Only the use of the continuity-mean kinetic pressure at Section 3 will be pursued further in this Note.

4.2 Coefficients Based on Continuity Mean Kinetic Pressure

Static-pressure loss coefficients using the continuity-mean kinetic pressure are defined as

$$K_{s,CM,1} = \frac{p_{s,1} - p_{s,2}}{\bar{q}_{CM,1}} = \frac{p_{s,1} - p_{s,2}}{\frac{1}{2} \rho \bar{V}_{CM,1}^2} \quad (4.4)$$

based on an upstream reference, as

$$K_{s,/CM,2} = \frac{P_{s,1} - P_{s,2}}{\bar{q}_{CM,2}} = \frac{P_{s,1} - P_{s,2}}{\frac{1}{2}\rho\bar{V}_{CM,2}^2}, \quad (4.5)$$

based on a downstream reference, and as

$$K_{s,/CM,3} = \frac{P_{s,1} - P_{s,2}}{\bar{q}_{CM,3}} = \frac{P_{s,1} - P_{s,2}}{\frac{1}{2}\rho\bar{V}_{CM,3}^2}, \quad (4.6)$$

based on an intermediate reference.

4.3 Coefficients Based on Reference Mean Kinetic Pressure

Static pressure loss coefficients using the reference-mean kinetic pressure are defined as

$$\tilde{K}_{s,1} = \frac{P_{s,1} - P_{s,2}}{\tilde{q}_1} = \frac{P_{s,1} - P_{s,2}}{J_{KE,u} \frac{1}{2}\rho\bar{V}_{CM,1}^2}, \quad (4.7)$$

based on an upstream reference, and as

$$\tilde{K}_{s,2} = \frac{P_{s,1} - P_{s,2}}{\tilde{q}_2} = \frac{P_{s,1} - P_{s,2}}{J_{KE,d} \frac{1}{2}\rho\bar{V}_{CM,2}^2}, \quad (4.8)$$

based on a downstream reference.

These values are the same as those based on mass-flow-weighted mean kinetic pressure, $K_{s,/mw,1}$ and $K_{s,/mw,2}$ respectively.

4.4 Coefficients Based on Momentum-mixed Mean Kinetic Pressure

Static pressure loss coefficients using the momentum-mixed-mean kinetic pressure are defined as

$$K_{s,/MM,1} = \frac{P_{s,1} - P_{s,2}}{\bar{q}_{MM,1}} = \frac{P_{s,1} - P_{s,2}}{J_{M,u}^2 \frac{1}{2}\rho\bar{V}_{CM,1}^2}, \quad (4.9)$$

based on an upstream reference, and as

$$K_{s,/MM,2} = \frac{P_{s,1} - P_{s,2}}{\bar{q}_{MM,2}} = \frac{P_{s,1} - P_{s,2}}{J_{M,d}^2 \frac{1}{2}\rho\bar{V}_{CM,2}^2}, \quad (4.10)$$

based on a downstream reference.

4.5 Coefficients Based on Area-weighted Mean Kinetic Pressure

Static pressure loss coefficients using the area-weighted-mean kinetic pressure are defined as

$$K_{s,/aw,1} = \frac{P_{s,1} - P_{s,2}}{\bar{q}_{aw,1}} = \frac{P_{s,1} - P_{s,2}}{J_{M,u} \frac{1}{2}\rho\bar{V}_{CM,1}^2}, \quad (4.11)$$

based on an upstream reference, and as

$$K_{s,aw,2} = \frac{P_{s,1} - P_{s,2}}{\bar{q}_{aw,2}} = \frac{P_{s,1} - P_{s,2}}{J_{M,d}^{1/2} \rho \bar{V}_{CM,2}^2}, \quad (4.12)$$

based on a downstream reference.

4.6 Effect of Averaging Method

Figure 2 shows the relationship between values of K_s from Equations (4.4), (4.7), (4.9) and (4.11) for a given static pressure drop and mass flow rate. The relationship is similar for values from Equations (4.5), (4.8), (4.10) and (4.12). For convenience, the notation with subscript *tilde* is used for reference-mean based values on the Figure.

The Reynolds numbers are based on $\bar{V}_{CM}$ for all options although, strictly, the Reynolds numbers used should be that based on the mean velocity appropriate to the mean-value-set from which the mean kinetic pressure is taken. However, Re_{CM} is appropriate for calculation of J_M and J_{KE} in all cases.

4.7 Pipe Friction Losses

Although this note is concerned primarily with pressure losses in components, similar considerations apply to pipe friction losses. The loss coefficient for a section of pipe is related to the friction factor by

$$K = 4f \frac{L}{D}. \quad (4.13)$$

Thus, friction factors corresponding to each of the static pressure loss coefficients described in Sections 4.2 to 4.5 could, in principle, be derived. It is considered unlikely, however, that historical values of friction factor have been based on any kinetic pressure other than the continuity mean value.

5. TOTAL-PRESSURE LOSS COEFFICIENTS

For fully-developed axial flow, the total pressure varies across a section and differences arise potentially in both numerator and denominator of the loss coefficient definition.

5.1 Coefficients Based on Continuity Mean Kinetic Pressure

To simplify matters, it will be assumed initially that the continuity-mean kinetic pressure is used in the denominator so that

$$K_t = \frac{\bar{p}_{t,1} - \bar{p}_{t,2}}{\bar{q}_{ref}} = \frac{\bar{p}_{t,1} - \bar{p}_{t,2}}{\frac{1}{2}\rho \bar{V}_{CM}^2}. \quad (5.1)$$

5.1.1 Coefficients for continuity mean total pressures

The total-pressure loss coefficient based on continuity-mean total pressures is given by

$$\begin{aligned} K_{t,CM/CM} &= \frac{\bar{p}_{t,CM,1} - \bar{p}_{t,CM,2}}{\bar{q}_{CM,ref}} = \frac{p_1 - p_2}{\bar{q}_{CM,ref}} + \frac{\frac{1}{2}\rho(\bar{V}_{CM,1}^2 - \bar{V}_{CM,2}^2)}{\bar{q}_{CM,ref}} \\ &= K_{s/CM} + \frac{\frac{1}{2}\rho(\bar{V}_{CM,1}^2 - \bar{V}_{CM,2}^2)}{\bar{q}_{CM,ref}}. \end{aligned} \quad (5.2)$$

With Equation (3.8), this can be expressed as

$$K_{t,CM/CM,1} = K_{s/CM,1} + \left[1 - \left(\frac{A_1}{A_2} \right)^2 \right], \quad (5.3)$$

based on an upstream reference, as

$$K_{t,CM/CM,2} = K_{s/CM,2} + \left[\left(\frac{A_2}{A_1} \right)^2 - 1 \right], \quad (5.4)$$

based on a downstream reference, and as

$$K_{t,CM/CM,3} = K_{s/CM,3} + \left[\left(\frac{A_3}{A_1} \right)^2 - \left(\frac{A_3}{A_2} \right)^2 \right], \quad (5.5)$$

based on an intermediate reference. Usually, $A_2 = A_1$ and Equation (5.5) reduces to

$$K_{t,CM/CM,3} = K_{s/CM,3}, \quad (5.6)$$

5.1.2 Coefficients for reference mean total pressures

The total-pressure loss coefficient based on the reference mean total pressures is given by

$$\begin{aligned} \tilde{K}_{t/CM} &= \frac{\tilde{p}_{t,1} - \tilde{p}_{t,2}}{\bar{q}_{CM,ref}} = \frac{p_1 - p_2}{\bar{q}_{CM,ref}} + \frac{\frac{1}{2}\rho(J_{KE,u} \bar{V}_{CM,1}^2 - J_{KE,d} \bar{V}_{CM,2}^2)}{\bar{q}_{CM,ref}} \\ &= K_{s/CM} + \frac{\frac{1}{2}\rho(J_{KE,u} \bar{V}_{CM,1}^2 - J_{KE,d} \bar{V}_{CM,2}^2)}{\bar{q}_{CM,ref}}. \end{aligned} \quad (5.7)$$