



ESDU TECHNICAL MEMORANDUM

Calculation of coefficients of rolling, braking and yawed rolling resistance

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ESDU TECHNICAL MEMORANDA

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The work of this Memorandum was partly sponsored by Douglas Equipment Ltd. The technical work involved in the assessment of the available information and the construction and subsequent development of the Memorandum were carried out under contract to ESDU by

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CALCULATION OF COEFFICIENTS OF ROLLING, BRAKING AND YAWED ROLLING RESISTANCE

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CALCULATION OF COEFFICIENTS OF ROLLING, BRAKING AND YAWED ROLLING RESISTANCE

1. NOTATION AND DIMENSIONS

		Dimension
c	correlating constant (see Equation (3.6))	–
d_E	effective size of surface constituents	L
d_{TEX}	depth of macro-texture	L
D	tyre diameter (inflated)	L
E	Young's modulus (of elasticity)	$M L^{-1} T^{-2}$
E'	storage modulus of elasticity	$M L^{-1} T^{-2}$
E''	loss modulus of elasticity	$M L^{-1} T^{-2}$
E^*	dynamic modulus	$M L^{-1} T^{-2}$
F_1	<i>micro</i> -texture sharpness parameter	–
F_2	function in correlation of side-force relation	–
G	modulus of rigidity or shear modulus	$M L^{-1} T^{-2}$
G_{TREAD}	modulus of rigidity of tread material	$M L^{-1} T^{-2}$
G'	storage modulus of rigidity	$M L^{-1} T^{-2}$
G''	loss modulus of rigidity	$M L^{-1} T^{-2}$
n	exponent in side-force model (see Equation (3.15))	–
N	speed number	–
p_{ABS}	inflation pressure (in absolute measure)	$M L^{-1} T^{-2}$
q	kinetic pressure in fluid	$M L^{-1} T^{-2}$
q_v	pressure in region of “viscous contact” between tyre and contaminated runway, Zone 2	$M L^{-1} T^{-2}$
S_1, S_2, S_3	areas in Zones 1, 2 and 3, respectively	L^2
s	slip ratio	–

Issued March 2010 – 18 pages
 With Amendments A and B, July 2015 – 18 pages
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t	time	T
V	speed	$L T^{-1}$
V_G	ground speed	$L T^{-1}$
V_{ROLL}	rolling speed	$L T^{-1}$
w	inflated width of tyre	L
X	decelerating force	$M L T^{-2}$
Z	normal load (to runway)	$M L T^{-2}$
α_3	constant in model, $\alpha_3 = 1.2$	—
X	function (see Equation (3.15))	—
δ	phase lag angle	radian
ϵ	strain	—
ϵ_C	shear strain induced by application of normal load	—
ϵ_F	shear strain induced by application of horizontal force	—
ϵ_P	shear strain induced by mean pressure acting on protuberances	—
ϵ_0	mean strain	—
$\eta_0, \eta_1, \eta_2, \eta_4, \eta_{40}, \eta_{50}$	constants in model	—
κ	ratio of hoop stress and inflation pressure	—
λ	tangent of phase angle (see Equation (4.1) of Reference 3)	—
Λ	tyre-loading parameter (see Equation (3.1))	—
μ	coefficient of decelerating force (friction)	—
ρ	density	$M L^{-3}$
ρ_C	density of contaminant	$M L^{-3}$
ρ_T	density of tread material	$M L^{-3}$
σ	stress	$M L^{-1} T^{-2}$
σ_0	mean stress	$M L^{-1} T^{-2}$

ϕ_E	effective porosity of surface ($= d_{TEX}/(d_{TEX} + d_E)$)	—
$\Phi_0, \Phi_1, \Phi_2,$ Φ_{01}, Φ_{10}	factors in model	—
$1/\tau$	ratio of surface water to total water ($= d/(d + d_{TEX})$)	—
ψ	yaw angle	radian
ω	frequency	T^{-1}

Subscripts

<i>ABS</i>	absolute
<i>DRY</i>	dry runway
<i>NOM</i>	nominal
<i>REF</i>	reference
<i>ROLL</i>	rolling wheel
<i>SKID</i>	skidding tyre
<i>SLIP</i>	slipping tyre
<i>STATIC</i>	static wheel
<i>TREAD</i>	tyre tread
<i>WET</i>	wet runway ($d < 3 \text{ mm}$)
<i>YAW</i>	yawed tyre

2. INTRODUCTION

2.1 General

This Technical Memorandum provides a background to part of ESDU 10015 (Reference 1), which deals with a 'Model for the performance of aircraft tyres rolling or braking on dry and precipitate contaminated runways'.

During the 17th and 18th centuries, only one kind of friction was recognised. This is what is now described as *mechanical* friction. Because this friction force is *apparently* dependent only on the applied normal load, the relationship between load, Z , and decelerating force, X , was described by the linear function, in which μ is known as the coefficient of friction, given by

$$X = \mu Z. \quad (2.1)$$

This relationship has been adopted by mechanical engineers as a touchstone of truth ever since it was set down in 1699 by Amontons as his first law of friction. The so-called laws of friction were set out as:

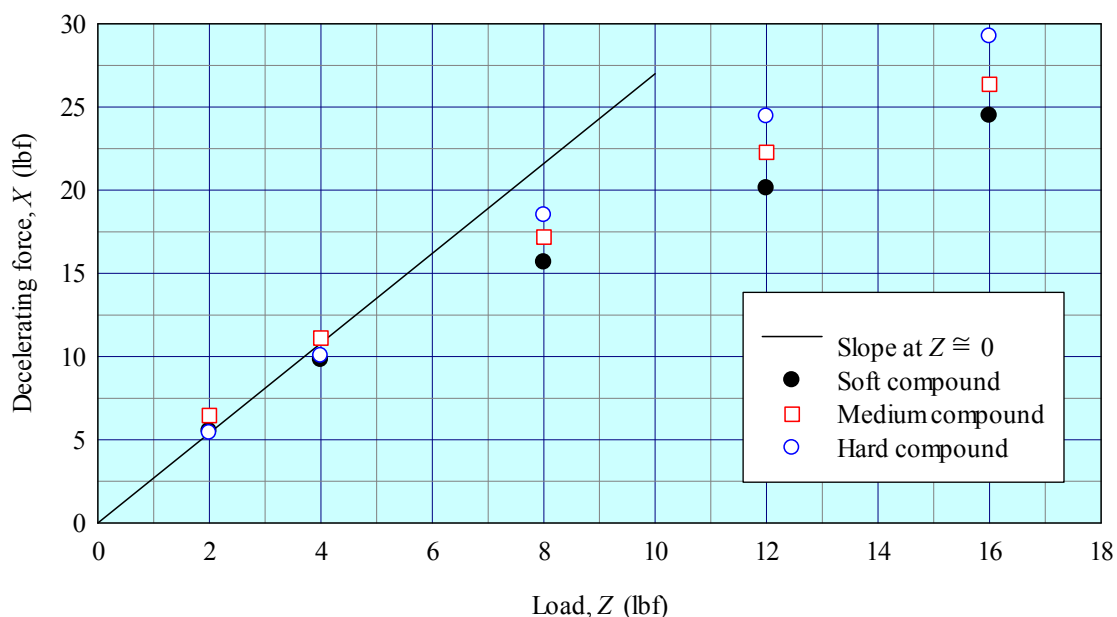
force of friction is directly proportional to the applied load (Amontons' 1st Law),

force of friction is independent of the apparent area of contact (Amontons' 2nd Law)

and kinetic friction is independent of the sliding velocity (Coulomb's Law).

These three laws apply only approximately even to the case that they were formulated to explain: that is, the mechanical friction between hard solid surfaces. For interactions between other types of materials the laws are not even approximately applicable.

Consider the case shown in Sketch 2.1. These data were collected in an experiment by Schallamach conducted in 1952 using blocks of rubber sliding on plate glass to "fill a gap in the understanding of rubber friction". The speed for all the testing was as close as was possible to zero so that dynamic modes were not excited.



Sketch 2.1 Effect of normal load on decelerating force for various rubber compounds