



ESDU TECHNICAL MEMORANDUM

Derivation of equations for the geometrical properties of straight-tapered and multi-panel wings

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ESDU TECHNICAL MEMORANDA

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DERIVATION OF EQUATIONS FOR THE GEOMETRICAL PROPERTIES OF STRAIGHT-TAPERED AND MULTI-PANEL WINGS

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DERIVATION OF EQUATIONS FOR THE GEOMETRICAL PROPERTIES OF STRAIGHT-TAPERED AND MULTI-PANEL WINGS

1. NOTATION AND UNITS

		SI	British
A	aspect ratio, $b^2/\bar{c}$		
b	wing span	m	ft
C_L	lift coefficient		
C_l	local lift coefficient		
c	local chord, see Sketch 4.1	m	ft
$\bar{c}$	standard (or geometric) mean chord	m	ft
$\bar{\bar{c}}$	aerodynamic mean chord	m	ft
$c_{centroid}$	chord at station of spanwise centroid, see Sketch 5.1	m	ft
c_i	chord at i^{th} crank, <i>i.e.</i> outboard chord of i^{th} half-wing panel, see Sketch 6.1	m	ft
c_0	wing centre-line chord, see Sketch 4.1	m	ft
c_t	wing tip chord, see Sketch 4.1	m	ft
d_1	length on c_0 shown in Sketch 5.1	m	ft
d_2	length on c_0 shown in Sketch 5.1	m	ft
F_i	parameter equal to right-hand side of Equation (6.6), see Table 6.2	m ³	ft ³
G_i	parameter equal to right-hand side of Equation (6.10), see Table 6.2	m ³	ft ³
L	wing total lift	N	lbf
M	moment about reference centre	N m	lbf ft
M_x	moment about x-axis for straight-tapered trapezoidal wing	N m	lbf ft
N	number of panels on multi-panel half-wing		
n	fraction of local chord measured from local wing leading edge (also see Subscripts list)		

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S	wing area, $b\bar{c}$	m^2	ft^2
S^*	area of half-wing	m^2	ft^2
S_i	area of i^{th} wing panel, $(s_i - s_{i-1})(c_i + c_{i-1})/2$, see Sketch 6.1	m^2	ft^2
s	wing semi-span, distance of wing tip measured from axis of symmetry and normal to wing centre-line, see Sketch 4.1	m	ft
s_i	spanwise distance, measured from axis of symmetry and normal to wing centre-line, of chord c_i , see Sketch 6.1	m	ft
V	free-stream velocity	m/s	ft/s
w_i	spanwise extent of i^{th} half-wing panel $(s_i - s_{i-1})$, see Sketch 6.1	m	ft
x	longitudinal coordinate, aft of x -datum and parallel to wing centre-line, see Sketch 4.1	m	ft
x_r	value of x for wing root leading edge, see Sketch 4.1	m	ft
x_t	value of x for wing tip leading edge, see Sketch 4.1	m	ft
x_0	value of x for wing leading edge at local chord c , see Sketch 4.1	m	ft
$\bar{x}_n$	longitudinal distance, aft of x -datum, of n^{th} chordwise station of reference chord $\bar{c}$	m	ft
$\bar{x}'_n$	longitudinal distance, aft of wing apex, of point at fraction n , of aerodynamic mean chord	m	ft
y	spanwise coordinate, measured from axis of symmetry and normal to wing centre-line, see Sketch 4.1	m	ft
$y_{centroid}$	spanwise distance of centre of area of straight-tapered wing, measured from wing centre-line, see Sketch 5.1	m	ft
η	spanwise distance from wing centre-line as fraction of wing semi-span, y/s		
η_i	spanwise extent of i^{th} half-wing panel as fraction of wing semi-span, $(s_i - s_{i-1})/s$		
λ	wing taper ratio, c_t/c_0		
λ_i	taper ratio of i^{th} wing panel, c_i/c_{i-1}		
Λ_{ni}	sweepback angle of n^{th} chord line, between spanwise stations y_{i-1} and y_i for $i = 1, 2, \dots, N$, see Sketch 6.1	deg	deg
Λ_0	sweepback angle of leading edge of straight-tapered wing, see Sketch 4.1	deg	deg
ρ	air density	kg/m^3	$slug/ft^3$

Subscripts

- i denotes properties associated with wing panel between chords c_{i-1} and c_i , i.e. i^{th} panel, or its outboard end, see Sketch 6.1
- j dummy panel identifier used in calculation of $\bar{\bar{x}}_{1/4}$ in Section 6.2.3
- n denotes properties associated with n^{th} chord line (n is fraction of local chord measured from leading edge, see main notation)
- $n = 0$ denotes leading edge
 $n = 1/4$ denotes quarter-chord line
 $n = 1$ denotes trailing edge

2. INTRODUCTION

The object of ESDU Technical Memo TM 168 is to supply analysis which provides the derivation and substantiation of equations given in ESDU 76003 for the geometrical properties of straight-tapered and multi-panel wings. In this Item, a 'wing' is taken to be the usual pair of symmetrically disposed identical half-wings. Note that, for the purposes of the definition of wing planform geometry, 'wing' here signifies the plan view of the wing.

Section 3 gives the basic definitions for the geometric (or standard) mean chord and aerodynamic mean chord for a wing, derived from first principles.

In Section 4, for a straight-tapered trapezoidal wing, determinations of the length of the aerodynamic mean chord and of the longitudinal position of its leading edge are provided. Also, an expression for the longitudinal position of any fraction of the aerodynamic mean chord is established.

Investigations in Sections 4.5 and 5 establish, for a straight-tapered trapezoidal wing, the relationships between the aerodynamic mean chord and the chord at the centroid location, with respect to their lengths and longitudinal locations.

In Section 6, equations are provided to determine the geometric properties of a multi-panel wing by consideration of the geometry of the individual panels. The calculation procedures are also tabulated in Section 6.1 to aid the user in the computation of multi-panel wing planform geometry. Section 6.2 presents, for a multi-panel wing, derivations from first principles of Equations (3.1) to (3.3) in ESDU 76003 for the aerodynamic mean chord, the chordwise location of the quarter-chord point of the aerodynamic mean chord, and the geometric mean chord, respectively.

Section 7 provides the Reference and Section 8 gives a worked Example illustrating the use of the calculation tables in Section 6.1.

3. DEFINITION OF WING MEAN CHORDS

For reference purposes, aerodynamic coefficients and other expressions frequently make use of the concept of a wing "mean chord". Two forms are commonly in use – the "Geometric" (or Standard) mean chord (SMC) (see Section 3.4 of Reference 1) and the "Aerodynamic" mean chord (AMC or MAC) (see Section 3.2 of Reference 1).

These chords both derive from the concept of a constant local (spanwise) lift coefficient C_l on the wing which is equal to the overall lift coefficient C_L , at all spanwise positions. Thus, the local lift grading (lift/unit span) is proportional to the local wing chord, c .

Hence, the total lift on the wing is obtained from an integration across the span

$$L = \frac{1}{2} \rho V^2 C_L 2 \int_0^{b/2} c \, dy . \quad (3.1)$$

3.1 Definition of Geometric (or Standard) Mean Chord

The geometric mean chord $\bar{c}$ is defined such that a wing of the same span with that chord generates the same lift as the actual wing, *i.e.*

$$L = \frac{1}{2} \rho V^2 C_L \bar{c} 2 \int_0^{b/2} dy = \frac{1}{2} \rho V^2 C_L 2 \int_0^{b/2} c \, dy , \quad (3.2)$$

so that

$$\bar{c} = \frac{\int_0^{b/2} c \, dy}{\int_0^{b/2} dy}, \quad (3.3)$$

as given in Section 3.4 of Reference 1.

Note that $2 \int_0^{b/2} c \, dy$ is, in fact, the wing area, S , and $2 \int_0^{b/2} dy$ is the wing span, b , and so the following familiar expression (as given, via the definition of S , in Section 1 of Reference 1) is obtained

$$\bar{c} = \frac{S}{b}. \quad (3.4)$$

Note also that the wing with the geometric mean chord has the same area as the true wing.

The above expressions do not define a longitudinal position for the geometric mean chord. To do that requires a consideration of pitching moments and introduces the concept of the aerodynamic mean chord.

3.2 Definition of Aerodynamic Mean Chord

The aerodynamic mean chord, $\bar{c}$, of a wing is defined such that a rectangular wing of the same span and with that chord generates the same pitching moment as the actual wing.

Consider the uniform C_l , as described above, to be acting at some chordwise position, nc , uniformly across the span, where $n = (0 \dots 0.25 \dots 1.0)$. At a local spanwise station y , let the leading edge be a distance x_0 behind the moment reference axis (the y -axis through the wing apex, say).

Thus, the moment about the reference axis is

$$M = \frac{1}{2} \rho V^2 C_l 2 \int_0^{b/2} c (x_0 + n c) dy. \quad (3.5)$$

Now define a corresponding n^{th} chordwise station of some (as yet unspecified) reference chord $\bar{c}$ to be situated at a point $\bar{x}_n$ aft of the moment reference axis, so that

$$M = \bar{x}_n \frac{1}{2} \rho V^2 C_l 2 \int_0^{b/2} c \, dy = \frac{1}{2} \rho V^2 C_l 2 \int_0^{b/2} c (x_0 + n c) dy, \quad (3.6)$$

from which

$$\bar{x}_n = \frac{\int_0^{b/2} c (x_0 + n c) dy}{\int_0^{b/2} c \, dy}. \quad (3.7)$$

By choosing specific values of n , the aerodynamic mean chord can be defined:

$$\text{For } n = 0 \text{ (leading edge), } \bar{x}_0 = \frac{\int_0^{b/2} c x_0 dy}{\int_0^{b/2} c dy}, \quad (3.8)$$

and

$$\text{for } n = 1 \text{ (trailing edge), } \bar{x}_1 = \frac{\int_0^{b/2} c (x_0 + c) dy}{\int_0^{b/2} c dy}. \quad (3.9)$$

The length of the aerodynamic mean chord, $\bar{c}$, is therefore $\bar{x}_1 - \bar{x}_0$, so that from Equations (3.8) and (3.9),

$$\bar{c} = \frac{\int_0^{b/2} c^2 dy}{\int_0^{b/2} c dy}, \quad (3.10)$$

which is as defined in Section 3.2 of Reference 1.