



ESDU TECHNICAL MEMORANDUM

Material damping of metals and alloys

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ESDU TECHNICAL MEMORANDA

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MATERIAL DAMPING OF METALS AND ALLOYS

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MATERIAL DAMPING OF METALS AND ALLOYS

1. NOTATION

		Units	
		SI	British
C_1, C_2	constants in vibrating string theory of dislocations		
c_p	specific heat per unit mass (specific heat capacity)	J/kgK	*
E	Young's modulus (isothermal or relaxed)	Pa	lbf/in ²
E_{ad}	adiabatic Young's modulus (same as unrelaxed)	Pa	lbf/in ²
E_c	complex Young's modulus	Pa	lbf/in ²
E_r	relaxed Young's modulus	Pa	lbf/in ²
E_{ur}	unrelaxed Young's modulus	Pa	lbf/in ²
E_ω	absolute dynamic Young's modulus	Pa	lbf/in ²
f	frequency	Hz	Hz
H	magnetic field strength	A/m	A/in
h	thickness of beam	m	in
K	constant of order unity (Equation (6.2))		
k_b	Boltzmann's constant	J/K	bf in/°F
k_t	transverse thermal conductivity	W/mK	lbf/s °F
l_i	distance between consecutive impurity atoms	m	in
L_n	characteristic length of dislocation line (dislocation loop length)	m	in
$N(\sigma_{loc})$	distribution function of the local internal stress barriers	Pa ⁻¹	(lbf/in ²) ⁻¹
n	power in relationship, $\Delta W \propto \sigma^n$		
Q	quality factor		
Q_h	activation energy	J	lbf in

T_0	absolute temperature at equilibrium	K	K
T_p	temperature corresponding to maximum damping	K	K
x	$\sigma/\sigma_i = \varepsilon/\varepsilon_i$		
α	linear coefficient of thermal expansion	K ⁻¹	(°F) ⁻¹
Δ_{rs}	relaxation strength		
ΔW	total energy loss density per cycle	Pa	lbf/in ²
ΔW_{loc}	local energy loss density per cycle	Pa	lbf/in ²
δ	logarithmic decrement		
ε	strain		
ε_i	average internal strain		
ε_0	maximum vibration strain (amplitude-dependent dislocation damping)		
ζ	equivalent viscous damping ratio		
η	loss factor		
η_{max}	maximum value of loss factor for thermoelastic damping		
λ	magnetostriction constant		
λ_s	saturation magnetostriction		
ν	Poisson's ratio		
ξ	dimensionless parameter (Section 4.4), Equation (4.7)		
ρ	density of beam material	kg/m ³	*
σ	stress	Pa	lbf/in ²
σ_c	maximum internal stress in the material	Pa	lbf/in ²
σ_i	average internal stress	Pa	lbf/in ²
σ_{loc}	local stress barrier opposing wall motion	Pa	lbf/in ²
σ_p	maximum stress	Pa	lbf/in ²
σ_s	stress amplitude referring to saturation damping	Pa	lbf/in ²
σ_{ts}	tensile strength	Pa	lbf/in ²

σ_y	yield stress	Pa	lbf/in ²
τ	relaxation time	s	s
τ_0	time constant, Equation (5.1)	s	s
φ	phase difference between stress and strain		
ψ	specific damping capacity		
ψ_{sat}	specific damping capacity of magnetically saturated material		
ψ_{um}	specific damping capacity of unmagnetized material		
$\psi_{0.1}$	specific damping index		
ω	frequency	rad/s	rad/s
ω_n	undamped natural frequency of beam	rad/s	rad/s
ω_{max}	natural frequency of beam corresponding to maximum thermoelastic damping	rad/s	rad/s

* In British units, for the system of units to be coherent, the mass in lb must be divided by the constant 386.4.

Abbreviations

ASTM	American Society for Testing and Materials
HIDAMET	high damping metallic material
MEMS	microelectromechanical systems
MMD	Metallic Materials Data
MMPDS	Metallic Material Properties Development and Standardization
SDC	specific damping capacity
SDI	specific damping index
bcc	body-centred cube
fcc	face-centred cube

2. INTRODUCTION

The ability to dissipate vibratory energy in structures, systems of structural components or structural materials is described by the term “damping”. The amplitude of the free vibration of a system with damping decays due to the loss of energy. The loss of the vibratory energy in a mechanical system is caused by the intrinsic and external friction (which leads to the release of heat), as well as by the radiation of sound into the surrounding medium (Reference 89).

The subject of this Memorandum is material damping, a type of damping which is intrinsic in a component of a structure subjected to a vibratory motion. As it is a form of damping which is associated with a dissipation of mechanical strain energy in the material, it is also referred to as internal friction. Material damping is independent of external structural factors. Whilst these external factors can often be controlled by proper structural design, the material damping remains unchanged in a given situation. Material damping may depend on environmental factors, such as temperature or a magnetic field. The way in which the material is prepared is significant and its heat treatment is an important factor which influences the material damping.

In general, damping of a structure depends on the geometrical properties of the structure, its loading, friction of joints, material characteristics and the type of process responsible for the dissipation of vibratory energy. Consequently, there are different ways of specifying damping (discussed in ESDU Item 09005, Reference 91), that can be selected to apply in a specific situation. The complex modulus (applicable to linear materials) and relative energy ratios, such as the loss factor and quality factor, as well as the specific damping capacity (applicable to both linear and nonlinear materials) are regarded as measures applied most often to material damping (Reference 30).

In the assessment of damping at the design stage reliance has to be placed on measured values of material damping. Measurements can be made in several ways on specimens with different shapes and for different types of vibration. For example, the decay of free torsional vibration of circular components can be analysed to obtain the logarithmic decrement, or in the case of forced excitation the bandwidth at resonance is estimated for vibrating cantilever beams in order to determine the loss factor. Specimens may be in the form of a wire or rod for torsional vibration measurement. Alternatively, the specimen may be a free-free beam or cantilever, when vibration in bending is investigated. There may be a tendency to overestimate the damping measurements, because damping from the supports has not been accounted for. If the specimen is a beam vibrating in air, the existence of damping due to sound radiation and local motion of the air may lead to higher results for the estimates of material damping.

Although material damping is, in general, small for most materials there are certain metals and alloys where the damping has comparatively large values. These materials include ferromagnetic alloys, manganese-copper alloys and grey cast iron. Materials with high damping of vibrations and high values of strength and elastic modulus, as well as good resistance to corrosion, are especially suitable for load carrying structures. It is also desirable that the material possesses high damping within a wide range of frequency and temperature. Such materials are widely used in applications to control fatigue and vibration. For many materials good damping can only be achieved for particular conditions, such as high temperature, and can be used to damp the vibrations of hot components, *e.g.*, turbine blades.

Material damping can be a significant factor for structures in space applications, since other types of damping, associated with such phenomena as sound radiation, or friction between different structural components do not exist, or are substantially reduced. Even in box structures of aircraft and in turbine blades of engines, material damping can make a worthwhile contribution to the overall damping (References 47 and 50). Apart from applications in the aerospace industries, the subject of damping is also important in microelectromechanical systems (MEMS), which are known to have damping that increases as the system size becomes smaller. MEMS devices, such as actuators and sensors, are required to operate with very little energy loss and, consequently, it is important to understand dissipation mechanisms in beam,

plate and ring components of small structures (References 71, 75, 80, 84, 85 and 88).

3. MECHANISMS OF MATERIAL DAMPING

Many processes responsible for linear material damping are relaxation processes that take place at both microscopic and macroscopic levels. The concept of relaxation time and the equations describing a relaxation process are presented in Appendix A. Another idea used in association with linear damping caused by a relaxation process is the anelastic solid (References 7 and 38), which is described in Appendix B. In general, during the process of the deformation of a solid at a certain finite rate, a deviation from the thermodynamic equilibrium takes place. This divergence from thermodynamic equilibrium triggers the relaxation process, *i.e.*, the process of returning to the equilibrium state, during which mechanical energy is converted irreversibly into heat. For materials which behave as anelastic solids, the deformation is not permanent. For such materials damping is amplitude-independent, but depends on the frequency and the relaxation time of the process. The measure of damping used, such as the loss factor (see ESDU 09006 for a definition), is a function of frequency in the form of the so-called Debye peak (Reference 38). A single peak indicates that only a single relaxation process takes place. If many relaxation processes with different relaxation times occur in the material, the loss factor as a function of frequency shows many peaks, each associated with a specific process. The response of the material can be also represented by a spectrum of relaxation times called a relaxation spectrum (Reference 38).

Since relaxation processes in solids occur mainly through thermally-activated phenomena, the relaxation time can be expressed by a function of temperature in the form of the Arrhenius equation (see Section 5). Hence, for thermally-activated processes the damping of the material can be considered either as a function of frequency at constant temperature or as a function of temperature at constant frequency. Point-defect relaxation processes, such as the Snoek type (References 38 and 87) and dislocation relaxations, such as the Bordoni relaxation (Reference 87) are particular examples amongst many thermally-activated relaxation processes. In general, defects in the crystal can be point defects, in the form of vacancies or interstitial impurities, or line defects, associated with dislocations (References 38 and 51). The applied cyclic stress causes a diffusive motion of point defects and the movement of dislocations, which lead to the dissipation of mechanical energy in the material and anelastic damping.

There are many mechanisms of material damping. In real structures several of the damping mechanisms act together and it may be impossible to separate the different effects. Hence, in the laboratory in order to investigate a particular mechanism, materials are chosen, such as pure metals, which allow that mechanism to be isolated and investigated. The mechanism investigated can still occur in more commonly used structural materials. Very often internal friction of materials is studied to provide information necessary for applications in non-destructive testing (Reference 22). For example, knowledge of internal friction can be useful in the study of defects and dislocations in nuclear reactor pressure vessels (Reference 72) and in detecting radiation damage.

The most important of the damping mechanisms occurring in metals and alloys are listed in Table 3.1, together with examples of materials which exhibit the considered mechanisms. The damping values quoted in Table 3.1 are specific damping capacities, expressed in percentages. 'High damping' implies values of specific damping capacity (SDC), see ESDU Item 09006 for a definition, above 10%, 'medium damping' 1 to 10% and 'low damping' is less than 1%. Thus, low damping corresponds to loss factors of 0.0015 and less. The definitions of the different measures of damping are presented in Item 09005 (Reference 91). High damping occurs in (a) materials with moveable dislocations, (b) ferromagnetic materials with moveable domain walls, (c) materials with heterogeneous structure where damping results from local plastic deformation of the soft phase or at interface boundaries (or associated with dislocations within one of the phases) and (d) materials with phase-related changes.

Grey cast iron is an example of a material with a damping mechanism specified by (c). Ferromagnetic stainless steel with 13% chromium and small amounts of other alloying elements, which possesses high damping, belongs to material (b). Defects and dislocations in material can be a source of damping which increases with the applied stress (examples of mechanisms (a)). Finally, manganese-copper alloy and nitinol are the best known examples of the mechanism (d). Low damping materials are mostly associated with relaxation processes. The majority of carbon steels, titanium alloys and aluminium alloys belong to this class of materials.

TABLE 3.1

	<i>Dislocation Damping</i>	<i>Magnetomechanical damping</i>	<i>Interface slip damping</i>	<i>Phase-related damping</i>
High Damping	Pure Mg	Fe, 3% Si	Cast iron (coarse flake)	Mn, Cu
	Al 1% Mg, 1% Si (>400 K)	Fe, 16% Cr	Zn, Al alloy	Nitanol (Ni, Ti)
	Mg, 2% Si	Nivco (Co, Ni)		Cu, Zn, Al alloy (thermoelastic mateusitic transformation)
Medium Damping	Al, 1% Mg, 1% Si (300-400 K)	Co, 30% Fe Armco Nivco (low stress)	Cast iron (spheroidal)	
	Al, 11.8% Si (heat treated at 810 K)	Ferromagnetics (peak values at a particular stress) Low carbon steels		
Low damping	High carbon, low alloyed commercial steels Most aluminium alloys (Al, 10% Zn; Al, 10% Si, Al, 5% Cu, etc.) Ti alloys			

4. THERMOELASTIC DAMPING

Of all the forms of material damping that exist, thermoelastic damping is the most easily quantifiable and is always present. On a basic level, the concept of thermoelastic damping is associated with a relaxation process of coupled temperature-strain fields during which dissipation of energy into heat takes place. Thermoelastic damping was first investigated theoretically by Zener (References 1, 2, 3 and 7). Zener was also interested in the physical interpretation of the properties of anelastic solids (Reference 7 and Appendix B). He established that thermoelastic coupling transforms an elastic solid, which obeys Hooke's law, into an anelastic solid. The Zener model of a standard anelastic solid (standard linear solid) incorporates the existence of an internal relaxation process in the solid. It has been applied to thermoelastic damping at a macroscopic as well as a microscopic level. The macroscopic thermoelastic relaxation takes place in a solid when a homogeneous structure, such as a beam of isotropic material, is under spatially inhomogeneous stress variations, *e.g.*, bending, which create gradients of temperature in the direction of the beam thickness. In the case of a microscopic thermoelastic relaxation, the inhomogeneity of the material and its heterogeneous mechanical properties can be a source of local gradients of temperature within the material while under an external applied stress. Consequently, in polycrystalline materials, such as metals and their alloys, thermal intercrystalline currents occur during the relaxation process, causing microscopic thermoelastic damping. Whilst all materials in a polycrystalline form have some degree of inhomogeneity and anisotropy, microscopic thermoelastic damping is always present, but is generally small. Especially in the case of aluminium the elastic anisotropy is very small and hence the damping is negligible (Reference 7). The damping is greater for iron and brass. For brass, loss factors of the order of magnitude 10^{-4} are possible at a frequency of 6000 Hz (Reference 4).

Zener's model of the standard anelastic solid provides an estimation of damping, due to a relaxation process. The loss factor, η , as a function of vibration frequency, ω , has the form:

$$\eta = Q^{-1} = \Delta_{rs} \left(\frac{\omega \tau}{1 + \omega^2 \tau^2} \right), \quad (4.1)$$

where τ is the relaxation time, *i.e.*, the characteristic time for the thermoelastic relaxation, see Appendix A. The thermal relaxation strength, Δ_{rs} , is given by

$$\Delta_{rs} = \frac{E_{ur} - E_r}{E_r} = \frac{E_{ad} - E}{E}. \quad (4.2)$$

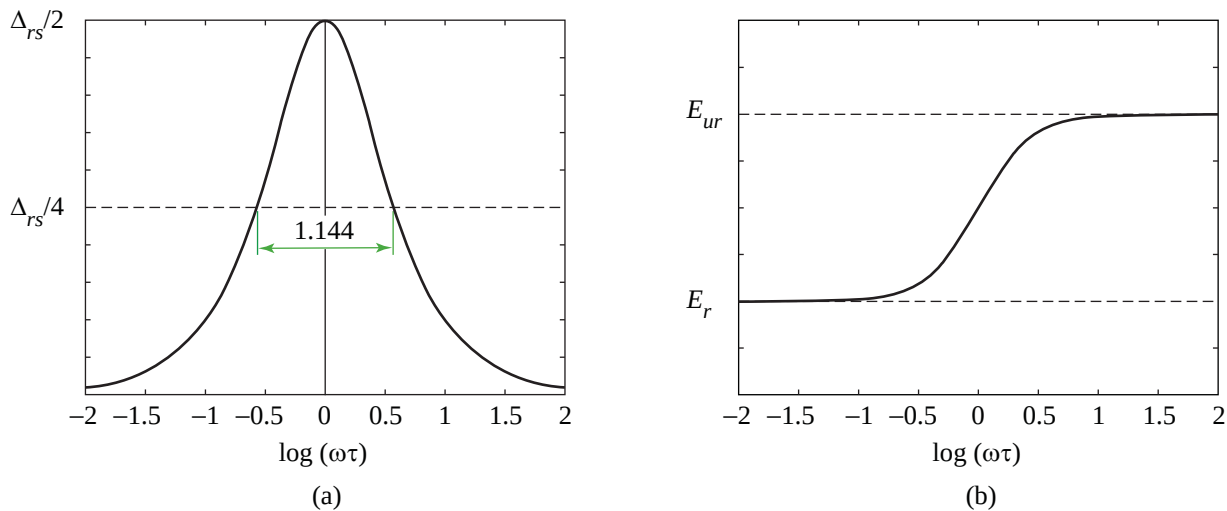
The unrelaxed or adiabatic Young's modulus is denoted by E_{ur} (or E_{ad}) and the relaxed or isothermal Young's modulus is denoted by E_r or E . Q is the quality factor, see ESDU Item 09005 (Reference 91) for the definition. The maximum damping depends upon the difference between the unrelaxed and relaxed moduli. The difference is small for metals and alloys. The Young's modulus which is normally measured (and quoted in handbooks) is the relaxed or isothermal modulus.

The dependence of the loss factor, η , as well as the absolute dynamic Young's modulus, E_ω upon the vibration frequency is shown schematically in Sketches 4.1a and b. The absolute dynamic Young's modulus, E_ω , is defined as the absolute value of the complex modulus, E_c ,

$$E_\omega = \sqrt{[\text{Re}(E_c)]^2 + [\text{Im}(E_c)]^2} = \text{Re}(E_c) \sqrt{1 + \eta^2}, \quad (4.3)$$

where $\text{Re}(E_c)$ is the storage modulus and $\text{Im}(E_c)$ is the loss modulus. The complex modulus for the standard anelastic solid depends on ω , the frequency of vibration, as well as on the relaxation time, τ (see Appendices A and B),

$$E_c = \frac{1 + i\omega\tau\sqrt{E_{ur}/E_r}}{1 + i\omega\tau\sqrt{E_r/E_{ur}}}E_r. \quad (4.4)$$



Sketch 4.1 Variation of (a) loss factor and (b) dynamic Young's modulus with $\omega\tau$

The relaxed and unrelaxed moduli are the asymptotic values of the absolute dynamic modulus, as well as the asymptotic values of the storage modulus. It is seen from Sketch 4.1b that for the case when $\omega\tau \gg 1$ the absolute dynamic modulus approaches asymptotically the value of the unrelaxed (adiabatic) Young's modulus. Similarly, when $\omega\tau \ll 1$ the absolute dynamic modulus tends to the value of the relaxed (or isothermal) Young's modulus. The relationship between the loss factor and $\log(\omega\tau)$ has for a single relaxation process the standard appearance of the Debye peak (References 38 and 87), as shown in Sketch 4.1a. Maximum damping at $\omega\tau = 1$ amounts to $\Delta_{rs}/2$, half of the relaxation strength. At a level of $\Delta_{rs}/4$ the width of the Debye peak is 1.444 on the logarithmic scale, as indicated in Sketch 4.1a. For $\omega\tau \gg 1$ and $\omega\tau \ll 1$, there is no dissipation of mechanical energy in the solid and the value of the loss factor is equal to zero. Hence, the material damping can only be detected around $\omega\tau \approx 1$. The approach adopted for thermoelastic damping applies only to small stresses.

The thermal relaxation strength, Δ_{rs} , is a characteristic of the material. Similarly, the relaxation time, τ , for the microscopic thermoelastic process depends only on the material properties. However, in the case of macroscopic thermoelastic relaxation, the relaxation time is also associated with the structural properties of the specimen and, strictly speaking, only when $\omega\tau = 1$ and the loss factor reaches the maximal value may the thermoelastic damping be treated as a material characteristic.

4.1 Application of Zener Theory to Thermoelastic Damping in Beams

As the beam vibrates in bending, the part of the beam in tension cools and the part in compression increases in temperature. A transverse gradient of temperature appears which triggers a relaxation process, during which thermal currents occur in the direction of the beam thickness and energy is dissipated. If the beam vibrates at a high frequency, where $\omega\tau \gg 1$, the relaxation time for transverse heat conduction in the beam is much longer than the period of oscillation. Hence, no noticeable equalization of the temperature difference, due to beam deformation, can take place during the oscillation period. Consequently, damping is negligible and vibrations in bending of the beam can be treated as an adiabatic process. In the case of

low frequencies, where $\omega\tau \ll 1$, the relaxation time for thermal conduction is much shorter than the period of oscillation. The equalization of the temperature difference can be treated as instantaneous and the process of bending as isothermal. Consequently, for the limiting case of low frequencies of vibration, damping is negligible.

Significant dissipation of mechanical energy can only take place at frequencies close to $\omega\tau \approx 1$, where the period of oscillation can be compared to the relaxation time for thermal conduction. Particularly, the condition $\omega\tau = 1$ indicates maximum energy dissipation and maximum damping.

In the case of a thin homogeneous and isotropic beam with a rectangular cross-section and uniform thickness, h , the relaxation time, τ , is given by

$$\tau = \frac{h^2 \rho c_p}{\pi^2 k_t}, \quad (4.5)$$

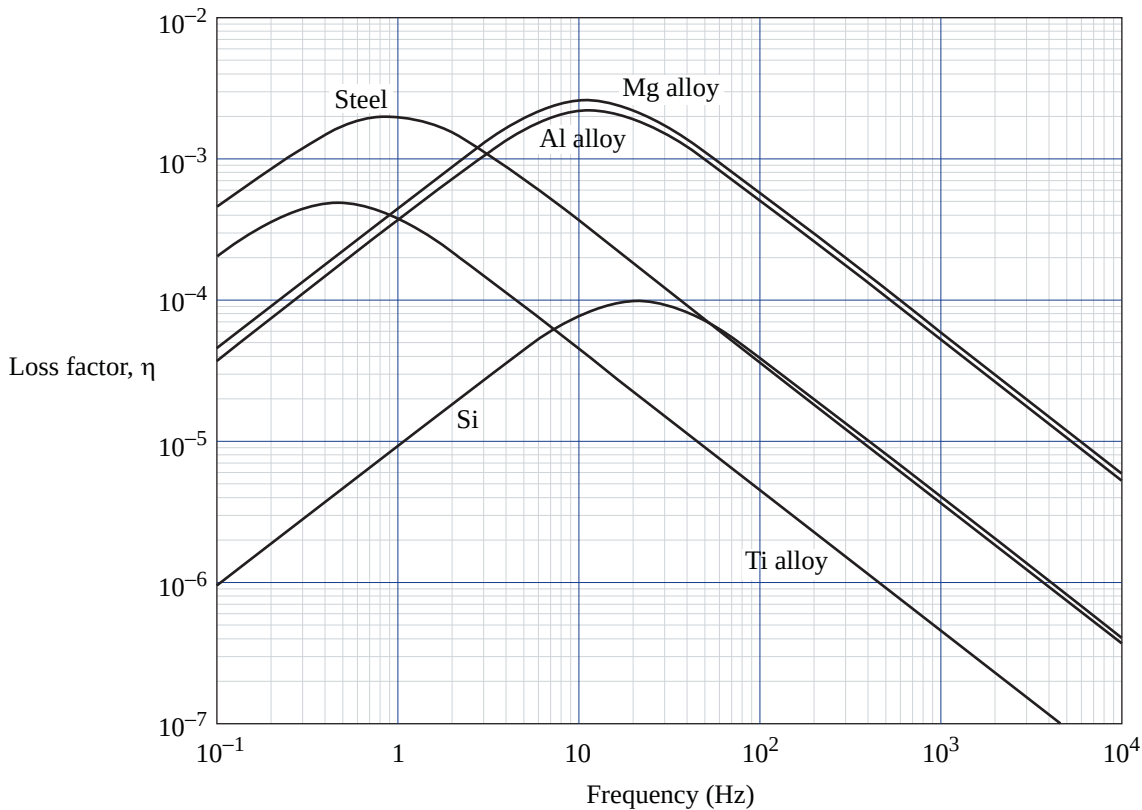
where ρ is the density of the beam material, c_p is the specific heat at constant pressure (or constant stress) and, k_t is the transverse thermal conductivity (in the direction of the beam thickness). The thermoelastic damping of the beam in terms of the loss factor, η , at frequency in rad/s is given by

$$\eta = \frac{\alpha^2 E T_0}{\rho c_p} \left(\frac{\omega\tau}{1 + \omega^2 \tau^2} \right), \quad (4.6)$$

where α is the linear coefficient of thermal expansion of the material, E is Young's modulus (isothermal modulus) of the material of the beam and T_0 is the absolute temperature at equilibrium.

Note that damping can be quantified in many ways (see ESDU Item 09005). The loss factor or equivalent viscous damping ratio are useful for designers. Hence, these damping measures are used predominately in design guides. For example, in ESDU Item 72015 the stress due to acoustic loading is estimated using damping in terms of the equivalent viscous damping ratio. The relationship between different measures of damping such as the loss factor, η , the equivalent viscous damping ratio, ζ , the quality factor, Q , the specific damping capacity, ψ , and logarithmic decrement, δ , are discussed in ESDU Item 09005. For small damping the relationship between the damping may be summarised as: $Q^{-1} = 2\zeta = \eta = \delta/\pi = \psi/2\pi$.

To illustrate how thermoelastic damping (Equation (4.6)) depends on the beam material, beams of different metallic materials but of the same thickness 2.54 mm (0.1 in), vibrating in bending, have been considered. Loss factors for each beam as a function of frequency are shown in Sketch 4.2. For comparison the loss factor of a silicon beam is also included, since silicon is used extensively in microdevices.



Sketch 4.2 Loss factors as a function of frequency for the thermoelastic damping of different materials

Values of material constants for different beam materials at room temperatures are shown in Table 4.1, where E is Young's modulus, ν is Poisson's ratio, ρ is volume density, α is the coefficient of linear thermal expansion, k_t is the thermal conductivity and c_p is the specific heat capacity.

TABLE 4.1

Material	E (GPa)	ν	ρ (kg/m ³)	α ($\mu\text{m/mK}$)	k_t (W/mk)	c_p (J/kgK)
Al alloy 2024-T3 clad	72	0.33	2770	23.3	125	920
Al alloy 7075-T6 clad	71	0.33	2800	20	134	920
Al alloy 6061-T6	69	0.33	2710	24	156	920
Ti alloy 6Al 4V	110	0.31	4420	9	5	595
Ti alloy 2Cu	107	0.28	4560	9.02	12.7	550
Mg alloy (L122)	44	0.35	1810	27.2	84	1000
Steel 18Cr 10Ni	193	0.29	7900	16.8	14.5	490
Silicon	165	—	2330	2.6	141	703.8

The MMD data files of ESDU (Reference 77) have been used to complete Table 4.1, apart from the data for silicon which originates from Reference 84. Note that thermal data can vary considerably from one source to the next and can also vary with temperature, so it is advisable to consult the original data bases for a particular application. Note that the data given in Table 4.1 is in SI units. In the handbook, Reference 78, British units are given. For an aluminium alloy, $c_p = 0.213 \text{ BTU}/(\text{lb } ^\circ\text{F})$, and in the inch-second system, $c_p = 0.213 \times 386 = 82.22 \text{ BTU}/(* ^\circ\text{F})$ – the mass unit has no recognised name. A typical value for the thermal conductivity is $k_t = 104 \text{ BTU}/(\text{h ft } ^\circ\text{F})$ which in terms of inches and seconds becomes $k_t = 104/(3600 \times 12) = 2.41 \times 10^{-3} \text{ BTU}/(\text{s in } ^\circ\text{F})$. The density, ρ , is given as 1.00 lb/in^3 and is converted to $1.00/386 = 2.59 \times 10^{-4}$. Thus, if Equation (4.5) is used to estimate the relaxation time in the case of a beam of thickness 0.1 in, $\tau = 0.1^2 \times 2.59 \times 10^{-4} \times 82.2/(\pi^2 \times 2.41 \times 10^{-3}) = 8.97 \times 10^{-3} \text{ s}$.

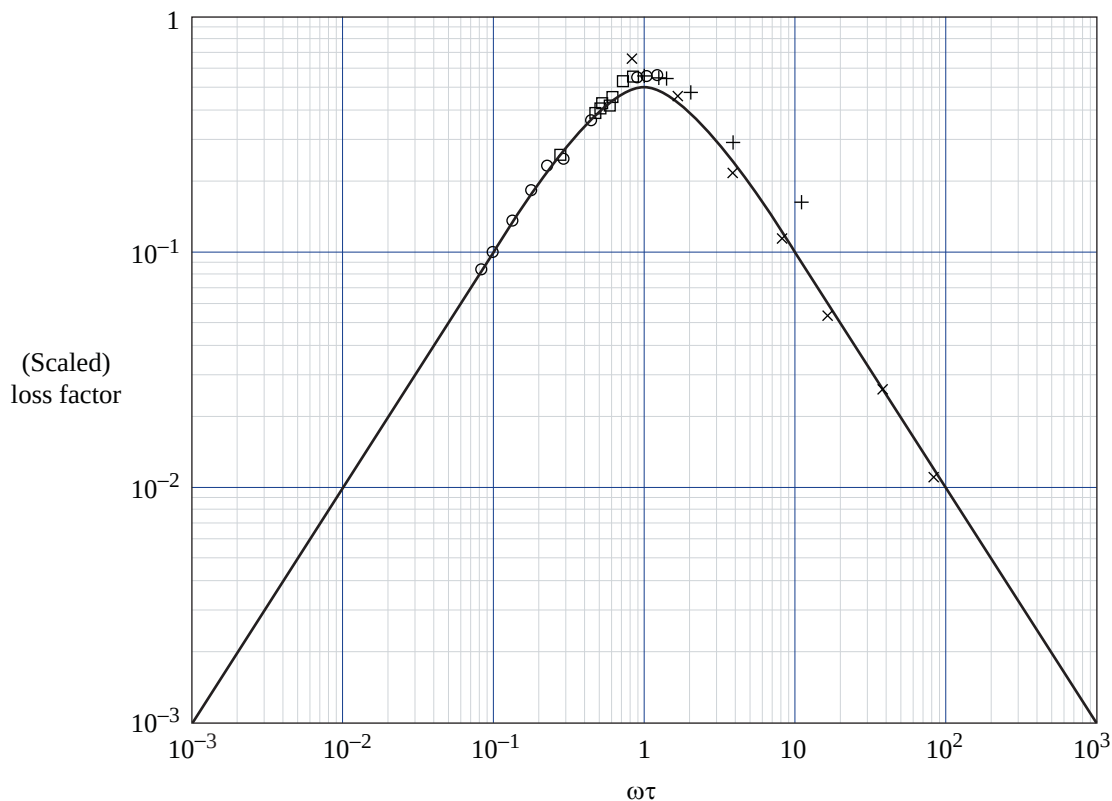
Thermoelastic damping depends upon the thickness of the structure, hence it cannot be treated as a material property. However, as has already been mentioned, its maximum value, reached at a frequency of $\omega_{max} = 1/\tau$ is a material property. The frequency ω_{max} is sometimes called the Zener relaxation frequency (Reference 49). The values of the maximum damping in terms of the loss factor (or equivalent viscous damping ratio), η_{max} , are shown for some materials in Table 4.2.

TABLE 4.2

Material	η_{max}
Al alloy 2024-T3 clad	0.0022
Ti alloy 6Al 4V	4.96×10^{-4}
Mg alloy (L122)	0.0026
Steel 18Cr 10Ni	0.0020
Silicon	10.0×10^{-5}

4.2 Experimental Verification of Zener Model of Thermoelastic Damping

Zener's model of thermoelastic damping has been confirmed experimentally (References 45, 47, 48, 49, 56 and 58). Thermoelastic (macroscopic) damping in terms of equivalent viscous damping ratio was estimated by Crawley and van Schoor (Reference 49) from recorded free decay responses of small beam specimens after they had been lofted into free fall inside a vacuum chamber. The specimens tested were small aluminium beams with rectangular cross sections, ten of which were made from 2024-T3 aluminium alloy and two from 6061-T4 aluminium alloy. The beams were tested in the fundamental mode of vibration with natural frequencies varying from one-tenth to ten times the Zener relaxation frequency. From the experimental results it was concluded that for small stresses the material damping for both types of aluminium alloy specimens is correctly predicted by the Zener model. In the case of high stress (high initial stress amplitude) it has been found that above the Zener frequency the damping was independent of stress up to approximately 50% of the yield stress ($\sigma_y = 290 \text{ MPa}$). However, for frequencies below the Zener relaxation frequency, it has been claimed that the damping is stress-dependent and increases approximately exponentially with stress (Reference 49). Similarly, experimental results by Grannick and Stern (Reference 25), performed on cantilever beams made from 2024-T4 aluminium alloy, implied that approximately up to 140 MPa the damping is stress-independent and within the range of frequencies 15 to 1500 Hz can be predicted from Zener's model. A comparison between the theoretical predictions obtained from the Zener theory in the form of the Debye peak and the experimental results (References 25, 49 and 58) is shown in Sketch 4.3, where the loss factor (or equivalent viscous damping ratio) is presented as a function of $\omega\tau$, as obtained on the basis of Equation (4.6).

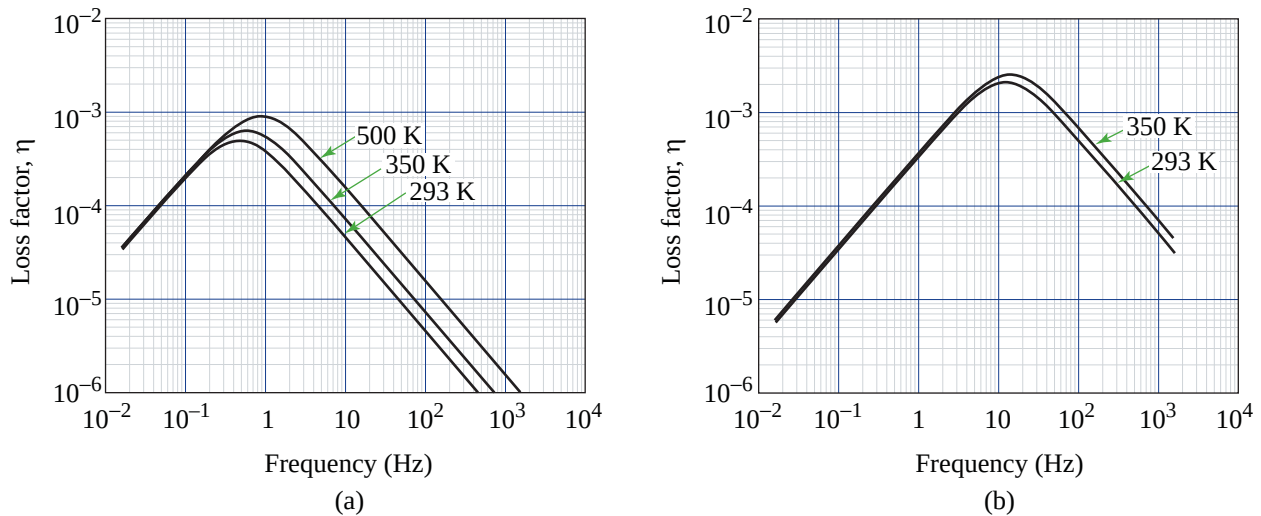


Sketch 4.3 Ratio of loss factor to relaxation strength (η/Δ_{rs}) for material damping of aluminium alloy in flexure. — theoretical results according to Zener theory (Equation (4.3)); experimental results × cantilever (Reference 25), + free-free beam (Reference 49), o cantilever and □ free-free beam (Reference 58)

In general, within a narrow range of low oscillation frequencies and for relatively low applied stress the thermoelastic damping in metals and their alloys can be estimated from Zener's model and identified correctly at experimental data distribution. However, at high frequencies many different processes contribute to the overall damping and it is difficult to differentiate individual peaks associated with the processes (Reference 23). Consequently, the loss factor varies comparatively little over a wide range of frequency. In such cases material damping can be treated as hysteretic, at least within a certain range of frequency.

4.3 Effect of Temperature on Thermoelastic Damping in Beams

The effect of temperature on material damping may be investigated by means of Equations (4.2) and (4.3). It is clear from Equation (4.3) that damping increases with temperature. However, there are additional effects, because c_p , k_t and α depend on temperature, see MMPDS handbook (Reference 78). The effect of temperature on the loss factor of a titanium alloy (6Al 4V) and an aluminium alloy (2024) is shown in Sketches 4.4a and 4.4b for a beam 2.54 mm thick. The effect is most noticeable with the titanium alloy where an increase in temperature causes the maximum to be shifted to higher damping values and higher frequencies.



Sketch 4.4 Effect of temperature on loss factors of a beam of (a) titanium alloy (6Al 4V) and (b) aluminium alloy (2024)

4.4 Refinement of Theory of Thermoelastic Damping on Macroscopic Level

Developments of the theory of thermoelastic damping in relation to structures have been made by various authors (References 19-21, 45, 80, 81 and 83). Landau and Lifshitz (Reference 52) have considered non-adiabatic transverse vibrations of beams (semi-infinite plates of thickness, h) and have calculated thermoelastic damping which, in terms of the loss factor, may be written as

$$\eta = \left(\frac{E \alpha^2 T_0}{\rho c_p} \right) \left(\frac{6}{\xi^2} - \frac{6}{\xi^3} \cdot \frac{\sinh \xi + \sin \xi}{\cosh \xi + \cos \xi} \right), \quad (4.7)$$

where

$$\xi = h \sqrt{\frac{\omega \rho c_p}{2k_t}},$$

a nondimensional parameter proportional to $\sqrt{\omega \tau}$.

It should be noted that the formula for ξ was originally presented as a function of the volumetric thermal expansion coefficient (Reference 52). The coefficient in Equation (4.3) denotes the linear thermal expansion coefficient which is one third the volumetric coefficient. The model has been considered in the context of the application of micro devices (MEMS) (Reference 71). Although Zener estimated and measured thermoelastic damping of beams, it is claimed that the theory can be extended to plates (References 80, 81 and 83).

5. OTHER MICROSCOPIC RELAXATION MECHANISMS

There are several relaxation mechanisms, other than thermoelastic, which bring about damping in metals and their alloys. Most of the mechanical relaxation phenomena in solids are triggered by thermally-activated processes and, as a consequence, the relaxation time, τ , can be described by the Arrhenius equation (Reference 38, page 58), *i.e.*,

$$\tau = \tau_0 \exp\left(\frac{Q_h}{k_b T}\right), \quad (5.1)$$

where τ_0 is a constant ($\tau_0 = \lim_{T \rightarrow \infty} \tau$), Q_h is the activation energy, k_b is Boltzmann's constant and T is the absolute temperature. The existence of the Arrhenius equation enables the investigation of damping at constant frequency as a function of temperature. From Equation (5.1) it follows that

$$\ln(\omega\tau) = \ln(\omega\tau_0) + \frac{Q_h}{k_b} \left(\frac{1}{T}\right). \quad (5.2)$$

Hence, there is a linear relationship between $\ln(\omega\tau)$ and the reciprocal of T , the absolute temperature. Consequently, damping (in terms of the loss factor), as well as absolute dynamic modulus, can be presented at a constant frequency as a function of temperature. Equation (4.1), which describes the loss factor in the form of the Debye peak as a function of frequency and relaxation time, can be transformed into the form

$$\eta = \frac{\Delta_{rs}}{2} \cdot \frac{2}{e^{-\ln(\omega\tau)} + e^{\ln(\omega\tau)}}, \quad (5.3)$$

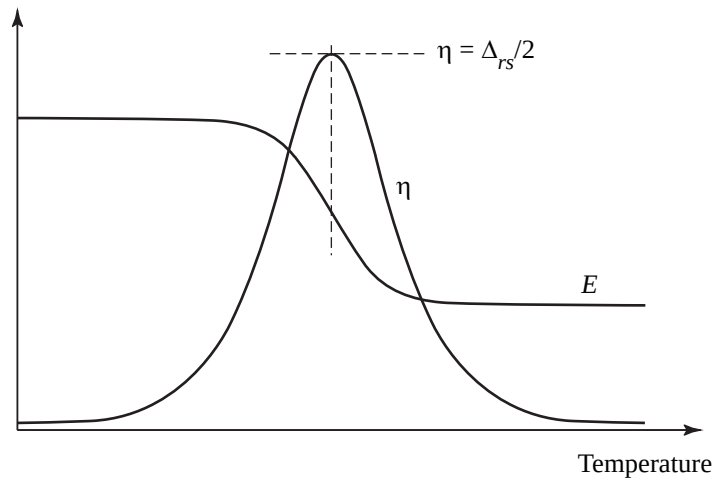
or,

$$\eta = \frac{\Delta_{rs}}{2} \cdot \operatorname{sech}[\ln(\omega\tau)], \quad (5.4)$$

where Δ_{rs} is the relaxation strength and $\Delta_{rs} = (E_{ur} - E_r)/E_r$. Combining Equations (5.4) and (5.2) leads to

$$\eta = \frac{\Delta_{rs}}{2} \cdot \operatorname{sech}\left[\ln(\omega\tau_0) + \frac{Q_h}{k_b} \cdot \frac{1}{T}\right]. \quad (5.5)$$

Sketch 5.1 shows the dependence of the loss factor, Equation (5.5), and the storage modulus as a function of temperature. See Appendix B and compare Sketch 5.1 with Sketch 4.1.



Sketch 5.1 Schematic variation of storage modulus and loss factor with temperature in a relaxation process

For a certain frequency ω the condition $\omega\tau = 1$ defines the peak temperature, T_p ,

$$\omega\tau_0 \exp\left(\frac{Q_h}{k_b} \cdot \frac{1}{T_p}\right) = 1. \quad (5.6)$$

Hence,

$$\ln(\omega\tau_0) = -\frac{Q_h}{k_b} \cdot \frac{1}{T_p}. \quad (5.7)$$

At different frequencies a series of Debye peaks, as a function of temperature, can be obtained in terms of the loss factor, see Equation (5.5). As is implied from Equation (5.7), there is a linear relationship between $\ln(\omega)$ and $(1/T_p)$. Hence, a plot of $\ln(\omega)$ against $(1/T_p)$ is a straight line with slope $-Q_h/k_b$ and, consequently, the activation energy can be estimated. Finally, combining Equations (5.5) and (5.7) leads to the Debye peak at constant frequency as a function of temperature:

$$\eta = \frac{\Delta_{rs}}{2} \cdot \operatorname{sech}\left[\frac{Q_h}{k_b}\left(\frac{1}{T} - \frac{1}{T_p}\right)\right]. \quad (5.8)$$

Characteristic features of thermally-activated relaxation processes are relatively narrow relaxation peaks in the frequency as well as in the temperature domains. For a given frequency a series of relaxation peaks occur at different temperatures. (It should be noted that damping can be achieved within a wide range of frequency, if it is not thermally-activated (Reference 86).)

Some examples of thermally-activated relaxation processes in alloys are tabulated in Appendix C. The maximum damping at particular temperatures is listed, together with the activation energy, as found in the Arrhenius equation.