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ESDU TECHNICAL MEMORANDUM

Effective phase angle in tyre-friction modelling

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EFFECTIVE PHASE ANGLE IN TYRE-FRICTION MODELLING

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EFFECTIVE PHASE ANGLE IN TYRE-FRICTION MODELLING

1. NOTATION AND DEFINITIONS

1.1 Notation

Dimensions

c	roughness function	–
D	inflated diameter of tyre	L
E^*	dynamic modulus for tread material	$ML^{-1}T^{-2}$
f_{λ}	parameter associated with λ_{EFF}	–
G_{TREAD}	effective shear modulus of tread material	$ML^{-1}T^{-2}$
p_{ABS}	inflation pressure in absolute measure	$ML^{-1}T^{-2}$
s	slip ratio	–
T_G	ground temperature	K
T_{REF}	reference temperature	K
T_{gt}	glass-transition temperature	K
T_r	reference temperature associated with f_{λ} term	K
V_G	ground speed	LT^{-1}
V_{SKID}	ground speed in skid	LT^{-1}
V_{YAW}	ground speed normal to plane of wheel	LT^{-1}
w	inflated width of tyre	L
Z	vertical load on tyre	MLT^{-2}
ϵ	strain	–
η	process-loss coefficient	–
η_4	empirical function	–
η_{40}	empirical constant	–
κ	constant of proportionality between hoop stress and inflation pressure	–

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Λ	tyre loading parameter	–
λ	phase angle between stress and strain in tyre compound	radian
λ_{EFF}	effective phase angle	radian
μ_{REF}	reference “frictional” force-coefficient	–
μ_{ROLL}	coefficient of resistance parallel to plane of wheel	–
$(\mu_{SKID})_{DRY}$	force-coefficient in skid on dry surface	–
$(\mu_{SLIP})_{DRY}$	force-coefficient in slip on dry surface	–
μ_{STATIC}	force-coefficient at inception of braked motion	–
$(\mu_{YAW})_{DRY}$	force-coefficient normal to plane of wheel on dry surface	–
$(\mu_{YAW})_{REF}$	reference force-coefficient normal to plane of wheel	–
ρ_{TREAD}	density of tyre compound	ML ⁻³
ψ	yaw angle	radian

1.2 Definitions

Effective phase angle is the angle that can be deduced in the process of modelling coefficients of force that are the consequence of stresses and strains in the *footprint* of a pneumatic tyre under load on a pavement. It is asserted that this angle is related to the phase angle between stress and strain for the materials and construction of the tyre.

Footprint is the jargon term for the region of contact between a pneumatic tyre and the pavement with which it is in contact.

Glass-transition is the reversible transition in amorphous materials from a hard and relatively brittle state into a molten or rubber-like state.

Glass-transition temperature is the temperature at which the transition occurs. In tyre compounds the transition temperature is in the region of 200K.

Loss tangent is the ratio of loss modulus, G'' , and storage modulus, G' , in a visco-elastic material. It is a consequence of the work absorbed by the material and that recovered in a dynamic process. It is the tangent of the phase angle between stress and strain.

Process-loss coefficient is the ratio of power lost and power stored as the footprint moves around the circumference of the tyre in a partial skid.