



ESDU TECHNICAL MEMORANDUM

Correlations for some aerodynamic characteristics of aerofoils in incompressible inviscid flow

P.D. Chappell

ESDU TECHNICAL MEMORANDA

ESDU Technical Memoranda are prepared from time to time to provide a formal means of presentation for work that is not issued as a Data Item. The main reasons for preparing an ESDU Technical Memorandum are as follows.

- Much of the work done in the preparation of a Data Item, such as data correlations or analytical studies, can be brought together in a Technical Memorandum which can then be quoted as one of the Derivations or References in the Data Item. Such a Memorandum therefore provides a formal record of the data or methods given in the Data Item itself.
- When ESDU takes on an external contract a Memorandum is used as the vehicle for formally reporting that work.

ESDU Technical Memoranda do not, in general, go through the same rigorous Committee validation process as ESDU Data Items, but they may be considered to be reliable sources of information supplementary to that in the Data Items.

The development and preparation of this Technical Memorandum were undertaken by

Mr P.D. Chappell

— Head of Aircraft Aerodynamics Group (1972).

1. NOTATION

a_{li}	lift-curve slope at zero lift in incompressible inviscid flow (degree ⁻¹)
C_{Li}	lift coefficient in incompressible inviscid flow
C_{moi}	zero-lift pitching moment coefficient in incompressible inviscid flow, based on aerofoil chord
c	aerofoil chord (see Section 2.1)
F	factor in Equation (5)
h	h_+ for $h_+ \geq h_- $, h_+ for $h_+ < h_- $ (see Section 2.4)
h_+	maximum positive camber line ordinate (see Section 2.4)
h_-	maximum negative camber line ordinate (see Section 2.4)
t	aerofoil maximum thickness (see Section 2.3)
x	chordwise distance measured from leading edge of aerofoil
x_{ai}	chordwise position of aerodynamic centre in incompressible inviscid flow
x_h	chordwise position of h (see Section 2.4)
x_{h+}	chordwise position of h_+ (see Section 2.4)
x_{h-}	chordwise position of h_- (see Section 2.4)
x_t	chordwise position of maximum thickness (see Section 2.3)
z	vertical coordinate at station x (see Section 2.2)
z_c	camber line ordinate at station x (see Section 2.2)
z_t	thickness ordinate at station x (see Section 2.2)
α	incidence (degree)

- α_{oi} zero-lift incidence in incompressible inviscid flow (degree)
- ρ aerofoil leading-edge radius, $\left[\sqrt{2\rho/c} = \lim_{x \rightarrow 0} (dz_t/d\sqrt{x}) \right]$
- τ aerofoil trailing-edge angle, $\left[\tau = 2 \tan^{-1}(-dz_t/dx)_{x=c} \right]$ (degree)
- ϕ_1, ϕ_2 factors in Equations (4a) and (4b)

Suffixes

COMP denotes a computed value

The following suffixes relate to x and z only:-

- 1 pertaining to $x = 0.005c$
- 2 pertaining to $x = 0.05c$
- 3 pertaining to $x = 0.2c$
- 4 pertaining to $x = 0.5c$
- 5 pertaining to $x = 0.9c$
- 6 pertaining to $x = 0.92c$

2. AEROFOIL GEOMETRICAL PARAMETERS

2.1 Definition of Aerofoil Chord

In the context of this Item the aerofoil chord is defined as the length of the line joining the trailing-edge (assumed sharp) and that unique point at which a circle centred at the trailing-edge is tangential to the aerofoil.

2.2 Definition of Aerofoil Geometry

z is measured perpendicular to the chord line. If the upper and lower surfaces of the aerofoil are defined by

$z = z_u$ and $z = z_l$ respectively,

then the thickness and camber line ordinates are defined by

$z_t = \frac{1}{2}(z_u - z_l)$ and $z_c = \frac{1}{2}(z_u + z_l)$ respectively.

3. INTRODUCTION

In Reference 10 the Lighthill¹ nominally-exact method for incompressible inviscid flow has been used to design a family of low-speed aerofoils having low-drag characteristics by virtue of the attainment of a roof-top pressure distribution on the upper and lower surfaces at the extremes of a specified incidence range. The method provides for independent modification of the leading-edge radius and trailing-edge shape. An appendix to the report contains details of the geometric and aerodynamic characteristics of 240 symmetrical and 720 cambered aerofoils derived by the method.

In view of the availability of such a large variety of aerofoils (see Tables III and IV) and their associated aerodynamic characteristics in incompressible inviscid flow it seemed pertinent to attempt to correlate the data with a view a) to check and extend existing correlations in the ESDU Aerodynamics Sub-series of Data Items and b) to develop correlations for parameters not previously covered. The particular aerodynamic parameters chosen for study were lift-curve slope, zero-lift incidence, zero-lift pitching moment coefficient and chordwise position of the aerodynamic centre.

Since the aerofoils derived in Reference 10 were all members of a single family there was a danger that correlations using these as a basis would be inapplicable to aerofoils not in that family. This was felt to be of particular significance with regard to those correlations which were camber-dependent, since there are only two basic camber-line shapes for the whole range of aerofoils in Reference 10. These are the simple-arc and reflex types and they are illustrated in Section 2.4. With these points in mind, data for a number of additional aerofoils (see Tables I and II) obtained from other sources^{3,11} were used in the correlations. The data from Reference 3 all relate to symmetrical aerofoils and were obtained using either exact or accurate approximate methods. The data referred to as Reference 11 were derived, using the nominally-exact method of Reference 9, for a range of aerofoils, both symmetrical and cambered; among these were members of the NACA 6 and 6A series⁵ and the RAE 100 to 104² and 2800 series¹². The camber-line shapes for these additional aerofoils were mainly of the simple-arc type but included examples of the reflex and "double-hump"^{*} types.

^{*}See sketch on Figure 4a.

4. LIFT-CURVE SLOPE

ESDU Data Item No. Aero W.01.01.05 provides curves for estimating the lift-curve slope at zero lift for any aerofoil in incompressible inviscid flow. The curves are derived⁴ from the empirical expression

$$a_{1i} = 0.10967 \left[1 + \frac{t}{c} (0.7559 + 0.00318\tau) \right] . \quad (1)$$

Figures 1a and 1b illustrate a comparison of the computed values of lift-curve slope at zero lift from Reference 10, for a large family of low-drag aerofoils, with the predictions of Equation (1). It will be noted that Equation (1) generally overestimates a_{1i} by up to 0.003, particularly for $t/c > 0.10$. Figure 2 shows a direct comparison between computed values of a_{1i} and the values predicted by Equation (1) for 46 of the aerofoils in Tables I and II. As can be seen from the Figure agreement is, for the most part, within ± 0.001 , although here again Equation (1) overestimates a_{1i} for the thicker aerofoils. Further investigation of the data revealed that the prime reason for the inability of Equation (1) to adequately represent the computed data of Reference 10 for other than aerofoils with low t/c lay in the neglect of any consideration of the effect of leading-edge shape. In other words Equation (1) simply represents the mean of the restricted range of leading-edge shapes of the aerofoils from which it was derived.

Consideration of leading-edge shape in the form of the parameter $2z_{t2}/t$ led to the expression

$$a_{1i} = 0.10967 \left[1 + \frac{t}{c} (0.549 + 0.400 \frac{2z_{t2}}{t} + 0.00212\tau) \right] . \quad (2)$$

Figures 3a to 3c indicate the measure of the improvement in the correlation of the data using Equation (2). Figure 3a, which is directly comparable to Figure 2, shows that the scatter band is halved to ± 0.0005 . Figures 3b and 3c, which relate to the GU-series of aerofoils⁵, indicate that Equation (2) represents the computed data equally well for both symmetrical and cambered aerofoils. The fact that the scatter band is ± 0.001 on these two Figures is, it is

⁵To reduce the amount of work involved only those aerofoils belonging to the GU-23, 43 and 63 series (i.e. one third of the total number in Reference 10) were considered. The members of these series are adequately representative of the family given in Reference 10.

felt, largely a reflection of the fact that the computed data in Reference 10 are quoted to only three decimal places.

5. ZERO-LIFT INCIDENCE

The use of thin-aerofoil theory suggested that those aerofoil aerodynamic characteristics which were camber-dependent could be correlated by means of three geometrical parameters, the camber line ordinates corresponding to the chordwise stations $x/c = 0.2, 0.5$ and 0.92 , i.e. z_{c3} , z_{c4} and z_{c6} respectively. By normalising two of the parameters with respect to the third (taken as z_{c4}) it was possible to characterise the camber-line "shape" in non-dimensional terms. The data for zero-lift incidence of the GU-series of aerofoils* and others listed in Table II were divided by z_{c4}/c and plotted against z_{c6}/z_{c4} . By this means it was established that $\alpha_{oi}/(z_{c4}/c)$ was essentially linear in z_{c3}/z_{c4} and that it was therefore possible to collapse all the data on to a single curve, as shown in Figure 4a. Furthermore, it is seen from this Figure that $\alpha_{oi}/(z_{c4}/c)$ is also linear in z_{c6}/z_{c4} for $z_{c6}/z_{c4} \leq 0.3$, which includes most of the data from Reference 10. The following expression adequately represents the data for $z_{c6}/z_{c4} \leq 0.3$:

$$-\frac{\alpha_{oi}}{z_{c4}/c} = 20 + 23\frac{z_{c3}}{z_{c4}} + 300\frac{z_{c6}}{z_{c4}} \quad (3)$$

The data for $z_{c6}/z_{c4} > 0.3$, which derive from a number of the simple-arc and reflex camber lines for the GU-series of aerofoils from Reference 10, the NACA $a = 1$ camber line and some "double-hump" camber lines, indicate a non-linear trend with increasing z_{c6}/z_{c4} . The scatter of the data in this region is rather greater than that for $z_{c6}/z_{c4} \leq 0.3$.

*To reduce the amount of work involved only those aerofoils belonging to the GU-23, 43 and 63 series (i.e. one third of the total numbers in Reference 10) were considered. The members of these series are adequately representative of the family given in Reference 10.

The accuracy of the correlation is indicated in Figures 4b and 4c. It was found that for $z_{c6}/z_{c4} \leq 0.5$ the curve of Figure 4a (defined by Equation (3) for $z_{c6}/z_{c4} \leq 0.3$) correlates 90% of the data for $\alpha_{oi} > 1.7$ degrees to within $\pm 3\%$, see Figure 4b. Since the computed data for α_{oi} given in Reference 10 were rounded out to the nearest 0.1 degree they are in general only correct to within ± 0.05 degree. This rounding error will become greater than the $\pm 3\%$ scatter of Figure 4b when $\alpha_{oi} \leq 1.7$ degrees. Therefore a separate plot is given, in Figure 4c, for $\alpha_{oi} \leq 1.7$ degrees. For these data an incremental scatter appears to be a more appropriate measure of the degree of success of the correlation curve of Figure 4a. It is stated on Figure 4c that 93% of the data for $\alpha_{oi} \leq 1.7$ degrees are correlated to within ± 0.08 degree, of which ± 0.05 degree is due to rounding error in the computed data. For values of $z_{c6}/z_{c4} > 0.5$ the curve of Figure 4a (based partly on data for aerofoils with "double-hump" camber lines) correlates the data to within $\pm 10\%$.

6. ZERO-LIFT PITCHING MOMENT COEFFICIENT

With regard to zero-lift pitching moment coefficient it was found that the correlation parameters suggested by thin-aerofoil theory (see Section 5) were not completely satisfactory in their ability to correlate the whole range of the data from Reference 10. Zero-lift pitching moment coefficient appeared to be much more sensitive to changes near the leading edge of the camber line than zero-lift incidence. What was required therefore was a parameter which reflected the leading-edge shape of the camber line and, after a trial-and-error process, this was established to be z_{c2} , the camber-line ordinate at $x/c = 0.05$. It was also established that two other parameters were required to satisfactorily define the camber-line with respect to zero-lift pitching moment coefficient and these were found to be h_+ and z_{c5} , the maximum positive ordinate of the camber line and the ordinate at $x/c = 0.9$. By normalising two of the parameters with respect to the third (taken as h_+) it was possible to characterise the camber-line "shape" in non-dimensional terms. The data for zero-lift pitching moment coefficient of the aerofoils in Table II* were divided by h_+/c and plotted against z_{c5}/h_+ . It was found that, unlike the correlation for α_{oi} , the data were non-linear in both the independent variables z_{c2}/h_+ and z_{c5}/h_+ , see Figure 5a. The data in this

*See footnote on page 5 concerning the aerofoils used from Reference 10.

Figure relate to aerofoils with camber lines of simple-arc or reflex shape only (Types I and II in Section 2.4); no data for other camber line shapes were available for comparison.

Figure 5b indicates that the curves of Figure 5a correlate 93% of the data to within $\pm 5\%$.

7. CHORDWISE POSITION OF AERODYNAMIC CENTRE

Previous theoretical work (see, for example, Reference 3) has suggested that the chordwise position of the aerodynamic centre of an aerofoil is related primarily to its thickness/chord ratio and the position of maximum thickness, t/c and x_t/c , with a secondary dependence on its trailing-edge angle and leading-edge radius, τ and ρ/c .

ESDU Data Item No. Aero A.08.01.05 provides a means of estimating the aerodynamic centre position for any aerofoil in incompressible inviscid flow. The basic equations used in that Item are

$$\frac{x_{ai}}{c} = 0.25 \left(1 + \tau \phi_1 + \frac{t}{c} \phi_2 \right) \quad (4a)$$

for $\tau \geq 100 \frac{t}{c}$, and

$$\frac{x_{ai}}{c} = \left[0.25 + 0.28 \left(\frac{t}{c} \right)^2 \right] \left(1 + \tau \phi_1 + \frac{t}{c} \phi_2 \right) \quad (4b)$$

for $\tau < 100 \frac{t}{c}$.

The factors ϕ_1 and ϕ_2 are given in A.08.01.05 as functions of x_t/c .

Figure 6 illustrates a comparison between the computed values of aerodynamic centre position for the symmetrical aerofoils of Reference 3 (see Table I) and the values calculated using Equations (4) (see Table 3 of Reference 3). Agreement is for the most part within ± 0.004 . It is observed in Reference 3 that for thick aerofoils the effect of trailing-edge angle is seriously overestimated by Equations (4) which imply a step change in x_{ai}/c as τ passes through the value given by $\tau = 100t/c$. In

addition to this Equations (4) take no account of the aerofoil leading-edge shape, because they simply represent the mean of the restricted range of leading-edge shapes of the aerofoils from which they were derived. Comparisons between computed values of x_{ai}/c for some of the aerofoils in Reference 10 with extremes of leading- and trailing-edge shape and values estimated by means of Equations (4) confirmed the necessity for a proper consideration of leading- and trailing-edge shapes via the parameters ρ/c and τ .

The geometrical data available for the GU-series of aerofoils in Reference 10 did not include leading-edge radius and it was therefore decided to replace this parameter by one based on the aerofoil thickness at a station as close to the leading edge as possible. The first tabulated thickness ordinate for the aerofoils of Reference 10 was that at $x/c = 0.005$ and this was normalized with respect to the maximum thickness to give the parameter $2z_{t1}/t$ as representative of the leading-edge shape.

Using the data of Reference 10 for a given value of τ it was established that the value of x_a/c for any given aerofoil was related to the value given by thin-aerofoil theory (i.e. 0.25) by the expression

$$\frac{x_{ai}}{c} = \frac{1}{4} \left(1 + f \frac{t}{c} \right) \quad (5)$$

where f is a factor dependent on x_t/c , $2z_{t1}/t$ and τ .

Figure 7a shows correlations for f based on computed data from Reference 10 for symmetrical and cambered aerofoils in the GU-series having trailing-edge angles within $\pm 1.5^\circ$ of 10° and 20° . Figure 7b compares computed values for x_{ai}/c with values obtained from the correlation curves of Figure 7a. These comparisons include aerofoils from Reference 10 with $\tau = 5^\circ$ and 30° in addition to those with $\tau = 10^\circ$ and 20° used to derive Figure 7a. Also included are the symmetrical aerofoils from Reference 3 which are listed in Table I. The comparison for these aerofoils is also shown separately in Figure 7c. To obtain values of x_{ai}/c from Figure 7a for aerofoils with trailing-edge angles other than 10° or 20° linear interpolation and extrapolation were employed. As noted on Figure 7b, 90% of the data were correlated to within ± 0.001 . Comparison of Figure 6 with Figure 7c illustrates the general measure of improvement obtained by a proper consideration of the secondary parameters related to the leading- and trailing-edges of the aerofoils. The scatter is in general reduced to ± 0.001 .

In order to use the correlation curves of Figure 7a for a given aerofoil it is necessary to know the value of the parameter $2z_{t1}/t$. The ordinate z_{t1}/c is often not given in the tabulations of aerofoil ordinates. A reasonably reliable estimate can be obtained by plotting z_t/c against $\sqrt{x/c}$, which is essentially linear near to the leading edge, and to read off the value corresponding to $x/c = 0.005$. Quite often the value of the leading-edge radius, ρ/c , is given in the tabulated geometrical data for an aerofoil and, where this is so, an alternative (and rapid) method of estimating $2z_{t1}/t$ from a direct correlation with leading-edge radius was felt to be desirable. Figure 8 illustrates such a correlation, and was derived from a large number of aerofoils including the blunt-nosed quasi-elliptical families from References 7 and 8, which allowed the correlation to be taken up to a value of $2z_{t1}/t$ of about 0.35. As an upper limit this is more than adequate for most purposes. It will be seen from Figure 8 that the expression

$$\frac{2z_{t1}}{t} = 0.201 \frac{(\rho/c)^{1/2}}{t/c} - 0.006 \frac{\rho/c}{(t/c)^2} \quad (6)$$

satisfactorily represents the data to within ± 0.01 , which is quite adequate for its intended purpose.

8. CONCLUSIONS

The correlations presented in this Memorandum provide a simple and rapid means of accurately estimating the lift-curve slope at zero lift, the zero-lift incidence, the zero-lift pitching moment coefficient and the position of the aerodynamic centre for any aerofoil in incompressible inviscid flow with a knowledge only of a few geometrical parameters describing the shape of the thickness distribution and camber line.