



ESDU TECHNICAL MEMORANDUM

Compendium of vibration damping data for aerospace structures

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ESDU TECHNICAL MEMORANDA

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THE PREPARATION OF THIS TECHNICAL MEMORANDUM

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COMPENDIUM OF VIBRATION DAMPING DATA FOR AEROSPACE STRUCTURES

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COMPENDIUM OF VIBRATION DAMPING DATA FOR AEROSPACE STRUCTURES

1. NOTATION

		Units	
		<i>SI</i>	<i>British</i>
A_p	area	m ²	in ²
a	length of panel in x -direction, length dimension for beam	m	in
b	length of panel in y -direction	m	in
C	dimensionless drag coefficient		
c	damping coefficient	Ns/m	lbfs/in
c_0	velocity of sound	m/s	in/s
c_p	specific heat per unit mass at constant pressure	J/kgK	*
$D_{11}, D_{22}, D_{12}, D_{66}$	flexural rigidities	Nm	lbf/in
E	Young's modulus	Pa	lbf/in ²
E_L	Young's modulus in longitudinal direction	Pa	lbf/in ²
E_T	Young's modulus in transverse direction	Pa	lbf/in ²
f	frequency	Hz	Hz
f_b	centre frequency	Hz	Hz
f_c	coincidence frequency	Hz	Hz
f_{mn}	natural frequency of plate at mode (m, n)	Hz	Hz
f_{11}	natural frequency of plate at mode (1, 1)	Hz	Hz
h	thickness, core thickness	m	in
G	shear modulus	Pa	lbf/in ²
G_{LT}	shear modulus in longitudinal/transverse plane	Pa	lbf/in ²
G_x, G_y	out-of-plane shear moduli in x - and y -axes	Pa	lbf/in ²

J_0	Bessel function of order zero		
k	wave number	m^{-1}	in^{-1}
k	spring stiffness	N/m	lbf/in
k_0	wave number (ω/c_0)	m^{-1}	in^{-1}
k_n	wave number of beam motion	m^{-1}	in^{-1}
k_t	transverse thermal conductivity	W/mK	lbf/s °F
L	length of beam	m	in
M	mass of structure	kg	*
m	modal number		
m	mass	kg	*
m_s	mass per unit area	kg/m ²	*
N	shear rigidity of panel	N/m	lbf/in
n	modal number; integer number for beam		
Q_{mn}	modal quality factor		
P	sound power	W	lbf in/s
R	radiation resistance	kg/s	*
R_r	radiation resistance for beam	kg/s	*
$R_{rad}^{2\pi}$	radiation resistance	kg/s	*
S_{11}	compliance in the direction of the longitudinal axis	m ² /N	in ² /lbf
s	number of fasteners per inch		
T	absolute temperature	K	
U	energy per cycle	J	lbf in/s
V	velocity amplitude of tip of cantilver	m/s	in/s
V_f	volume fraction		
v	velocity of structure	m/s	in/s
w	width of beam	m	in

α	linear coefficient of thermal expansion	K^{-1}	$(^{\circ}F)^{-1}$
δ	logarithmic decrement		
η	loss factor		
η_{av}	average loss factor		
ζ	viscous damping ratio		
ζ_F	equivalent viscous damping ratio of panels with fasteners		
ζ_M	equivalent viscous material damping ratio		
ζ_{mn}	equivalent viscous damping ratio for mode (m, n)		
ζ_{11}	equivalent viscous damping ratio for mode $(1, 1)$		
ρ	density (of beam)	kg/m^3	*
ρ_0	density of air	kg/m^3	*
$\Psi_{LT}, \Psi_L, \Psi_T$	longitudinal shear, longitudinal and tensile damping of composite layer (SDC)		
Ψ_m	SDC for matrix		
σ	radiation efficiency		
σ_{av}	average radiation efficiency		
σ_{corner}	radiation efficiency of corner modes		
σ_{edge}	radiation efficiency of edge modes		
σ_{unbaf}	radiation efficiency of unbaffled plate		
σ_{mn}	modal radiation efficiency at mode (m, n)		
σ_{rad}	radiation efficiency		
θ	orientation of fibres		
τ	relaxation time	s	s
ω	circular frequency	rad/s	rad/s
ω_n	natural frequency	rad/s	rad/s

Abbreviations

ACAP	advance composite airframe program
APWT	acoustic progressive wave tunnel
B/AI	boron in aluminium matrix
BFRP	boron fibre reinforced plastic
CFRP	carbon fibre reinforced plastic
ET LOX	external tank liquid oxygen
FML	fibre-metal laminates
GFRP	glass fibre reinforced plastic
GLARE	glass aluminium reinforced plastic
HCF	high cycle fatigue
HM	high modulus
HT	high temperature
IGV	inlet guide vane
MMC	metal matrix composites
PEEK	polyether ether ketone
PMC	polymer matrix composites
PMR	polymerization of monomer reactants
PWT	progressive wave tube
RT	room temperature
SDC	specific damping capacity
SDOF	single degree of freedom
SEA	statistical energy analysis
SFRP	synthetic fibre reinforced plastic
STS	space transportation system
SYNT	syntactic
TPS	thermal protection system

2. INTRODUCTION

This Item is concerned with energy dissipation from vibrating systems and how that dissipation can be quantified. Forces due to damping are particularly important in controlling the response of a vibrating structure at its resonances when the structure undergoes force, displacement or acoustic excitation. An increase in damping reduces the displacement response and hence the associated stresses. For a structure subjected to high intensity acoustic loading, a reduction in stresses leads to a longer fatigue life. In this data Item damping mechanisms are briefly described and some guidance is provided on the range of damping likely to be found in typical aerospace structures. The damping of fundamental components such as beams, plates and shells is first considered. Subsequently, wings, fuselages and control surfaces, which are built-up from the fundamental components, are also described.

Information on damping is mostly provided in terms of the equivalent viscous damping ratio or loss factor, but other means of specifying damping are given. It is expected that this data Item will be used in conjunction with Items such as ESDU 72005, 'The Estimation of r.m.s. stress in stiffened skin panels subjected to random acoustic loading'. Nomographs are used in ESDU 72005 which are based on a viscous damping ratio of 0.017 (1.7%), a value regarded as typical for aerospace structures, and, if a different value is used, a correction factor is applied. Alternatively, for a program available with ESDU 72005 a value of the damping ratio is required as input. Useful information on the damping of a wide range of structures is provided in ESDU 73011, Damping in acoustically excited structures. Numerous results have been collected together for (a) flat skin and stringer panels, (b) curved skin and stringer panels, (c) built-up structures with integrally machined skins and (d) honeycomb structures. The present item considers individual structures in more detail and in combination with ESDU 73011 is intended to enable the user to select an appropriate value for the damping ratio of the structure under consideration.

Although damping depends upon laws, such as the viscous force proportional to velocity or the friction force proportional to the normal force, in many cases the empirical laws are not obeyed. The phenomena of damping are varied and the controlling parameters difficult to formulate. There is a need for simplified models and, in the analysis of vibrating systems, there are two models, viscous and hysteretic, which are normally used to represent damping, even though damping as it occurs in structures is not exactly explained by these models. With the viscous damping model, damping is expressed in terms of the viscous damping ratio and with the hysteretic model the loss factor is used to define the damping. Damping from sources which do not have viscous or hysteretic origins will be included in the viscous damping ratio or loss factor. Material and acoustic radiation damping, which are of different origin and can, for certain situations, be predicted from the geometrical and material properties, should be added to the damping based on the empirical laws. Thermoelastic material damping and acoustic radiation damping are, in general, frequency dependent. The viscous and hysteretic models lead to a damping which is frequency independent.

Damping at resonance for various structures, even though the structures may be similar, shows considerable scatter when plotted as a function of frequency, or when both damping ratio and frequency are plotted logarithmically (Hay, 1972). Hay made the arbitrary decision to show the logarithm of the viscous damping ratio as proportional to the logarithm of the frequency with a negative slope, although the data could equally have been shown as frequency-independent. In effect the damping ratio was shown in the presentation by Hay as inversely proportional to frequency. In the presentation of damping data for different structures ESDU has followed Hay and shown the damping data as inversely proportional to frequency (ESDU, 1973). As may be seen from the data presented within, there is some experimental evidence to justify Hay's approach. The inverse proportionality of the damping data has been incorporated into models for acoustic fatigue, for example, by Wallace (Wallace, 1985). The assumption of inverse proportionality implies that the damping values approach zero at high frequencies, although a zero value is never attained. More information is obtained, if linear regression is applied to the logarithmic values of both the damping and frequency. In the case of linear regression of order one, the viscous damping ratios are in general inversely proportional not just to frequency but to frequency to a certain power. When damping values occur over a

small range of frequencies, it may be appropriate to apply linear regression not to the logarithmic values of the damping ratio and frequency, but to the linear values. If linear regression of order one is applied, it is important to state the range of frequencies involved, as it cannot be implied that damping values reach zero. Quite often in this Item linear regression of order one or two is used to model the damping data.

The other aspect of interest is the dependence of viscous damping ratio, or other methods of specifying damping, on the amplitude of vibration. As the viscous and hysteretic models of damping are based on linear theory, no dependence on amplitude is indicated. Similarly, there is no dependence on amplitude in the case of friction damping. The testing of structures at high amplitudes probably brings in nonlinearities that invalidate the usual methods of measuring the viscous damping ratio, which is a linear concept. Nonetheless, attempts are regularly made to measure the viscous damping ratio under laboratory conditions with the amplitude increasing. The general trend is that damping increases with amplitude, although the scatter in the results is considerable.

When considering values of damping published in the technical literature, it is important to know about the type of test and how the measurements were made. The most frequently used methods are based on the half-power bandwidth of a mode in a forced vibration test or on the envelope of the decay of a signal, which leads to the logarithmic decrement, but other methods have been used. It is equally important to establish whether the damping is measured as a viscous damping ratio, a loss factor, or as some other quantity. It is also important to know about the loading on the structure, in terms of, for example, the amplitude of the electromagnetic shaker table or the strain in the component under test. Damping measurements can be carried out in air at normal pressure or in a vacuum. Measurements made in air include a contribution from acoustic radiation damping, which in some cases can be comparable to damping from other sources. If measurements are carried out in air, the loss factor is a total loss factor, which includes the radiation loss factor. Unfortunately, in many cases the complete information regarding the tests may not be available.

Boundary conditions are important factors in controlling the damping values of a structure. If damping is measured in the laboratory, the boundary conditions may be specified, although realisation of the conditions may be difficult to achieve in practice. A structure, such as a panel, in, for example, an aircraft will have supports which will not be possible to define precisely. The estimation of damping values for the panel is difficult. Approximate prediction methods are available, if the panel is riveted to the rest of the structure. However, boundary conditions introduce considerable uncertainty into the estimation of damping values.

It is also necessary to quantify damping in Statistical Energy Analysis (SEA), a statistical method normally used at high frequencies. In SEA the energy transmission between subsystems is considered and any dissipation of energy has to be accounted for (Lyon and DeJong, 1995; ESDU 1999). The loss of energy from a subsystem is assessed in terms of a dissipation loss factor (directly related to the viscous damping ratio) and the accuracy of SEA depends upon good estimates of the loss factor. As SEA is often carried out in frequency bands, such as one-third octave bands, averaged loss factors in the bands are required, rather than modal loss factors. Special experimental techniques, based on steady-state power flow, have been developed to measure the averaged loss factors (Clarkson and Pope, 1981) and where available such loss factors are provided in the compendium. If averaged loss factors are not available, the damping data assembled in this Item for damping ratios or loss factors at a particular modal frequency may be used for an approximate value in the power balance equations required in SEA.

Note that, although the expression viscous damping ratio is often used in connection with damping, viscosity is rarely directly involved and the term 'equivalent viscous damping ratio' is more applicable, see Section 6.3, ESDU 09006, Introduction to damping.

The relationship between the different damping measures, specific damping capacity, viscous damping ratio, *etc.*, are given in Appendix.

2.1 VISCOUS MODEL

In the viscous model, the damping force is proportional to the relative velocity of vibrating components, see the Item on Mobilities and impedances of structures (ESDU, 2004, Section 2.2.3) and Introduction to damping (ESDU a, 2009, Section 6). In the case of a single degree of freedom system (SDOF) with viscous damping, a linear differential equation with constant coefficients may be derived which can be easily solved. The most convenient way of specifying damping is by means of the viscous damping ratio, the definition of which follows from the auxiliary equation formed from the differential equation. With a viscous damper the force-displacement curve forms a hysteresis loop during a cycle of vibration, and the energy dissipated, the area within the loop, is proportional to both the square of the amplitude of the displacement and the frequency. From the equations for a SDOF system the damping ratio may be shown, in terms of the mass m and the stiffness k , to be equal to $c/\{2\sqrt{(km)}\}$ (ESDU a, 2009). As the theory used is linear, the damping ratio is independent of amplitude. In this Item damping is often specified by the viscous damping ratio. On some graphical information illustrating damping for different components a line is drawn showing ζ inversely proportional to frequency, after Hay (Hay, 1972). In some sketches in this Item linear regression of order one has been applied to the data and the equation of the regression line given.

2.2 HYSTERETIC MODEL

With hysteretic damping the stiffness of a SDOF system is represented as complex with the form $k(1 + i\eta)$ (ESDU, 2004, Section 3.3; ESDU, 2009 a, Section 5). The ratio of the imaginary to the real part of the stiffness is the loss factor, η . The loss factor of a system with hysteretic damping is equal to twice the damping ratio of a system with viscous damping which has the same resonance frequency, although this relationship is strictly only applicable at resonance. The damping ratio, loss factor and other terms used to specify damping are listed in the Appendix of this Item. In the hysteretic model of a damper, force versus displacement exhibits hysteresis, as the force is applied and removed during a cycle. The area within the hysteresis loop is the energy dissipated by the damper and is proportional for harmonic excitation to the square of the amplitude of the displacement, but is independent of frequency. The ratio of the energy dissipated to the peak potential energy (proportional to the loss factor) is independent of amplitude, as both terms are proportional to the square of amplitude. This ratio (or the loss factor) is inversely proportional to the square of the frequency at resonance. The independence of loss factor or damping ratio with respect to amplitude is a characteristic of a linear system.

Hysteretic damping is thought to represent the damping of materials and structures better than a model based on viscous damping. There are, however, some theoretical problems with the hysteretic model. It predicts a finite amount of energy absorbed at zero frequency, as the energy absorbed is proportional to amplitude squared and independent of frequency. At zero frequency there is a finite phase angle ($\arctan 1/\eta$, see ESDU 04009, Section 3.3). It is said to contradict the concept of causality (Crandall, 1970). The differential equations involving hysteretic damping only have meaning when the excitation is harmonic and cannot be applied to arbitrary motion (Milne, 1970). Thus, difficulties occur when attempts are made to transform from the frequency to the time domain. Problems with hysteretic damping do not arise if attention is confined to frequencies in the region of resonance.

A discussion of viscous damping, called the Kelvin-Voigt model, and hysteretic damping is found in ESDU 09005, Introduction to damping (ESDU a, 2009).

3. DAMPING SOURCES

The vibration response of a component, such as a panel in an aircraft, may be caused by transmission of energy from other parts of the aircraft, by air flow in the boundary layer or by acoustic means from the sound generated by the engines. In any estimate of the response, by, for example, Statistical Energy

Analysis, damping is a significant parameter. Damping of the vibration of the panel or any other structure can be achieved by four main methods.

- (1) Material damping in which the dissipation of energy is distributed through the bulk of the material of the structure.
- (2) Boundary damping where the dissipation is associated with friction at junctions, joints or interfaces between parts of the structure.
- (3) Fluid damping where fluid is in contact with the structure and dissipation is by viscous effects, turbulent flow or other local effects caused by vibration of the structure (near field motion). Cantilevered beams can undergo large amplitude vibrations which are damped by turbulent flow. Compressor blades are subjected to aerodynamic forces, which provide damping when out of phase with the blade motion.
- (4) Radiation damping in which sound energy is propagated into the far field away from the structure.

Material and boundary damping may be present simultaneously in a structure and is often collectively referred to as structural damping. Some forms of fluid damping are closely related to boundary damping. For example, some joints may have lubrication which is an integral part of the structure. There may be a vibratory motion of the fluid between bolted surfaces, see Section 6.1.

4. MATERIAL DAMPING

In all materials there is some conversion of energy into heat when a stress is applied, a process which is referred to as material damping (ESDUB, 2009). Even in the absence of stress there are several micro-mechanisms in metals involving defects in the crystalline structure that cause relaxation processes. The stress-strain cycle in a relaxation process forms a hysteretic loop, the area of which is proportional to the energy lost per cycle. With relaxation phenomena there is no permanent deformation and the associated damping ratio is generally independent of amplitude but dependent on frequency.

4.1 Thermoelastic Damping

Thermoelastic damping is an example of a relaxation process. There are several relaxation mechanisms that cause thermoelastic damping, but the most important is due to the transport of thermal currents (Zener, 1948). In the case of a beam in bending, as it vibrates the part of the beam in tension cools rapidly. Similarly, the part in compression heats up. Heat flows from the hot to the cold zone across the neutral axis, according to the laws of thermal conductivity, and thermal currents occur in the direction of the beam thickness. For a beam the loss factor at frequency ω in rad/s is given by

$$\eta = \frac{\alpha^2 E T}{\rho c_p} \left(\frac{\omega \tau}{1 + \omega^2 \tau^2} \right), \quad (4.1)$$

where α is the linear coefficient of thermal expansion,
 E is Young's modulus of the material of the beam,
 T is the absolute temperature,
 ρ is the density of the beam material,
 c_p is specific heat per unit mass at constant pressure and
 τ is a relaxation time.

The relaxation time is given by

$$\tau = \frac{h^2 \rho c_p}{\pi^2 k_t}, \quad (4.2)$$

where h is the thickness of the beam and k_t is the transverse thermal conductivity (in the direction of the thickness). The equivalent viscous damping ratio is half the loss factor.

When $\omega = 1/\tau$ the damping is a maximum and the frequency is sometimes called the Zener relaxation frequency. The Zener-type damping is very small, but has been shown to provide a lower limit for material damping for both 2024-T3 and 6061-T4 aluminium alloys over a wide frequency range, applicable at low strain rates (Crawley and Sheen, 1984). Experimental confirmation of thermoelastic damping has been provided for small beams made of an aluminium alloy (Granick and Stern, 1965; Gibson and Plunkett, 1977). Further discussion of thermoelastic damping is found in ESDU 09006 (ESDU b, 2009). The damping is strain dependent, and increases in a particular component, such as an aluminium alloy beam, as the strain increases. Damping is dependent on the surrounding air pressure. If measurements are carried out at atmospheric pressure, damping values are greater than for equivalent measurements in a vacuum, indicating that radiation damping is contributing to the overall damping. Experimental evidence has been provided that the material damping of aluminium is independent of amplitude when the pressure is reduced to 1 mm Hg (Gibson and Plunkett, 1977).

With the application of stress the damping in a structural component increases. The process is irreversible and the component suffers a permanent deformation after the applied stress is removed. The increase in damping with stress is greater at frequencies less than the Zener relaxation frequency (obtained from Equation (4.2) as $1/\tau$ rad/s) (Crawley and Sheen, 1984; Wren and Kinra, 1989).

4.2 Magnetomechanical Damping

In general, ferromagnetic materials such as steel have better damping than non-ferrous materials, such as aluminium and titanium alloys. Ferromagnetic materials can achieve additional damping from magneto-mechanical damping, an irreversible process caused by the stress-induced motion of magnetic domain walls. Magnetomechanical damping is dependent on the applied cyclical stress, but is independent of frequency. The damping of steels depends upon the heat treatment (it is greater for annealed than for cold-worked materials), the temperature (the magneto-mechanical effect disappears above the Curie temperature, the upper limit for ferromagnetism) and the presence of a magnetic field (a saturation magnetic field removes the magneto-mechanical damping). Further complicating effects are related to a 'cycle stress sensitivity limit', above which stress the damping can increase considerably (Lazan, 1968; Adams, 1972). In the case of stainless steels those which are ferromagnetic, such as type 03 (12% Cr and 5% Ni), have greater damping than steels which are not ferromagnetic, such as 18/8 steel (18% Cr, 8% Ni) or Sandvik steel (1% Cr, 0.2% Si, 1% C, 0.26% Mn and 0.24% Mo) (Adams, 1972; Birchak, 1977; Goodman, 1988).

Although there are some relaxation processes, such as thermoelastic damping, which are frequency-dependent, metals and alloys which are subjected to engineering stresses have damping which is stress-dependent and it is not realistic to quote a single value for the loss factor or viscous damping ratio. The dependence of the damping of metals and alloys on stress has been described graphically in ESDU 09006, Sketch 10.1. Similar information may be found in other references (Adams, 1972; Birchak, 1977; Soovere and Drake, 1985).

4.3 Average Damping Properties of Metals and Structural Alloys

Although damping of alloys can depend on frequency and applied stress, it is sometimes necessary to assign an average value of the damping that can be used when detailed information is not available or not required. In Table 4.1 average values of loss factors are provided for various metals and alloys under low stress levels and at room temperature (Rawal and Misra, 1986). The values given in the Table are approximate, but may be useful as an initial guide.

TABLE 4.1 Damping properties of metals and structural alloys

<i>Material</i>	<i>Loss factor</i>	<i>Frequency range</i>
Aluminium alloy, 2024 T-3	2.4×10^{-3}	10 – 100 Hz
Aluminium alloy, 6061 T-6	2.0×10^{-3}	10 – 100 Hz
Aluminium alloy, Fe-12, Cr-3	3.6×10^{-2}	kHz range
Steel, 1020	5.0×10^{-4}	50 – 500 Hz
Steel, 310	1.0×10^{-3}	kHz range
Stainless steel, 403	1.4×10^{-3}	kHz range
Cast iron (coarse graphite flake)	5.7×10^{-2}	kHz range
Titanium alloy, Ti-6Al-4V	2.0×10^{-4}	@ 40 kHz
Brass	9.0×10^{-5}	kHz range
NIVCO	3.0×10^{-2}	kHz range
Nitinol, Ni,Ti-45	2.8×10^{-2}	kHz range
Sonoston, Mn, Cu-36, Al-4.5, Ni-2, Fe-3	4.0×10^{-2}	kHz range
Ingramute, Cu, Mn-44, Al-1.8,	3.1×10^{-2}	kHz range
Cast pure Mg	3.9×10^{-2}	kHz range

Nitinol, Sonoston and Ingramute are examples of Hidamets, *i.e.*, they posses high material damping (ESDUb, 2009).

5. DAMPING OF COMPOSITES

Composites are formed from fibres embedded in a matrix. The main types of fibre are glass, aramid (Kevlar), carbon and boron. Carbon is frequently used, as it is stable and has a high stiffness. One function of the matrix is to transfer the external load to the fibres, whilst providing protection for the fibres. Composites may be classified according to the type of resin used for the matrix. Polymer matrix composites have thermoset or thermoplastic resins as the matrix. Epoxy resin is commonly used as a thermosetting plastic for the matrix in fibre-reinforced plastics (FRP). The polymer materials used are viscoelastic in nature. The material is described as glassy at temperatures below the transition temperature, which defines the upper limit for structural applications. Other possible matrix materials include metals, ceramics and carbon. Each composite type may be subdivided according to whether the reinforcement is formed from continuous fibres, discontinuous woven fibres or particulates. Additionally, there are laminated composites which consist of thin layers of different materials bonded together with each layer at different angles. One source

of internal damping in composites derives from microplastic and viscoelastic phenomena associated with the matrix and relative slipping between the matrix and the fibre reinforcement (Adams, 1986; ESDUb, 2009). There is also a contribution from thermoelastic damping (Sun *et al*, 1986).

Section 5 is concerned with the basic damping of composites, mostly from measurements of composites in form of small beams with unidirectional fibres and with the beams subjected to low levels of strain. It is meant to represent, as far as possible, the damping of composites, rather than structures, such as beams, plates and cylinders, under engineering strains.

5.1 Polymer Matrix Composites

The best known polymer matrix composites are carbon fibre reinforced plastics (CFRP) and those that have glass fibre as reinforcement. Carbon (graphite), boron and aramid (Kevlar) fibres are additionally used. Epoxy and polyester resins are frequently used for the matrix. Experiments with CFRP beams have shown that where the fibres are randomly distributed or short fibres at right angles to the beam axis the damping is greater than with continuous fibres along the axis of the beam (White, 1975). On the other hand the higher damping is associated with low elastic modulus values. The damping is influenced by the material damping of the resin. Some typical values of viscous damping ratios are presented in this Section for single layer and unidirectional composites and represent the composites in their most basic form. The data have been obtained from tests on small beams. Information on the damping of laminates, in which there are multiple layers with the fibres in different directions, is provided in a later section.

In general carbon fibre reinforced plastic has greater damping than glass fibre reinforced plastics (GFRP) and Kevlar has greater damping than GFRP. Comparative damping results can be obtained from the work of White, White and Palmer and Gibson *et al*. For a CFRP beam, with an epoxy matrix, the equivalent viscous damping ratio, ζ , is 0.45×10^{-3} for continuous fibres aligned in the direction of the beam and 4.4×10^{-3} for the fibres aligned in the transverse direction (White, 1975). For short fibres (0.5 mm long) which are aligned with the beam ζ increases to 0.95×10^{-3} (White and Palmer, 1984). For a glass fibre reinforced plastic (epoxy) beam the viscous damping ratio is 1.1×10^{-3} for continuous fibres and 3.3×10^{-3} for short fibres (1.6 mm long), with both sets of fibres in line with the beam (Gibson *et al*, 1984). Values of ζ for Kevlar in epoxy are greater, 5.7×10^{-3} for continuous fibres and 7.7×10^{-3} for short fibres (1.6 mm) (Gibson *et al*, 1984).

5.1.1 Damping of resins

Some typical values of viscous damping ratios for resins, which form the matrix, are available and are given in Table 5.1. In the case of some results the shear modulus, G , is available (White, 1975). According to Georgi, when the epoxy resin is made into a beam, which is subjected to bending vibrations, the viscous damping ratio increases from 11×10^{-3} for mode 1 (at 300 Hz) to 26×10^{-3} for mode 6 at 5 kHz (Georgi, 1979). The damping was dependent on temperature as well as frequency and reached a maximum value at 0°C for all modes tested. In longitudinal vibration the viscous damping ratio was found to be independent of the mode number (and frequency) with a value 13×10^{-3} . Young's modulus of the epoxy is about 3.8 GPa at 0°C , decreases to a value of 2.3 GPa as the temperature increases to 120°C and increase to a value of about 7 GPa at -120°C (Georgi, 1979). A viscous damping ratio of 35×10^{-3} has been quoted for a resin referred to as polypropylene (density 910 kg/m^3 , Young's modulus 1.5 GPa) and 40×10^{-3} for a resin referred to as copolymer EPF30R (density 900 kg/m^3 , Young's modulus 1.25 GPa (Balis Crema *et al*, 1989). The results of Georgi and White forepoxy are summarised in Table 5.1.

TABLE 5.1 Viscous damping ratios of polymer matrices

Type	Resin name	Test	f (Hz)	G (GPa)	ζ (10^{-3})	Reference
Epoxy	ERLA 4617 DDM	Torsion		4.68	8.3	White, 1975
Epoxy	DX 210	Torsion		3.43	5.56	
Epoxy	CY209/HT972	Bending	240	—	11	Georgi, 1979
Epoxy	CY209/HT972	Bending	5000	—	26	
Epoxy	CY209/HT972	Longitudinal	250 to 25 000	—	13	

Extensive properties of resin beams are available (Willway and White, 1989). Information related to the damping of the beams is given in the form of loss factors for different mixtures of resins.

5.1.2 General results and comments on damping of composites

More damping results are shown in Table 5.2 for boron fibre reinforced composites (BFRP) (Georgi, 1979), carbon fibre reinforced composites (CFRP) (White, 1975; Georgi, 1979; White and Palmer, 1984), graphite reinforced composites and Kevlar (Gibson *et al*, 1984). Note that the carbon and graphite reinforced composites are essentially the same. The results listed were carried out on small specimens, usually in the form of beams, under low strain. The type of test used to obtain the results is listed, *i.e.*, either bending, torsion or longitudinal vibration at frequency f . The orientation of the fibres, θ degrees, is included in the table, together with the temperature at which the test was carried out. The volume fraction, V_f , the ratio of fibres to the total volume of the composite, is listed.

TABLE 5.2 Viscous damping ratios of unidirectional polymer matrix composites

PMC	Test	θ (deg.)	Temperature (°C)	f (Hz)	V_f	ζ (10^{-3})	Remarks
BFRP	Bending	0	RT	480	0.55	0.72	mode 1, Georgi, 1979
BFRP	Bending	0	RT	8500	0.55	4.3	mode 6, Georgi, 1979
BFRP	Bending	90	RT	130	0.55	9.6	mode 1, Georgi, 1979
BFRP	Bending	90	RT	2000	0.55	12.1	mode 6, Georgi, 1979
CFRP	Bending	0	RT	1200	0.5	0.64	mode 1, Georgi, 1979
CFRP	Bending	0	RT	6000	0.5	2.1	mode 6, Georgi, 1979
CFRP	Bending	90	RT	90	0.5	7.3	mode 1, Georgi, 1979
CFRP	Bending	90	RT	1700	0.5	8.9	mode 6, Georgi, 1979
CFRP	Longitudinal	0	20	3200	0.5	1.3	
CFRP	Longitudinal	90	20	7000	0.5	13.5	
CFRP	Longitudinal	0	0	3200	0.5	1.3	

TABLE 5.2 Viscous damping ratios of unidirectional polymer matrix composites (Continued)

PMC	Test	θ (deg.)	Temperature (°C)	f (Hz)	V_f	ζ (10^{-3})	Remarks
CFRP	Longitudinal	90	0	7000	0.5	18.3	
CFRP	Bending	0	RT	140-600	0.5	0.45	White, 1975
CFRP	Bending	90	RT	50	0.5	4.2	Continuous fibres
CFRP	Bending	90	RT	140	0.5	4.4	ERLA 4617 DDM
CFRP	Bending	0	RT	100	0.5	4.0	Random
CFRP	Bending	0	RT	400	0.5	4.7	Random
CFRP	Bending	0	RT	50	0.5	4.6	Short
CFRP	Bending	0	RT	50	0.5	4.6	Short
CFRP	Bending	0	RT	200	0.5	4.9	Short
CFRP	Bending	0	RT	—	0.55	0.95	Short fibres, 0.5 mm White & Palmer, 1984, HM-S/DX210
CFRP	Bending	0	RT	—	0.64	0.5	Short fibres, 3 mm
CFRP	Bending	0	RT	—	0.48	0.65	
CFRP	Bending (+45/-45) ₄	0	40	—	0.60	6.0	
GFRP	Bending	0	RT	—	0.50	0.64	Eglass DX210 (Gibson, 1984)
(Gr/epoxy) ₁₂	Bending	0	RT	60	0.654	1.1	Continuous fibres
Gr/epoxy	Bending	0	RT	1050	0.654	1.4	Continuous fibres
Gr/epoxy	Bending	0	RT	60	0.654	1.6	3.2 mm fibres
Gr/epoxy	Bending	0	RT	950	0.654	3.0	3.2 mm fibres
Gr/epoxy	Bending	0	RT	60	0.654	3.3	1.6 mm fibres
Gr/epoxy	Bending	0	RT	850	0.654	5.0	1.6 mm fibres
Kevlar/epoxy	Bending	0	RT	40	0.661	5.7	Continuous fibres (Gibson, 1984)
Kevlar/epoxy	Bending	0	RT	730	0.661	7.5	Continuous fibres
Kevlar/epoxy	Bending	0	RT	40	0.661	6.6	3.2 mm fibres
Kevlar/epoxy	Bending	0	RT	680	0.661	7.6	3.2 mm fibres
Kevlar/epoxy	Bending	0	RT	40	0.661	7.7	1.6 mm fibres
(Kevlar/epoxy) ₁₂	Bending	0	RT	620	0.661	10.4	1.6 mm fibres

For composites with glass fibres (HT-S/epoxy) the viscous damping ratio is between 1.3 and 2.0×10^{-4} , whilst for E glass/epoxy the damping ratio is greater at 9×10^{-4} (Rawal and Misra, 1986).

In general, the damping ratio is greater for short fibres than for continuous fibres, but there is a reduction in Young's modulus (Soovere and Drake, 1985). White found that there was an increase in damping of a factor of 10 with continuous fibres as the orientation of the unidirectional fibres change from 0° to 90°

(White, 1975). The increase in damping with the change in orientation is confirmed by some quoted results for Gr/epoxy, namely, Gr/epoxy (0)₈ has a peak viscous damping ratio of 0.64×10^{-3} , Gr/epoxy (90)₈ a value of 5.5×10^{-3} and Gr/epoxy (± 45)₂₅ a viscous damping ratio of 5.5×10^{-3} for a frequency range from 40 to 140 Hz for an applied stress which resulted in 700 microstrain (Rawal and Misra, 1986). However, there was a corresponding decrease in the shear modulus by a factor of 10 (White, 1975).

Just as short fibres, compared to continuous fibres, resulted in a small increase in the damping, randomized fibres result in the same damping as short fibres in the same direction (0°) (White, 1975). There is some evidence of a slight decrease in the viscous damping ratio as the fibre volume, V_f , increases in the case of graphite and glass fibres in a polyester matrix (Soovere and Drake, 1985). The effect of an increase of temperature on CFRP is to decrease the damping, whereas moisture ingress causes the damping to increase (Galea, 1989).

Attempts have been made to maximize the damping of composites by the selection of highly dissipative resins and fibres which dissipate more energy (Benčekchou and White, 1997). The PMR15 prepreg was selected as a composite likely to have high damping (Benčekchou and White, 1997). This material was of (0°/90°) woven-glass form and had a 58% volume fraction. The specimens were 20 mm long by 12 mm wide. It was reported to have good fatigue resistance to acoustic loading. As an example, a sample with a stacking sequence of (0/90)₄ had a viscous damping ratio of 5.9×10^{-3} at 40°C. The damping increased with temperature and at the glass transition temperature of the resin the viscous damping ratio was 64.5×10^{-3} (Benčekchou and White, 1997).

Whilst high damping is very desirable, it is often the case that stiffness is low, a point which is demonstrated by some results provided by Georgi (Georgi, 1979). The equivalent viscous damping ratios and corresponding values of Young's moduli are given in Table 5.3 for different composites and show the general trend. In each case they were obtained from experiments on a simply-supported beam, 1 m long, by 67 mm wide and 1 mm thick, with the fibres aligned along the beam axis and a volume fraction of between 0.5 and 0.6. The beams were vibrating in bending at the fundamental frequency with amplitude less than 5µm.

TABLE 5.3 Viscous damping ratio as related to Young's modulus

Material	Viscous damping ratio $\times 10^{-3}$	Young's modulus Pa $\times 10^9$	Frequency Hz	Density kg/m ³
Epoxy	11.1	3.5	1.69	1275
SFRP	6.4	22.0	4.23	1279
CFRP/SFRP	1.5	97.0	9.33	1160
GFRP	0.97	45.0	4.88	1967
BFRP	0.56	190.0	10.02	1970
CFRP	0.48	160.0	10.28	1576
B/Al	0.38	235.0	9.68	2610
Al	0.016	69.0	5.06	2804

Ideally, when damping values for composites are listed, the corresponding moduli should be quoted.

6. STRUCTURAL DAMPING

Material damping is small, but more damping is likely to result when two components are fastened together by rivets or bolts. In the case of two surfaces pressed together where the relative motion is normal to the surfaces the damping is small, if the joint is dry. In a lubricated joint there is more damping which increases with the viscosity and quantity of the lubricant, but does not vary much with frequency (Hother-Lushington and Johnson, 1963). More damping occurs in a lap joint where there is a shearing action as two parallel surfaces move relative to each other. In general, energy dissipation increases with the friction force which depends upon the coefficient of friction and the pressure clamping the two plates together. As the externally applied force separating the plates increases, a theoretical investigation shows that the damping increases to a maximum value and then decreases (Earles, 1966). The maximum value of the energy loss occurs when the applied force is only transmitted by friction through the joint. Normally, the plates are kept together by a bolt or rivet and the effective clamping pressure depends upon the degree of tightening of the bolt or the method of inserting the rivet. When the applied force tending to shear the plates apart is greater than the friction force, a shear force is applied to the bolt or rivet joining the plates and the damping effect decreases. Joints that achieve the optimum condition for damping are likely to be too loose for structural requirements. Lubricants applied to the mating surfaces increase the damping, as do viscoelastic inserts (Mentel, 1967).

Optimum values of damping, as the normal force is changed, can be achieved in the laboratory. Experiments on a cantilevered steel beam, $508 \times 25.4 \times 6.4$ mm, made from two 3.2 mm thick beams screwed together to make a single beam by two rows of socket-head cap screws have shown that the viscous damping ratio, as the tightening torque increased, increased from zero to a maximum of 0.11 when the torque applied to the screws was 0.0423 N m and then decreased to a constant value of about 4.6×10^{-3} for torque values between 0.226 and 2.26 N m. (McWithey and Hayduk, 1965) The results were confirmed at both normal pressure and in a vacuum. In a vacuum with two clean surfaces in contact there is a danger of cold welding, but this was not observed by McWithey and Hayduk.

6.1 Gas Pumping Effect

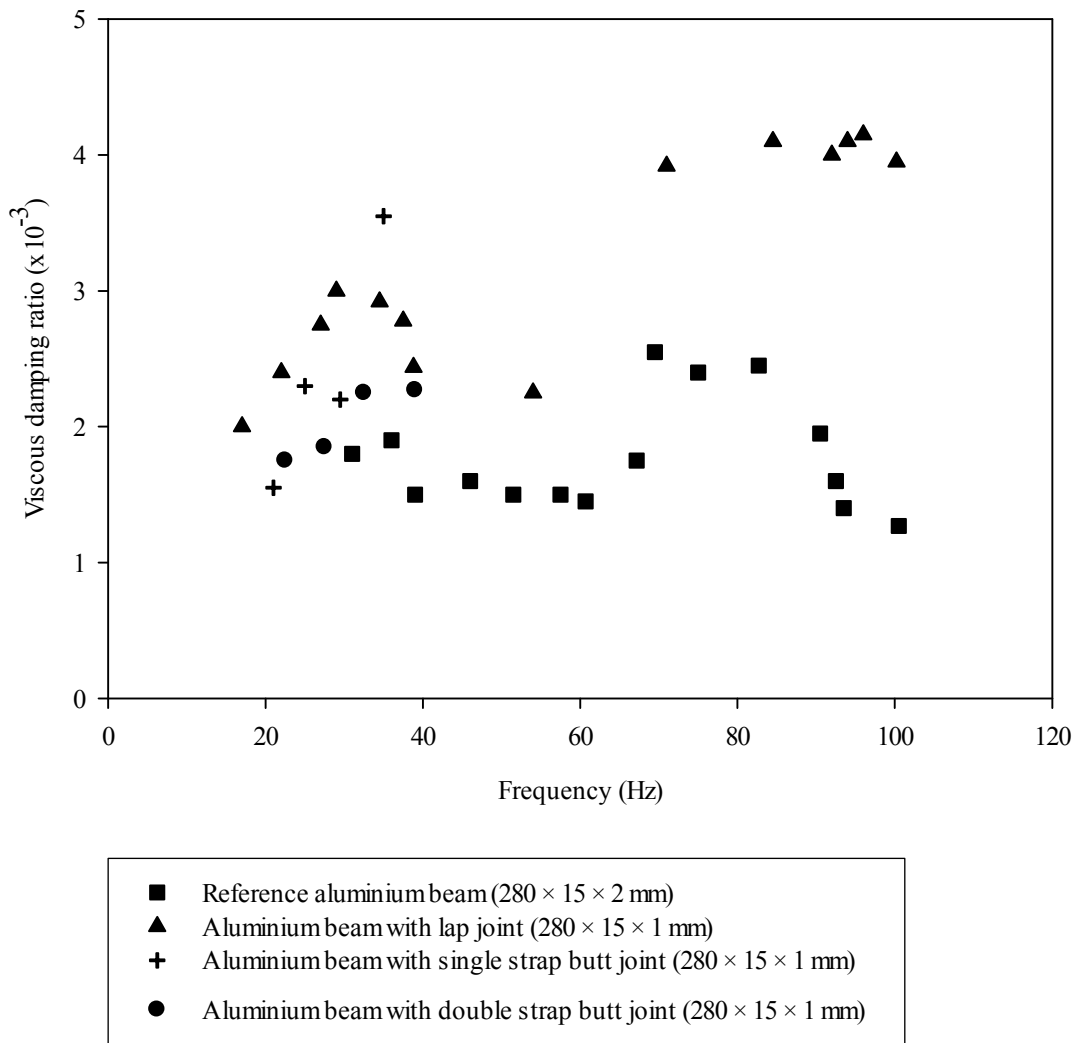
A beam attached to a panel by rivets, bolts or spot welds provides more damping than a beam continuously welded to a plate. It is claimed that with such a built-up structure interface slip is not as important as a 'gas pumping' effect in which the relative motion of the beam and plate, normal to the surface of the plate, creates an oscillatory movement of the trapped fluid (Maidanik, 1966; Ungar, 1973; Maidanik, 1966; Ungar and Carbonell, 1966; Ungar and Maidanik, 1968). In the case of a stringer riveted to a panel the air between the stringer and the plate is pumped out as the surfaces move together and rushes in again as the surfaces open during the vibration cycle. This type of damping is reduced if the air pressure is decreased. It depends upon the ratio of bolt or rivet spacing to the panel flexural wavelength and generally increases with this ratio, *i.e.*, the gas pumping effect is more important at high frequencies. The damping is independent of the materials of the panel and beam. In general the damping is not affected by the stiffnesses of either the panel or beam. Maidanik has developed a theory to predict the damping caused by gas pumping (Maidanik, 1966). The theory depends on parameters, such as the average separation between the beam and plate. This separation is very small (typically about 0.04 mm) and is difficult to measure or predict and, consequently, their theory has limited use for design purposes.

Quite often viscoelastic jointing compounds are used with fasteners so that the compound forms a layer between the rivet or bolt head and the mating surface. In this case Mentel found that damping mostly derived from a rocking motion of the rivet head relative to the surface rather than from normal motion (Mentel, 1967). Damping results from a viscous shear in the compound and a pumping action into and out of the space between the mating surfaces. In general a rocking of the rivet or bolt relative to the mating surfaces is a better source of damping than an oscillating normal pressure (Beards, 1977).

6.2 Structural Damping in Bonded Joints

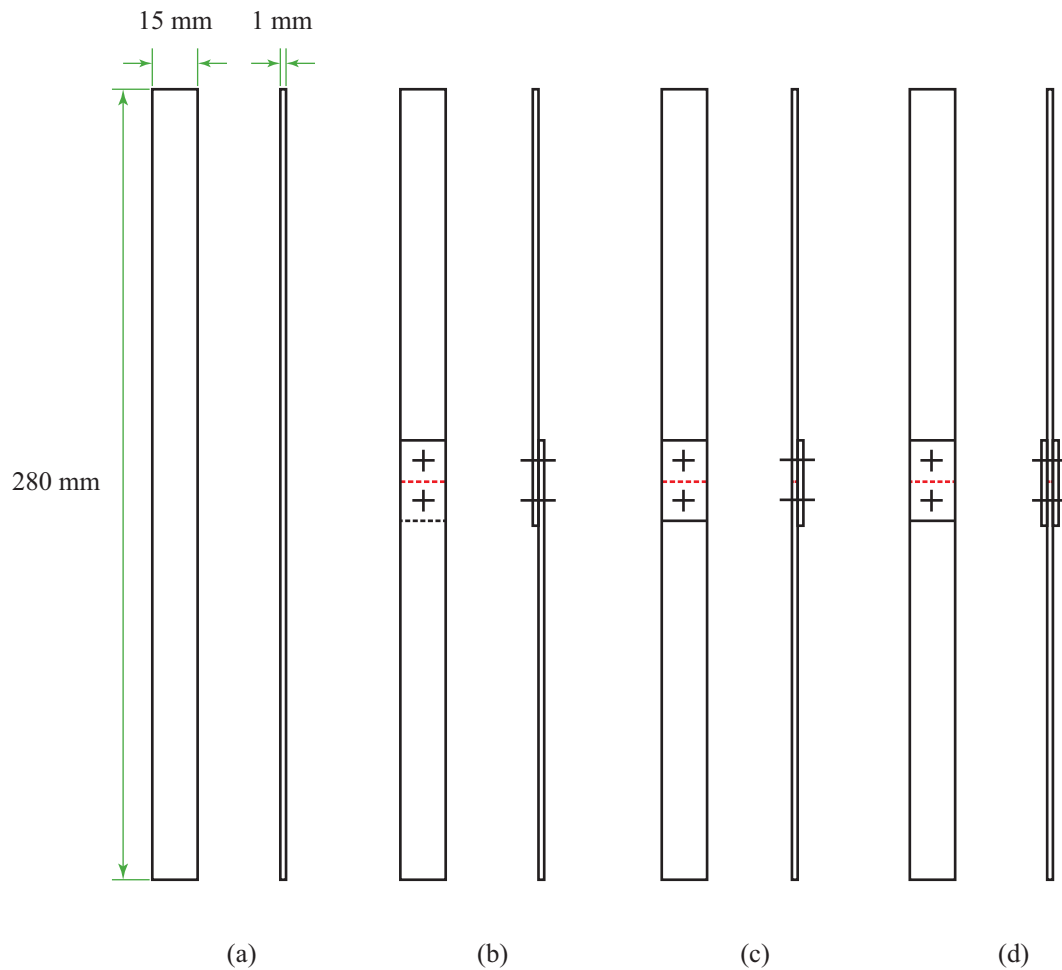
6.2.1 Structural Damping with Lap and Butt Joints in aluminium beams

The overall effect of riveted joints on the equivalent viscous damping ratio can be seen in Sketch 6.1a where a plain beam is compared with beams with a lap joint, a butt joint with a single strap and a butt joint with a double strap (Balis Crema *et al*, 1979). The beams were made from aluminium alloy (2024-T3-QQ-A-250/4) and had dimensions $280 \times 15 \times 2$ mm. For the lap joints the overlap was 30 mm and for the butt joints the strap lengths were also 30 mm. In all cases two rivets were used as fasteners. The beams were clamped at both ends and set into resonance by an electrodynamic vibrator. The damping ratios were measured by both the bandwidth and the logarithmic decrement method. As a reference the Zener-type damping for an aluminium alloy beam has been added to the results in Sketch 6.1a.



Sketch 6.1a Viscous damping ratio as a function of frequency for aluminium alloy beams, plain, and riveted with lap joint, single butt joint and double butt joint (Balis Crema *et al*, 1979)

The diagrams of the joint described in Sketch 6.1a are shown in Sketch 6.1b.



**Sketch 6.1b (a) plain beam (b) beams joined by lap joint
(c) single butt joint and (d) double butt joint**

The riveted joints are responsible for approximately doubling the damping of the beams. The results of Sketch 6.1 and other results (Crema *et al*, 1979) indicate that the lap joint provides about the same amount of damping as the butt joints. Apart from at the higher frequencies the joints are contributing about 0.001 to the damping ratio. Tests in a vacuum reduced the damping ratio by about 20% (Crema *et al*, 1979). Radiation damping would not be expected to be large for a thin beam. As the gas pumping effect is greater for a line of rivets, the effect for the beams described in Sketch 6.1b would be limited. The amplitudes for the tests were small, thus aerodynamic or air damping would not play a great part.

6.2.2 Structural damping with adhesively bonded lap joints in steel beams

Adams, Cooper and Pearson have measured the damping in steel beams, adhesively bonded with a variable overlap. The steel used was high strength and low damping with a Young's modulus of 200 GPa. The length of the beam was maintained at 200 mm and the overlap ratio varied from about 0.1 to 1.0. Of the four adhesives tested the two most successful were a structural bonding tape (9245 manufactured by 3M) and a two-part polyurethane (3532 manufactured by 3M). Damping values were obtained from the free decay of the beams in flexure. The greatest damping was obtained when the overlap ratio (length of bonded joint to total length) was about 0.2 to 0.25 where the specific damping ratio was about 9 to 12%.

6.2.3 Composite beams with lap joints

Some measurements have been made on composite beams with a lap joint and an epoxy adhesive used as the fastener (Rao *et al*, 1989). The overlapping beams were made from a graphite epoxy material and the length of each unbonded beam was 122.5 mm, the width 28 mm and the thickness 2.5 mm. The overlap ratio of the beams was either 0.2 or 0.4 (overlap of 24.5 or 49.0 mm). The beams were excited by an impact hammer and the viscous damping ratio obtained from the half power points of the frequency response function. Simply-supported boundary conditions were assumed. For an overlap ratio of 0.2 the viscous damping ratio was experimentally determined as 1.51% for a natural frequency of 207 Hz in the first mode, 1.61% at 986 Hz for mode no. 2 and 2.29% at 1984 Hz for mode 3. For an overlap ratio of 0.4 the viscous damping ratio was 0.68% at 270.6 Hz, 1.43% at 1075 Hz and 0.83% at 2684 Hz. The data is summarised in Table 6.1.

TABLE 6.1 Viscous damping ratios for simply-supported composite beam with lap joints with overlap ratios of 0.2 and 0.4

Overlap ratio 0.2		Overlap ratio 0.4	
Frequency (Hz)	Viscous damping ratio (%)	Frequency (Hz)	Viscous damping ratio (%)
207	1.51	270.6	0.68
986	1.61	1075	1.43
		2684	0.83

With the same beams free-free conditions were simulated by mounting the beam on an electrodynamic shaker. With free-free boundary conditions the viscous damping ratios were reduced to 0.20%, 0.16% and 0.10% for the first three modes with an overlap ratio of 0.2. For an overlap ratio of 0.4 the corresponding viscous damping ratios were 0.43, 0.12 and 0.15. The results appear to indicate that friction effects at the supports increased the damping considerably.

A further experiment on two graphite/epoxy beams joined together by a double butt joint showed that the butt joint had only a small influence on the damping, compared with a single composite beam with the same dimensions (Rao *et al*, 1989). The beams were supported in the middle on an electrodynamic shaker in order to simulate free-free boundary conditions. For the double-butt-jointed specimens the average viscous damping ratio from results on five beams was 0.114% at 259 Hz for mode 1 and 0.155% for mode 3 at 1376 Hz. For the plain graphite/epoxy beams the corresponding results for the viscous damping ratios were 0.107% at 337 Hz for mode 1 and 0.138% at 1779 Hz for mode 3.

Although the composite beam and the aluminium beams are not directly comparable, the trends are that the damping of the composite beam is less than that of the aluminium, mainly because the composite beam uses rivets as fastener, rather than adhesives.

6.3 Estimation of Structural Damping

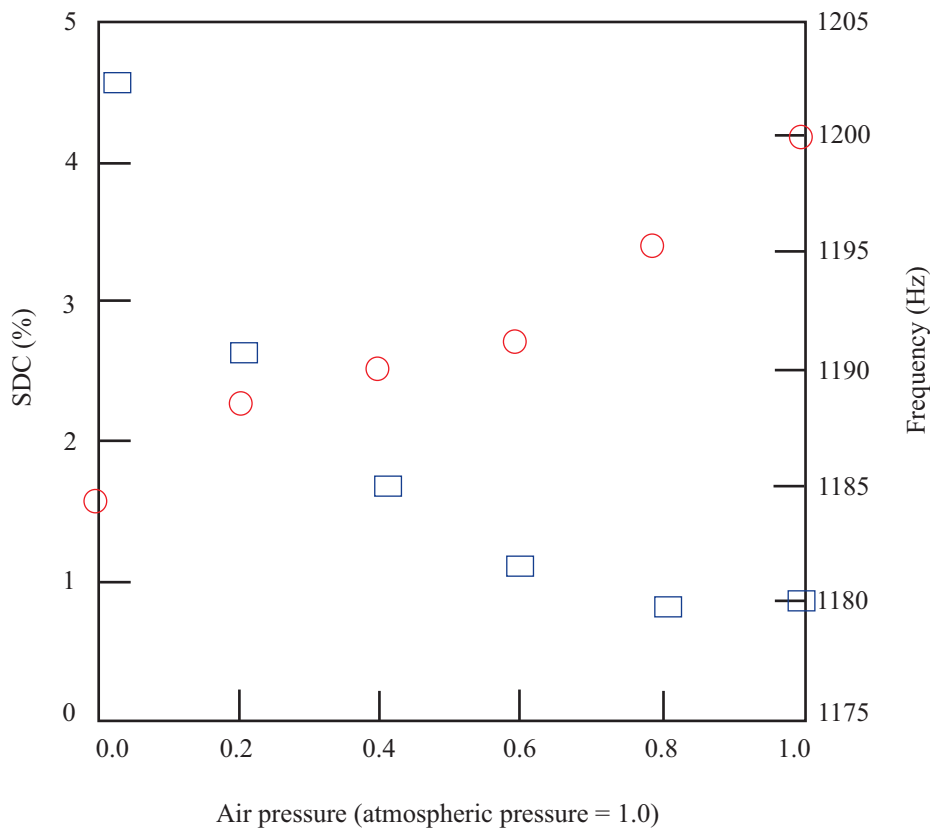
A semi-empirical formula has been proposed for the equivalent viscous damping ratio, ζ_F , of panels with fasteners, such as rivets, along the periphery (Soovere and Drake; 1985, Soovere, 1986). The formula was developed in terms of inch units and is:

$$\zeta_F = 0.0253 \frac{\left(s(a + b) - \frac{n}{s} - \frac{m}{s} \right)}{ab}, \quad (6.1)$$

where s is the number of fasteners per inch and the dimensions of the plate, a and b , are in inches. On the basis of Equation (6.1), ζ_F increases as the number of fasteners increases, particularly in the case of the low mode numbers. The friction damping ratio also increases as the size of the plate decreases. For a plate vibrating in the fundamental mode ($m = 1, n = 1$) with dimensions $a = b = 20$ in. and with one fastener every inch ($s = 1$), ζ_F has a value of 0.0024, according to Equation (6.1). This value of the viscous damping ratio for riveted joints is typical of the results obtained experimentally with strap joints (Crema *et al*, 1979).

7. ACOUSTIC RADIATION DAMPING

Damping can be achieved as a consequence of the radiation of sound. Acoustic radiation damping is particularly significant in the case of lightweight structures, such as composite and honeycomb sandwich panels. For example, some experimental results for the second mode of vibration of a honeycomb sandwich panel (aluminium core with CFRP face plates) are shown in Sketch 7.1, as measured in air and in a vacuum (Maheri *et al*, 2008).



Sketch 7.1 Variation of damping and frequency with air pressure for the second mode of honeycomb and sandwich plate 400×400 , 24.92 mm thick) with CFRP skins (+60, 0, -60) and aluminium honeycomb core, $\circ$ SDC; $\square$ frequency (Maheri *et al*, 2008)

It can be seen from Sketch 7.1 that damping in air is considerably greater than in a complete vacuum, thus demonstrating the significance of radiation damping. The natural frequency of the mode increases as the air pressure decreases.

Radiation of sound implies that energy is lost from the structure. The sound power, P , radiated by the structure, the radiation efficiency, σ , and the radiation loss factor, η , are all inter-related. For identifiable

structures, such as flat or curved panels, η is given by

$$\eta = \frac{P}{\omega M \langle |v|^2 \rangle}, \quad (7.1)$$

where $|v|^2$ is the time-averaged normal velocity of the structure, the brackets $\langle \rangle$ indicate averaging over the surface of the structure, M is the mass of the structure and ω is the circular frequency of the sound radiation. Alternatively,

$$\eta = \frac{\rho_0 \sigma}{m_s k}, \quad (7.2)$$

where m_s is the mass per unit area and k is the wave number of the radiated sound. Additionally, the radiation loss factor is related to the radiation resistance, R_r , by

$$\eta = \frac{R_r}{\omega M}. \quad (7.3)$$

Equations (7.1), (7.2) and (7.3) are general expressions in relation to panel-like structures.

7.1 Radiation Damping of Beams

Radiation damping of beam has been studied in previous work (Blake, 1974, Jeyapalan and Richards, 1979, Johnston and Barr, 1969; Wallace, 1972).

In order to analyse the radiation from a beam of rectangular section in transverse vibration, Blake modelled the beam as a finite, long, slender strip with a thin elliptical cross section (Blake, 1974). His expression for the radiation resistance, R_r , of a thin, unbaffled beam with a rectangular cross-section, was given as

$$R_r = \rho_0 c_0 (2wL) \left[\frac{8\pi^2}{1536} \right] (k_0 w)^2 \frac{w}{\pi L} \left(\frac{k_0}{k_n} \right)^2 \left\{ (1 - (-1)^n) J_0(k_0 L) \right\} \left\{ 1 + (-1)^n \frac{\sin k_n L}{k_n L} \right\}, \quad (7.4)$$

where w is the width of the beam, L is its length and J_0 is Bessel's function of the first kind of order zero. The boundary conditions for the beam are free-free. The wave number of the beam motion, k_n , is given by

$$k_n = \frac{(2n + 1)}{2L} \pi, \quad (7.5)$$

and

$$k_0 = \frac{\omega}{c_0}. \quad (7.6)$$

For a baffled beam, which is free-free, the corresponding expression for the radiation resistance is (Blake, 1974)

$$R_r = \frac{2\rho_0 c_0 (k_0 w)^2}{\pi k_n} [(1 - (-1)^n) J_0(k_0 L)] \left[1 + (-1)^n \frac{\sin k_n L}{k_n L} \right]^{-1}. \quad (7.7)$$

The radiation loss factor may then be obtained from Equation (7.3). The expressions for the radiation resistances were obtained by considering the beam as a finite, long, slender strip with a thin elliptical cross-section and the results apply for, $k_0/k_n < 1$. For $k_0/k_n > 1$, i.e., above acoustic coincidence, the radiation resistance is constant. Wallace has also provided extensive results for the radiation damping of baffled beams in the form of charts for different modal number, n (Wallace, 1972).

Jeyapalan and Richards considered a simplified method of estimating the radiation efficiency of unbaffled beams with either circular or rectangular cross-sections. Their method is based on radiation from beams with an elliptical cross-section; a circular beam is a special case and a rectangular beam may be approximated by an elliptic section. They claim that their method can be applied to beams with quite different cross sections, such as an I-beam (Jeyapalan and Richards, 1979).

7.2 Radiation Damping of Plates

The viscous damping ratios due to sound radiation from a plate at a particular mode are closely related to the modal radiation efficiencies (Wallace, 1987). For a simply-supported rectangular plate in an infinite baffle, vibrating at the resonance frequency, f_{mn} , corresponding to the mode (m, n), the damping ratio, ζ_{mn} , is related to the modal radiation efficiency, σ_{mn} , for single-sided radiation by the expression (Wallace, 1987; Lyon and DeJong, 1995; Norton and Karczub, 2003):

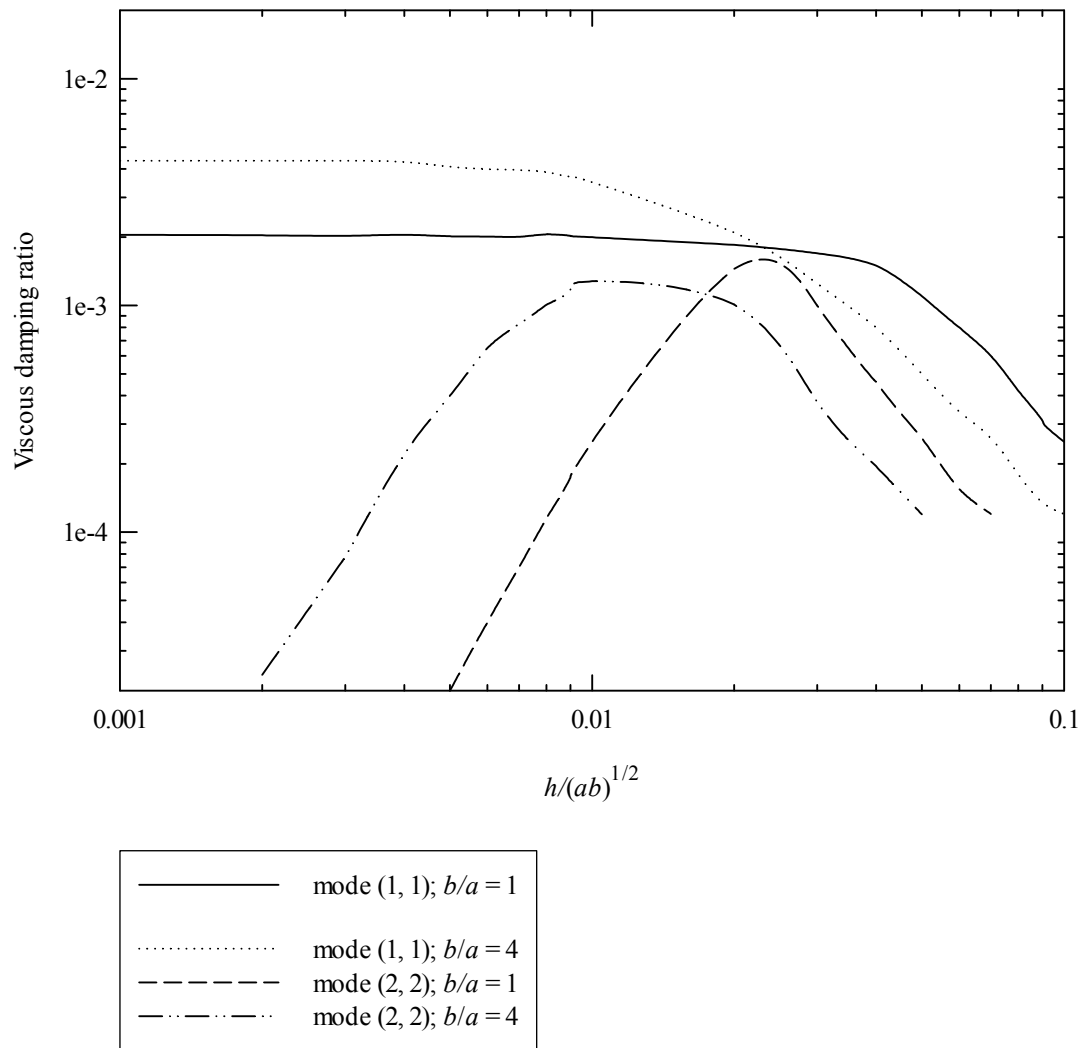
$$\zeta_{mn} = \frac{\rho_0 c_0 \sigma_{mn}}{4\pi \rho h f_{mn}}, \quad (7.8)$$

or, in terms of the modal loss factor

$$\eta_{mn} = \frac{\rho_0 c_0 \sigma_{mn}}{2\pi \rho h f_{mn}}, \quad (7.9)$$

where the plate has density ρ and thickness h . The density of air is ρ_0 and c_0 is the velocity of sound. The modal radiation efficiency, σ_{mn} , for a simply-supported plate may be obtained by the method described by Wallace or ESDU 02009 (Wallace, 1987; ESDU, 2002).

In general the modal damping ratio is less as the integers m and n increase, particularly for small plate structures. The greatest damping is for thin plates in the (1, 1) mode. Additionally, the damping increases with the plate aspect ratio, b/a . It is shown in Sketch 7.2 how the equivalent viscous damping ratio varies for the symmetrical modes (1, 1) and (2, 2) for aspect ratios of 1 and 4 for an isotropic, aluminium alloy plate as a function of the non-dimensional ratio $h\sqrt{ab}$. Sketch 7.2 and Equation (7.1) apply to a plate radiating sound from one side of the plate; if both sides radiate sound, ζ_{mn} is multiplied by 2.



Sketch 7.2 Viscous damping ratios for sound radiation from an isotropic, aluminium alloy plate vibrating in the (1, 1) and (2, 2) modes

In ESDU 07011 radiation damping is considered in more detail and equivalent viscous damping ratios for the first 16 modes are presented for plates of any material.

7.2.1 Simplified expression for modal radiation damping

A simplified expression for the viscous damping ratio of a simply-supported plate due to radiation damping can be obtained, if the wavelength of the radiated sound is much greater than the plate dimensions, namely, (Soovvere, 1985)

$$\zeta_{mn} = \frac{64}{\pi^4} \frac{\rho_0}{\rho h} \frac{f_{mn}}{c_0} \frac{a}{m^2} \frac{b}{n^2} \quad (7.10)$$

For the fundamental mode of a simply-supported plate, Equation (7.10) can be simplified to

$$\zeta_{11} = \frac{64}{\pi^4} \frac{\rho_0}{\rho h} \frac{f_{11}}{c_0} ab \quad (7.11)$$

Equations (7.10) and (7.11) are obtained by taking the first term in an integral in the form of a series and overestimate the radiation damping by a small amount. By considering the second term in the series a more accurate expression for Equation (7.11) can be established for the radiation damping of the fundamental mode of the simply-supported plate (Soovvere, 1985), namely,

$$\zeta_{11} = \frac{64}{\pi^4} \frac{\rho_0}{\rho h} \frac{f_{11}}{c_0} ab \left\{ 1 - \frac{\pi^2 - 8}{3} \left(\frac{f_{11}}{c_0} \right)^2 (a^2 + b^2) \right\}. \quad (7.12)$$

Equations (7.10) to (7.12) were found to give a reasonable agreement for the damping of composite stiffened panels, which are light-weight and for which most of the damping is from radiation.

7.3 Averaged Radiation Loss Factors

The averaged radiation resistance of baffled, simply-supported, single-layer panels, which are rectangular and radiating on one side, has been given by Maidanik in terms of corner and edge modes (Maidanik, 1962), as

$$R_{rad}^{2\pi} = A_p \rho_0 c_0 (\sigma_{corner} + \sigma_{edge}) \text{ for } f < f_c, \quad (7.13)$$

where the superscript 2π indicates radiation on one side of the panel and

$$\begin{aligned} \sigma_{corner} &= \left(\frac{8c_0^2}{\pi^4 f_c^2 A_p} \right) \left(\frac{1 - 2f/f_c}{f/f_c} \right) \sqrt{1 - f/f_c} \text{ for } f > f_c/2 \\ &= 0 \text{ for } f < f_c/2 \end{aligned} \quad (7.14)$$

$$\sigma_{edge} = \left(\frac{Pc_0}{4\pi^2 f_c A_p} \right) \left(\frac{\ln[(1 + f/f_c)/(1 - f/f_c)] + 2f/f_c}{\sqrt{1 - f/f_c}} \right) \quad (7.15)$$

$$R_{rad}^{2\pi} = \sqrt{af_c/c_0} + \sqrt{bf_c/c_0} \text{ for } f = f_c \quad (7.16)$$

$$R_{rad}^{2\pi} = (1 - f_c/f)^{-1/2} \text{ for } f > f_c. \quad (7.17)$$

In the above formulae f_c is the coincidence frequency, A_p is the area of the panel and P is the total length of the sides of the panel. From the radiation resistance the averaged radiation loss factor can be obtained from Equation (7.3). Maidanik's formulae have been used to estimate the radiation resistance, which was subtracted from total loss factor to give the dissipation loss factor of a panel (Renji and Shankar Narayan, 2002, Renji *et al*, 2002). In order to estimate the averaged radiation resistance for fully clamped panels, Heron suggests that Equation (7.13) is multiplied by a factor of 2 (Heron, 1979).

In the case of unbaffled panels the fluid flow around the edges reduces the sound radiation. An expression for the radiation efficiency of unbaffled, simply-supported panels has been provided (Oppenheimer and

Dubowsky, 1997)

$$\sigma_{unbaf} = \left(\frac{c_0^4 A_p^2 / (48 \pi^2 f^4)}{1 + c_0^4 A_p^2 / (48 \pi^2 f^4)} \right) \left\{ \left(\frac{13 f / f_c}{2(1 + 13 f / f_c)} \right) \sigma_{corner} + \left(\frac{49 f / f_c}{2(1 + 49 f / f_c)} \right) \sigma_{edge} \right\} \text{ for } f < f_c. \quad (7.18)$$

Charts for the estimation of the radiation losses of flat, isotropic, baffled plates are given in ESDU 02009. The charts are provided in terms of the equivalent viscous damping ratios (multiply by two for the radiation loss factors) for rectangular plates of different aspect ratios and various thicknesses in one-third octave bands. Note that the values given assume that the plate is radiating sound on one side of the plate only, a condition that would apply, if the panel were radiating freely on one side and into a box formed by stiffeners and frames on the other. If the panel is radiating freely on both sides, the above equations should be multiplied by a factor of two.

From the expressions for radiation efficiency and radiation resistance by Maidanik and others, or from measurements, it is possible to estimate an averaged radiation efficiency, σ_{av} , which can be related to the averaged radiation loss factor, η_{av} , by the expression,

$$\eta_{av} = \frac{\rho_0 c_0 \sigma_{av}}{2 \pi \rho h f_b}, \quad (7.19)$$

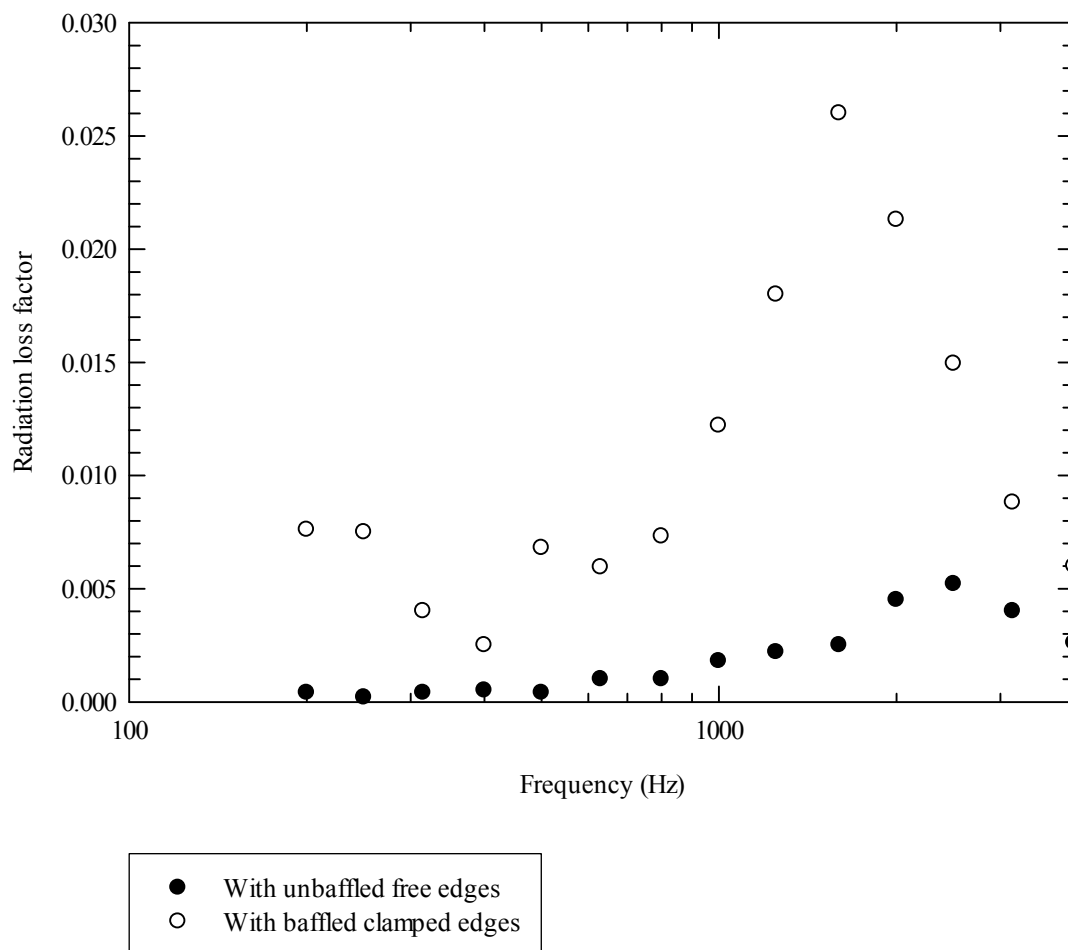
where f_b is the centre frequency of the band, such as a one-third octave band, within which estimations or measurements have been made. Equation (7.19) is comparable with Equation (7.9).

Experimental values of band-limited radiation loss factors for honeycomb sandwich panels have been provided (Zhou and Crocker, 2010). Measurements were made on two sandwich panels with plane weave fabric-reinforced graphite composite face sheets and polyurethane (PUR) foam-filled honeycomb core panels. The panels were designated B and C and the properties are listed in Table 7.1.

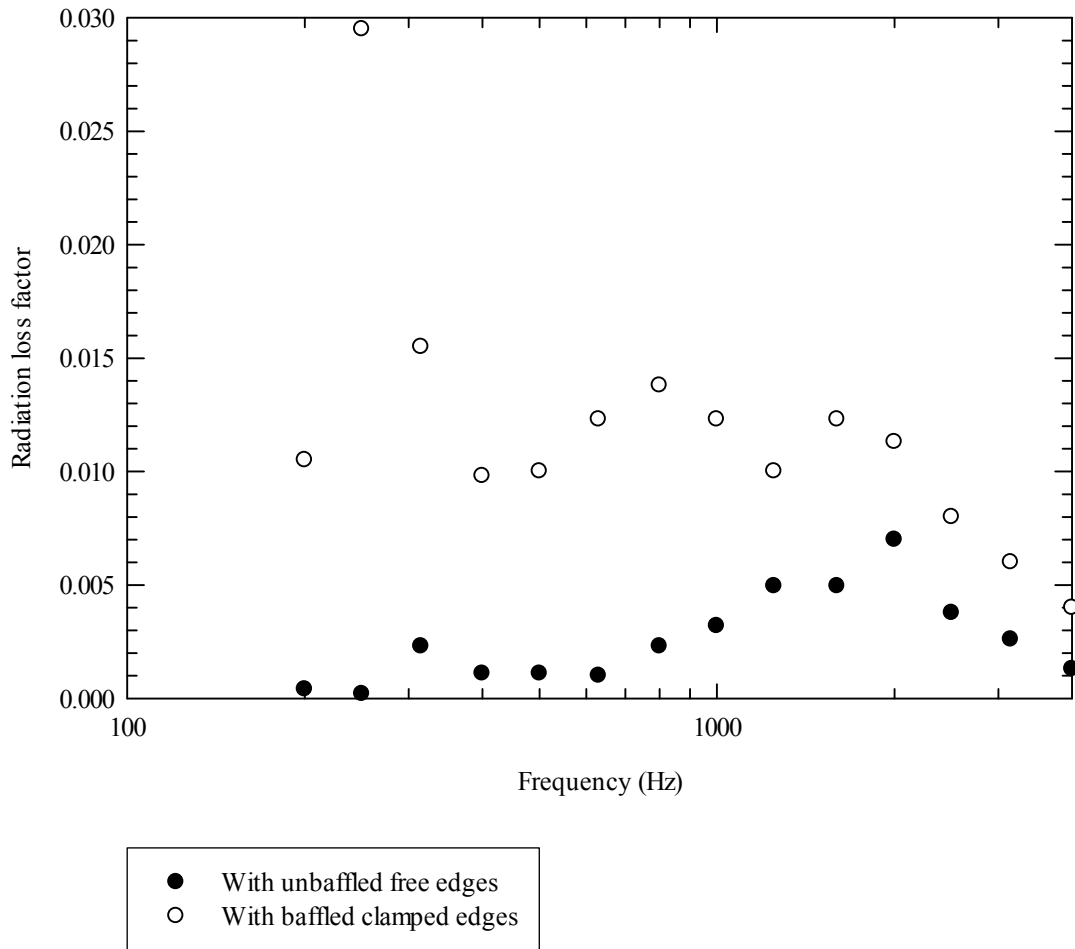
TABLE 7.1 Material properties of honeycomb sandwich panels (Zhou and Crocker, 2010)

Property	Panel B	Panel C
Face sheet		
Density (kg/m ³)	1600	1600
Young's modulus E_t (GPa)	49	39
Poisson's ratio	0.15	0.15
Thickness t (mm)	0.5	0.5
Core		
Density (kg/m ³)	160	120
Thickness h (mm)	6.35	12.7
Out-of-plane shear modulus G_x (MPa)	90	100
Out-of-plane shear modulus G_y (MPa)	140	60
Poisson's ratio	0.15	0.15
Dimensions $a \times b$ (m $\times$ m)	1.12 $\times$ 0.62	1.12 $\times$ 0.62

In order to measure the radiation loss factors each panel was placed between two reverberation chambers and the panel was excited by an electromagnetic vibration shaker. The estimation of the radiation losses is based on SEA and is essentially the method described by Crocker and Price (Crocker and Price, 1969). The radiation loss factors for panels B and C are given in Sketches 7.3 and 7.4, respectively. In both cases results are given for the panel with clamped edges with a baffle and for the panel with free-free boundary conditions without a baffle. The results show that the panels with baffled clamped edges have greater radiation damping than the free unbaffled panels. This is in line with previous results which have demonstrated that baffled panels have greater radiation efficiencies than unbaffled panels (Laulagnet, 1998).



Sketch 7.3 Radiation loss factor for honeycomb sandwich panel B (Zhou and Crocker, 2010)



Sketch 7.4 Radiation loss factor for honeycomb sandwich panel C (Zhou and Crocker, 2010)

The critical frequencies of wave coincidence of the panels B and C were estimated theoretically by Zhou and Crocker to be 1600 and 1000 Hz, respectively.

Further details on honeycomb to cores are available (Adam and Maheri, 1993; ESDU, 2011).

8. DAMPING OF SIMPLE STRUCTURES

Simple structures are beams, plates and cylinders.

8.1 Beams

Material damping of beams has been studied by several investigators (Crawley and van Schoor, 1986; Granick and Stern, 1965; Wren and Kinra, 1989; Zener, 1948). It has been reviewed in ESDU documents (ESDUB, 2009, ESDU 2013). Acoustic radiation damping has been considered in Section 7. Additionally, Baker *et al.* have studied the air damping of beams. Cantilever beams, in particular, can be subjected to considerable drag forces, as a result of large displacements at the tip of the beam. The drag force can be assumed to be proportional to either the transverse velocity amplitude of the tip of the cantilever beam, V , or to $|V|$ (Baker *et al.*, 1967). In the case of proportionality to V , Baker *et al.* showed that the logarithmic decrement, δ , was given by

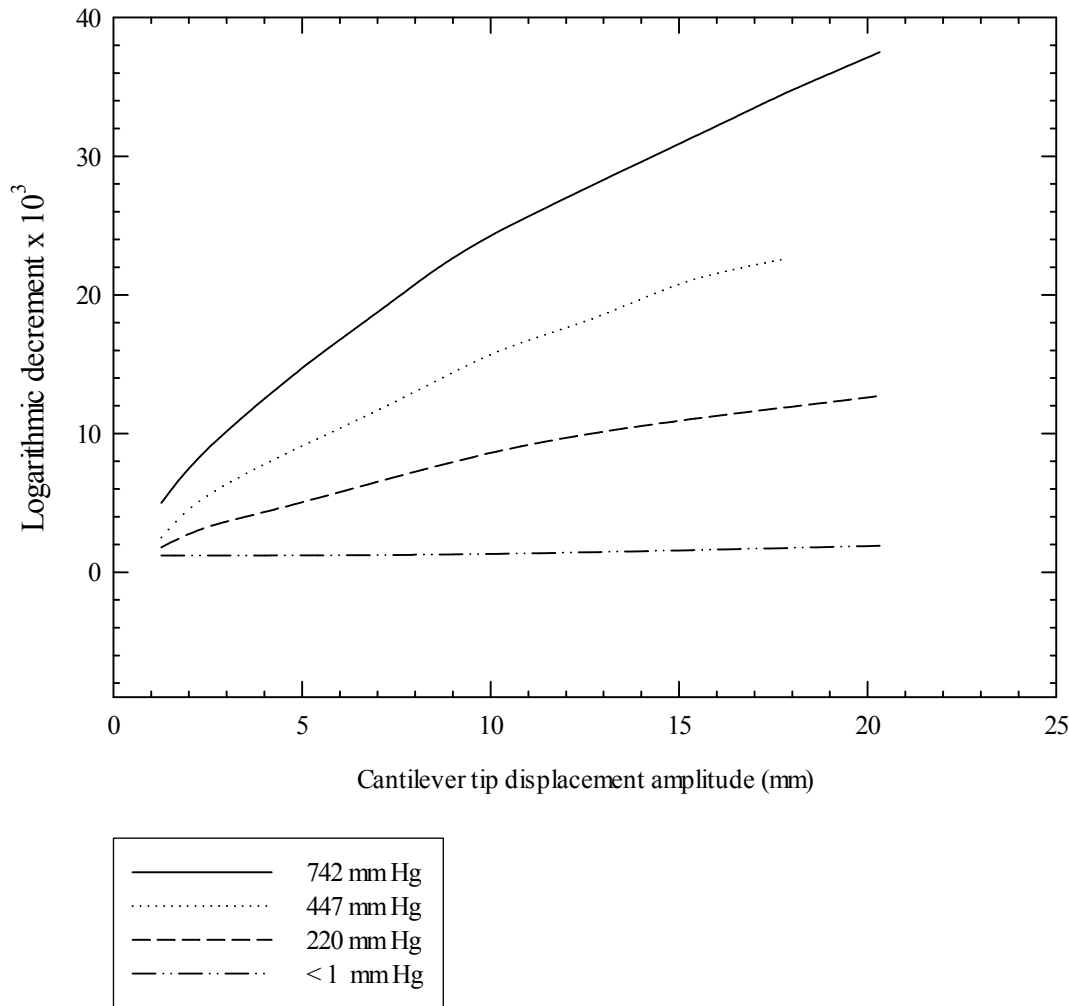
$$\delta = \pi C \frac{\rho_0}{\rho} \frac{l}{h}, \quad (8.1)$$

where C is a dimensionless drag coefficient, l is the length of the beam and h is the beam thickness. In terms of an equivalent viscous damping ratio,

$$\zeta = 0.5\pi C \frac{\rho_0}{\rho} \frac{l}{h}. \quad (8.2)$$

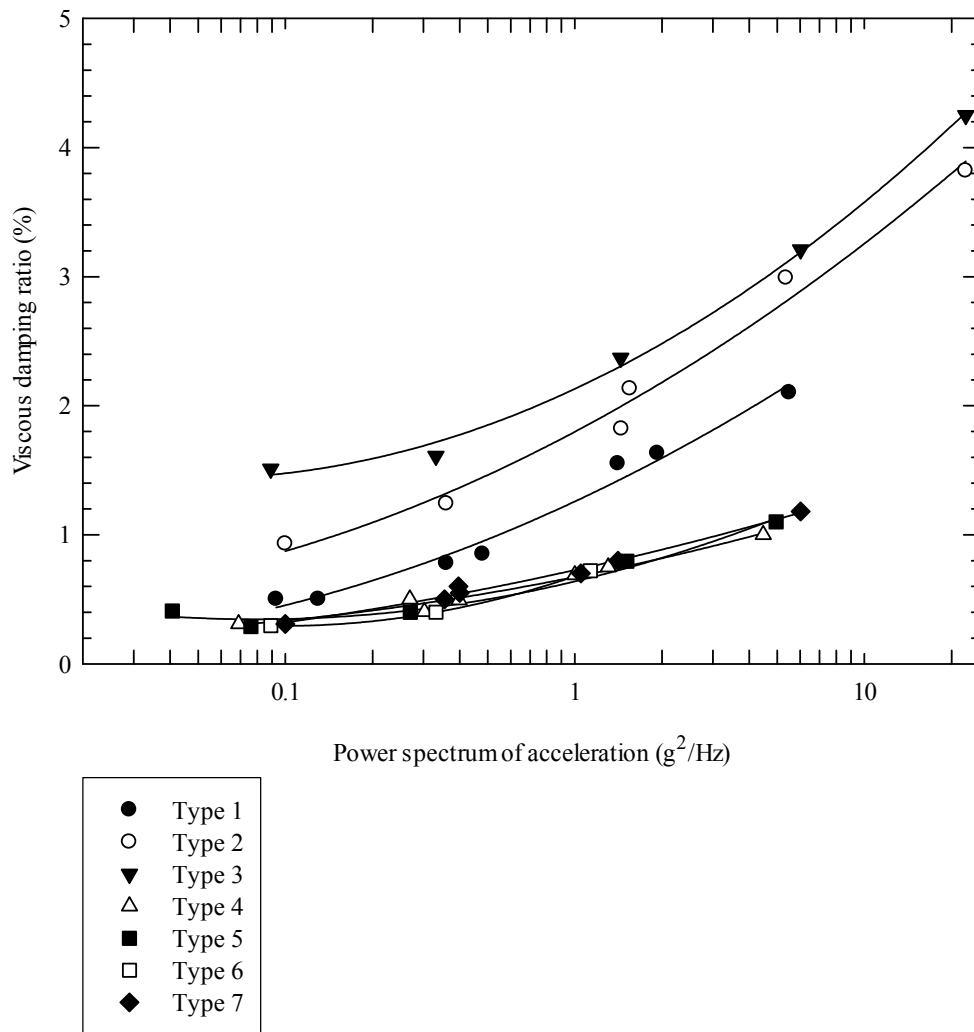
Equations (8.1) and (8.2) represent air damping for small amplitudes. Damping is independent of the tip amplitude and was found to be unaffected by the beam width, w . For the beams investigated experimentally by Baker *et al.*, it may be deduced that $C = 0.0038$.

For what Baker *et al.* refer to as air damping with large amplitudes, the transverse drag force is proportional to the square of the velocity of the tip of the beam. Simple expressions for the damping are no longer possible, although they present some evidence that the damping is controlled by the ratio of the tip displacement to the beam thickness. The results in Sketch 8.1 illustrate how the movement of air can influence the damping of a cantilever beam. When the experiment is conducted in a vacuum (< 1 mm Hg), the damping is low and almost independent of the amplitude of the tip of the cantilever. At normal atmospheric pressure the damping is considerably greater and increases with tip amplitude. The results shown are for a cantilever beam which is vibrating in the first mode at 6.25 Hz and is of length 9.06 in (230.1 mm), width 1.0 in (25.4 mm) and 0.019 in (0.48 mm). The beam is described as isoelastic (36% Ni, 8% Cr, 0.5% Mo, balance Fe).



Sketch 8.1 Damping for the first mode of a cantilever beam (Baker *et al.*, 1967)

Cantilevers are often used as coupons in acoustic fatigue tests. Some results from such tests are shown in Sketch 8.2, where the cantilever has been vibrated on an electromagnetic shaker in the first mode of vibration (Koenig *et al.*, 1998). The excitation is narrow band random, centred on the frequency of the first mode. As the amplitude of the excitation was increased for each coupon, the damping increased, as shown in Sketch 8.2. Coupon type 1 is a plain cantilever, manufactured from a standard aluminium alloy LN 3.1364 (clad), of length 70 mm, thickness 1.2 mm and width 35 mm. Types 2 and 3 have separate doublers of length 10 mm and 17 mm, respectively, which increased the thickness to 1.8 mm. Types 4, 5, 6 and 7 were chemically milled and had integral doublers of lengths 10, 17 12 and 17 mm, respectively. All the beams were 1.2 mm thick, apart from type 6, which was 1.5 mm thick and type 7, which was 0.9 mm thick (Koenig *et al.*, 1998).



Sketch 8.2 Damping of cantilever coupons used in acoustic fatigue tests (Koenig *et al*, 1998)

Damping was greater for the coupons with separate doublers, where dry friction between the two components played a part. Type 3, which had the longest doubler, provided the greatest amount of damping. Types 4, 5, 6 and 7 which had the integral doublers had very similar damping.

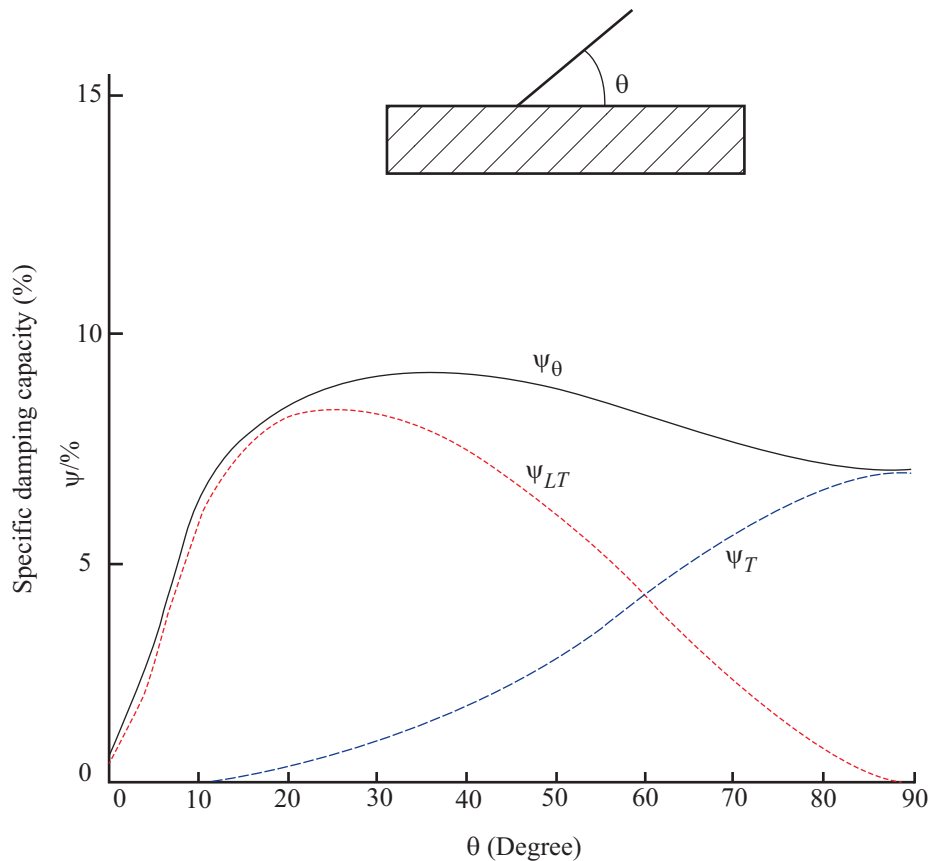
8.1.1 Composite beams

In the case of composite beams where high-strength fibres are embedded in a matrix, the main sources of damping arise from the material damping of the matrix and from relative slipping between the matrix and the fibres. For unidirectional fibres in a polymer matrix, *e.g.*, carbon fibre reinforced plastics damping arises from longitudinal shear, which involves the twisting of the composite, from the longitudinal tension/compression and from the transverse tension/compression of the fibres (Adams, 1973; Adams and Bacon, 1974). The damping depends upon the angle θ that the fibres make with the axis of the beam. The specific damping capacity ψ_θ is given by

$$\psi_\theta = \frac{1}{S_{11}} \left[\sin^4 \theta \left(\frac{\psi_T}{E_T} \right) + \cos^2 \theta \sin^2 \theta \left(\frac{\psi_{LT}}{G_{LT}} \right) \right], \quad (8.3)$$

where S_{11} is the compliance in the direction of the specimen axis, ψ_T is the SDC in the transverse direction,

ψ_{LT} in the SDC form shear, E_T is the Young's modulus in the transverse direction and G_{LT} is the shear modulus (Adams and Bacon, 1974). The specific damping capacities ψ_θ , ψ_T and ψ_{LT} are shown as a function of θ in Sketch 8.3. The total damping capacity ψ_θ is mostly the sum of the damping in shear ψ_{LT} and in the transverse direction ψ_T .



Sketch 8.3 Variation of total specific damping capacity with fibre orientation, ψ_θ , for a beam of carbon fibre in DX209 epoxy resin with a volume fraction of 0.5; SDC in shear ψ_{LT} and SDC in transverse direction ψ_T are also shown

A significant contribution to the damping comes from the longitudinal shear (ψ_{LT}), where the direct stresses are in the direction of the fibres and from transverse tension/compression (ψ_T). Similar results are obtained for laminates of several layers where the fibre angles are symmetrically disposed at $\pm\theta$ (Adams and Bacon, 1974). Equation (8.3) and Sketch 8.3 have significance for the damping of laminates in general (Adams and Lambert, 1986).

There is experimental evidence that a small fibre volume fraction (*i.e.*, a large fraction of matrix) results in increased amounts of damping and, additionally, the damping increases as the fibre aspect ratio (length to diameter) becomes less (White and Palmer, 1984). A similar decrease of damping with fibre aspect ratio has been noted for graphite/epoxy (Gibson *et al*, 1984).

TABLE 8.1 Material data, including damping, for composite layers

Material	E_L GPa	E_T GPa	G_{LT} GPa	ν_{LT}	ρ kg/m ³	V_f	Ψ_L	Ψ_T	Ψ_{LT}
PP55Gr/6061Al (Wren and Kinra, 1989)	157.9	36.7	16.5		2430	0.5	0.004	0.0155	0.039
HMS/DX-209 (Adams and Bacon, 1974, HMS carbon fibre/epoxy resin DX-209)	189	6.08	2.72	0.30			0.0064	0.069	0.1
HMS/DX-210 (Lin <i>et al</i> , 1984, carbon fibre/epoxy resin DX-210)	172.7	7.20	3.76	0.30	1566	0.50	0.0045	0.0422	0.0705
Glass/DX-210 (Lin <i>et al</i> , 1984; Ni and Adams, 1984, glass fibre/epoxy resin DX-210)	37.8	10.90	4.91	0.30		0.50	0.0087	0.0505	0.0691
CF/PEEK (Maheri and Adams, 2003, carbon fibre reinforced PEEK)	128.1	9.18	6.50	0.30		0.61	0.0042	0.035	0.0440
913C-TS (Maheri and Adams, 2003, C-TS carbon fibre in 913 epoxy resin)	110.1	8.97	3.90	0.34	1513	0.54	0.0075	0.0595	0.0679
913C-HTA (Maheri and Adams, 2003)	124.5	10.2	6.30	0.34	1532		0.0055	0.0498	0.0592
Fibredux 913G-E-5-30 (Adams and Maheri, 1994) E-glass fibre in Fibredux resin	41.5	12.5	5.03	0.2		0.54	0.0161	0.067	0.073

8.1.2 Polymer–matrix composite beams

Some results for polymer-matrix composites for beams subjected to a strain rate of 700 microstrain are given in Table 8.2 (Rawal and Misra, 1986).

TABLE 8.2 Viscous damping ratios for polymer matrix composite beams subjected to strain of 700 μ strain (Rawal and Misra, 1986)

PMC	Test	Strain ($\mu\epsilon$)	T ($^{\circ}\text{C}$)	f (Hz)	ζ (10^{-3})	Remarks
Gr/epoxy (0) ₈	—	700	—	40-140	0.64	peak
Gr/epoxy (90) ₈	—	700	—	40-140	5.5	
Gr/epoxy (± 45) ₂₅	—	700	—	40-140	5.5	
Kevlar/epoxy	—	700	—	40-140	10	
HT-S/epoxy-F	—	700	—	40-140	0.15	
HT-S/epoxy-Ly	—	700	—	40-140	0.2	
E glass/epoxy DX21	—	700	—	40-140	0.9	

Some further results for polymer-matrix composites are provided in Table 8.3 (Gibson *et al*, 1984).

TABLE 8.3 Viscous damping ratios for graphite/epoxy and Kevlar/epoxy composite beams

PMC	Test	θ (deg.)	Strain ($\mu\epsilon$)	T ($^{\circ}\text{C}$)	f (Hz)	V_f	ζ (10^{-3})	Remarks
(Gr/epoxy) ₁₂	Bending	0		RT	60	0.654	1.1	cont fibres Gibson <i>et al</i> , 1984
Gr/epoxy	Bending	0	–	RT	60	0.654	1.6	¼ in fibres
Gr/epoxy	Bending	0	–	RT	60	0.654	3.3	1/16 in fibres
Gr/epoxy	Bending	0	–	RT	1050	0.654	1.4	cont fibres
Gr/epoxy	Bending	0	–	RT	9.50	0.654	3.0	¼ in fibres
Gr/epoxy	Bending	0	–	RT	8.50	0.654	5.0	1/16 in fibres
Kevlar/epoxy	Bending	0	–	RT	40	0.661	5.7	cont Gibson <i>et al</i> , 1984 49fibres/934 Ep
Kevlar/epoxy	Bending	0	–	RT	40	0.661	6.6	¼ in fibres
Kevlar/epoxy	Bending	0	–	RT	40	0.661	7.7	1/16 in fibres
Kevlar/epoxy	Bending	0	–	RT	730	0.661	7.5	cont
Kevlar/epoxy	Bending	0	–	RT	680	0.661	7.6	¼ in fibres
(Kevlar/epoxy) ₁₂	Bending	0	–	RT	620	0.661	10.4	1/16 in fibres

The above measurements for graphite/epoxy (Rawal and Misra, 1986) have been largely corroborated by Crawley and Sheen who found that on tests with free-free beams the viscous damping ratio for Gr/epoxy (0)₈ was independent of frequency (test range from 45 to 237 Hz) and independent of applied strain (range from 20 -775 μstrain) and had an average value of 0.56×10^{-3} (Crawley and Sheen, 1984). They also found that for Gr/epoxy (90)₈ in the frequency range 43 to 143 Hz and strain range 7 to 675 μstrain the viscous damping ratio increased with frequency from 5.5 to 6.6×10^{-3} . For Gr/epoxy (45)₈ damping ratios from 5.0 to 6.7×10^{-3} were obtained.

8.1.3 Metal-matrix composite beams

Metal matrix composites (MMC) are characterised by a metallic matrix in which is embedded a ceramic or a refractory metal as reinforcing agents. The composites can be classified according to whether the reinforcement is in the form of continuous fibres, short fibres or particles. Examples are continuous graphite fibres in an aluminium alloy, graphite fibres in a magnesium matrix, aluminium oxide (Al_2O_3) particles in aluminium alloy matrix and silicon carbide (SiC) filaments in titanium. They are used in applications, such as space structures, where high stiffness and low density are required (Clyne and Withers, 1993)

Many of the damping mechanisms in MMCs are similar to those in the unreinforced metal. Additionally, interfacial mechanisms for damping may arise at the fibre and metal boundaries. Factors influencing the damping include the strain, frequency, alignment of the fibres, fibre diameter and volume fraction. Measurements of the damping of Al_2O_3 fibres in aluminium showed that there was no increase in damping relative to the unreinforced matrix and that, unlike the matrix alone, there was no increase of damping with increasing strain (Wolfenden and Wolla, 1989). On the other hand similar measurements for tungsten fibres in aluminium have shown the damping increased by a factor of three relative to the aluminium and a considerable increase in damping with strain (Wolfenden and Wolla, 1989)

For graphite in 6061 aluminium the damping has been recorded up to 15 times that of the aluminium alloy alone (Zhang *et al*, 1993). For unidirectional graphite fibres in 6061-T6 aluminium alloy a typical average value of the viscous damping ratio with the fibres in line with the beam axis is 5.2×10^{-3} , when the aluminium alloy alone tested under similar conditions had a value of 2.6×10^{-3} (Misra and Lagreca, 1984). (The beam was cantilevered and the damping estimates were based on decrement of the time signal.) The viscous damping ratio increased to 12×10^{-3} when the fibres are at right angles to the beam axis. Table 8.4 gives some examples of MMCs.

TABLE 8.4 Viscous damping ratios for continuous fibre-reinforced MMCs

MMC	Test	Strain ($\mu\epsilon$)	T ($^{\circ}\text{C}$)	f (Hz)	ζ (10^{-3})	Remarks
$\text{Al}_2\text{O}_3/6061\text{Al}$	Bending	0.01-100	—	80000	0.15	
B/6061Al	—	~ 1	25-250	2000	0.2-1.2	
B/6061Al	—	—	450	2000	7.5	0°
B/6061Al	—	—	20	1	2.5	0°
P55Gr/6061Al	Axial	200	—	1-10	5.5-12.5	0°
P55Gr/6061Al	Axial	200	—	1-10	2.5-7	90°
P55Gr/AZ91C Mg	Bending	—	—	13.1	7.1	0°
P55Gr/AZ91C Mg	Bending	—	—	11	7.9	90°
P55Gr/Mg-0.6%Zr	Bending	~ 0.1	170-430	2000	0.3-0.75	
P55Gr/6061al	Axial	55	25	10-120	2.2	$\pm 60^{\circ}$
55MSIGr/6061Al	Bending	150-325	—	0.4-1.4	10-15	0°
P100Gr/6061Al	Bending	—	25	36-48	5.2	0° , $V_f = 0.34$
P100Gr/6061Al	Bending	—	25	37-39	12.3	90° , $V_f = 0.34$
P100Gr/AZ91C Mg	Bending	—	—	13.6	6.1	0° , $V_f = 0.265$
P100Gr/AZ91C Mg	Bending	—	—	8.6	9.7	90° , $V_f = 0.265$
P55Gr/AZ91C Mg	Bending	—	—	13.0	4.9	0° , $V_f = 0.127$
P55Gr/AZ91C Mg	Bending	—	—	11.1	6.8	90° , $V_f = 0.127$
SiC/Ti-6Al-4V	Bending	~ 1	25-600	1200	0.1-0.5	$V_f = 0.25$
SiC/6061Al	Bending	0.01-100	—	80000	0.1	$V_f = 0.42$
W/6061Al	Bending	0.01-10	—	80000	0.2-0.3	$V_f = 0.44-0.50$

Metal matrices reinforced with discontinuous fibres or particulates appear to provide similar improvements. For example, graphite particulates in 6061 aluminium alloy have damping values 3 to 18 times that of the alloy alone. Silicon carbide particulates in aluminium alloys have similar damping properties (Zhang *et al*, 1993). Some results are summarised in Tables 8.5 and 8.6.

TABLE 8.5 Viscous damping ratios for short fibre or whisker reinforced MMCs

MMC	Test	Strain ($\mu\epsilon$)	T ($^{\circ}\text{C}$)	f (Hz)	ζ (10^{-3})	Remarks
SiC _w /6061T6	Bending	—		32.1	2.5	$V_f = 0.17$
Gr/6061Al	Bending	—	RT	700-1000	1-3.5	$V_f = 0.1-0.35$
SiC _w /Al	Torsion	10-1000	RT	—	0.2	$V_f = 0.22$
SiC _w /CT90Al	—	—	—	53	3.8	$V_f = 0.20$

TABLE 8.6 Viscous damping ratios for particulate reinforced MMCs

MMC	Test	Strain ($\mu\epsilon$)	T ($^{\circ}\text{C}$)	f (Hz)	ζ (10^{-3})	Remarks
SiC/6061-F Al	Bending	—	—	120	4.6	$V_f = 0.20$
SiC/6061-F Al	Bending	—	—	20	3.7	$V_f = 0.30$
SiC/6061-F Al	Bending	—	—	105	1.3	$V_f = 0.30$
SiC/6061-F Al	Bending	—	—	105	1.5	$V_f = 0.30$
SiC/6061-T6 Al	—	—	—	20	1.9	$V_f = 0.20$
SiC/CT90Al	—	—	—	62	3.2	$V_f = 0.20$
Al ₂ O ₃ /6061Al	Bending	260	-10-250	0.1-10	3.5-6	$V_f = 0.20$
Gr/Al-2.2Cu-0.42Si	—	—	—	—	2.4	Flake Gr, $V_f = 0.011$
Gr/Al-2.1Cu-0.42Si	—	—	—	—	3.3	Flake, $V_f = 0.024$
Gr/AL-3.2Cu-0.42Si	—	—	—	—	5	Flake, $V_f = 0.0005$
Gr/6061Al	Bending	260	25-250	0.1-10	3.4-8	$V_f = 0.05$, sprayed
Gr/6061Al	Bending	260	25-250	0.1-10	4.8-11.8	$V_f = 0.05$, extruded
Gr/6061Al	Bending	260	25	0.1-10	4.2-11	$V_f = 0.07$, $d_{50} = 6\mu\text{m}$
Gr/6061Al	Bending	260	25-250	0.1-10	7-18	$V_f = 0.1$, $d_{50} = 26\mu\text{m}$
Gr/SiC/6061Al	Bending	260	25	290	3.7	4% SiC, 4% Gr

The effect of strain on the damping of a graphite/6061 Al metal-matrix composite has been investigated at low frequencies (0.4 to 10Hz) for space applications (Ray *et al*, 1986). The specimens were in the form of beams and were subjected to an oscillating axial tension which resulted in a strain with both an amplitude and mean value from 150 to 566 μstrain . With continuous graphite fibres in the direction of the force (0°) the viscous damping ratio was about 1.5×10^{-3} at 150 μstrain and increased to about 3×10^{-3} at 320 μstrain . For graphite fibres at right angles to the force (90°) the viscous damping ratio was almost independent of strain at a typical value of about 0.8×10^{-3} .

Some values of the damping ratio have been obtained for glass fibre in magnesium which are different from those of Misra and LaGreca, given in the table. For P100/AZ91C/Mg (a single ply magnesium MMC with