



ESDU TECHNICAL MEMORANDUM

Analysis of tests on a convergent nozzle in an altitude test facility

K.J. Balkwill

ESDU TECHNICAL MEMORANDA

ESDU Technical Memoranda are prepared from time to time to provide a formal means of presentation for work that is not issued as a Data Item. The main reasons for preparing an ESDU Technical Memorandum are as follows.

- Much of the work done in the preparation of a Data Item, such as data correlations or analytical studies, can be brought together in a Technical Memorandum which can then be quoted as one of the Derivations or References in the Data Item. Such a Memorandum therefore provides a formal record of the data or methods given in the Data Item itself.
- When ESDU takes on an external contract a Memorandum is used as the vehicle for formally reporting that work.

THE PREPARATION OF THIS TECHNICAL MEMORANDUM

The work on this particular Memorandum was monitored and guided by the Performance Committee. This Committee first met in 1946 and now has the following membership:

Chairman	
Mr G.J.R. Skillen	— Independent, UK
Vice-Chairman	
Mr K.J. Balkwill	— Independent, UK
Members	
Mr P.V. Aidala*	— Northrop Grumman ESID, Bethpage, New York, USA
Dr J.D. Avery	— Independent, UK
Mr M. Baxter	— BAE SYSTEMS, Brough, UK
Mr M. Broad	— Independent, UK
Mr E. Burdak*	— Pilatus Aircraft Ltd, Switzerland
Mr A. Dalton	— MOD, Boscombe Down, UK
Mr G. van Es	— National Aerospace Laboratory NLR, The Netherlands
Mr P. Giesman*	— Federal Aviation Administration, USA
Mr S.J. Hyde	— BAE SYSTEMS, Prestwick, UK
Mr L. Kornstaedt	— Airbus Industrie, Toulouse, France
Mr D.A. Law	— Independent, UK
Mr C. J. Loughton	— Independent, UK
Mr P.E.C.S. Magalhaes*	— Embraer, Brazil
Mr R.J. Orłowski*	— Boeing Commercial Airplane Company, Seattle, Washington, USA
Dr P. Render	— Loughborough University, UK
Mr G.E. Smith	— Independent, UK
Mr R. Storey	— Independent, UK
Mr S. Tsach*	— Israel Aircraft Industries, Jerusalem, Israel
Mr C.J. Turner*	— Boeing Commercial Airplane Company, Seattle, Washington, USA.
Mr J. Werner*	— Cessna Aircraft, Wichita, Kansas, USA.

* Corresponding Member

The development and preparation of this Technical Memorandum were undertaken by

Mr K.J. Balkwill	— Consultant to ESDU
------------------	----------------------

The person with overall responsibility for the work in this subject area is Mr C.J. Whittaker, Head of Performance.

ANALYSIS OF TESTS ON A CONVERGENT NOZZLE IN AN ALTITUDE TEST FACILITY

CONTENTS

	Page
1. NOTATION AND DEFINITIONS	1
1.1 Notation	1
1.2 Values of Constant Terms	3
1.3 Definitions	3
2. INTRODUCTION	4
2.1 General	4
2.2 Computational Constraints	4
2.3 Assumptions	4
3. TESTING AND CORRELATION	5
3.1 General	5
3.2 Correlation of Momentum-transfer Effective Area	5
3.3 Correlation of Mass-transfer Effective Area	6
4. DISCUSSION	6
4.1 General	6
4.2 Momentum-transfer Effective Area	6
4.3 Mass-transfer Effective Area	7
4.4 Statistics	7
5. CONCLUDING REMARKS	10
6. REFERENCES	10
FIGURES 1 - 6	11 - 16

ANALYSIS OF TESTS ON A CONVERGENT NOZZLE IN AN ALTITUDE TEST FACILITY

1. NOTATION AND DEFINITIONS

1.1 Notation

The values of constant terms are listed in Table 1.1.

		Units
$(A_9)_{REF}$	reference area	L^2
A_9	generic term representing either A_{9C} or A_{9E}	L^2
A_{9C}	mass-flow effective area	L^2
A_{9E}	momentum-transfer effective area	L^2
$(A_{9C})_{ADJ}$	mass-flow effective area adjusted for between-test variability	L^2
$(A_{9E})_{ADJ}$	momentum-transfer effective area adjusted for between-test variability	L^2
a_0	constant used to calculate effective mass-transfer area	—
a_1	constant used to calculate effective mass-transfer area	—
a_2	constant used to calculate effective mass-transfer area	—
h	test altitude	L
i	number of values in a test series	—
i_j	number of values in j th test series	—
j	test series number	—
k	total number of test values	—
M	flight Mach number	—
M_9	Mach number in nozzle throat	—
m	constant used to calculate momentum-transfer effective area	—
n	constant used to calculate momentum-transfer effective area	—
$(p_S)_0$	free-stream static pressure	$ML^{-1}T^{-2}$
$(p_T)_0$	nozzle stagnation pressure	$ML^{-1}T^{-2}$

Re	Reynolds number based on free-stream conditions and diameter of nozzle throat	—
R_{09}	nozzle pressure ratio	—
r_{09}	reciprocal of nozzle pressure ratio	—
$\text{var}[\]$	variance of quantity in square brackets	—
X	term related to free-stream Mach number	—
x	total number of test series	—
y	index for individual term in test series	—
$z_{0.95}$	95 per cent partition value of the standard Normal distribution	—
Δ	deviation of measurements, approximately equal to both Δ_B and Δ_W	L^2
Δ_B	mean value of deviations of test-points from correlation values for a test series (between-test)	L^2
$(\Delta_B)_j$	mean value of deviations of test-points for any test series j	L^2
Δ_T	deviation of an individual measured value from the corresponding correlation value (overall)	L^2
$(\Delta_T)_{j, y}$	deviation of y -th measured value in j -th test series from corresponding correlation value	L^2
Δ_W	deviation of a test point within a test series from mean value of deviations for the series (within-test)	L^2
$E[\]$	expected value of quantity in $[\]$	—
$\Gamma_{0.95}$	95 per cent confidence interval	—
γ	ratio of specific heats	—
$M[\]$	measured value of quantity in $[\]$	—
$\sigma[\]$	standard deviation of quantity in square brackets	—

1.2 Values of Constant Terms

TABLE 1.1 Values of constant terms

Term	Value	Unit
a_0	2.5×10^{-2}	—
a_1	2.59×10^{-4}	—
a_2	2.75×10^{-3}	—
m	4	—
n	2	—
$z_{0.95}$	1.96	—

1.3 Definitions

Mass-transfer effective area (A_{9C}) is the product of the geometric nozzle area and the ratio of measured nozzle-flow rate to the “ideal” (one-dimensional isentropic) nozzle-flow rate. This quantity is analogous to the customary “nozzle-discharge coefficient”.

Momentum-transfer effective area (A_{9E}) is the product of the geometric nozzle area and the ratio of measured stream thrust to the “ideal” (one-dimensional isentropic) stream thrust. This quantity is analogous to the customary “nozzle-velocity coefficient”.

Ratio of specific heats ($\gamma = 1.36$) for calculated products of combustion is asserted to be constant through-out the operational envelope.

2. INTRODUCTION

2.1 General

In-flight deduction of propulsion forces from a turbo-jet engine requires separate assessment of the stream thrust (X_G) resulting from the passage of fluid through the propelling nozzle and the force (X_D) due to the passage of (air) mass-flow through the inlet to the engine. For at least two decades after the introduction of gas turbine engines to aviation, it was customary to determine values of inlet momentum and thrust forces in-flight using correlations of some form of “mass-flow” and “thrust” functions with a semi-dimensional engine operating quantity. Such an approach was found to lead to a variety of difficulties. Distortion of the flow at the face of the inlet was perceived to be a distinct problem. Alternative methods had therefore been sought; one option to overcome that particular situation had been identified. It relied solely on calibration of the final propelling nozzle in conjunction with a specific engine in an altitude test facility.

As part of a research programme to determine precise values for airframe forces and moments on a twin-engine military aircraft, each engine and accompanying flight jet-pipe (but a “bell-mouth” inlet) were tested in Cell 3 of the National Gas Turbine Establishment (NGTE) at Pyestock (Hants). The purpose of the testing was to establish calibrations of the final nozzles in terms of two effective areas; the one for mass transfer and the other for momentum (force) transfer. Unpublished evidence for the effectiveness of such an approach had shown that the method eliminated effects of intake distortion on nozzle calibration and thus improved the precision and/or accuracy of in-flight thrust determination.

The propelling nozzles for the aircraft are convergent; thus in the standard nomenclature, the throat and the exit plane are at the same engine station. For convenience, this is nominated to be station 9.

2.2 Computational Constraints

The computer available for analysis of flight-test data was an ICL1900. Not only was the machine slow but, because of memory limits, all programs were restricted in size to no more than some twenty *kilobytes* per “page”. In order to solve the in-flight system equations for net thrust it is necessary to find an iterative solution over a set of nine equations. Because of the “paging” requirements, the resource-cost of such an effort over the proposed project would have led to at least a ten-fold overspend; a solution was suggested by the staff of NGTE (restrain the ratio of specific heats) that brought costs within budget. Furthermore, the cost in terms of precision was estimated to be marginal and probably undetectable within the estimated limits of the precision of measurement and correlation.

2.3 Assumptions

The following assumptions apply to both the ground-test facility and the in-flight cases. Thus, any systematic errors introduced are likely to be either self-cancelling or, at the very worst, marginal in their effect.

- One-dimensional isentropic flow in the jet pipe and propelling nozzle
- For calculation of temperature the compressors and turbine operate at 100% mechanical efficiency
- For calculation of temperature the (measured) fuel-flow burns in the intake mass-flow with 100% combustion efficiency at a temperature equal to the intake total temperature
- Products of combustion so generated represent the actual combustion products mixed with the by-pass flow