



CFD validation studies for pressure loss and flow characteristics in sudden contractions

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CFD VALIDATION STUDIES FOR PRESSURE LOSS AND FLOW CHARACTERISTICS IN SUDDEN CONTRACTIONS

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CFD VALIDATION STUDIES FOR PRESSURE LOSS AND FLOW CHARACTERISTICS IN SUDDEN CONTRACTIONS

1. NOTATION AND UNITS

		SI Unit
A_C	Couette coefficient, defined in Equation (4.4)	–
A_H	Hagenbach coefficient, defined in Equation (4.4)	–
A_{vc}	Flow area at the <i>vena contracta</i> plane	m ²
A_1	Upstream pipe cross-sectional area	m ²
A_2	Downstream pipe cross-sectional area	m ²
A_2/A_1	Contraction area ratio	–
C_c	Jet contraction ratio, $C_c = A_{vc}/A_2$	–
D_1	Upstream pipe diameter	m
D_2	Downstream pipe diameter	m
J_{KE}	Kinetic-energy profile factor, defined in Equation (3.8)	–
J_M	Momentum profile factor, defined in Equation (3.9)	–
K_{si}	Static pressure loss coefficient, defined in Equation (3.4)	–
$K_{ti,aw}$	Area-weighted mean total pressure loss coefficient, defined in Equation (3.7)	–
$K_{ti,CM}$	Continuity-mean total pressure loss coefficient, defined in Equation (3.5)	–
$K_{ti,mw}$	Mass-flow-weighted mean total pressure loss coefficient, defined in Equation (3.6)	–
l_e	Edge chamfer length	m
L_r	Downstream recovery length	m
L_{s1x}	Upstream separation length	m
L_{s1r}	Upstream separation height	m
L_{s2x}	Downstream separation length	m

		SI Unit
L_{s2r}	Downstream separation height	m
L_{u1}	Upstream pipe length	m
$L_{x,vc}$	<i>Vena contracta</i> axial location	m
L_{2d}	Downstream pipe length	m
p	Static Pressure	Pa
p_u	Static Pressure at station u	
p_t	Total Pressure	Pa
r	Edge radius	m
R_1	Upstream pipe radius	m
R_2	Downstream pipe radius	m
Re_1	Upstream Reynolds Number	–
Re_2	Downstream Reynolds Number	–
u	Axial local fluid velocity	m/s
u_{cl}	Axial local centre-line velocity	m/s
v	Radial local fluid velocity	m/s
V_1	Upstream bulk velocity, $V_1 = m/(\rho A_1)$	m/s
V_2	Downstream bulk velocity, $V_2 = m/(\rho A_2)$	m/s
x	Axial distance from contraction plane	m
y	Radial distance	m
β	Diameter contraction ratio, $\beta = D_1/D_2$	–
ϕ	Edge chamfer angle	degree
$\lambda_{r,CM}$	Continuity-mean round-edge factor, defined in Equation (3.10)	–
$\lambda_{c,CM}$	Continuity-mean chamfer-edge factor, defined in Equation (3.11)	–
μ	Fluid dynamic viscosity	Pa s
ρ	Fluid density	kg/m ³

2. INTRODUCTION

This Technical Note describes the CFD studies conducted by ESDU for the validation of the predictions for pressure loss and flow characteristics in circular pipes with sudden contractions. These studies were carried out in support of the correlations developed in ESDU 05024. The CFD calculations were performed using the commercial CFD package CFX10 by ANSYS Inc.

Pipes with sudden contractions exist in a variety of engineering applications, such as process and chemical plants, power plants, automotive hydraulic lines, water pipelines, heat exchangers, aircraft ducts, etc. In order to design such systems, it is required to determine the pressure loss across the pipe fittings and the extent of the flow separations to avoid placing sensitive equipment in the recirculation regions and, hence, generate unexpected equipment performance or non reliable measurements. There are few reliable experimental data on pressure loss in sudden contractions, especially for round-edged and chamfer-edged sudden contractions. The correlations in ESDU 05024 provide rigorously validated data for pressure loss and flow characteristics in sudden contractions for a wide range of geometrical configurations and flow conditions.

In order for CFD to predict reliably pressure losses and flow characteristics in sudden contractions, it is essential to capture the important flow mechanisms for a wide range of geometrical configurations, defined by the contraction area ratio (A_2/A_1) and the edge type (sharp, rounded, or chamfered), and flow conditions, defined by the flow Reynolds number, Re . In this Technical Note, the validation studies for four Tests Cases are presented, covering the following geometrical configurations and flow Reynolds numbers:

- contraction area ratio: $0.135 \leq A_2/A_1 \leq 0.629$
- sharp edges; rounded edges: $0 \leq r/D_2 \leq 0.2$; chamfered edges: $0 \leq l_e/D_2 \leq 0.2$, $0^\circ \leq \phi \leq 90^\circ$,
- laminar, transitional, and turbulent flow regimes: $1 \leq Re_1 \leq 10^6$.

A summary of the four Tests Cases is presented in Table 2.1. In the CFD calculations, the following assumptions were made:

- axisymmetric geometrical configurations with circular cross-section,
- incompressible, single-phase, Newtonian fluid,
- fully-developed flow conditions upstream and downstream of the contraction.

Pressure loss and flow characteristics in sudden contractions are briefly described in Section 3. A review of the data in the literature for sharp-, round-, and chamfer-edged sudden contractions is presented in Section 4. A detailed description of the Test Cases is given in Section 5. A description of the CFD methodology is given in Section 6. The validation test results are presented and discussed in Section 7. Best Practice Guidelines are given in Section 8.

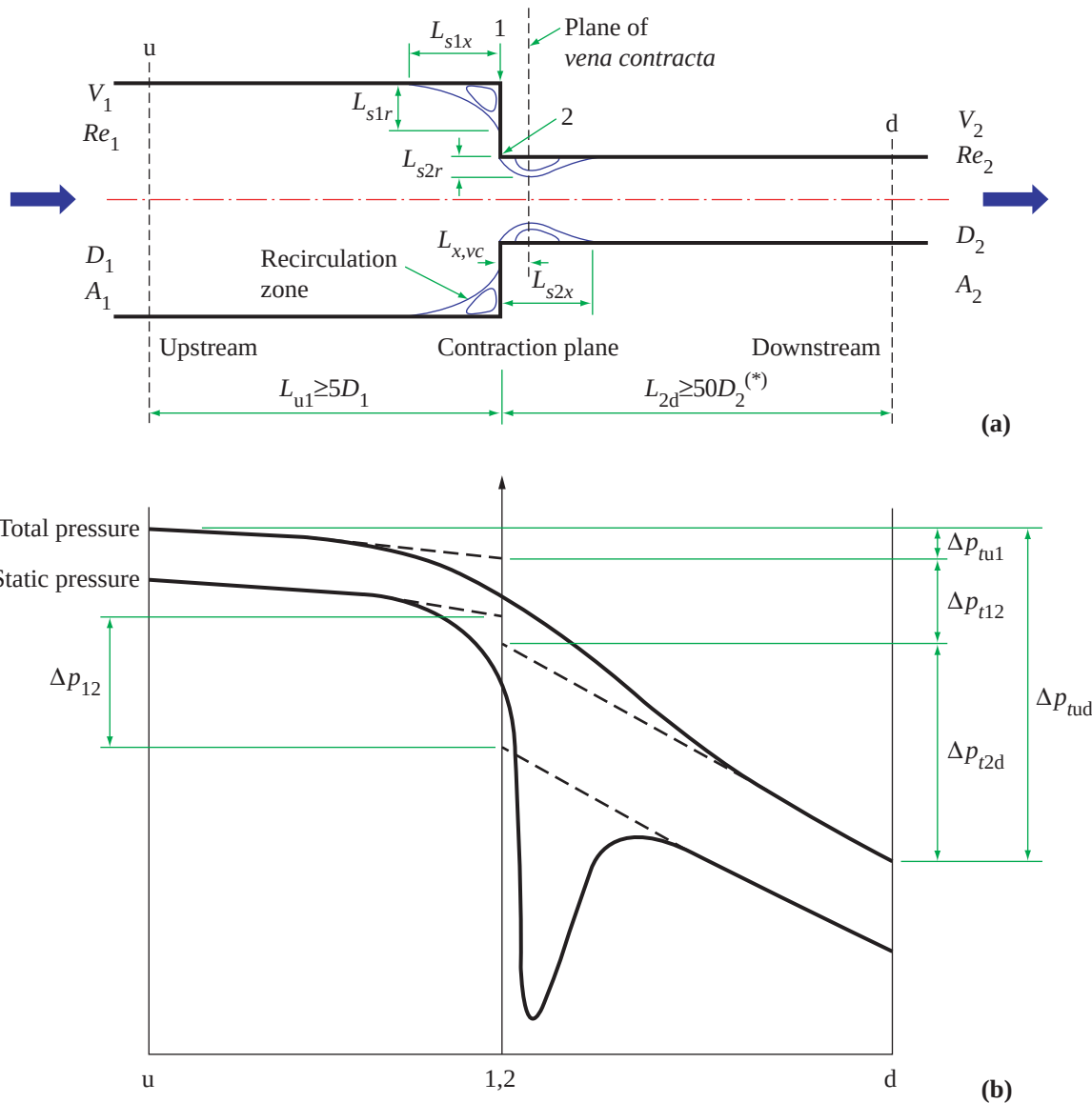
TABLE 2.1 Summary of the Validation Test Cases

Test Case	Validation Data	A_2/A_1	Re_1	Edge Type	r/D_2	l_e/D_2	ϕ
1	Axial and radial velocities Pressure distribution Separation size Pressure loss coefficient	0.285	$1-10^6$	Sharp	–	–	–
2	Axial and radial velocities Separation core extent Downstream recovery length Pressure loss coefficient	0.135, 0.332, 0.629	$1-10^6$	Sharp	–	–	–
3	Round-edge factor	0.135	10^5	Rounded	0.0 – 0.2	–	–
4	Chamfer-edge factor	0.135, 0.332, 0.629	1.5×10^5	Chamfered	–	0 – 0.2	$0^\circ - 90^\circ$

3. PRESSURE LOSS AND FLOW CHARACTERISTICS IN SUDDEN CONTRACTIONS

A detailed description of the pressure loss and flow characteristics in sudden contractions is given in ESDU 05024. The information given in this Section allows the engineer to understand the CFD modelling strategy and the derivation of the pressure loss coefficient used in the CFD validation studies. Definitions for the pressure loss coefficient are given in Section 3.1. The key flow patterns are briefly described in Section 3.2.

The main feature of the flow in sudden contractions is represented by the flow separations in the vicinity of the contraction plane (see Sketch 3.1). These recirculation regions induce a pressure loss across the contraction as a result of two consecutive processes: (1) contraction of the flow to the *vena contracta*, and (2) enlargement of the flow cross-section in an adverse pressure gradient. The smaller the contraction area ratio, A_2/A_1 , the larger is the loss.



Sketch 3.1 (a) Schematic of flow through a sharp-edged sudden contraction.
(b) Centreline static- and total-pressure distributions near plane of contraction

(*) This is valid for turbulent flow. In laminar flow, $L_{2d} \geq 110D_2$ may be required

3.1 Pressure Loss Coefficients

To calculate the pressure loss in a pipe with a sudden contraction, it is required to determine the total-pressure loss coefficient. This is defined as a function of the downstream bulk-mean velocity, $V_2 = \frac{\dot{m}}{\rho A_2}$, as

$$K_{ti} = \frac{\Delta p_{t12}}{\frac{1}{2}\rho V_2^2} \quad (3.1)$$

where Δp_{t12} (see Sketch 3.1) is the mean-total-pressure loss due to the contraction between station 1, upstream of the contraction, and station 2, downstream of the contraction plane, and is defined as the difference between the total-pressure loss between stations u and d, Δp_{tud} , and the losses due to friction for fully-developed flow conditions in the upstream duct (Δp_{tu1}) and downstream duct (Δp_{t2d}), extrapolated to the contraction plane:

$$\Delta p_{t12} = \Delta p_{tud} - \Delta p_{tu1} - \Delta p_{t2d} . \quad (3.2)$$

Total pressures at the upstream and downstream stations u and d are derived from wall static pressures. These must be measured or obtained from CFD calculations in regions of fully-developed flow outside the region of influence of the contraction. An exhaustive discussion on the correct location of pressure measurements can be found in Section 3.1 of ESDU 05024. To ensure fully-developed flow upstream of the contraction plane, a duct length of $50D_1$ is typically required. However, a length of $5D_1$ is usually acceptable. Downstream of the contraction plane, the recovery length, L_r , required to ensure a return to fully-developed flow varies with Reynolds number. In laminar flow, L_r may be as large $110D_2$; in turbulent flow, $L_r \leq 50D_2$.

The total-pressure loss coefficient can be derived from the static-pressure loss coefficient, K_{si} , as

$$K_{ti} = K_{si} - J_d + J_u (A_2/A_1)^2 . \quad (3.3)$$

where J_d and J_u are the profile factors that take into account the effect of non-uniform velocity profiles on total pressures across sections u and d. The static-pressure loss coefficient K_{si} is defined as

$$K_{si} = \frac{\Delta p_{12}}{\frac{1}{2}\rho V_2^2} . \quad (3.4)$$

Depending on the type of average applied to the velocity profile, different mean-total pressure loss coefficients can be defined. In most engineering applications, the following mean total pressure coefficients are used:

- the continuity-mean total-pressure loss coefficient:

$$K_{ti,CM} = K_{si} - (1 - (A_2/A_1)^2) \quad (3.5)$$

- the mass-flow-weighted mean total-pressure loss coefficient:

$$K_{ti,mw} = K_{si} - J_{KE,d} + J_{KE,u} (A_2/A_1)^2 \quad (3.6)$$

- the area-weighted mean total-pressure loss coefficient:

$$K_{ti,aw} = K_{si} - J_{M,d} + J_{M,u} (A_2/A_1)^2 \quad (3.7)$$

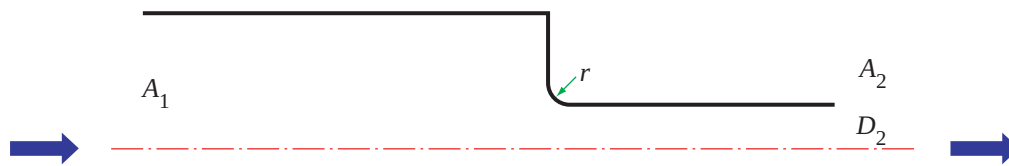
where $J_{KE,d}$ and $J_{KE,u}$ are the kinetic-energy profile factors at stations d and u, respectively. $J_{M,d}$ and $J_{M,u}$ are the momentum profile factors at stations d and u, respectively. These are defined by:

$$J_{KE} = \frac{1}{m} \int \left(\frac{u}{V} \right)^2 dm = \frac{1}{A} \int \left(\frac{u}{V} \right)^3 dA \quad (3.8)$$

$$J_M = \frac{1}{A} \int_A \left(\frac{u}{V} \right)^2 dA . \quad (3.9)$$

Total and static pressure loss coefficients are a function of the Reynolds number and the contraction characteristic geometrical parameters. Correlations for such coefficients, based on the CFD calculations presented in this Technical Note, are in ESDU 05024.

The characteristic geometrical parameter for a sharp-edged sudden contraction, shown in Sketch 3.1, is the contraction area ratio, A_2/A_1 . The characteristic geometrical parameters for a round-edged sudden contraction, shown in Sketch 3.2, are the contraction area ratio A_2/A_1 and the radius to diameter ratio r/D_2 .



Sketch 3.2 Geometrical parameters of a round-edged contraction

Edge rounding is used to reduce the pressure drop across the contraction. This effect can be quantified using the continuity-mean round-edge factor:

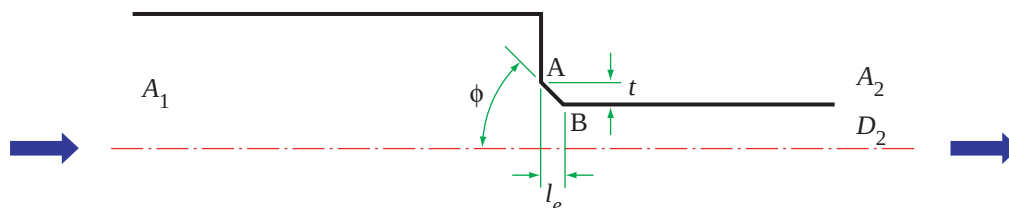
$$\lambda_{r,CM} = \frac{K_{ti,CM}}{K'_{ti,CM}} \quad (3.10)$$

where $K_{ti,CM}$ is the continuity-mean total-pressure loss coefficient for the round-edged sudden contraction, and $K'_{ti,CM}$ is the continuity-mean total-pressure loss coefficient for the sharp-edged sudden contraction with the same contraction area ratio.

Edge chamfering also reduces the pressure loss across a contraction. The characteristic geometrical parameters for a chamfer-edged sudden contraction, shown in Sketch 3.3, are the contraction area ratio A_2/A_1 , the chamfer length to diameter ratio, l_e/D_2 , and the chamfer angle, ϕ . The effect on pressure loss of edge chamfering can be quantified using the continuity-mean chamfer-edge factor:

$$\lambda_{c,CM} = \frac{K_{ti,CM}}{K'_{ti,CM}} \quad (3.11)$$

where is $K_{ti,CM}$ the total pressure coefficient for the chamfer-edged sudden contraction, and $K'_{ti,CM}$ is the total-pressure loss coefficient for the sharp-edged sudden contraction with the same contraction area ratio.



Sketch 3.3 Geometrical parameters of a chamfer-edged contraction

3.2 Flow Patterns

Detailed information on the flow characteristics in sharp-, round-, and chamfer-edged sudden contractions is provided in Section 4.3.1 of ESDU 05024.

In a sharp-edged sudden contraction, the extent of the upstream and downstream flow separations vary with contraction area ratio, A_2/A_1 , and Re . For a given contraction area ratio, A_2/A_1 , a flow separation occurs upstream of the contraction even at low Re . A downstream flow separation and reattachment develops at a particular Reynolds number depending on A_2/A_1 . In laminar flow, the upstream separation size (length and height), decreases slightly with increase in Re , until it reaches a minimum value. Then it increases with Re until the fully turbulent flow regime is reached. In turbulent flow it reduces slightly with Re . The size of the downstream separation increases with Re in laminar and transitional flow. In turbulent flow, it reduces slightly with Re .

Separation sizes in round-edged sudden contractions are generally smaller than for sharp-edged contractions. In chamfer-edged sudden contractions, the flow separation size and location are a complex function of the chamfer geometry (chamfer length ratio and angle) and flow Re . The flow may separate: (a) at only the upstream corner of the chamfer, (b) at only the downstream corner of the chamfer, (c) at both the upstream and the downstream corners of the chamfer, (d) or it may never separate. The separated flow may then reattach: (a) downstream of the chamfer, (b) on the chamfer with further downstream separation, (c) on the chamfer with no further downstream separation. Detailed information on the flow separation characteristics in chamfer-edged sudden contractions can be found in Appendix D of ESDU 05024.

4. REVIEW OF THE DATA IN THE LITERATURE

4.1 Sharp-edged Sudden Contractions

A number of experimental and numerical studies have been carried out to investigate the flow in pipes with sudden contractions. A literature survey on sharp-edged sudden contractions can be found for example in (Boger, 1982). Investigations in the published literature report on measurements and numerical predictions of wall static pressure drop, pressure loss coefficients, pressure profiles, velocity measurements, flow separation sizes, and downstream recovery lengths. However, reliable measurements for CFD validation are scarce. For example, the wall pressure measurements published by Astarita and Greco (1968), and Sylvester and Rosen (1970) differ by more than 100% in some ranges of Re . Velocity measurements using Laser Doppler Anemometry (LDA) are generally reliable as the flow field is undisturbed and the temporal and spatial resolution is high. A summary of the most relevant published data for sharp-edged sudden contractions is given in Tables 4.1 to 4.3. The pressure loss data in Table 4.2 include pressure loss coefficient data and pressure profile data.

TABLE 4.1 A Summary of the Velocity Data for Sharp-edged Sudden Contractions

Reference	Technique	A_2/A_1	Re_1
Christianen <i>et al.</i> (1972)	Experimental and numerical	0.047	65, 108
Vrentas and Duda (1973)	Numerical	0.063, 0.444	200
Durst and Loy (1985)	Experimental and numerical	0.285	23 – 1213
Xia <i>et al.</i> (1992)	Experimental and numerical	0.19, 0.31	12
Bullen <i>et al.</i> (1990)	Experimental and numerical	0.332	1.538×10^5
Buckle and Durst (1993)	Experimental and numerical	0.285	23 – 968
Bullen <i>et al.</i> (1996)	Experimental and numerical	0.332	1.538×10^5

TABLE 4.2 A Summary of Pressure Loss Data for Sharp-edged Sudden Contractions

Reference	Technique	A_2/A_1	Re_1
Harris (1928)	Experimental and numerical	0.125	10^5
Kays (1950)	Experimental	0.280	550 – 7500
Kays (1950)	Semi-empirical correlation	0.0 – 1.0	500 – 20000
Crane (1965)	Correlation	0.0 – 1.0	10^5
Benedict <i>et al.</i> (1966)	Experimental correlation	0.0 – 1.0	10^5
Astarita and Greco (1968)	Experimental	0.162	20 – 2000
Sylvester and Rosen (1970)	Experimental	0.0156	4 – 1100
Levin and Clermont (1970)	Experimental	0.223, 0.476, 0.885	$10^5 - 10^6$
Kaye and Rosen (1971)	Experimental correlation	0.041 – 0.636	4 – 1100
Benedict (1980)	Experimental correlation	0.0 – 1.0	10^5
Durst and Loy (1985)	Numerical	0.285	563
Durst and Loy (1985)	Experimental	0.0156	20 – 1200
Miller (1990)	Correlation	0.0 – 1.0	10^5
Idel'chik (1994)	Correlation	0.0 – 1.0	10^5
Bullen <i>et al.</i> (1996)	Experimental	0.14 – 0.63	$4 \times 10^4 - 2 \times 10^5$
ESDU (2001)	Correlation	0.0 – 1.0	$\geq 10^4$
Curzons (2004)	Semi-empirical correlation	0.0 – 1.0	$10 - 10^6$

TABLE 4.3 A Summary of Separation Size Data for Sharp-edged Sudden Contractions

Reference	Technique	A_2/A_1	Re_1
Durst and Loy (1985)	Experimental and numerical	0.285	23 – 1213
Buckle and Durst (1993)	Numerical	0.285	23 – 968
Bullen <i>et al.</i> (1996)	Experimental	0.332	1.538×10^5

Among these data, those published by Durst and co-workers (Durst and Loy, 1985; Buckle and Durst, 1993), and by Bullen and co-workers (Bullen *et al.*, 1990, 1996) are considered the most consistent and reliable.

Durst and Loy (1985) and Buckle and Durst (1993) carried out experimental and numerical investigations in laminar flow ($23 \leq Re_1 \leq 1213$) for a contraction area ratio $A_2/A_1 = 0.285$. Reliable LDA measurements data are provided, and compared with their numerical predictions obtained using a finite volume computer code. Pressure profile predictions at $Re_1 = 563$ are provided, and comparisons of the numerical pressure loss coefficient are displayed for $A_2/A_1 = 0.0156$ against the experimental predictions of Sylvester and Rosen (1970). Differences up to 50% can be estimated from the graphs. Numerical predictions of the upstream separation size are published in Durst and Loy (1985). Estimated measurements and numerical predictions of the downstream separation size are compared for different CFD model mesh sizes. Durst's data have been used in Test Case 1.

Bullen and co-workers have published a number of papers on sharp-edged sudden contractions (Bullen *et al.*, 1984; 1987; 1990; 1996). The flow regime is turbulent with Re in the range of 4×10^4 to 2×10^5 . The geometry is characterized by contraction area ratios A_2/A_1 between 0.13 and 0.63. Measurements of the pressure loss coefficients are provided for the above geometrical configurations and flow conditions. LDA measurements of mean velocity, turbulence kinetic energy, and downstream separation extent estimation are provided for $A_2/A_1 = 0.332$ and $Re_1 = 1.54 \times 10^5$ (Bullen *et al.*, 1990, 1996). Measurements of the turbulent kinetic energy are not considered reliable (Bullen, 2003; Rodi, 2003). Numerical predictions were obtained by Bullen *et al.* (1990) using the CFD code FLUENT and the standard $\kappa - \epsilon$ turbulence model with logarithmic wall function. Bullen's data have been used in Test Case 2.

Measurements and semi-empirical correlations (analytical approach with the use of coefficients determined from experimental data) for the prediction of the pressure loss coefficient have been available since early 1950s with Kays' work (Kays, 1950). Before then, Harris (1928) carried out static pressure measurements and numerical studies for the determination of the total pressure loss coefficient in a sudden contraction with $A_2/A_1 = 0.125$ and $Re_1 = 10^5$. Harris reported his experiments in great detail, and is considered a reliable source. His numerical results agreed with his measurements. Harris' data have been used in Test Case 2.

Kays (1950) developed a semi-empirical correlation (Equation (4.1)) for the total pressure loss coefficient in sudden contractions and compared it with measurements carried out for $A_2/A_1 = 0.28$ in water. Both theory and experiments showed a double transition step of the static pressure in the transitional flow regime, with a wider gap for the laminar-to-transitional flow step than for the transitional to turbulent. The correlation agreed well with the experiments in transitional and turbulent flows. In laminar flow, the correlation started to disagree significantly for $Re < 1500$. Kays' have been used in Test Cases 1 and 2.

Benedict *et al.* (1966) carried out measurements of wall static pressure for $A_2/A_1 = 0.3, 0.525, 0.703$, and 0.821 and $Re_2 = 10^5$, and generated an empirical correlation for the total pressure loss coefficient (see Equation (4.8)). The location at which the downstream static pressure measurements were carried out is reported (Benedict, 1980) to be at the flow reattachment point, and assumed to be in fully-developed flow.

This assumption is not correct as the flow becomes fully-developed at least $20D_2$ length downstream of the contraction plane, while it reattaches at about $1D_2$ length, as discussed in Section 4.3.2 of ESDU 05024. Benedict's data have been used in Test Case 2.

Sylvester and Rosen (1970) and Kaye and Rosen (1971) carried out wall static pressure measurements in laminar flow at different Reynolds number ($Re_1 = 4 - 1100$), and for $A_2/A_1 = 0.041 - 0.636$. Different glycerol-water solutions were used at different Reynolds numbers. Based on the pressure measurements, the correlations in Equations (4.6) and (4.7) were generated. Kaye and Rosen's data have been used in Test Cases 1 and 2.

The more recent experimental data for the pressure loss coefficient by Bullen *et al.* (1996) are for turbulent flow and $A_2/A_1 = 0.135 - 0.629$. These data are considered reliable and have been used in Test Cases 1 and 2.

4.1.1 Correlations for pressure loss coefficients and profile factors

The semi-empirical correlation by **Kays** (1950) is based on the 1-D equations for (1) the momentum conservation at the inlet and outlet of the contraction, and for (2) the energy conservation at the contraction plane:

$$K_{ti,CM} = \frac{1 - J_{KE,1}(A_2/A_1)^2 C_c^2 - 2C_c^2 + 2C_c^2 J_{M,2}}{C_c^2} - (1 - (A_2/A_1)^2) \quad (4.1)$$

This correlation is valid for laminar, transitional and turbulent flows. The jet contraction ratio, $C_c = A_{vc}/A_2$ is determined analytically from Rouse's data (Rouse, 1946). Semi-empirical correlations for J_{KE} and J_M are given for transitional and turbulent flows in smooth tubes:

$$J_{KE} = -1.735(4f)^{3/2} + 3.272(4f) + 0.0883(4f)^{1/2} + 1 \quad (4.2)$$

$$J_M = -1.09068(4f) + 0.05884(4f)^{1/2} + 1 \quad (4.3)$$

where the friction factor f is $f = 0.46Re^{-2}$. The values of J_{KE} and J_M in laminar flow can be calculated as 2.0 and 1.33, respectively. Correlations in Equations (4.1) to (4.3) have been used in Test Cases 1 and 2.

Based on dimensional analysis, **Holmes** (1967) proposed the following equation for the static pressure loss coefficient in laminar flow:

$$K_{si} = A_H + A_C/Re_2 \quad (4.4)$$

where A_H is the Hagenbach coefficient and A_C is the Couette coefficient. Data in the literature for the Hagenbach and Couette coefficients are summarized in Section 4.1.2.

Based on pressure measurements, **Astarita and Greco** (1968) generated the following correlation for the static pressure loss coefficient in laminar flow:

$$K_{si} = 5.48 + 795/Re \quad (4.5)$$

This correlation is believed to be unreliable as measurements were not carried out in fully-developed flow conditions.

Again in laminar flow, **Sylvester and Rosen** (1970) generated the following correlation for the static pressure loss coefficient based on pressure measurements:

$$K_{si} = 2.4 + 295/Re_2 . \quad (4.6)$$

This correlation was improved by **Kaye and Rosen** (1971):

$$K_{si} = 2.32 + 159/Re_2 \quad (4.7)$$

and has been used in Test Cases 1 and 2.

In turbulent flow, **Benedict et al.** (1966) determined the following experimental correlation valid for the pressure loss at the flow reattachment point (see Section 4.1) and for $Re_2 \geq 10^5$:

$$K_{ti,CM} = 0.57806 + 0.39543(D_2/D_1) - 4.53854(D_2/D_1)^2 \\ + 14.24265(D_2/D_1)^3 - 19.22214(D_2/D_1)^4 + 0.854038(D_2/D_1)^5 . \quad (4.8)$$

A correlation for the kinetic-energy profile factor, J_{KE} , was provided by Benedict (1980) as a function of the friction factor f for transitional and turbulent flow:

$$J_{KE} = 1 + 2.7f . \quad (4.9)$$

Correlations in Equations (4.8) and (4.9) have been used in Test Case 2.

The ESDU correlations in **ESDU 01016** were based on: (a) Kaye and Rosen (1971) for laminar flow, and (b) Benedict et al. (1966), Levin and Clermont (1970), Bullen et al. (1987, 1988) for turbulent flow:

$$K_{ti,CM} = 0.54 + 0.004(A_2/A_1) - 1.285(A_2/A_1)^2 + 0.74(A_2/A_1)^3 . \quad (4.10)$$

In the transitional region ($2000 \leq Re_2 \leq 4000$), the flow condition was considered uncertain and no data were provided. The correlation in Equation (4.10) is biased towards the experimental data in Benedict et al. (1966) which are typically slightly higher than most other sources as they overestimate pressure drop (see Section 7.2.4). The correlation in Equation (4.10) has been used in Test Cases 1 and 2.

An alternative correlation for the total pressure loss coefficient, first proposed by Spiers (1955) and since updated by **Curzons** (2004), is based on a semi-empirical flow analysis:

$$K_{ti,CM} = \max \left[\frac{1}{2}(1 - A_2/A_1), \sqrt{\left(F \frac{l}{d} \right)^2 + F_2 \frac{l}{d}} - F \frac{l}{d} \right] \quad (4.11)$$

where $F = 64/Re$, $F_2 = 171/Re$ and $l/d = 4(1 - A_2/A_1)^4$. This correlation is valid for $10 < Re_2 < 10^6$, and has been used in Test Cases 1 and 2.

Commonly-used engineering references, such as **Idel'chik** (1994), **Miller** (1990), and **Crane** (1965) have published correlations for the pressure loss coefficient in sharp-edged, round-edged and chamfer-edged sudden contractions. Mathematical expressions are not available for these correlations and the data used

for comparison in this Technical Note have been extracted from graphs. The flow Reynolds number is not specified, but it is believed to be around $Re_2 \geq 10^5$, and, hence, in the turbulent flow regime. These correlations are based on experimental data and include the effects of fitting different mathematical equations, extrapolation where data were not available and application of the knowledge of the physics of the flow where data were not believable. Miller (1990) correlation for sharp-edged sudden contractions is based (Miller, 2004) on Harris (1928) and Benedict *et al.* (1966) experimental data. Idel'chik (1994) correlation is based on his own measurements, but details are not available. Correlations in **Idel'chik** (1994), **Miller** (1990), and **Crane** (1965) have been used in Test Case 2.

4.1.2 Hagenbach and Couette coefficients

As proposed by Holmes (1967) (Section 4.1.1) in laminar flow, the static pressure loss coefficient can be expressed as a function of the Couette coefficient, A_C , and the Hagenbach coefficient, A_H (see Equation (4.4)). The Couette coefficient is the controlling factor for pressure losses at very low Reynolds numbers, flow conditions. The Hagenbach coefficient is the controlling factor for pressure losses at higher, sub-transition Reynolds numbers. Theoretical and experimental data for A_C and A_H available in the literature are tabulated in Tables 4.4 and 4.5, respectively. These data have been used in Test Case 2.

TABLE 4.4 Values of Couette Coefficient in the Literature

Reference	Technique	Re_2	A_2/A_1	Couette Coefficient
Weissberg (1962)	Theoretical	0	0	37.7 – 43.6
Holmes (1967)	Theoretical	0	0	38.3
LaNieve and Bogue (1968)	Experimental	0.0001	0.00718	36.8
Astarita and Greco (1968)	Experimental	10 – 2000	0.1616	795
Sylvester and Rosen (1970)	Experimental	4 – 2000	0.015625	295 ± 50
Kaye and Rosen (1971)	Experimental	4 – 2000	0.04 – 0.636	159 ± 30

TABLE 4.5 Values of Hagenbach Coefficient in the Literature

Reference	Technique	Re_2	A_2/A_1	Hagenbach Coefficient
Schiller (1922)	Theory	1 – 2000	0 – 1	2.16
Collins and Schowalter (1962)	Theory	2000	0	2.32
Lundgren et al. (1964)	Theory	1 – 2000	0 – 1	2.34
Sparrow and Lin (1964)	Theory	1 – 2000	0	2.24
Astarita and Greco (1968)	Experimental	10 – 2000	0.1616	5.48
Sylvester and Rosen (1970)	Experimental	4 – 2000	0.015625	2.4 ± 0.1
Kaye and Rosen (1971)	Experimental	4 – 2000	0.04 – 0.636	2.32 ± 0.05

4.1.3 Downstream recovery length

The downstream recovery length, L_r , sometimes referred to as entry length (Durst and Loy, 1985; Boger, 1987), is the distance along the downstream duct from the contraction plane at which the flow can be considered fully-developed. Experimental and numerical studies on the downstream recovery length in laminar flow have been carried out by Durst and Loy (1985) and Vrentas and Duda (1973), respectively. Correlations for the downstream recovery length are expressed as a function of the downstream Re_2 .

The boundary layer asymptote by Hornbeck (1965) is given by:

$$L_r = 0.55 Re_2 . \quad (4.12)$$

This is based on the boundary-layer analysis which assumes no dissipation upstream of the contraction plane, flat velocity profile at the contraction plane, and $A_2/A_1 = 0$.

In laminar flow, Durst and Loy (1985) provided measurements of the downstream recovery length for $A_2/A_1 = 0.285$, and the following experimental correlation:

$$L_r = 0.54 Re_2 + 0.2 . \quad (4.13)$$

For $A_2/A_1 < 0.25$, Boger (1987) provided the following numerical correlation:

$$L_r = \frac{0.49 + 0.11 Re_2}{2} . \quad (4.14)$$

The correlations in Equations (4.12) to (4.14) and the experimental data by Durst and Loy (1985) have been used in Test Case 2.

4.2 Round-edged Sudden Contractions

Experimental and numerical data for round-edged sudden contractions are scarce. A summary of the data found in the literature is given in Table 4.6. Bullen and co-workers (Bullen *et al.*, 1988) have conducted reliable measurements for round-edged sudden contractions with $A_2/A_1 = 0.21 - 0.52$ and $Re_1 = 1.5 \cdot 10^5 - 2.5 \cdot 10^5$. Their data have been used by ESDU (2001) to develop the following correlation for the continuity-mean round-edge factor defined in Equation (3.10):

$$\lambda_{r,CM} = 0.455 \left[e^{-17.5(r/D_2)} + e^{-19.5(r/D_2)} + 0.198 \right] \quad (4.15)$$

This correlation fits Bullen's measurements within their estimated uncertainty of $\pm 10\%$.

TABLE 4.6 Round-edge Factor Data in the Literature

Reference	Technique	r/D_2	A_2/A_1	Re_1
Hamilton (1929)	Experimental	0 – 0.3	0.125	$\geq 10^5$
Harris (1930)	Experimental	0 – 0.3	0.125	$\geq 10^5$
Crane (1965)	Correlation	0 – 0.5	0	$\geq 10^5$
Bullen et al. (1988)	Experimental	0 – 0.14	0.21 – 0.52	$1.5 \cdot 10^5 - 2.5 \cdot 10^5$
Miller (1990)	Correlation	0 – 0.3	0	$\geq 10^5$
Idel'chik (1994)	Correlation	0 – 0.3	0	$\geq 10^5$
ESDU (2001)	Correlation	0 – 0.2	0.0 – 1.0	$\geq 10^4$

Among the data available in engineering reference books for round-edged sudden contractions, the correlations of Crane, Miller and Idel'chik are probably the most commonly-used. Crane does not provide specific values for roundness, and it reports two values of pressure loss coefficient for a “slightly rounded inlet” and for a “well rounded inlet”. Idel'chik's correlation is based on his own measurements. Miller's correlation is based (Miller, 2004) on Harris' data (Harris, 1930), which are based on the measurements carried out by Hamilton (1929). Hamilton's data are not reliable as the reported values of $\lambda_{r,CM} < 0$ for $r/D_2 \geq 0.2$ are not realistically possible. The experimental data of Bullen and Hamilton, and the correlations of Crane, Miller and Idel'chik have been used in Test Case 3.

4.3 Chamfer-edged Sudden Contractions

Data for chamfer-edged sudden contractions are even fewer. Idel'chik and Miller's data for the chamfer-edge factor are described in Table 4.7. Idel'chik's correlation is reported to be “approximate” and is based on the authors' experiments for flow through an entry from a large space or inlet, *i.e.* $A_2/A_1 = 0$. Miller did not report the source of data used to develop his correlations, and noted it as Class 2; that is, based on experimental data “where no detailed cross checking is possible against other sources”. Miller's correlation is also for flow in inlets, *i.e.* $A_2/A_1 = 0$. Miller and Idel'chik data have been used in Test Case 4.

TABLE 4.7 Chamfer-edge Factor Data in the Literature

Reference	Technique	l_e/D_2	ϕ	A_2/A_1	Re_1
Miller (1990)	Correlation	0 – 0.5	30°, 45°	0	$\geq 10^5$
Idel'chik (1994)	Correlation	0 – 0.6	0° – 90°	0	$\geq 10^5$

5. DESCRIPTION OF THE TEST CASES

5.1 Test Case 1: Sharp-edged Sudden Contractions with $A_2/A_1 = 0.285$ and $1 \leq Re_1 \leq 10^6$

The geometrical parameters for this Test Case are based on the work by Durst and co-workers (Durst and Loy, 1985; Buckle and Durst, 1993):

$$A_2/A_1 = 0.285 \text{ and } D_1 = 19.1 \text{ mm}.$$

The flow conditions were extended to transitional and turbulent flow:

$$1 \leq Re_1 \leq 10^6.$$

A summary of the experimental and numerical data used in this Test Case is given in Table 5.1. Validation data for the axial and radial velocities, pressure distribution and separation size in laminar flow were based on Durst's data. Validation data for the pressure loss coefficient were from Kaye and Rosen (1971), Kays (1950), Bullen *et al.* (1996) experimental data, and Kays (1950) and Curzons (2004) correlations. A description of the experimental techniques and their uncertainties is given in Section 5.1.1. The numerical methods are discussed in Section 5.1.2.

TABLE 5.1 Summary of the validation data for Test Case 1

Reference	Technique	Data	Data Format	Re_1
Durst and Loy (1985), Buckle and Durst (1993)	Experimental and Numerical	Axial and radial velocities	Tables and graphs	23, 196, 372, 968, 1213
	Numerical	Pressure profiles	Graphs	568
	Experimental and Numerical	Separation size	Graphs	23 – 1213
Kaye and Rosen (1971)	Experimental	Pressure loss coefficient	Correlation	4 – 1100
Kays (1950)	Experimental and semi-empirical ($A_2/A_1 = 0.280$)	Pressure loss coefficient	Graphs and correlation	550 – 20 000
Bullen <i>et al.</i> (1996)	Experimental ($A_2/A_1 = 0.297$)	Pressure loss coefficient	Graphs	$4 \cdot 10^4 - 2 \cdot 10^5$
ESDU (2001)	Correlation	Pressure loss coefficient	Correlation	$\geq 10^4$
Curzons (2004)	Semi-empirical	Pressure loss coefficient	Correlation	$10 - 10^6$

5.1.1 Experimental techniques

LDA measurements ($Re_1 = 23 - 1213$)

The LDA test section in Durst and co-workers' measurements consisted of a glass pipe with sudden change in cross section, mounted inside a channel filled with same fluid. The fluid was oil-Palatinol mixture with the same refractive index as the glass of the pipe wall, kept under controlled temperature conditions. A three-dimensional traversing system was used to allow precise positioning of the measuring point at any location inside the pipe. The Laser Doppler system consisted of a 15 mW-Helium-Neon-Laser and a compact optical system manufactured by OEI incorporating a double Bragg cell device for frequency shifting the laser beams. The effective measuring control volume was 200 μm in diameter and about 1600 μm in length. Measurements were carried at various sections upstream and downstream of the contraction plane.

The reported estimated global uncertainty of the measurements was within 2% with 95% confidence. This was derived from the following reported uncertainties:

- Signal processing electronics of LDA-system: 1%
- Traversing unit: 1%
- Mass flow rate oscillations: 0.5%

Separation size measurements ($Re_1 = 23 - 1213$)

Durst and co-workers carried out observations of the separation regions by the illumination of a laser light sheet. Measurements of the separation bubble developing downstream of the contraction plane were provided as length and height of the separation. Details of the flow inside the separation could not be obtained, as a control volume of 20 μm is required. Based on the information provided and the scatter of the experimental data, the uncertainty of the measurements can be estimated as $\pm 30\%$. The upstream separation region could not be resolved and only visual observations were carried out to analyse the variation of the recirculation size with Re .

Pressure loss coefficient ($Re_1 = 4 - 2 \times 10^5$)

The experimentally based correlations of Kaye and Rosen (1971) and Sylvester and Rosen (1970) for the static pressure loss coefficient K_{si} (see Equations (4.6) and (4.7)) are for $A_2/A_1 = 0.041 - 0.636$ and $Re_1 = 4 - 1100$. The experimental uncertainty was not reported by Kaye and Rosen, but could be estimated from the minimum precision reported: ± 0.02 inHg. Different glycerine-water solutions were used for different Reynolds number but only the minimum and maximum values of 30% to 95% solutions of glycerine in water were specified. It can be assumed that the solution with the highest glycerol content was used for lowest Reynolds number ($Re_2 = 6$) and that with the smallest glycerol content was used at the highest Reynolds number ($Re_2 = 2004$). If it is then assumed that the experimental error was only due to measurements at the inlet of the contraction, the resulting estimated uncertainty is about $\pm 80\%$ at $Re_2 = 6$ and $\pm 20\%$ at $Re_2 = 2004$.

Measurement uncertainties for Kays' data are not reported. Measurement uncertainties for Bullen's data are discussed in Section 5.2.1.

5.1.2 Numerical method

Durst and co-workers used the CFD software TEACH-T to generate their numerical velocity and pressure data. The mass and momentum equations were solved for a 2-D axisymmetric model using a finite volume method. The pressure correction method used was SIMPLE (Semi-Implicit Method for Pressure-Linked Equation). The differencing scheme was Hybrid (combination of Central and Upwind solution schemes). The grid distribution allowed for mesh refinement at the wall and at the contraction plane. The largest mesh size used in (Durst and Loy, 1985) was made of 4740 cells. This was increased to 38400 in (Buckle and Durst, 1993). The boundary conditions used were: inlet velocity with parabolic profile, and outlet zero gradients for all variables. The lengths of the upstream and downstream pipes were $L_{u1} = 1.3D_1$ and $L_{d2} = 1D_1$, respectively.

5.2 Test Case 2: Sharp-edged Sudden Contractions with $A_2/A_1 = 0.135, 0.332, 0.629$ and $1 \leq Re_1 \leq 10^6$

The geometrical parameters for this Test Case are based on the work by Bullen and co-workers (Bullen *et al.*, 1984; 1987; 1990; 1996)

$$A_2/A_1 = 0.135, 0.332, 0.629 \text{ and } D_1 = 110.17 \text{ mm}.$$

The flow conditions were extended to laminar and transitional flow:

$$1 \leq Re_1 \leq 10^6.$$

A summary of the experimental and numerical data used in this Test Case is given in Table 5.2. Validation data for the axial and radial velocities, and separation core extent in turbulent flow were based on Bullen's data for $A_2/A_1 = 0.332$. Validation data for the pressure loss coefficient are from Harris (1928), Benedict *et al.* (1966), Kaye and Rosen (1971) and Bullen *et al.* (1996) experimental data, and Kays (1950), Crane (1965), Miller (1990), Idel'chik (1994), ESDU (2001), and Curzons (2004) correlations. Validation data for the Couette and Hagenbach coefficients are from the sources described in Tables 4.4 and 4.5, respectively. Validation data for the downstream recovery length are from the sources described in Section 4.1.3 and Equations (4.12) to (4.14). Validation data for the profile factors are from the sources described in Section 4.1.1 and Equations (4.2), (4.3) and (4.9). A description of the experimental techniques and their uncertainties is given in Section 5.2.1. The numerical methods are described in Section 5.2.2.