



CFD studies for the validation of friction losses and flow characteristics in circular straight pipes with smooth walls

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CFD STUDIES FOR THE VALIDATION OF FRICTION LOSSES AND FLOW CHARACTERISTICS IN CIRCULAR STRAIGHT PIPES WITH SMOOTH WALLS

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CFD STUDIES FOR THE VALIDATION OF FRICTION LOSSES AND FLOW CHARACTERISTICS IN CIRCULAR STRAIGHT PIPES WITH SMOOTH WALLS

1. NOTATION AND UNITS

		SI Unit
A	pipe cross-sectional area	m^2
D	pipe internal diameter	m
f	friction factor, defined in Equation (3.16), also termed Fanning friction factor	-
J_{KE}	kinetic-energy velocity-profile factor, defined in Equation (3.49)	-
J_M	momentum velocity-profile factor, defined in Equation (3.50)	-
K_{en}	non-dimensional additional entrance pressure drop, defined in Equation (3.15)	-
$K_{en,t}$	non-dimensional additional (negative) pressure drop in the turbulent section, defined in Sketch 3.2	—
$K_{en,tot}$	non-dimensional pressure drop between pipe entrance and fully-developed flow axial location, defined in Sketch 3.2	—
K_τ	Kármán number, defined as $K_\tau = u_\tau \delta / \nu$	-
k_s	average sandgrain height	m
k_s^+	non-dimensional average sandgrain height, defined as $k_s^+ = k_s u_\tau / \nu$	-
L	pipe length	m
L_d	pipe length required to reach fully-developed flow	m
m	mass flow rate	kg/s
p	static pressure	Pa
R	pipe internal radius	m
Re	pipe Reynolds number, defined as $Re = u_{mean} D / \nu$	-
Re_θ	momentum-thickness Reynolds number, defined as $Re_\theta = \theta u_\infty / \nu$	-
$Re_{\theta t}$	transition-onset momentum-thickness Reynolds number	—
r	pipe radial coordinate	m
U_{max}	theoretical centre-line velocity in laminar flow	m/s

u	axial local velocity in laminar flow or time-mean axial local velocity in turbulent flow	m/s
u^+	non-dimensional local axial velocity, defined as $u^+ = u / u_\tau$	-
u_{cl}	axial local centre-line velocity	m/s
$u_{cl,fd}$	axial centre-line velocity in fully-developed flow	m/s
u_{max}	maximum axial local centre-line velocity in turbulent flow	m/s
u_{mean}	bulk-mean velocity, defined as $u_{mean} = m/(A\rho)$	m/s
u_τ	friction velocity, defined as $u_\tau = \sqrt{\tau_w / \rho}$	m/s
u_∞	local free-stream velocity	m/s
x	pipe axial coordinate	m
$Yplus$	wall y^+ , defined as $Yplus = y_w u_\tau / \nu$	-
y	distance from wall	m
y^+	non-dimensional wall distance, defined as $y^+ = y u_\tau / \nu$	-
y_w	distance from wall of closest mesh node	m
$\bar{y}_w$	average distance from wall of closest mesh node	m
Δf	percentage difference between CFD predictions or measurements of friction factor and ESDU 66027 ⁴³ friction factor	-
Δp_{en}	additional pipe pressure drop due to entry of fluid into pipe from a large tank, defined in Sketch 3.2	Pa
$\Delta p_{en,t}$	additional (negative) pressure drop due to entry of fluid into a circular pipe in turbulent section, defined in Sketch 3.2	Pa
$\Delta p_{en,tot}$	pressure drop between pipe entrance and fully-developed flow axial location, defined in Sketch 3.2	Pa
Δu_{cl}	percentage difference between u_{max} predicted using CFD and theoretical axial centre-line velocity in laminar flow, U_{max}	-
δ	boundary layer thickness	m
δ_l	thickness of laminar sub-layer, defined in Equation (3.41)	m
ε	effective roughness height or turbulence eddy dissipation	m $m^2 s^{-3}$
γ	intermittency factor, defined as the fraction of time that motion is turbulent in the transition process	-
κ	turbulence kinetic energy	$m^2 s^{-2}$

λ	alternative friction factor, defined as $\lambda = 4f$	-
μ	fluid dynamic viscosity	Pa s
μ_t	turbulent viscosity	Pa s
μ_t^*	turbulent viscosity in transitional flow	Pa s
ν	fluid kinematic viscosity	m ² /s
θ	boundary layer momentum thickness	m
ρ	fluid density	kg/m ³
τ	fluid shear stress	Pa
τ_w	wall fluid shear stress	Pa
ω	turbulent frequency	s ⁻¹

Abbreviations Glossary

AUTOMATIC	ANSYS CFX proprietary near-wall treatment which switches automatically from wall-functions to a low-Reynolds-number treatment near-wall formulation as the mesh is refined
CFD	computational fluid dynamics
DNS	direct numerical simulation
LDA	laser Doppler anemometry
LES	large eddy simulation
RANS	Reynolds-averaged Navier Stokes
SCALABLE	ANSYS CFX proprietary near-wall treatment which allows solution on arbitrarily fine mesh near the wall when using the $\kappa-\varepsilon$ turbulence model
SST	shear stress transport

2. INTRODUCTION

In this Technical Note, the ESDU CFD validation studies are presented for the prediction of friction losses and flow characteristics in straight circular pipes with smooth walls.

Understanding of pipe friction and associated flow characteristics in different flow regimes is fundamental to the understanding of boundary layer development and turbulence modelling in internal flow engineering applications such as viscous drag and heat-transfer processes.

Typical industrial CFD applications of pipe flow include systems where pipes are part of a more complex domain, such as aircraft ducts, pipe fittings, heat exchangers, various components of power generation plants, etc. In more general CFD applications, straight long pipes are often used to set inlet and outlet boundary conditions. At inlet boundaries, they are used as an artificial extension of the inlet to the investigated flow domain to allow the natural development of duct boundary layers. At outlet boundaries, straight pipes are used as an artificial extension to the outlet of the investigated flow domain to reduce the influence of the outlet local fluid dynamics on the CFD predictions. In all such applications, it is essential to reduce the sensitivity of CFD predictions of friction losses to mesh density and distribution, pipe length required for fully-developed flow, boundary conditions and turbulence modelling.

Accurate prediction of the friction losses and flow characteristics in straight pipes is one of the most challenging problems in CFD. Although direct numerical simulation (DNS) and large eddy simulation (LES) are used in research, in industry Reynolds-averaged Navier-Stokes (RANS) methods are largely used.

The purpose of this work is to provide guidance on the CFD modelling of friction losses and flow characteristics in:

- straight circular pipes with smooth walls,
- laminar, transitional and turbulent flow regimes,
- swirl-free, uniform inlet velocity conditions,
- fully-developed exit flow conditions,
- steady-state flow conditions,
- incompressible flow of single-phase, Newtonian liquids and gases.
- employing commercial CFD packages typically used in the industry, *i.e.* finite volume RANS code with low-Reynolds-number turbulence and transition correlation-based ‘bypass’ modelling capabilities.

Extensions to rough pipes and to compressible flow will be considered in future work.

Three types of meshes were considered: 2D-axisymmetric hexahedral meshes, 3D purely tetrahedral meshes and 3D tetrahedral meshes with prismatic layers. Pipes with different lengths were tested in laminar, transitional and turbulent flow at Reynolds numbers ranging $1 < Re < 10^7$.

Three different turbulence models were considered with different near-wall treatments: the $\kappa-\varepsilon$ with SCALABLE near-wall treatment, $\kappa-\omega$ with AUTOMATIC near-wall treatment and SST with AUTOMATIC near-wall treatment⁹⁵. Three transition models were considered: the SST specific- γ , SST γ and SST $\gamma-Re_0$ models⁹⁵. Detailed CFD validation studies on transitional flow are presented in ESDU TN 08009¹⁰⁵.

A brief description of the fundamental fluid mechanics in straight pipes is given in Section 3. In this section, correlations, measurements and DNS predictions in the literature for friction losses and flow characteristics in straight pipes are reviewed. The methodology used in the CFD predictions is described in Section 4.

The ESDU CFD predictions for the different meshes, pipe lengths, Reynolds number and turbulence models are discussed in Section 5 and compared to experimental data and correlations in Section 6. Best Practice Guidelines on mesh density and distribution, setting of boundary conditions and turbulence modelling are given in Section 7. Overviews on the CFD modelling of turbulent and transitional flows are given in Appendices A and B, respectively.

3. VALIDATION DATA IN THE LITERATURE

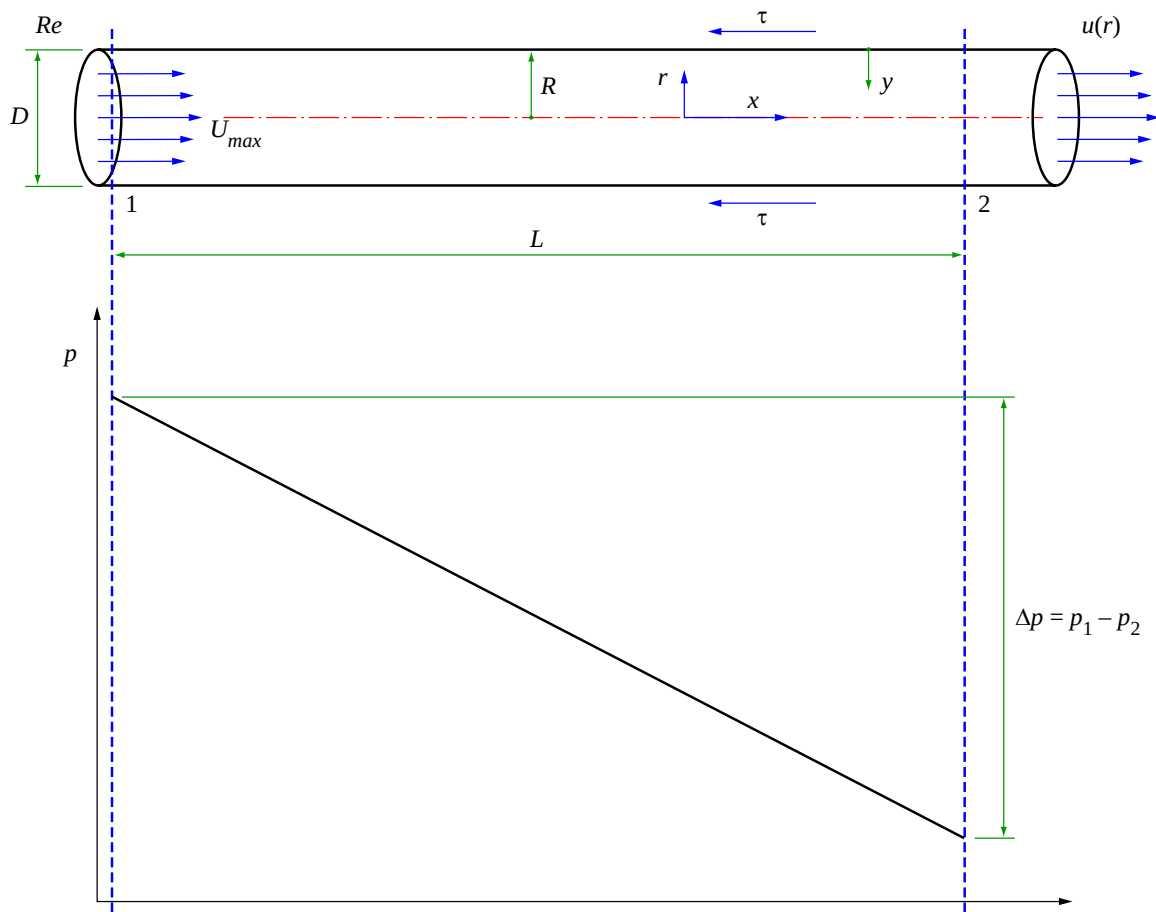
The flow in a straight pipe has been the focus of extensive experimental and CFD investigations for many years. At the time of writing, CFD predictions of turbulent flows has become an important research tool in studying the basic physics of turbulence and work was conducted using direct numerical simulations (DNS)^{56,57,60}. Whilst, in general, experimental and CFD findings agreed well on the flow structure away from the wall, inconsistencies in the turbulence properties at the wall were reported. This motivated recent experimental work on non-invasive LDA measurements in the near-wall region^{61,62,63}.

At the time of writing, the main research investigations reported in the literature included:

- the experimental work conducted at the “superpipe” facility at Princeton University led by Smits^{66,73,82,83},
- the experimental and CFD work conducted at the Center for Turbulence Research⁶⁹(consortium between Stanford University and NASA),
- the experimental and CFD work conducted by Durst and co-workers at the University of Erlangen-Nürnberg^{61,62,86}, and
- the experimental and CFD work conducted at University of Delft^{60,63,75,79}.

3.1 Flow in a Straight Pipe

The flow characteristics through a straight pipe with circular cross-section are illustrated in Sketch 3.1. The axial local velocity, $u(r)$, is zero at the pipe wall and maximum at the centreline. At a distance from the entrance section, the velocity distribution across the section becomes independent of the axial coordinate and hence reaches fully-developed flow conditions.



Sketch 3.1 Schematic of fully-developed flow in a straight pipe

3.1.1 Laminar flow

In laminar flow, the velocity remains constant on cylindrical surfaces concentric to the axis and individual cylindrical laminae slide over each other. In a coaxial fluid cylinder of length, L , between sections 1 and 2 (Sketch 3.1), and radius, r , the fluid motion is controlled by the force balance between the pressure gradient in the axial direction, and the frictional shear stress, τ , at a distance y from the wall:

$$(p_1 - p_2) \pi r^2 = 2\pi y L \tau \quad (3.1)$$

Hence,

$$\tau = \frac{(p_1 - p_2)}{L} \frac{r}{2}$$

or for a length dx ,

$$\tau = \frac{dp}{dx} \frac{r}{2} \quad (3.2)$$

Using the definition of the shear stress,

$$\tau = -\mu \frac{du}{dr}, \quad (3.3)$$

upon integration, a parabolic velocity distribution over the radius is found:

$$u(r) = \frac{p_1 - p_2}{4\mu L} (R^2 - r^2) \quad (3.4)$$

From Equation (3.4) and considering the mean velocity over the cross-section, u_{mean} , it is found that the flow rate through a pipe in laminar flow is given by the following Hagen-Poiseuille equation:

$$Q = \frac{\pi R^4}{8\mu L} (p_1 - p_2). \quad (3.5)$$

It is also found that the pressure drop, $\Delta p = (p_1 - p_2)$, increases linearly with u_{mean} and the flow rate through the pipe (or Re):

$$\Delta p = (p_1 - p_2) = 8 \mu \frac{L}{R^2} u_{mean} \quad (3.6)$$

Based on Schlichting⁵¹ observations, for example, it can be assumed that laminar flow exists for

$$Re \leq 2300 \quad (3.7)$$

where

$$Re = \frac{u_{mean} D}{\nu} \quad (3.8)$$

For $Re > 2300$, as the Reynolds number increases, the regularity of the fluid motion starts to break down and the flow regime becomes transitional. At higher Reynolds numbers, as the flow regime becomes turbulent, irregular radial fluctuations occur that superimpose on the axial motion and cause an exchange of the transversal momentum and affect mixing. As a consequence, the velocity distribution over the cross section is more uniform than in laminar flow.

3.1.2 Transitional flow

At low Reynolds number the flow is laminar. As the Reynolds number is increased above around $Re = 2300$, the flow regime becomes transitional. The transition from laminar to turbulent flow is a relatively complex process which is far from being completely known and documented. Experimental observations^{15,47,64,77} have shown that the transition process in pipe flow is characterized by the appearance of turbulent spots in the viscous wall layer which subsequently grow and merge downstream. This is due to the presence of the pipe wall boundary and, depending on the shape of the pipe entry and on the presence of separated, recirculating flow, can lead to a transition that is rapid compared to that observed in zero-pressure-gradient flat plate boundary layer flow. This latter external flow application is characterised by the sequential formation of typical flow instability features, *i.e.* Tollmien-Schlichting waves, 3D vortices and bursting vortices, all of which precede the generation of turbulent spots.

In transitional pipe flow, turbulence spots can be generated by localized disturbances and are favoured by wall roughness, *i.e.* transition occurs at lower Reynolds number for a rough wall than for a smooth wall. These are bursts of turbulent-like fluctuations. Following their appearance and depending on the pipe Reynolds number, the transition process can progress to form local regions of turbulent motion, or turbulent plugs or slugs, which travel and grow downstream in the pipe.

Over the pipe length where flow transition is in progress, alternate regions of non-stable (turbulent) and stable (laminar) flow may exist. Transition to turbulence occurs if disturbances are sufficiently amplified in the non-stable regions. At any specific location in the transition zone of a pipe, laminar and turbulent flow motions alternate in time. The fraction of time that motion is turbulent is known as the intermittency factor, γ . At the onset of transition, $\gamma = 0$. The intermittency factor increases with distance downstream in the pipe as a result of the turbulent slug growth at the expense of the non-turbulent regions. The region of intermittently turbulent flow is often hundreds of diameters long. As turbulence propagates into neighbouring non-turbulent regions at each end of the slug, the upstream and downstream slugs merge and the flow becomes fully turbulent. At this condition, $\gamma = 1$.

At the pipe entrance, the appearance of local turbulent regions depends on a critical disturbance level being attained. It has been observed that for $Re < 1760$, no turbulence structures can be sustained, *i.e.* turbulent spot production can be triggered, but transition to turbulence does not result⁹⁷. By taking great care to minimize the disturbance level, experimenters have been able to maintain laminar flow up to $Re = 6.4 \times 10^4$, reported by Draad *et al.*⁶⁴, or even $Re = 10^5$, reported by Lachmann³⁴. It is likely that in these experiments, local unstable regions forming in the entry boundary layer decay instead of leading to transition to turbulent flow.

It has been reported⁷⁷ that the lower Reynolds-number limit for transition to slug growth, even for very large disturbances, is $Re = 2.1 \pm 0.3 \times 10^3$. For $Re \geq 10^5$, the flow always becomes turbulent through amplification of small disturbances.

3.1.3 Turbulent flow

Turbulent flow is a complex three dimensional, time-dependent phenomenon which consists of multiple-scale fluctuations in time and space. At a given physical location in the flow, velocity and pressure are not constant in time but exhibit very irregular, high frequency fluctuations. These can only be considered constant at a given physical location when averaged over a long period of time⁵¹.

Much of the knowledge of turbulent flows is based on experiments. A statistical description for turbulence considers two parts for pressure and velocity components, the time-mean and fluctuating parts. When using a statistical approach, some of the quantities can be measured from which measurements an understanding of the physical process and a theoretical approach can be developed. Based on the further development of the theoretical method, certain quantities are then measured in its support.

In principle, the time-dependent Navier-Stokes equations describe both laminar and turbulent flows. However, turbulent flows generally involve length scales much smaller than the smallest finite volume mesh that can be practically used in a CFD analysis. Turbulence models are commonly used in industry that model turbulence effects without solving the turbulent fluctuations. The direct numerical simulation (DNS) of these flows requires computing power which is many orders of magnitude greater than that available in the foreseeable future.

One of the classical analyses of pipe turbulent flow was conducted by Laufer²⁹ using hot-wire velocity measurements. His measurements are still used to validate experimental and CFD investigations. They are also used to set velocity and turbulence intensity boundary conditions in pipes where experimental data are not available. Laufer²⁹ showed that over the pipe cross-section the rates of turbulent energy production, dissipation and diffusion have sharp maxima near the edge of the laminar sub-layer and that there exists a strong movement of kinetic energy away from this radius and an equally strong movement of pressure energy towards it. For the turbulence structure, three regions were identified:

- a wall proximity region where turbulence production, diffusion and viscous dissipation are equally important,
- a central pipe region where energy diffusion plays a predominant role,
- an intermediate region where the local rate of change of turbulent-energy production dominates the energy received by dissipation.

These findings form the basis of the understanding of turbulence production, diffusion and dissipation used to validate measurements and CFD models of pipe turbulent flows. An overview on the CFD modelling of turbulent flow using RANS methods is given in Appendix A.

3.2 Pipe Entrance

When a fluid enters a circular pipe from a large reservoir, the cross-sectional velocity distribution along the pipe length varies with the distance from the inlet until the pressure gradient becomes constant and the flow is said to be fully-developed. The distance over which the velocity development occurs is often called the hydrodynamic entrance length or entrance region.

In turbulent flow, since the fluid near the wall is retarded, there must be a small radial component of velocity away from the wall³⁷. Low-momentum fluid is thus convected to regions of higher velocities and a momentum exchange between high and low-momentum fluid particles occurs. New turbulence is initiated in the form of axial velocity fluctuations. The turbulence spread continues until the shear flow fills the pipe. After the fluid has become completely turbulent the velocity distribution reaches the fully-developed conditions by turbulent mixing transverse to the flow.

The flow problem for the entrance region does not yield to an exact solution. This is also true for laminar flow where various approximate analytical solutions have been devised to provide information on the flow development and entrance pressure drop.

3.2.1 Development Length

The pipe length required to achieve fully-developed flow conditions is referred as to the development length, L_d . This is usually defined as the length required for the centreline velocity, u_{cl} , to achieve 99% of the theoretical or calculated fully-developed flow value.

The importance of knowing the development length for the design of pipe flow systems has long been recognized. In laminar and transitional flows, development lengths vary widely depending on the Reynolds number. In turbulent flow, transition to turbulence at the pipe entrance needs to be taken into account.

A summary of the data in the literature for the development length, L_d , is given in Table 3.1 and Figure 3.1.

TABLE 3.1 Summary of the development length data in the literature

Reference	Technique	Data Format	Re
Schiller ⁹	Theory	Equation (3.10)	$< 10^5$
Nikuradse ²⁶	Velocity measurements	Equation (3.12)	Laminar flow
Nikuradse ¹³	Measurements	Values	Turbulent flow
Collins and Schowalter ³⁵	Theory	Table 3.2	Laminar flow
Barbin and Jones ³⁷	Hot wire measurements	Plots	$Re = 3.88 \times 10^5$
Hornbeck ³⁸	Theory	Equation (3.11)	$< 10^4$
Sparrow <i>et al.</i> ⁴⁰	Theory	Table 3.2	Laminar flow
Fargie and Martin ⁴⁵	CFD	Table 3.2	Laminar flow
Chen ⁴⁶	Theory	Table 3.2	Laminar flow
Lien <i>et al.</i> ⁸¹	Measurements (Channels)	Plots	$4 \times 10^4 < Re < 2 \times 10^5$
Durst <i>et al.</i> ⁸⁶	CFD	Equation (3.14)	$0.01 < Re < 10^4$
ESDU ^{87,91}	CFD	CFD predictions	$1 < Re < 10^7$
Poole and Ridley ⁹⁹	CFD	Plots	$0 < Re < 1000$

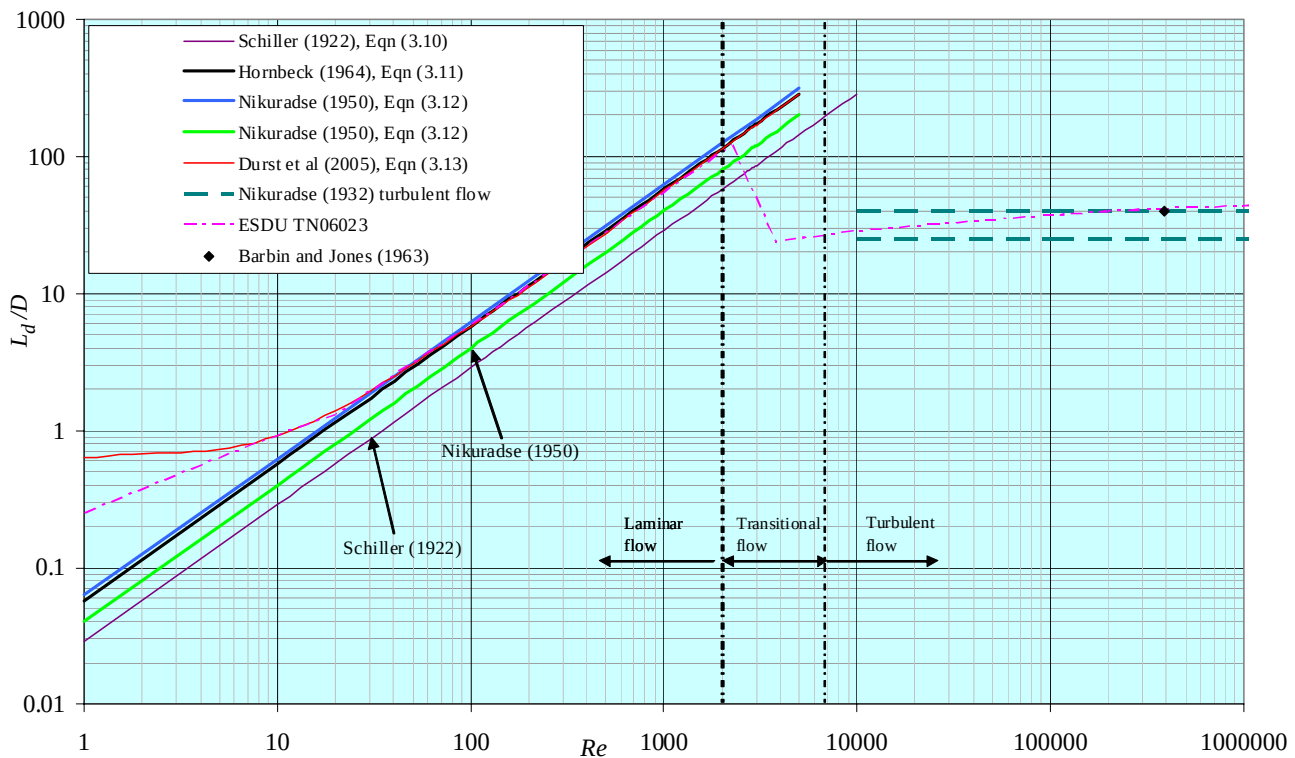


FIGURE 3.1 Summary of the development length data in the literature

3.2.1.1 Laminar and transitional flow

In laminar flow, there have been various investigations into the pipe development length using analytical, experimental and CFD methods (see Tables 3.1 and 3.2). Most of these investigations provide a linear correlation for L_d / D with the Reynolds number in the following general form:

$$\frac{L_d}{D} = C Re \quad (3.9)$$

where C is a constant. Values for the constant C reported by various sources in the literature are shown in Table 3.2 and Figure 3.2.

TABLE 3.2 Values of the constant C from various sources in the literature

Reference	Technique	C
Schiller ⁹	Theory	0.02875
Nikuradse ²⁶	Velocity measurements	0.04 to 0.0625
Collins and Schowalter ³⁵	Theory	0.061
Hornbeck ³⁸	Theory	0.0575
Sparrow <i>et al.</i> ⁴⁰	Theory	0.056
Fargie and Martin ⁴⁵	CFD	0.049 to 0.068
Chen ⁴⁶	Theory	0.052 to 0.068
ESDU ⁹¹	CFD	0.053 to 0.067

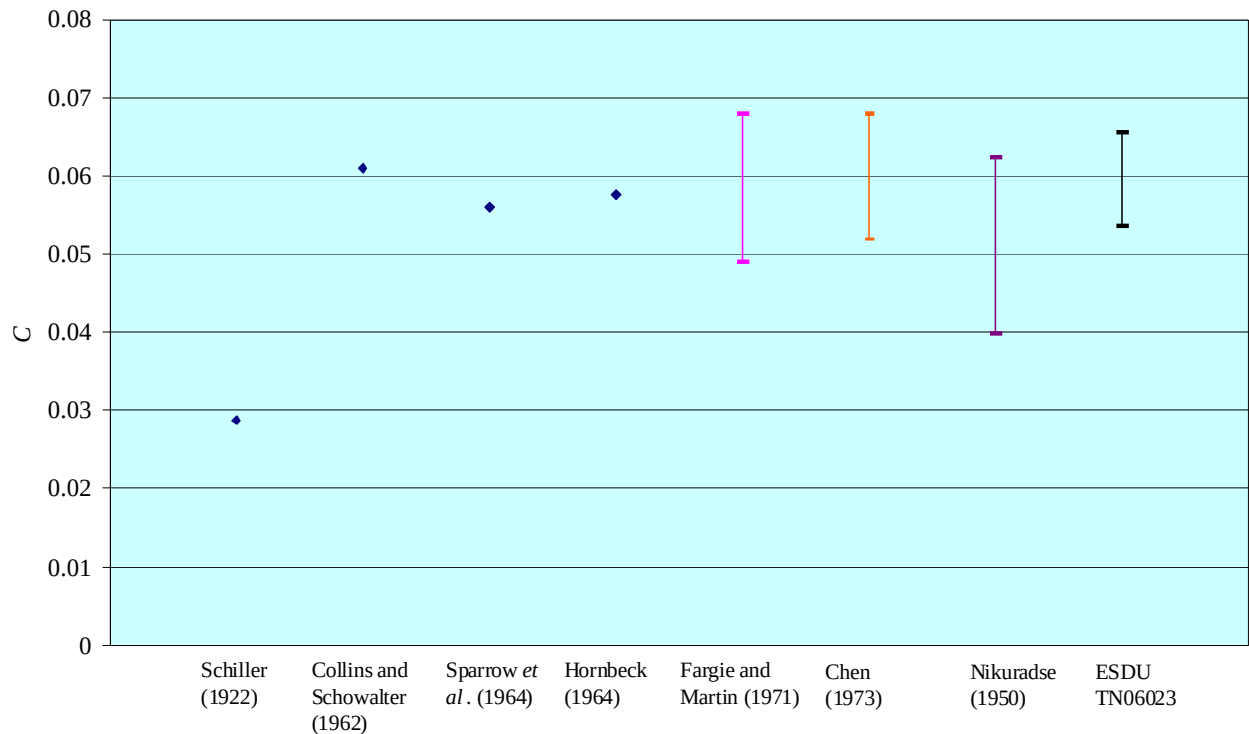


FIGURE 3.2 Values of the constant C from various sources in the literature

In laminar to transitional flow, Schiller⁹, for example, provided the following equation based on the theoretical integral method valid for $Re \leq 10^5$:

$$L_d = 0.02875DRe \quad (3.10)$$

Equation (3.10) gives for $Re = 2000$ and $10\,000$ lengths of $L_d = 57.5D$ and $287D$, respectively. These values are much smaller than those reported by other investigators in Table 3.2. According to Hornbeck³⁸, for example:

$$L_d = 0.057DRe \quad (3.11)$$

i.e. $L_d = 114D$ for $Re = 2000$ and $L_d = 570D$ for $Re = 10000$. Equation (3.11) is based on the boundary-layer analysis which assumes no dissipation upstream of the pipe entrance and a flat velocity profile at pipe entrance. Velocity measurements by Nikuradse²⁶ also show that fully-developed flow-conditions are reached more slowly than those predicted in Equation (3.11):

$$L_d = C D Re \quad (3.12)$$

$$C = 0.04 \text{ to } 0.0625$$

Hot wire measurements by Durst *et al.*⁸⁶ have revealed that L_d is very sensitive to small measurement inaccuracy. This could explain the wide range of values reported for the constant C , see Figure 3.2 and Table 3.2. Recent experimental and CFD studies have shown that at Reynolds numbers $Re < 100$, L_d is

independent of Reynolds number⁸⁶. The following form for L_d has been suggested:

$$\frac{L_d}{D} = C_0 + C_1 Re \quad (3.13)$$

However, based on CFD simulations, Durst *et al.*⁸⁶ found that for $0.01 < Re < 10^4$ the following non-linear correlation predicted their CFD results within 3%:

$$L_d = D \left[0.619^{1.6} + (0.0567 Re)^{1.6} \right]^{1/1.6} \quad (3.14)$$

This correlation was consistent with the CFD predictions of Poole and Ridley (2007).

As shown in Figure 3.1, values of L_d from ESDU CFD studies in sudden contractions are in agreement with Durst and Hornbeck's equations⁹¹.

For non-circular pipes, Fleming and Sparrow⁴⁴, provide data for L_d from analytical solutions and experiments in rectangular and triangular ducts. These show that for rectangular ducts with aspect ratios greater than 2:1, L_d was smaller than for circular ducts. For rectangular ducts with aspect ratios smaller or equal to 2:1 and for triangular ducts, L_d was greater than for circular ducts.

3.2.1.2 Turbulent flow

In turbulent flow, enhanced mixing among fluid layers causes a more rapid growth of the boundary layer. Measurements by Nikuradse¹³ for $Re \geq 10^5$ have shown that L_d varies between 25 to 40 D . This range is in agreement with ESDU CFD predictions for entry to sudden contractions for turbulent flow⁹¹.

Measurements by Barbin and Jones³⁷ showed that centreline velocity reaches fully-developed conditions at about 40 pipe diameters downstream the pipe entrance. They also found that turbulent quantities become fully-developed at a pipe length much greater than 40 D . However, measurements of the friction factor indicated that only 15 D are required to reach fully-developed conditions.

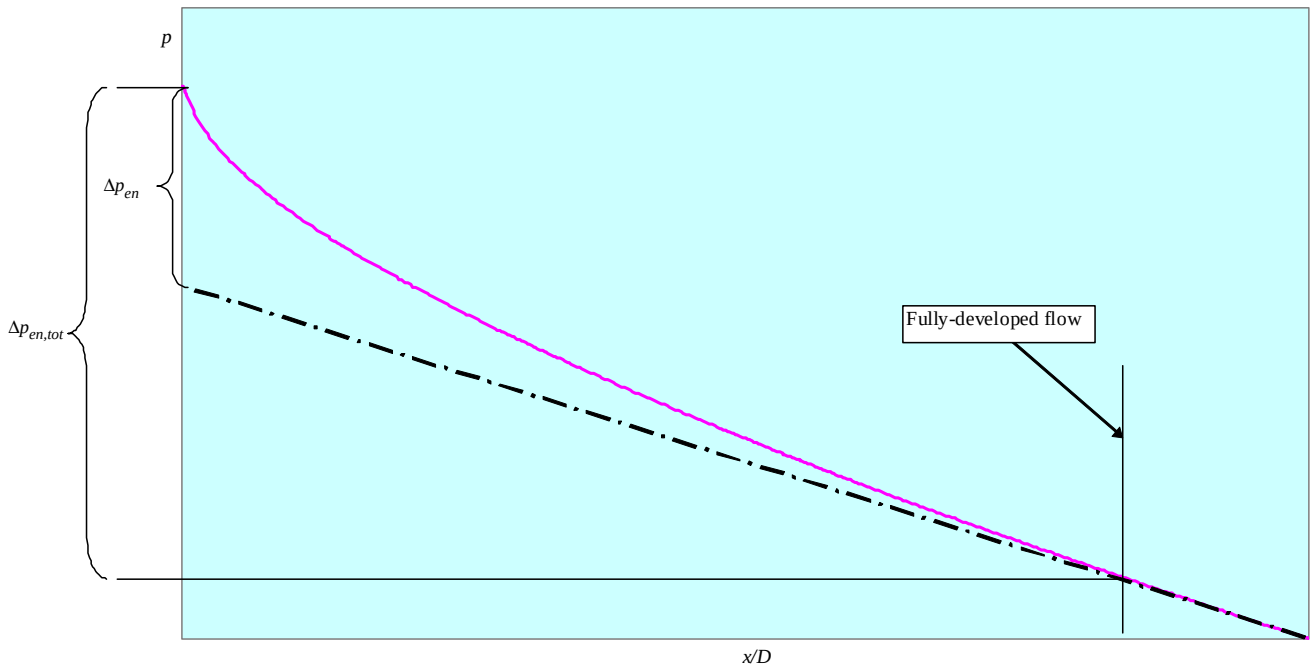
3.2.2 Entrance Pressure Drop

At a pipe entrance, the pressure drop is higher than in the fully-developed region. In fact, if the velocity distribution at the pipe entrance is considered approximately uniform, as the fluid enters the pipe the boundary layer is very thin at entrance and grows in the downstream direction. Consequently, the velocity gradient at the wall, and hence the wall shear stress, decreases from a high value near the pipe entrance to the fully-developed flow value. The acceleration of the flow central core and the higher wall shear cause a larger pressure gradient than that in fully developed flow.

This additional entrance pressure drop, Δp_{en} , can be expressed as

$$\Delta p_{en} = K_{en} \frac{\rho u_{mean}^2}{2} \quad (3.15)$$

where K_{en} is the non-dimensional additional pressure drop (Sketch 3.2).



Sketch 3.2 Definition of entrance pressure drop

In laminar flow, a number of analytical solutions can be found in the literature for circular pipes that calculate K_{en} , see Table 3.3. The computed values for K_{en} obtained from the analytical solution were validated against measurements of Schiller¹⁴ and Riemann¹¹. The quoted values^{2,14,22,23,35,39,40} for K_{en} ranged from 1.16 to 1.41. The experimental values quoted by Ward-Smith⁵² range between $K_{en} = 1.115$ and 1.45 with most data concentrated between $K_{en} = 1.25$ and 1.35. A summary of the values for K_{en} found in the literature is presented in Table 3.3. These are in agreement with the values recommended for the pressure loss coefficient in ESDU 05024⁸⁷ for entry to sudden contractions downstream of a large container and those used by Curzons¹⁰⁴.

For non-circular pipes, Fleming and Sparrow⁴⁴, for example, provide data for K_{en} from analytical solutions and experiments in rectangular and triangular ducts. These show that for rectangular ducts with aspect ratios greater than 2:1, K_{en} was smaller than for circular ducts. For rectangular ducts with aspect ratios smaller or equal to 2:1 and triangular ducts, K_{en} was greater than for circular ducts.

In turbulent flow, Curzons¹⁰⁴ provided the following correlation:

$$K_{en} = 12f$$

This correlation is compared in Figure 3.3 with the values recommended for the pressure loss coefficient in ESDU 05024⁸⁷ for entry to sudden contractions downstream of a large reservoir and the values cited in Table 3.3 for laminar flow.

TABLE 3.3 Values of the constant K_{en} from various sources in laminar flow for circular pipes

Reference	Technique	K_{en}
Boussinesq ²	Theory	1.24
Riemann ¹¹	Experimental	1.24
Schiller ¹⁴	Theory	1.16
Schiller ¹⁴	Experimental	1.32
Goldstein ²²	Theory	1.41
Langhaar ²³	Theory	1.28
Collins and Schowalter ³⁵	Theory	1.32
Lundgren <i>et al.</i> ³⁹	Theory	1.34
Sparrow and Lin ⁴⁰	Theory	1.24
Ward-Smith ⁵²	Experimental	1.115 to 1.45
ESDU ⁸⁷	CFD and Experimental	1.4175
Curzons ¹⁰⁴	Theory	1.33

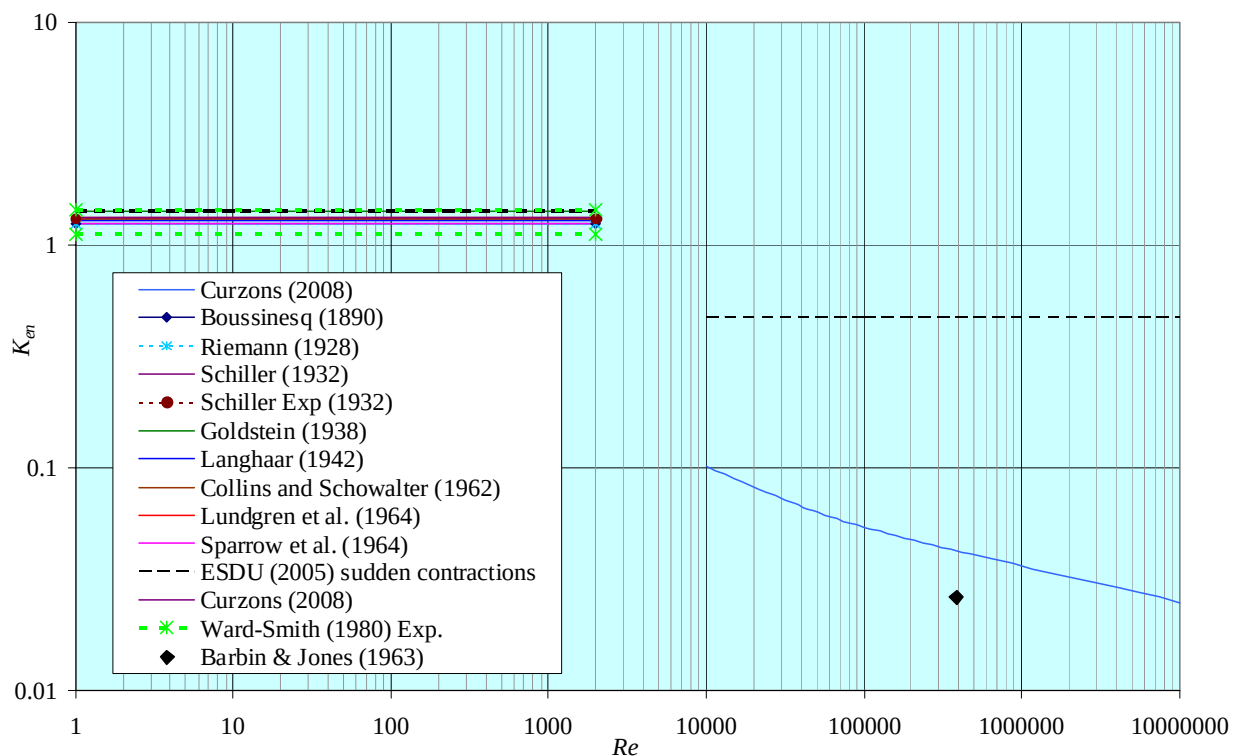


FIGURE 3.3 Values of K_{en} from various sources in the literature

3.3 Friction

In a straight circular pipe, the pressure drop, $\Delta p = p_1 - p_2$ (Sketch 3.1), due to friction can be expressed in terms of the friction factor, f , as

$$\Delta p = 4f \frac{L}{D} \frac{1}{2} \rho u_{mean}^2 \quad (3.16)$$

Different definitions of the friction factor are used in different engineering areas. In civil engineering, for example, a friction factor, λ , defined as

$$\lambda = 4f,$$

is typically used.

As shown in Equation (3.2) both in laminar and turbulent flow, the shear stress distribution is linear across the pipe cross section and its largest value occurs at the wall:

$$\tau_w = \frac{p_1 - p_2}{L} \frac{R}{2} \quad (3.17)$$

The shear stress at the wall, τ_w , can be determined directly measuring the pressure gradient along the pipe. From Equations (3.16) and (3.17), τ_w can be expressed in terms of the friction factor as

$$\tau_w = \frac{1}{2} f \rho u_{mean}^2 \quad (3.18)$$

In **laminar flow**, the friction factor can be derived from Equations (3.6), (3.8) and (3.16) as

$$f = \frac{16}{Re} \quad (3.19)$$

In **turbulent flow**, Blasius⁵ developed the following empirical correlation for the friction factor based on a large number of measurements:

$$f = 0.0791 / Re^{0.25} \quad (3.20)$$

This is valid for smooth circular pipes and $Re \leq 10^5$.

In 1933, Nikuradse¹⁶ published some friction measurements on artificially roughened pipes on which Prandtl's theoretical analysis of pipe flow is based. Nikuradse measurements did not cover the transition region between smooth and rough pipes, or "region of incomplete turbulence". The correlation by Colebrook and White^{20, 21} provides a practical bridge for this gap.

At very large Reynolds numbers, Prandtl¹⁹ derived what is known as the "universal law of friction" for smooth pipes based on Prandtl's universal velocity-distribution law (Equation (3.32)):

$$\frac{1}{\sqrt{f}} = 4 \log \left[Re \sqrt{f} \right] - 0.4 \quad (3.21)$$

This equation is in close agreement with the measurements shown in Figures 3.4 and 3.5.

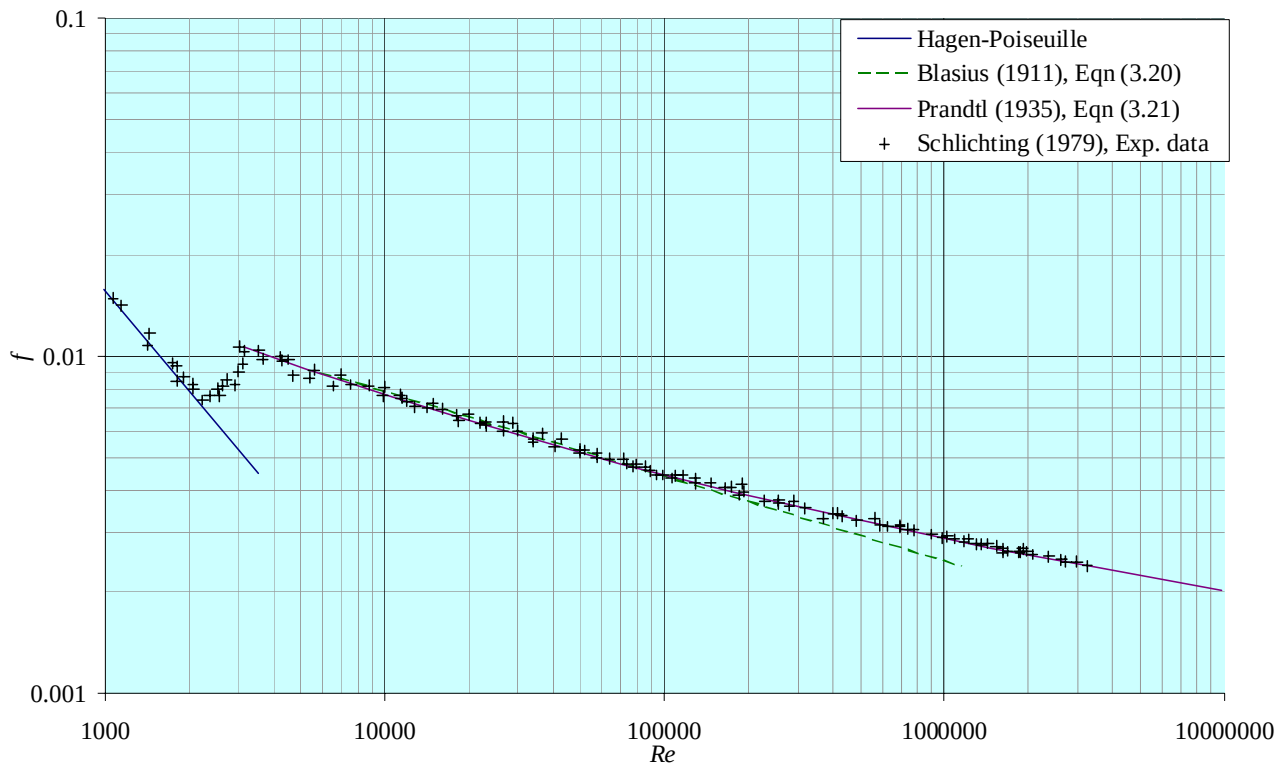


FIGURE 3.4 Experimental data from Nikuradse¹⁶, Saph and Schoder³, Nusselt⁴, Ombeck⁶, Jacob and Erk¹⁰, Stanton and Pannel⁷, and Hermann¹², and correlations for friction factor in a smooth circular pipes.

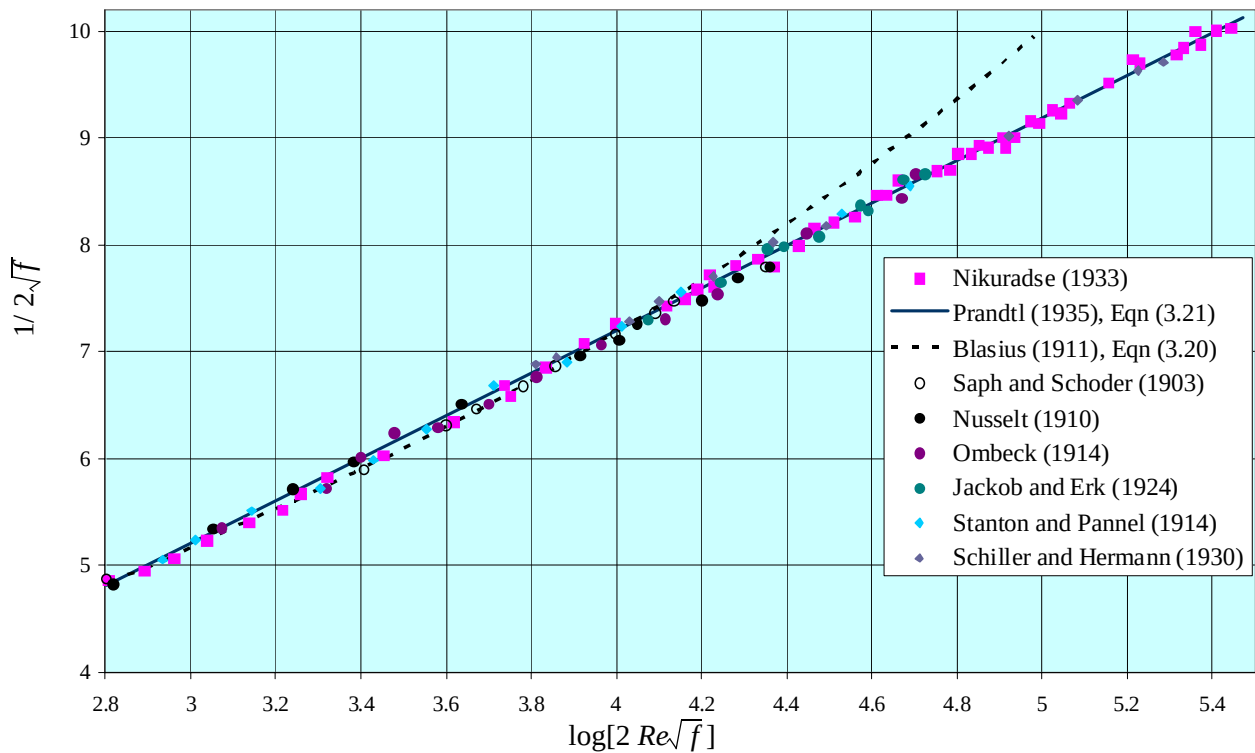


FIGURE 3.5 Universal law of friction for smooth pipe vs measurements.

An explicit form of Prandtl's implicit equation, Equation (3.21), is provided in ESDU 66027⁴³

$$f = \left[4 \log \left[\frac{Re}{4.35 \log Re - 3.82} \right] \right]^{-2} \quad (3.22)$$

This is based on Techo *et al.*⁴¹ which represents Equation (3.21) to within 1% in the range $10^4 < Re < 2.5 \times 10^8$. A simpler expression is also presented in ESDU 66027⁴³ based on Colebrook²¹:

$$f = \left[3.6 \log \left[\frac{Re}{7} \right] \right]^{-2} \quad (3.23)$$

Equation (3.23) represents Equation (3.21) within 1% in the range $10^4 < Re < 10^7$.

For non-circular pipes, Hartnett *et al.*³⁶ found that the circular-pipe correlations for the friction factor are valid for rectangular pipes with a wide range of aspect ratios for Reynolds numbers between 6×10^3 and 5×10^5 .

3.3.1 Roughness effect

In laminar flow, the friction factor is a function of the Reynolds number only. In turbulent flow, the surface roughness affects friction losses and the friction factor is a function both of Reynolds number and of the relative surface roughness²⁴:

$$\frac{\varepsilon}{D}$$

where ε is the effective surface roughness height. Pipes are then considered smooth when the surface irregularities are small compared to the thickness of the laminar sub-layer. They are considered rough when the roughness protrudes sufficiently to break up the laminar sub-layer and the flow is completely turbulent. At low Reynolds numbers, the flow is not affected by the surface roughness, *i.e.* it is hydraulically smooth. As the Reynolds number increases, the flow becomes transitionally rough, *i.e.* friction is a function of both roughness height and of Reynolds number, and the friction factor increases above its smooth-wall value. At higher Reynolds numbers, the flow is fully hydraulically rough, *i.e.* friction is independent of Reynolds number, and the friction factor increases only with the surface roughness. In terms of the viscous length scale, ν/u_τ where ν is the kinematic viscosity and u_τ is the friction velocity defined as $u_\tau = \sqrt{\tau_w/\rho}$, the flow is smooth when $\varepsilon u_\tau / \nu$ is small, and fully rough when $\varepsilon u_\tau / \nu$ is so large that viscosity is no longer important.

For a given roughness type, the behaviour of the transitional roughness regime is known only from experiments⁹⁴. For a surface with closely packed, mono-disperse sand-grain roughness, Nikuradse²⁶ found that the flow can be hydraulically smooth, transitional or fully rough depending on the values taken by the non-dimensional average sandgrain height,

$$k_s^+ = k_s u_\tau / \nu$$

where k_s is the average sandgrain height:

- hydraulically smooth for $k_s^+ = k_s u_\tau / \nu \leq 5$,
- hydraulically transitionally rough for $5 < k_s^+ < 70$, and
- hydraulically fully rough for $k_s^+ \geq 70$.

Many other types of roughnesses have been studied and the behaviour in the transitional region can vary widely⁹⁴.

For rough pipes, ESDU 66027 provides two correlations. One valid for the hydraulically rough regime (see Section 3.4.1) based on von Kármán¹⁸,

$$\frac{1}{\sqrt{f}} = 4 \log \left(3.7 \frac{D}{\epsilon} \right), \quad (3.24)$$

and one valid for the intermediate smooth-rough regime based on **Colebrook and White**²⁰,

$$\frac{1}{\sqrt{f}} = -4 \log \left(\frac{1.25}{Re \sqrt{f}} + \frac{\epsilon}{3.7D} \right) \quad (3.25)$$

The Colebrook-White Equation (3.25) is based on a large collection of measurements in commercial pipes. It is used in the Moody diagram where a monotonic roughness behaviour is assumed. The roughness distribution in commercial pipes typically is not uniform. This may account for the different behaviour in the transitional region reported by Moody²⁵ and Nikuradse²⁶. Shockling *et al.*⁹⁴ found that the Colebrook-White Equation (3.25) gives a poor prediction for the transitionally rough behaviour of honed surfaces as it overestimates the friction factor. They also found that: (i) the transitional roughness regime is characterized by an inflectional curve as also observed by Nikuradse²⁶ and not by the monotonic behaviour represented in the Moody diagram, (ii) and that the departure from hydraulically smooth regime occurs at a relative roughness approximately two orders of magnitude larger than that predicted by the Moody diagram.

One of the earliest explicit correlations for smooth and rough pipes was provided by **Moody**²⁵:

$$f = 0.001375 \left[1 + \left(2 \times 10^4 \frac{\epsilon}{D} + \frac{10^6}{Re} \right)^{1/3} \right], \quad (3.26)$$

This is valid for $4 \times 10^3 < Re < 10^8$ and $0 \leq \epsilon/D \leq 0.05$ and provides friction factor values between -16% and +13% of Equation (3.25)⁵⁴.

Jain⁴⁹ developed an explicit correlation which is within ± 1 % of the Colebrook-White Equation (3.21) for $5 \times 10^3 < Re < 10^8$ and $10^{-6} \leq \epsilon/D \leq 10^{-2}$:

$$\frac{1}{\sqrt{f}} = 2.28 - 4 \log \left(\frac{\epsilon}{D} + \frac{21.25}{Re^{0.9}} \right) \quad (3.27)$$

For Reynolds number $Re \geq 4000$, **Chen** Equation⁵⁰ is suggested in ESDU 83037 for rough straight pipes:

$$\frac{1}{\sqrt{f}} = -4 \log \left(\frac{1}{3.7605} \left(\frac{\epsilon}{D} \right) - \frac{5.0452}{Re} \log \left[\frac{1}{2.8257} \frac{\epsilon^{1.1098}}{D} + \frac{5.8506}{Re^{0.8981}} \right] \right) \quad (3.28)$$

For smooth and rough pipes, **Haaland**⁵⁴ proposed the following general correlation:

$$\frac{1}{\sqrt{f}} = -\frac{3.6}{n} \log \left(\left[\frac{6.9}{Re} \right]^n + \left[\frac{\varepsilon}{3.75D} \right]^{1.11n} \right) \quad (3.29)$$

where n is an integer constant which can take values 1 and 3. For $n = 1$, Equation (3.29) is in close agreement with Colebrook-White Equation (3.25). However, the experiments by Smith *et al.*³¹ and Uhl⁴² have shown that in gas pipelines the transition from the smooth to the rough regime is much more abrupt than that indicated by the Colebrook-White Equation (3.25). To accommodate this behaviour Haaland⁵⁴ suggested using $n = 3$ in Equation (3.29):

$$\frac{1}{\sqrt{f}} = -1.2 \log \left(\left[\frac{6.9}{Re} \right]^3 + \left[\frac{\varepsilon}{3.75D} \right]^{3.33} \right) \quad (3.30)$$

Sonnad and Goudar^{90, 101, 102} developed two explicit correlations for the friction factor in turbulent flow for smooth and rough pipes. The correlation for smooth pipes⁹⁰ is based on the work of Nikuradse¹³, Prandtl¹⁷ and von Kármán¹⁸ and is given by:

$$\frac{1}{\sqrt{f}} = 1.7372 \ln \left(\frac{Re}{1.964 \ln(Re) - 3.8215} \right) \quad (3.31)$$

This correlation is valid for turbulent flow in the region $4000 < Re < 10^8$. The correlation for rough pipes^{101, 102} is based on Colebrook-White²⁰ Equation (3.25):

$$\frac{1}{\sqrt{f}} = 0.4343 \ln \left(\frac{0.4587 Re}{\frac{s}{s+1}} \right) \quad (3.32)$$

$$s = 0.124(\varepsilon/D)Re + \ln(0.4587 Re)$$

This correlation is valid for turbulent flow in the range $4000 < Re < 10^8$.

Recent friction factor measurements were reported by McKeon *et al.*⁸³ in smooth pipes for a wide range of Reynolds number: $10 < Re < 3.6 \times 10^7$. Their measurements were carried out at the “**Superpipe**” facility at Princeton University using compressed air up to very high Reynolds numbers and at Oregon University using helium, oxygen, nitrogen and sulphur hexafluoride. The reported experimental error was 2 to 4% for the Oregon measurements and 1.1% for the Princeton measurements. Measurements by McKeon and co-workers⁸³ have shown the Prandtl law of flow in smooth pipes in Equation (3.21) is not accurate for $Re > 10^6$. The following correlation was developed to fit the experimental data within 1.25% in the range $3 \times 10^4 < Re < 3.6 \times 10^7$:

$$\frac{1}{\sqrt{f}} = 0.965 \log(Re \sqrt{f}) - 0.2685 \quad (3.33)$$

To take into account the effect of viscous deviation from the log law at the wall, McKeon *et al.*⁸⁹ proposed the following equation:

$$\frac{1}{\sqrt{f}} = 0.965 \log(Re\sqrt{f}) - 0.2375 - \frac{3.25}{(Re\sqrt{f})^{0.55}} \quad (3.34)$$

This equation fits friction factor measurements within 1.4% for $10^4 < Re < 3 \times 10^7$. The effect of surface roughness was investigated by the same group of investigators in hydraulically smooth, transitionally rough and fully rough regimes in honed surfaces⁹⁴. They found that a hydraulic smooth behaviour was exhibited for $Re \leq 2.7 \times 10^7$ for these surfaces.

Measurements of friction factor in transitional flow have recently been published by Draad *et al.*⁶⁴ for $2 \times 10^3 < Re < 6.6 \times 10^4$. They found that when extreme precaution is taken to minimize disturbances, parabolic fully developed flow profiles are observed for $Re \leq 14\,300$ and that laminar flow can be sustained for $Re \leq 60\,000$ as shown in Figure 3.6.

The correlations in Equations (3.20), (3.22) to (3.32) are compared with the data from the Oregon and the Princeton measurements in Figures 3.6 and 3.7, respectively. Comparisons with the measurements of Draad *et al.*⁶⁴ are displayed in Figures 3.6(a) and (b). Most of the cited correlations fit within the experimental uncertainty of McKeon's measurements⁸³ except for Moody's Equation (3.26).

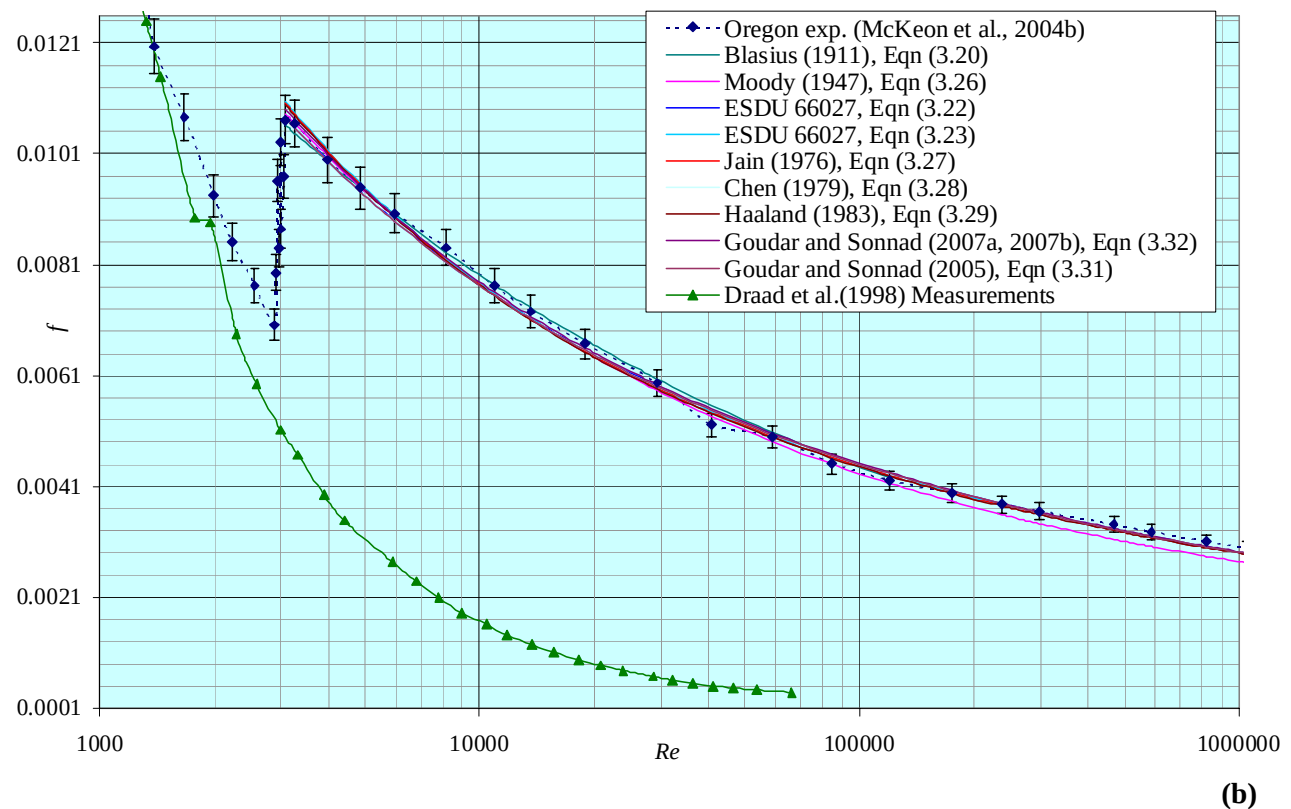
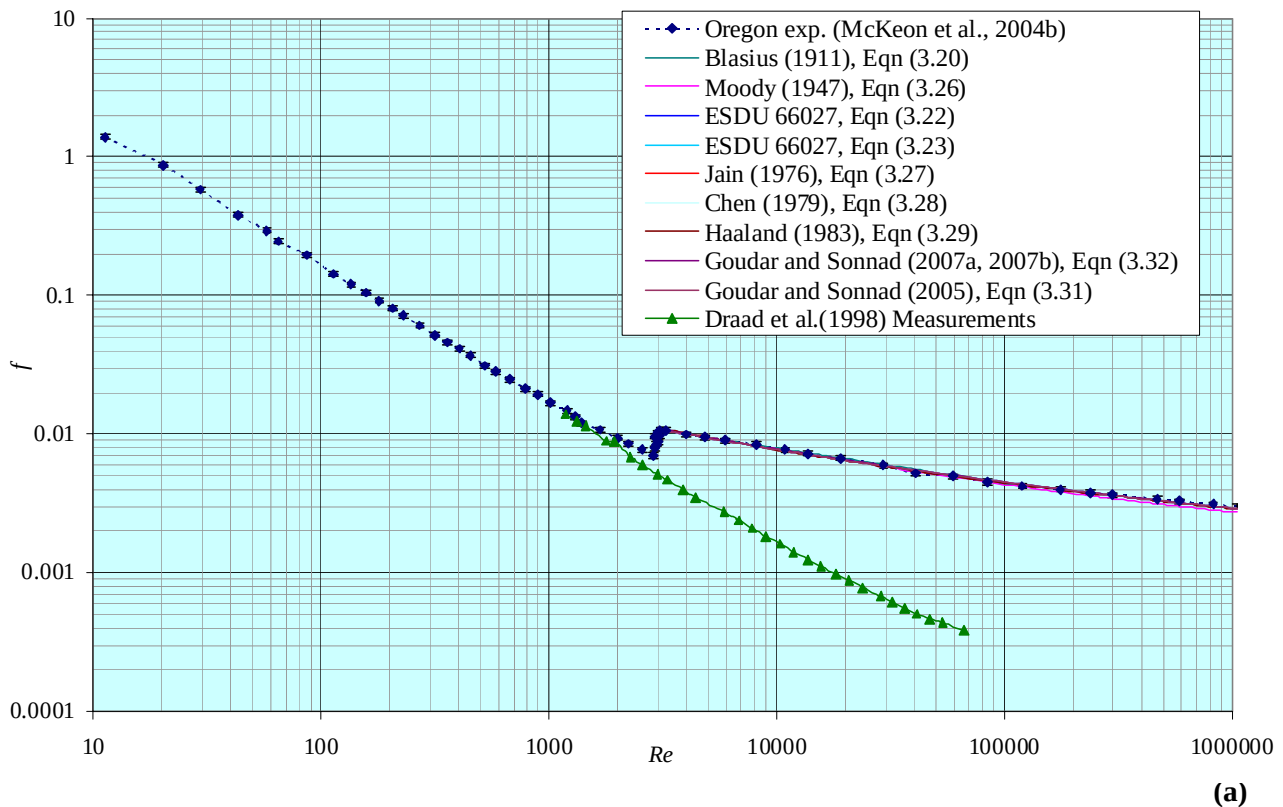


FIGURE 3.6 Comparisons of the different correlations with measurements

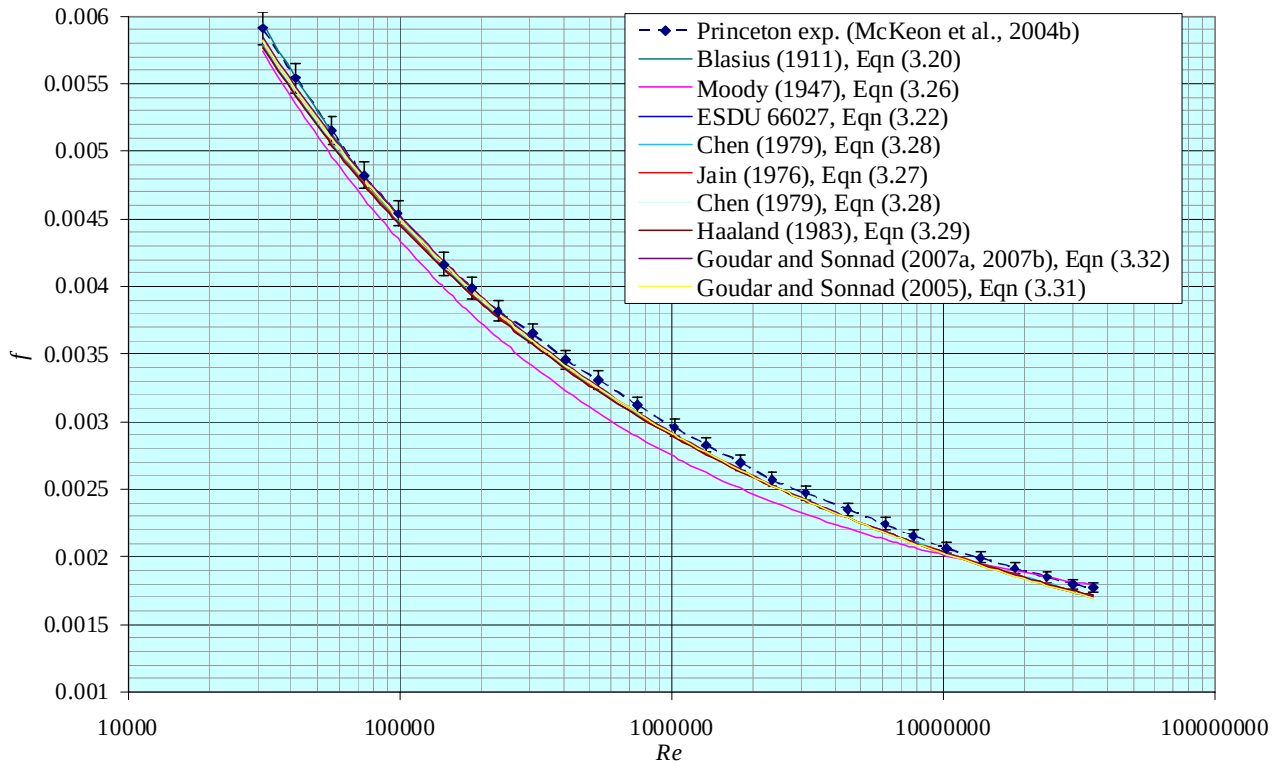


FIGURE 3.7 Comparisons of the different correlations with McKeon *et al.*⁸³ measurements

TABLE 3.4 Summary of the friction factor data in the literature

Reference	Technique	Re	Wall Surface	Data Format
Blasius ⁵	Correlation	$Re \leq 10^5$	Smooth	Equation (3.20)
von Kármán ¹⁸	Correlation	Turbulent flow	Rough	Equation (3.24)
Prandtl ¹⁹	Correlation	$Re > 4000$	Smooth	Equation (3.21)
Colebrook and White ¹⁹	Correlation	Turbulent flow	Smooth-rough	Equation (3.25)
Moody ²⁵	Correlation	$4 \times 10^3 < Re < 10^8$	Smooth and rough	Equation (3.26)
ESDU 66027 ⁴³	Correlation	$10^4 < Re < 2.5 \times 10^8$	Smooth	Equation (3.22)
ESDU 66027 ⁴³	Correlation	$10^4 < Re < 10^7$	Smooth	Equation (3.23)
ESDU 66027 ⁴³	Correlation	$10^4 < Re < 10^7$	Rough	Equation (3.24)
ESDU 66027 ⁴³	Correlation	$10^4 < Re < 10^7$	Smooth and rough	Equation (3.25)
Jain ⁴⁹	Correlation	$5 \times 10^3 < Re < 10^8$	Rough	Equation (3.27)
Chen ⁵⁰	Correlation	$Re \geq 4000$	Rough	Equation (3.28)
Haaland ⁵⁴	Correlation	$4 \times 10^3 < Re < 10^8$	Smooth and rough	Equation (3.29) Equation (3.30)
Draad <i>et al.</i> ⁶⁴	Experiment	$2 \times 10^3 < Re < 6.6 \times 10^4$	Smooth	Plots
Sonnad and Goudar ⁹⁰	Correlation	$4 \times 10^3 < Re < 10^8$	Smooth	Equation (3.31)
Sonnad and Goudar ^{101,102}	Correlation	$4 \times 10^3 < Re < 10^8$	Rough	Equation (3.32)
McKeon <i>et al.</i> ⁸³	Experiment and correlation	$10 < Re < 3.6 \times 10^7$	Smooth	Tabulated data and Equation (3.33)
McKeon <i>et al.</i> ⁸³	Correlation	$10^4 < Re < 3 \times 10^7$	Smooth	Equation (3.34)

3.4 Turbulent Velocity

One of the earliest attempt to measure mean velocity profiles was carried out by Nikuradse¹³ in straight smooth pipes for a wide range of Reynolds number: $4000 < Re < 3.2 \times 10^6$. These are plotted in Figure 3.8 and show that profiles become fuller with increasing Reynolds number and fit the power-law equation:

$$\frac{u}{u_{cl,fd}} = \left(\frac{y}{R} \right)^{1/n} \quad (3.35)$$

where y is distance from the wall and the exponent n varies with the Reynolds number taking the values of $n = 6$ for $Re = 4000$, $n = 7$ for $Re = 10^5$ and $n = 10$ for $Re = 3.2 \times 10^6$. The ratio of the bulk-mean velocity, u_{mean} to the centre-line velocity in fully developed flow, $u_{cl,fd}$ can be calculated as

$$\frac{u_{mean}}{u_{cl,fd}} = \frac{2n^2}{(n+1)(2n+1)} \quad (3.36)$$

Velocity measurements that followed Nikuradse's work have shown that power-law correlations provide a good representation of the velocity profile except close to the wall and on the centreline.

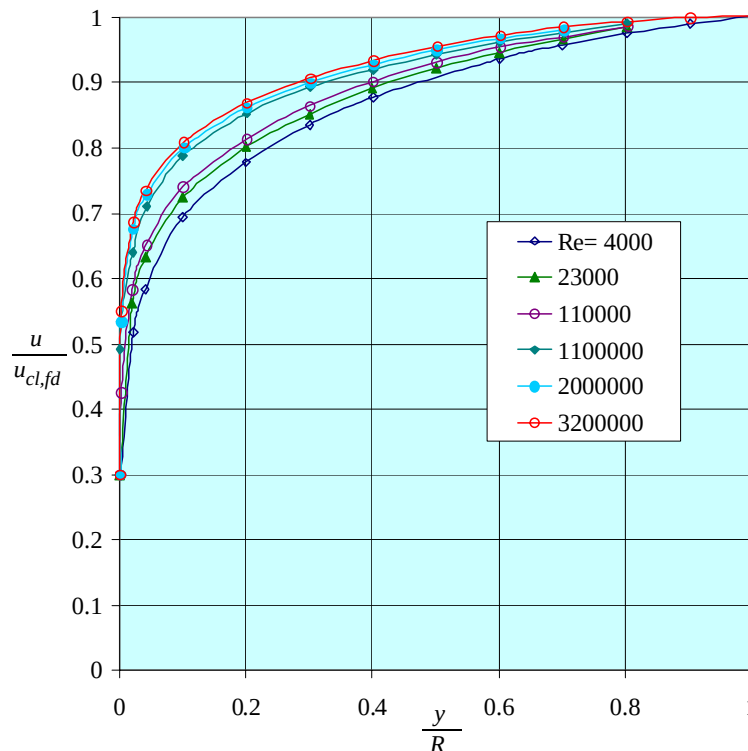


FIGURE 3.8 Velocity measurements for smooth pipes at different Reynolds numbers.
From Nikuradse¹³

3.4.1 Universal law of velocity distribution for smooth pipes

Prandtl¹⁷ discovered that Equation (3.21) is related to Blasius' friction law in Equation (3.20) and showed that the $1/7^{\text{th}}$ -power law can be derived from Blasius' equation at $Re = 10^5$. It can be shown that using Equation (3.18) and introducing the friction velocity u_τ defined as

$$u_\tau = \sqrt{\tau_w / \rho}. \quad (3.37)$$

a 'universal law' for the velocity distribution exists between (u/u_τ) and the non-dimensional wall distance,

$$y^+ = y u_\tau / \nu,$$

$$u^+ = \frac{u}{u_\tau} = C(n) \left(\frac{y u_\tau}{\nu} \right)^{1/n} = C(n) [y^+]^{1/n} \quad (3.38)$$

where $C(n)$ takes the values in Table 3.5. As referred to in Section 3.4, the coefficient n is typically taken as $n = 7$ for $Re = 10^5$ and as $n > 7$ for higher Reynolds numbers.

TABLE 3.5 Coefficients for Equation (3.38)

n	7	8	9	10
$C(n)$	8.74	9.71	10.6	11.5

The 'universal law' for velocity-distribution for smooth pipes is plotted in Figure 3.9.

At very large Reynolds numbers, it has been suggested that the expressions in Equations (3.35) and (3.38) tend asymptotically to a logarithmic function:

$$u^+ = \frac{u}{u_\tau} = 5.75 \log y^+ + 5.5 \quad (3.39)$$

This assumption is confirmed by experimental data as shown in Figure 3.9 and is valid in flows where the laminar shear stress is negligible compared with the turbulent shear stress. In laminar flow, the universal law for velocity distribution is given by:

$$u^+ = y^+ \quad (3.40)$$

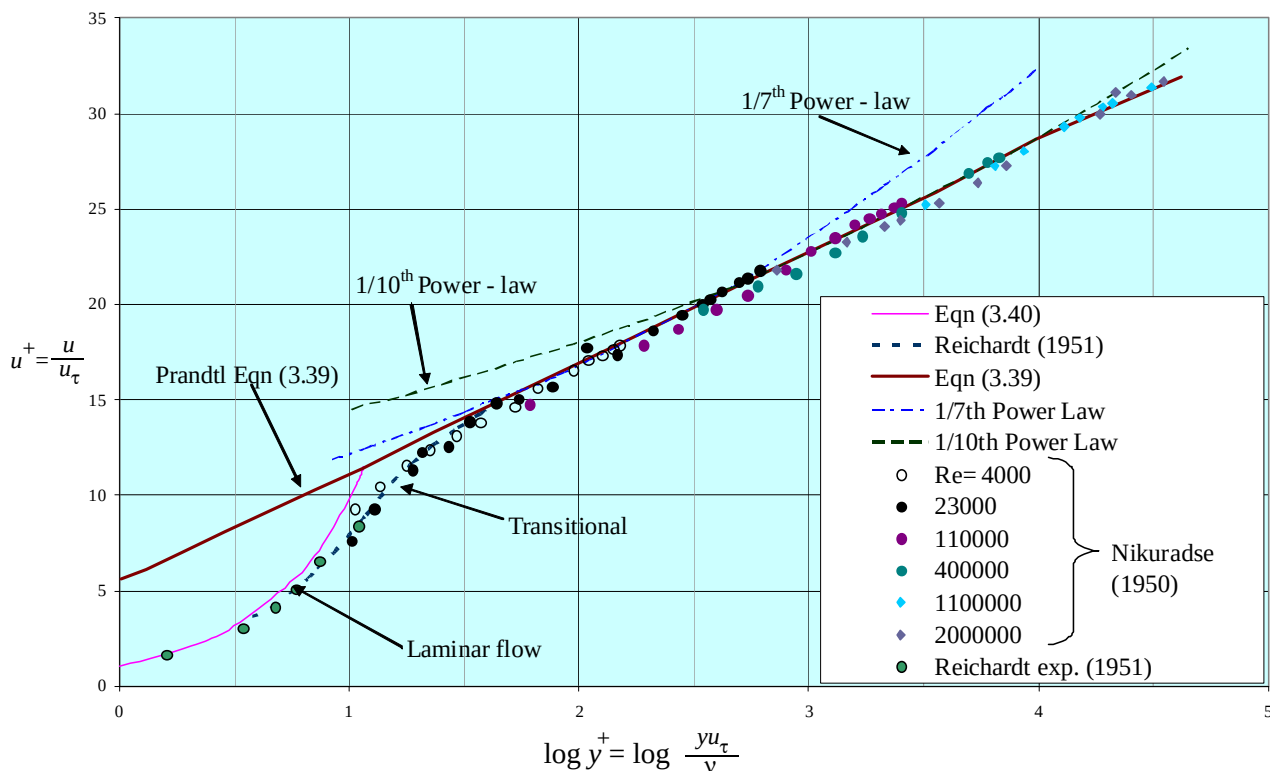


FIGURE 3.9 Universal law of velocity distribution for smooth pipes and comparisons with measurements from Nikuradse²⁶ and Reichardt²⁸

As illustrated in Figure 3.9, three regions can be identified:

- $y^+ < 5$ purely laminar friction (laminar sub-layer)
- $5 < y^+ < 70$ laminar-turbulent friction
- $y^+ > 70$ purely turbulent friction

Hence the thickness of the laminar sub-layer can be derived as

$$\delta_l = 5 \frac{\nu}{u_\tau} \quad (3.41)$$

At a distance sufficiently far from the wall, in the fully-developed turbulent layer, the velocity distribution can be represented by the logarithmic law in Equation (3.39). In the laminar sublayer near the wall, the linear law in Equation (3.40) is applicable. In the mixed laminar-turbulent zone, the velocity distribution can be represented by Reichardt's law in Figure 3.9. These three laws are referred as the '**law of the wall**'.

3.4.2 Universal velocity distribution laws for smooth and rough pipes

At low Reynolds numbers and small roughness heights, the effect of roughness on the velocity distribution is negligible. At sufficiently high Reynolds numbers or large relative roughness ε/D , the viscous length scale, ν/u_τ , may become comparable the roughness height, ε , and the surface roughness disrupts the wall viscous layer. This causes the logarithmic velocity distribution u^+ in Figure 3.9 to shift down without changing its slope and the velocity profile to be more peaked. With further increase in Reynolds number or for large relative roughness, the velocity distribution becomes independent of viscosity, although it retains its logarithmic distribution.

Prandtl¹⁷ derived a universal velocity-distribution law for rough and smooth walls under the assumption that laminar friction is negligible compared to turbulent friction:

$$\frac{u_{cl,fd} - u}{u_\tau} = 5.57 \log\left(\frac{R}{y}\right) \quad (3.42)$$

Equation (3.42) can be extrapolated to large Reynolds numbers and is plotted in Figure 3.10 where it is compared to experimental data and **von Kármán's** universal velocity-distribution law based on the similarity law,

$$\frac{u_{cl,fd} - u}{u_\tau} = -\frac{1}{0.36} \left\{ \ln \left[1 - \sqrt{1 - \frac{y}{R}} \right] + \sqrt{1 - \frac{y}{R}} \right\}, \quad (3.43)$$

and **Darcy**¹ empirical equation based on measurement of velocity distributions,

$$\frac{u_{cl,fd} - u}{u_\tau} = 5.08 \left(1 - \frac{y}{R} \right)^{3/2} \quad (3.44)$$

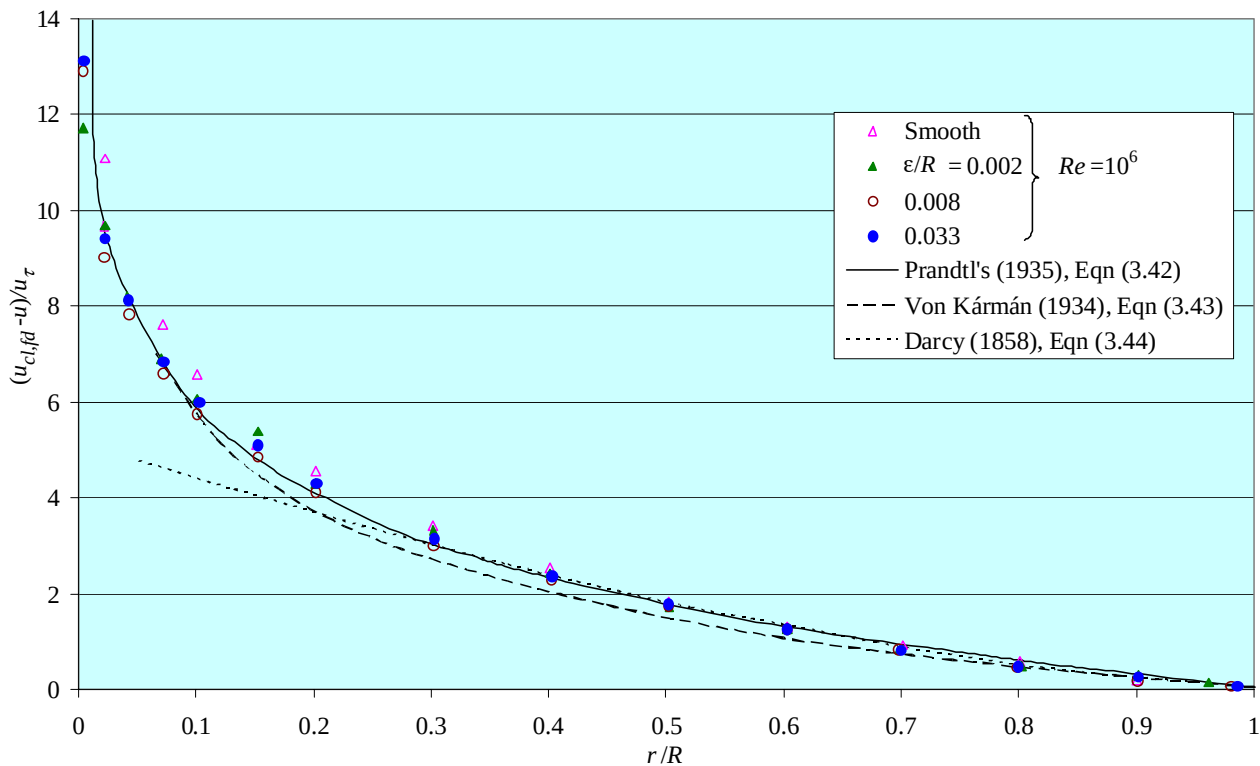


FIGURE 3.10 Universal velocity distribution law for smooth and rough pipes.
Measurements are from Schlichting⁵¹

3.4.3 Velocity measurements

Laufer²⁹ was the first to document detailed turbulence statistics using hot wire anemometry for Reynolds numbers, $Re = 41000$ and 426000 . Laufer's measurements are compared in Figures 3.11 to 3.13 with the recent LDA measurements at the National Institute of Standards and Technology of Mattingly⁷⁶ and the DNS predictions conducted at the Center for Turbulence Research by Poroseva *et al.*⁶⁹. Comparisons with $1/7^{\text{th}}$ and $1/9^{\text{th}}$ power-law profiles are also shown in these Figures.

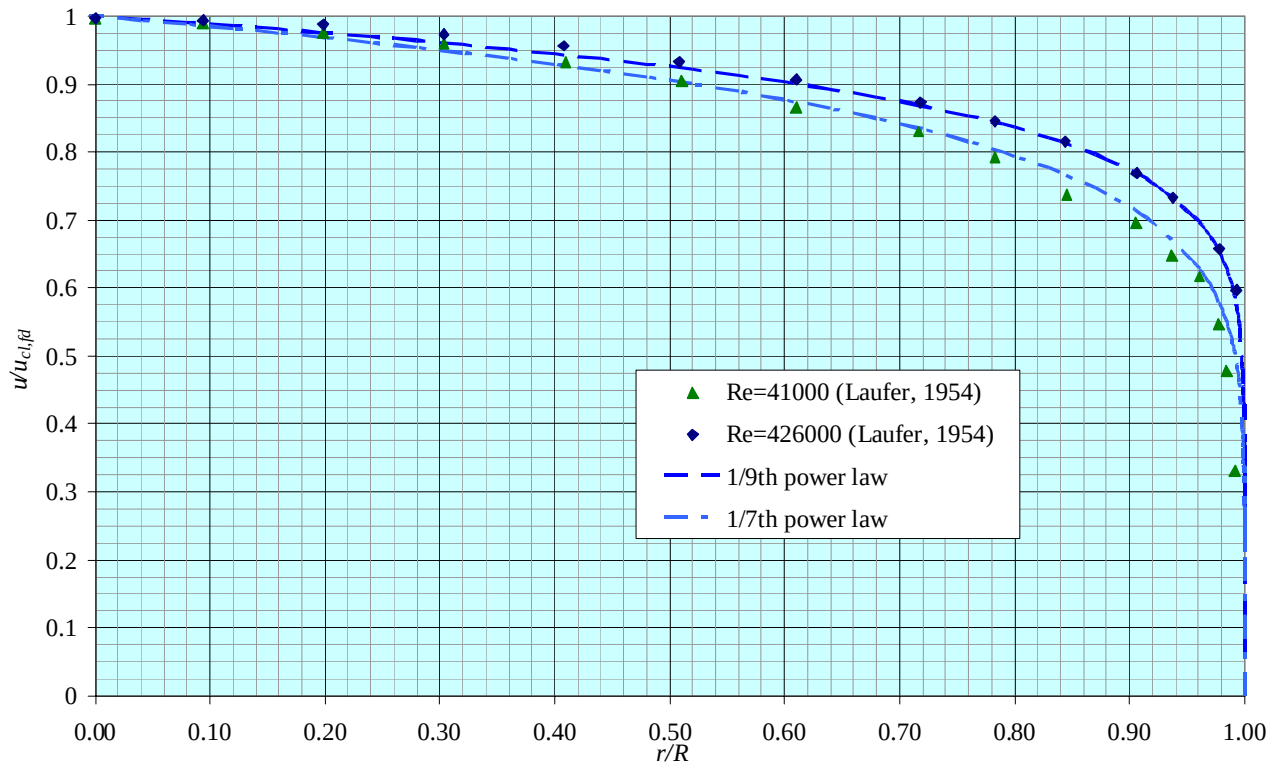


FIGURE 3.11 Comparisons of Laufer's²⁹ data with power-law distributions

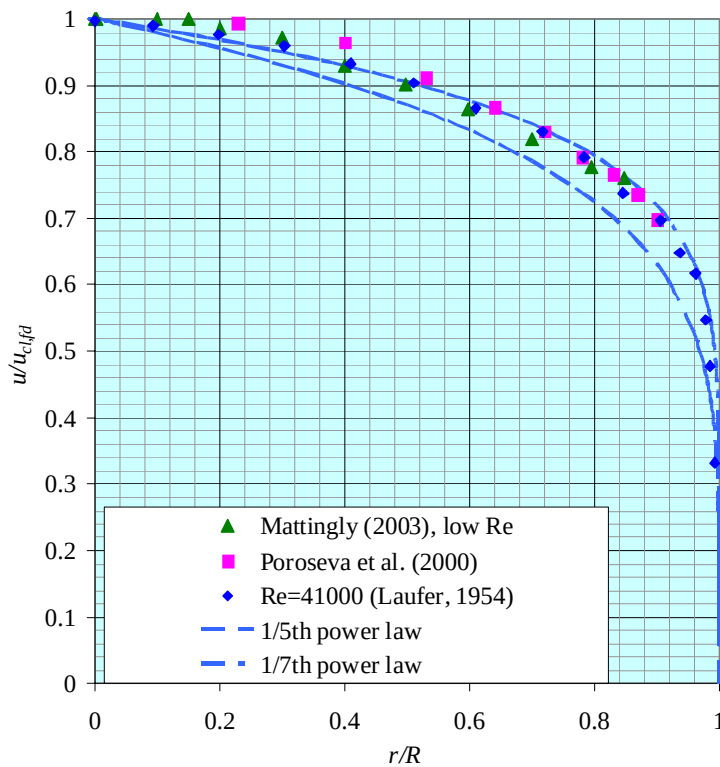


FIGURE 3.12 Comparisons of Laufer²⁹, Mattingly⁷⁶ and Poroseva's⁶⁹ data with power-law distributions

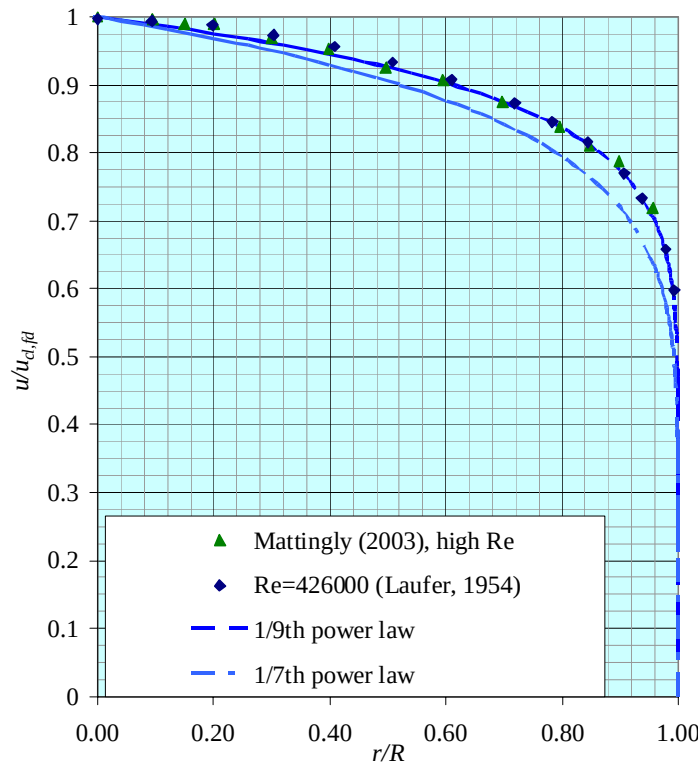


FIGURE 3.13 Comparisons of Laufer²⁹, Mattingly⁷⁶ and Poroseva's⁶⁹ data with power-law distributions

Hot wire velocity measurements have recently been carried out by Smits and co-workers at the 'Superpipe' facility at Princeton University designed for very high Reynolds numbers^{66,82,83,94}. Zagarola and Smits⁶⁶ carried out measurements of the mean velocity profiles for $3.1 \times 10^4 < Re < 3.5 \times 10^7$. They found that in fully-developed pipe flow velocity profiles display two overlap regions: a power-law region for $60 < y^+ < 500$ and a log-law region for $600 < y^+ < 0.07 K_\tau$. The Kármán number, K_τ , represents the boundary-layer thickness, δ , expressed in viscous wall units, defined as

$$K_\tau = \frac{u_\tau \delta}{\nu} \quad (3.45)$$

The power-law dependence of the mean velocity on y^+ expresses the direct influence of viscosity outside the laminar sub-layer. The logarithmic - law dependence is an expression of a self-similar region in which the mean velocity scales on the wall friction velocity, u_τ , at the distance from the wall, y , only.

More recent measurements from McKeon *et al.*⁸² using high-Reynolds-number static-pressure correction confirmed the presence of power-law and logarithmic regions found by Zagarola and Smits⁶⁶ but found different limits for such regions, *i.e.* the power-law region for $50 < y^+ < 500$ and the log-law for $600 < y^+ < 0.12 K_\tau$, and the following correlations for the power-law and logarithmic regions:

$$u^+ = \frac{u}{u_\tau} = 8.48 \left[y^+ \right]^{0.142} \quad (3.46)$$

$$\frac{u_{cl,fd} - u}{u_\tau} = - \frac{1}{0.421} \ln(y/R) + 1.2, \quad (3.47)$$

respectively (see Figure 3.14). It was also found that the surface roughness affects velocity profiles only at high Reynolds number, *i.e.* $Re > 1.36 \times 10^7$.

A relationship between the centreline velocity and the friction factor was found by McKeon *et al.*⁸⁹:

$$\frac{u_{cl,fd}}{u_\tau} = 2\sqrt{2} \left(1.934 \log \left[Re 2\sqrt{f} \right] + 0.958 \right) \quad (3.48)$$

This equation fits the measurements within $\pm 3.2\%$ for $3 \times 10^5 \leq Re \leq 1.8 \times 10^7$, within 0.52% for $Re > 1.8 \times 10^7$ and within 1% for $Re \leq 3 \times 10^5$.

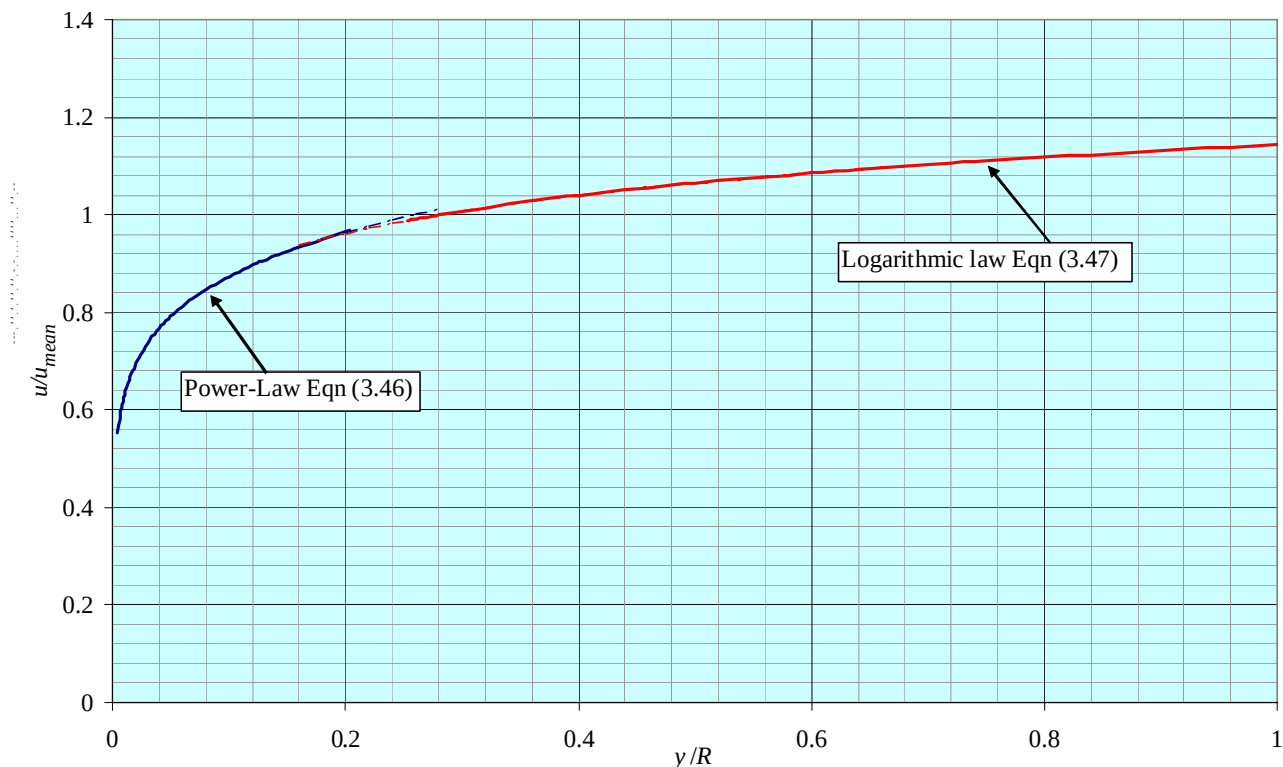


FIGURE 3.14 McKeon *et al.*^{82,83} correlations in Equations (3.46) and (3.47) and their overlap at $Re = 10^5$

Durst and co-workers^{61,62} conducted high-spatial-resolution LDA measurements for the mean velocity and turbulence intensity in the **near-wall region** for $7500 < Re < 20500$ (Figures 3.15 and 3.16). Measurements are in agreement with the DNS predictions of Kim *et al.*⁵⁶ and show a linear behaviour of u^+ with y^+ in the laminar viscous sub-layer for $y^+ \leq 2$ and a logarithmic profile for $2 < y^+ < 20$ (Figure 3.15). For $y^+ > 100$ a fully turbulent layer is reached and measurements are in agreement with the universal law of the wall. The measurement uncertainty for mean velocity was $\pm 0.28\%$. The maximum uncertainty for the measurement of shear stress at the wall was 1%.

Durst *et al.*⁶² also carried out high quality turbulence intensity measurements near the wall. It was also found that the axial velocity component of the turbulence intensity at the pipe wall tends to a limiting constant value which is Reynolds number independent. The findings are confirmed by the DNS predictions of Kim *et al.*⁵⁶ (Figure 3.16).