



CFD validation studies for transitional flow in circular straight pipes with smooth walls

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THE PREPARATION OF THIS DATA ITEM

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CFD VALIDATION STUDIES FOR TRANSITIONAL FLOW IN CIRCULAR STRAIGHT PIPES WITH SMOOTH WALLS

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CFD VALIDATION STUDIES FOR TRANSITIONAL FLOW IN CIRCULAR STRAIGHT PIPES WITH SMOOTH WALLS

1. NOTATION AND UNITS

		SI Unit
A	pipe cross-sectional area	m^2
D	pipe internal diameter	m
f	friction factor, defined in Equation (3.4), also termed Fanning friction factor	–
J_{KE}	kinetic-energy velocity-profile factor, defined in Equation (3.5)	–
J_M	momentum velocity-profile factor, defined in Equation (3.6)	–
K_{en}	non-dimensional additional entrance pressure drop (Sketch 3.5), defined in Equation (3.2)	–
$K_{en,t}$	non-dimensional additional (negative) pressure drop in the turbulent section, defined in Sketch 5.1	–
$K_{en,tot}$	non-dimensional pressure drop between pipe entrance and fully-developed flow axial location, defined in Sketch 5.1	–
L	pipe length	m
L_d	pipe length required to reach fully-developed flow (Section 3.2.1)	m
L_p	pipe length required to reach constant pressure gradient (Section 3.2.2)	m
m	mass flow rate	kg/s
p	static pressure	Pa
R	pipe internal radius	m
r	pipe radial coordinate	m
Re	pipe Reynolds number, defined as $Re = u_{mean}D/\nu$	–
Re_θ	momentum-thickness Reynolds number, defined as $Re_\theta = \theta u_\infty/\nu$	–
$Re_{\theta t}$	transition-onset momentum-thickness Reynolds number	–
u	local axial velocity	m/s
u^+	non-dimensional local axial velocity, defined as $u^+ = u/u_\tau$	–

u'^{+}	non-dimensional local turbulence intensity	–
u_{cl}	local axial centre-line velocity	m/s
$u_{cl,fd}$	centre-line axial velocity in fully-developed flow	m/s
u_{mean}	bulk-mean velocity, defined as $u_{mean} = m/(A\rho)$	m/s
u_{τ}	friction velocity, defined as $u_{\tau} = \sqrt{\tau_w/\rho}$	m/s
u_{∞}	local free-stream velocity	m/s
x	pipe axial coordinate	m
Y_{plus}	wall y^+ , defined as $Y_{plus} = y_w u_{\tau}/\nu$	–
y	distance from wall	m
y^+	non-dimensional distance from wall, defined as $y^+ = y u_{\tau}/\nu$	–
y_w	distance from wall of closest mesh node	m
Δf	percentage difference between CFD or measured friction factor and ESDU 66027(a) friction factor	–
Δp_{en}	additional pipe pressure drop due to entry of fluid into pipe from a large tank, defined in Sketch 3.5	Pa
$\Delta p_{en,t}$	additional (negative) pressure drop due to entry of fluid into a circular pipe in turbulent section, defined in Sketch 5.1	Pa
$\Delta p_{en,tot}$	pressure drop between pipe entrance and fully-developed flow axial location, defined in Sketch 5.1	Pa
ε	effective roughness height of pipe internal surface	m
γ	intermittency factor, defined as the fraction of time that motion is turbulent in the transition process (Section 3.1)	–
κ	turbulence kinetic energy	$m^2 s^{-2}$
μ	fluid dynamic viscosity	Pa s
μ_t	turbulent viscosity	Pa s
μ_t^*	turbulent viscosity in transitional flow	Pa s
ν	fluid kinematic viscosity	m^2/s
θ	boundary-layer momentum thickness	m

ρ	fluid density	kg/m^3
τ_w	wall shear stress	Pa
ω	turbulent frequency	s^{-1}

Abbreviations Glossary

AUTOMATIC	ANSYS CFX proprietary near-wall treatment which switches automatically from wall-functions to a low-Reynolds-number treatment near-wall formulation as the mesh is refined
CFD	computational fluid dynamics
DNS	direct numerical simulation
LDA	laser Doppler anemometry
LES	large eddy simulation
RANS	Reynolds-averaged Navier Stokes
SCALABLE	ANSYS CFX proprietary near-wall treatment which allows solution on arbitrarily fine mesh near the wall when using the $\kappa-\epsilon$ turbulence model
SST	shear stress transport

2. INTRODUCTION

In this Technical Note, the ESDU CFD validation studies are presented for the prediction of transitional flow in straight, circular pipes with smooth walls.

The onset and extent of transitional flow are of major importance in the design of many flow devices. The transition process from laminar to turbulent flow is accompanied by significant changes in flow-related processes such as mixing, heat transfer and friction, with important consequences for the design parameters.

Prediction of transitional pipe flow is one of the most fundamental and long-standing problems in fluid mechanics. There is no theory describing the transition process in pipe flow. Accurate prediction of transitional flow in straight pipes is one of the most challenging CFD applications and direct numerical simulation (DNS) is considered the most useful method. However, the high computational cost of DNS has limited its use in industry. At the time of writing, Reynolds-averaged Navier-Stokes (RANS) methods are most typically used in industry for internal flow applications.

It is not advisable to design fluid equipment to operate in transitional flow regime because of the high uncertainty associated with the prediction of the flow behaviour. Nevertheless, in spite of the modelling difficulties in generating accurate CFD predictions for transitional flow, these are required in many industrial flow applications where transition controls drag or heat transfer.

The purpose of this work is to provide guidance on the CFD modelling of transitional flow for:

- straight, circular pipes with smooth walls
- swirl-free, uniform inlet velocity (pipe entrance from a large tank),
- fully-developed exit flow conditions,
- steady-state flow conditions,
- incompressible flow of single-phase, Newtonian liquids and gases,
- use of commercial CFD packages typically used in the industry, *i.e.* finite volume RANS code with low-Reynolds-number turbulence and transition correlation-based 'by-pass' modelling capabilities.

The CFD modelling of transitional flow has only become available in commercial RANS CFD packages in recent years and has mainly been developed for boundary layers in internal flow, turbomachinery applications. In such applications, the transition process is characterized by the 'bypass' of transition features such as Tollmien-Schlichting waves, spanwise vorticity and three-dimensional breakdown typical of external flows and, similarly to pipe flow, turbulent spots are initiated in the viscous wall layer which grow and merge to form turbulent slugs.

In the validation studies presented here, five different turbulence and transition models were used:

- the $\kappa-\omega$ and SST low-Reynolds-number turbulence models with AUTOMATIC near-wall treatment⁶⁶, and
- the SST specific- γ , SST γ and SST $\gamma-Re_\theta$ transition models⁶⁶.

A description of the fundamental fluid mechanics of transitional flow in straight smooth pipes is given in Section 3 where correlations, measurements and DNS predictions from the literature are reviewed. The methodology used in the ESDU CFD predictions is described in Section 4. Although these predictions were performed using the commercial package ANSYS CFX11⁶⁶, the methodologies can be applied to any RANS finite volume CFD code.

The ESDU CFD results are discussed in Section 5 and compared with the data in the literature in Section 6. Best Practice Guidelines are given in Section 7. An overview on the CFD modelling of transitional flow is given in Appendix A.

3. VALIDATION DATA IN THE LITERATURE

3.1 Transitional Flow in Circular Pipes

In circular pipes, flow is characterised by the pipe Reynolds number, Re , defined as

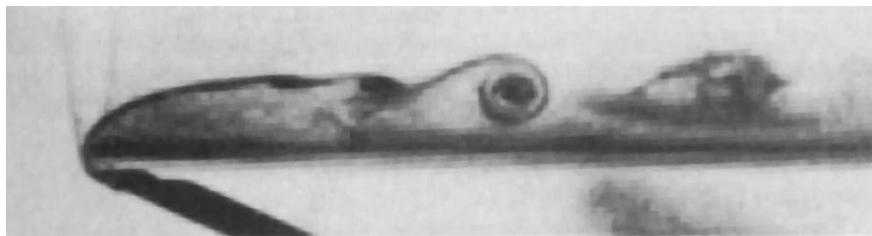
$$Re = \frac{u_{mean} D}{\nu},$$

where u_{mean} is the pipe bulk-mean velocity, D is the pipe internal diameter and ν is the fluid kinematic viscosity.

At low Reynolds number the flow is laminar. In laminar flow, the velocity remains constant on cylindrical surfaces concentric to the axis and individual cylindrical laminae slide over each other. As the Reynolds number is increased above around $Re = 2300$, the flow regime becomes transitional. At still higher Reynolds numbers the flow regime becomes turbulent.

The transition from laminar to turbulent flow is a relatively complex process which is far from being completely known and documented. Knowledge on transitional flow comes primarily from experimental investigations, see, for example, Wygnanski *et al.*³⁰ and Wygnanski and Champagne³¹. Whilst there are some similarities with flat plate flow, the lack of a free boundary in the pipe flow leads to some important differences in transitional flow behaviour and the associated development length⁷⁶.

Experimental observations by, for example, Kurzweg¹², Wygnanski *et al.*³⁰, Draad *et al.*⁴⁵, and Tritton⁵², have shown that the transition process in circular pipes is characterized by the appearance of turbulent spots, in the viscous wall layer which subsequently grow and merge downstream. This is due to the presence of the pipe wall boundary and, depending on the shape of the pipe entry and on the presence of separated, recirculating flow, can lead to a transition that is rapid compared to that observed in zero-pressure-gradient flat plate boundary-layer flow. In this latter external flow application, the transition process is characterised by the sequential formation of typical flow instability features *i.e.* Tollmien-Schlichting waves, 3-D vortices and bursting vortices, all of which precede the generation of turbulent spots (see Sketch 3.1).



Sketch 3.1 Transitional flow at sharp entry to pipe showing turbulent spots and vortex formation using dye to trace flow. From Kurzweg¹²

In transitional pipe flow, turbulence spots can be generated by localized disturbances and are favoured by wall roughness, *i.e.* transition occurs at lower Reynolds number for a rough wall than for a smooth wall. These are bursts of turbulent-like fluctuations. Following their appearance and depending on the pipe Reynolds number, the transition process can progress to form local regions of turbulent motion, or turbulent plugs or slugs, which travel and grow downstream in the pipe.

Over the pipe length where flow transition is in progress, alternate regions of non-stable (turbulent) and stable (laminar) flow may exist. Transition to turbulence occurs if disturbances are sufficiently amplified in the non-stable regions. At any specific location in the transition zone of a pipe, laminar and turbulent flow motions alternate in time. The fraction of time that motion is turbulent is known as the intermittency factor, γ . At the onset of transition, $\gamma = 0$. The intermittency factor increases with distance downstream in the pipe as a result of the turbulent slug growth at the expense of the non-turbulent regions. The region of intermittently turbulent flow is often hundreds of diameters long. As turbulence propagates into neighbouring non-turbulent regions at each end of the slug, the upstream and downstream slugs merge and the flow becomes fully turbulent. At this condition, $\gamma = 1$.

At the pipe entrance, the appearance of local turbulent regions depends on a critical disturbance level being attained. It has been observed that for $Re < 1760$, no turbulence structures can be sustained, *i.e.* turbulent spot production can be triggered but transition to turbulence does not result⁶⁸. By taking great care to minimize the disturbance level, experimenters have been able to maintain laminar flow up to $Re = 6.4 \times 10^4$, reported by Draad *et al.*⁴⁵, or even $Re = 10^5$, reported by Lachmann²⁰. It is likely that in these experiments, local unstable regions forming in the entry boundary layer decay instead of leading to transition to turbulent flow.

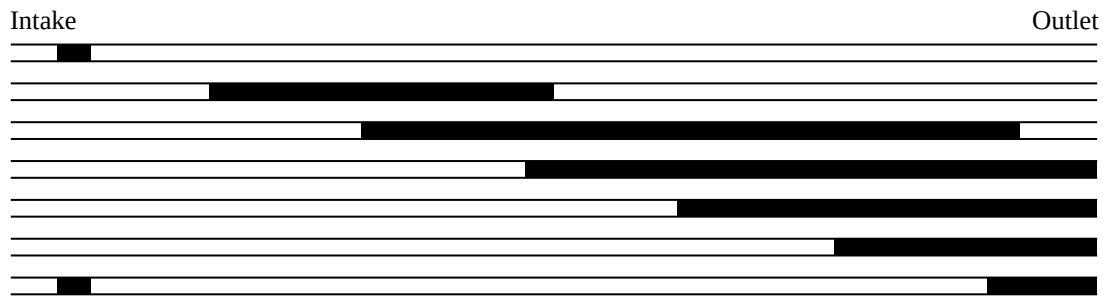
It has been reported⁵² that the lower Reynolds-number limit for transition to slug growth, even for very large disturbances, is $2.1 \pm 0.3 \times 10^3$. For $Re \geq 10^5$, the flow always becomes turbulent through amplification of small disturbances.

The entrance geometry can radically influence the transitional flow behaviour and entry length. Depending on the entry edge geometry, two different flow regimes may establish:

- For sharp edges, a flow separation occurs at the entrance and turbulent spots appear in the wall boundary layer, grow and merge with a relatively short development length to give fully-developed flow. This type of behaviour is typical of “bypass transition”, which is characterised by a strongly amplifying mechanism of the flow instability (see Appendix A).
- For rounded edges (‘bellmouth’ entry), there is no-significant flow separation. Turbulent flow slugs however, may appear at the entry and travel intermittently through the pipe. The development length for reaching fully-developed turbulent flow conditions can be very long.

For sharp entry geometries with consequent flow separation regions, shear-layer instability develops at lower Reynolds number than any attached boundary layer and rapid amplification of disturbances occurs as shown in Sketch 3.1. However, the entry flow behaviour is not dependent on the entry geometry alone. Other disturbances, such as an adjacent turbulent region, can also cause transition to occur.

There is a short Reynolds range over which localized regions of turbulence neither spread nor decay. The resulting features are called “turbulent puffs”⁵². This particular transitional flow behaviour has been reported to occur in experimental arrangements of transitional flow where the pressure difference is kept constant; as a turbulent slug grows, the pressure drop across the pipe length section where the turbulent flow exists increases, hence the flow rate is reduced. This inhibits instability at the pipe entry and reduces the probability of production of another slug. Only when the front of the slug reaches the pipe end does the fraction of pipe length in laminar motion increase. This leads to an increase in flow rate and consequently the production of another slug and the cycle recommences (see Sketch 3.2). This periodic transition happens only over a limited Reynolds number range at around $Re = 5000$ and for pipes with lengths $L > 60D$ ⁵².



Sketch 3.2 Periodic transitional pipe flow in increasing time sequence from top to bottom.
From Tritton⁵²

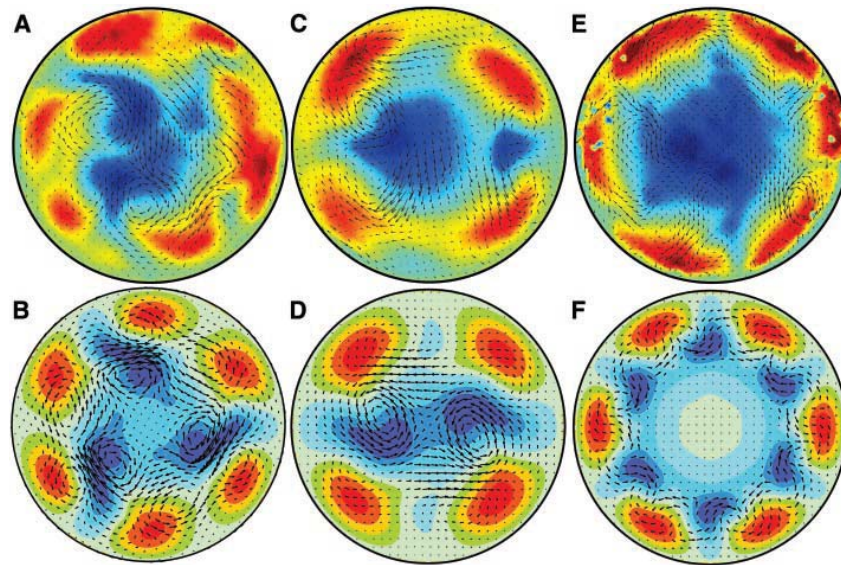
3.1.1 Recent Studies

Recent experimental and theoretical studies, carried out within an international collaboration project between Delft University of Technology (The Netherlands), Philipps University of Marburg (Germany), Bristol University, the University of Manchester (UK) and the University of Wisconsin (USA), have provided a better understanding of transitional pipe flow. Using LDV measurements and dynamical system theory applied to the Navier-Stokes equations, growth rate and lifetimes of turbulent structures were examined.

In contrast to other laminar-to-turbulent transitional flow situations, such as the flow over a flat plate, where a mathematical evolution of disturbances superposed on a laminar base flow (stability theory) can be used to predict primary and secondary instabilities from laminar flow state, in pipes turbulence sets in suddenly and fully without intermediate states and without a clear stability boundary. According to the stability theory, in a pipe flow should stay laminar for all Reynolds numbers because infinitesimal perturbations do not trigger turbulence. In practice, however, flow becomes turbulent even at moderate Re .

Mullin and co-workers at Manchester showed that larger amplitude perturbations are needed to establish turbulence. An experimental setup with a novel perturbation system was designed to produce step-profile pressure perturbations of controlled magnitude and duration caused by pulses of fluid injected tangentially. This allowed the replication of the perturbation analysis in the theoretical studies by Hof at Delft⁵¹. It was found⁵¹ that the minimum amplitude required to destabilize the flow scales with Re^{-1} , in agreement with the theoretical studies. As the Reynolds number is increased an increasing number of unstable solutions arise.

It was also discovered that unstable travelling waves and periodic solutions to the governing Navier-Stokes equations could provide a new scenario for pipe flow transition^{51,54}. These waves travel in the streamwise direction at a phase speed typically slightly larger than the bulk velocity. The experimental studies at Delft in pipes with lengths up to $L = 800D$ confirmed that the dynamics of turbulent flow at moderate Reynolds number is governed by unstable non-linear travelling waves. These were found to be 2- to 6-fold transient structures which consist of flow-wise swirls and streaks with rotational symmetry about the pipe axis (see Sketch 3.3).



Sketch 3.3 Experimental (A, C, E) and theoretical (B, D, F) streak patterns by Hof *et al.*⁵⁴ at (A) $Re = 2000$, (B) $Re = 1250$, (C) $Re = 2500$, (D) $Re = 1360$, (E) $Re = 5300$ and (F) $Re = 2900$.

3.1.2 Measurements in Transitional Flow

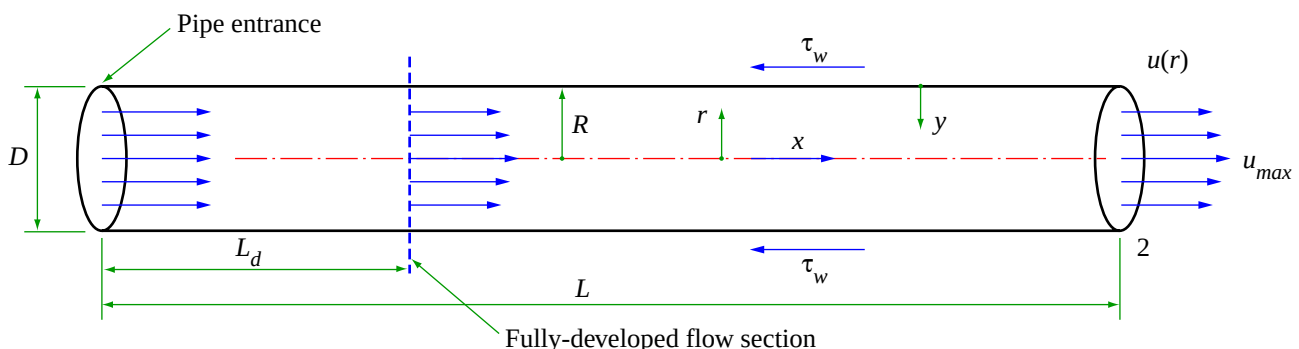
When measuring pressure and flow characteristics in transitional flow it is important to take into account their unstable/intermittent behaviour, and to use pipes with great length to diameter ratio⁵², i.e. $L \geq 200D$.

A review of data in the literature for transitional flow is provided in the next sections.

3.2 Development Length

The characteristics of laminar or turbulent flow through a straight circular pipe are illustrated in Sketch 3.4. The time-averaged axial local velocity, $u(r)$, is zero at the pipe wall and maximum at the centre-line. At the pipe entrance, the velocity profile can be considered uniform as is typically assumed for a pipe entrance from a large tank. In practice, this profile can be closely achieved using a 'bellmouth' entry.

Under steady-state conditions, at a distance from the entrance, the velocity distribution across the section becomes independent of the axial coordinate and, hence reaches a condition known as a 'fully-developed' in which the pressure gradient is constant. This distance is termed the development length. In transitional flow, the flow is laminar in the entry section of the pipe and becomes turbulent and fully-developed downstream at generally greater lengths than for either laminar or fully-turbulent flows.



Sketch 3.4 Schematic of flow in a straight pipe with uniform inlet velocity profile

3.2.1 Velocity development length

The development length required to achieve fully-developed velocity and pressure conditions, L_d , can also be referred to as the velocity development length. This is usually defined as the length required for the centre-line velocity, u_{cl} , to achieve 99 % of the fully-developed flow value.

A summary of the data in the literature for L_d in laminar to turbulent flow is given in Table 3.1 and Figure 3.1. Transitional flow correlations which are an extension of the laminar flow correlations provide a linear relation for L_d/D with Reynolds number in the following general form:

$$\frac{L_d}{D} = C \cdot Re \quad (3.1)$$

where C is a constant. Values for the constant C reported by various sources in the literature are reported in Table 3.2. Note that, in laminar to transitional flow, the equation by Schiller⁷, which is based on the theoretical integral method, provided much lower values for L_d and is considered not reliable. The other analytical and experimental data by, for example, Hornbeck²³ and Durst *et al.*⁵⁸, and CFD predictions by ESDU^{59,63}, are more consistent.

TABLE 3.1 Velocity development length data in the literature

Reference	Technique	Re
Schiller ⁷	Theory	$< 10^5$
Hornbeck ²³	Theory	$< 10^4$
Collins and Schowalter ²¹	Theory	Laminar flow
Barbin and Jones ²²	Hot wire measurement	$Re = 3.88 \times 10^5$
Durst <i>et al.</i> ⁵⁸	CFD	$0.01 < Re < 10^4$
ESDU ^{59,63}	CFD	$1 < Re < 10^7$
Poole and Ridley ⁶⁹	CFD	$0 < Re < 1000$

Finally, measurements reported by Campbell⁷⁴ provided values for the development length of $L_d = 100$ for $Re = 5000$, *i.e.* between typical laminar and turbulent values in Figure 3.1, and $L_d = 40$ for $Re = 10^4$, *i.e.* typical for turbulent flow in Figure 3.1.

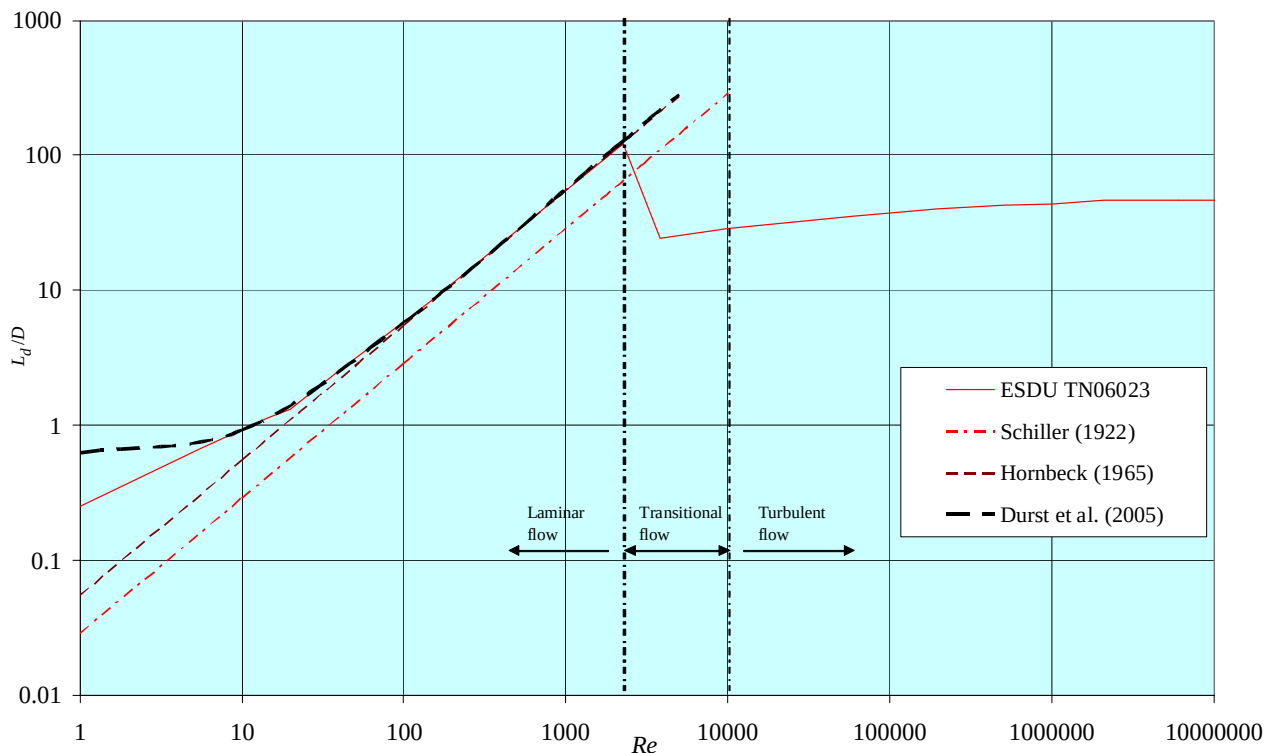


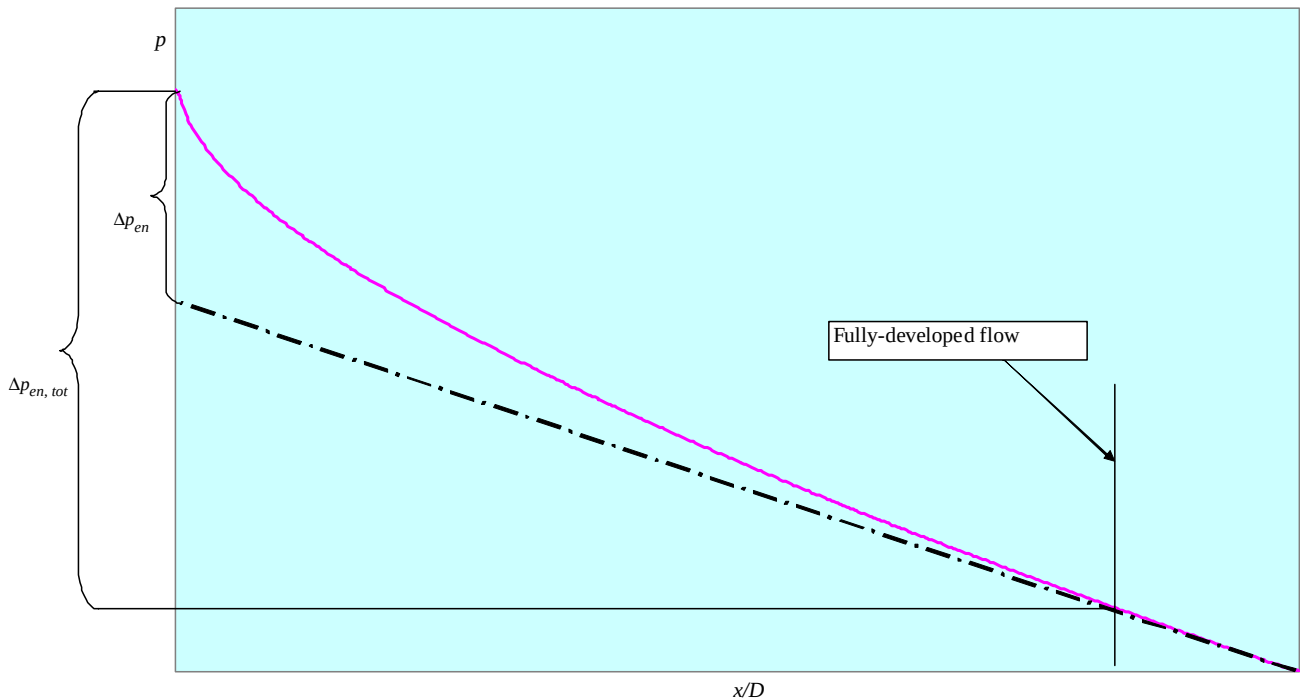
FIGURE 3.1 Summary of the development length data in the literature

TABLE 3.2 Constant C from various sources in the literature

Reference	Technique	C
Schiller ⁷	Theory	0.02875
Collins and Schowalter ²¹	Theory	0.061
Sparrow <i>et al.</i> ²⁵	Theory	0.056
Hornbeck ²³	Theory	0.0575
Fargie and Martin ²⁸	CFD	0.049 to 0.068
Chen ²⁹	Theory	0.052 to 0.068

3.2.2 Pressure development length

In situations where the flow does not reach the fully-developed velocity condition defined in Section 3.2.1, due to flow instabilities and to intermittent transitional behaviour, but the pressure gradient along the pipe length becomes constant, pressure measurements in the pipe section where the pressure gradient is constant can still be used to derive friction factor. A pressure development length, L_p , can then be defined as the pipe length required for the pressure gradient to become constant (Sketch 3.5).



Sketch 3.5 Definition of additional entrance pressure drop in laminar or turbulent flow

3.3 Entrance Pressure Drop

In a pipe entrance where the inlet velocity profile is assumed to be uniform, the velocity gradients experienced by the fluid as it enters the pipe and hence the high shear stress cause a larger pressure gradient than in fully-developed flow. This additional entrance pressure drop, Δp_{en} , can be expressed in laminar or turbulent flow as

$$\Delta p_{en} = K_{en} \frac{\rho u_{mean}^2}{2} \quad (3.2)$$

where K_{en} is the non-dimensional additional pressure drop (Sketch 3.5).

While a number of sources report on the experimental and theoretical values for K_{en} in laminar flow (see Table 3.3), no data are available in the literature for transitional flow. In turbulent flow, Curzons⁷² provided the following correlation:

$$K_{en} = 12 f . \quad (3.3)$$

Correlations and measurements in the literature for laminar and turbulent flow are given in Figure 3.2.

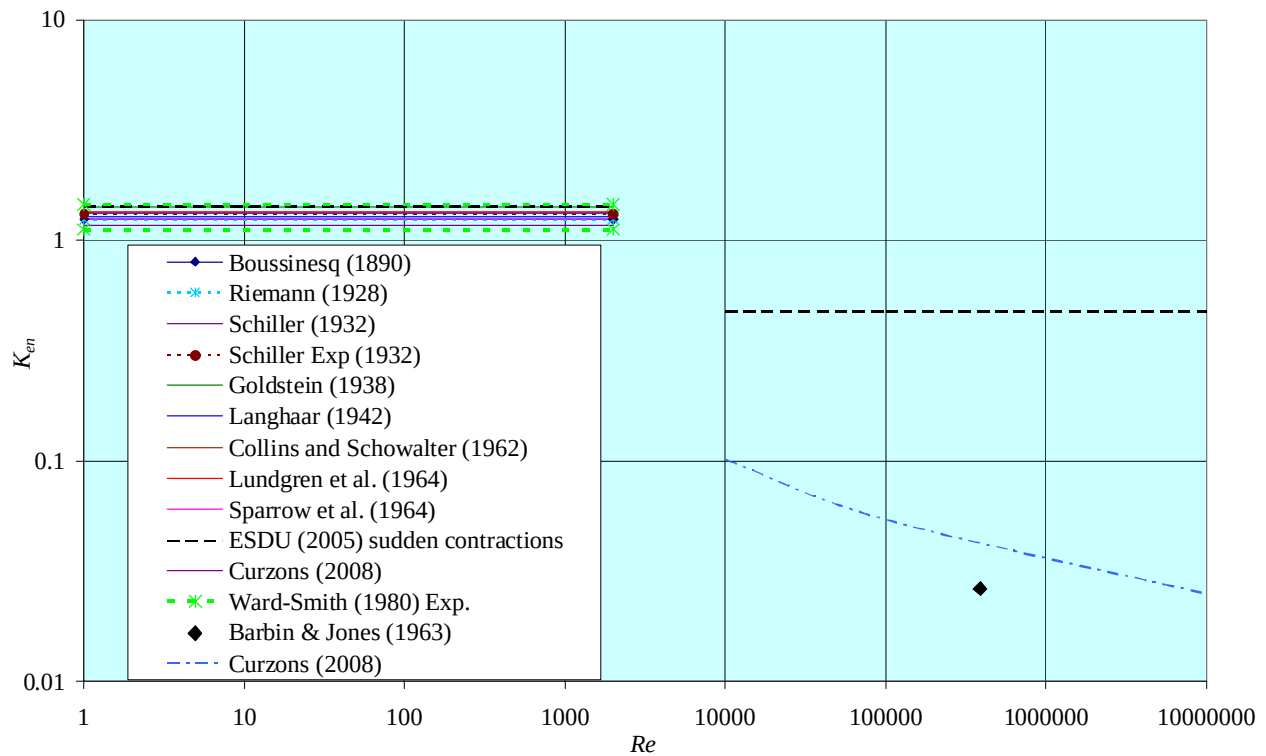


FIGURE 3.2 Values of K_{en} from various sources in the literature

TABLE 3.3 Values of K_{en} for laminar flow in circular pipes from various sources

Reference	Technique	K_{en}
Boussinesq ¹	Theory	1.24
Riemann ⁹	Experiment	1.24
Schiller ¹¹	Theory	1.16
Schiller ¹¹	Experiment	1.32
Goldstein ¹⁴	Theory	1.41
Langhaar ¹⁵	Theory	1.28
Collins and Schowalter ²¹	Theory	1.32
Lundgren <i>et al.</i> ²⁴	Theory	1.34
Sparrow and Lin ²⁵	Theory	1.24
Ward-Smith ³⁵	Experiment	1.115 to 1.45
ESDU ⁵⁹	CFD and Experiment	1.4175
Curzons ⁷²	Theory	1.33

3.4 Friction Factor

In a straight circular pipe, the pressure drop, $\Delta p = (p_1 - p_2)$ (Sketch 3.4), due to friction can be expressed in terms of the friction factor, f , as

$$\Delta p = 4f \frac{L}{D} \frac{1}{2} \rho u_{mean}^2 \quad (3.4)$$

where L is the pipe length, D the diameter and u_{mean} the bulk-mean velocity of the fluid.

A summary of the correlations and measurements available in the literature for laminar and turbulent flow is given in Table 3.4. Details for each correlation are given in ESDU TN 08008⁷⁵.

The correlations in Table 3.4 are compared in Figure 3.3 in the Reynolds number range $2000 < Re < 10^5$. Percentage differences from the ESDU 66027(a)²⁶ correlation are given in Figure 3.4.

Figure 3.5 shows early experimental data for friction factor in smooth circular pipes produced by Nikuradse¹³, Saph and Schoder², Nusselt³, Ombeck⁵, Jacob and Erk⁸, Stanton and Pannel⁶ and Hermann¹⁰ as quoted by Schlichting³⁴. It also includes recent measurements by McKeon *et al.*⁵⁶ and Draad *et al.*⁴⁵ and comparisons with the ESDU 66027²⁶ correlations. The experimental uncertainty of McKeon's data⁵⁶ is also shown in this Figure.

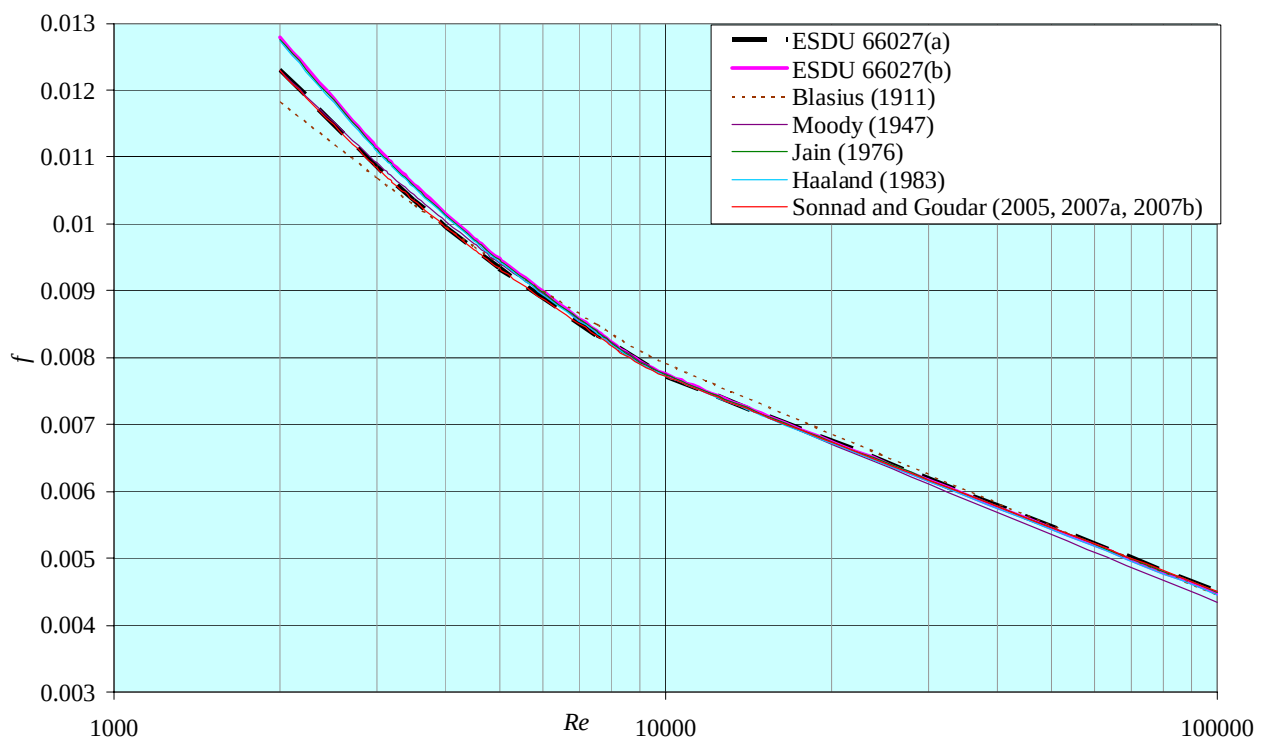


FIGURE 3.3 Friction factor correlations in the literature

TABLE 3.4 Summary of data in the literature for friction factor

Reference	Data/Equation	Re
ESDU 66027(a) ²⁶	$f = \left[4 \log \left[\frac{Re}{4.53 \log Re - 3.82} \right] \right]^{-2}$	$10^4 < Re < 2.5 \times 10^8$
ESDU 66027(b) ²⁶	$f = \left[3.6 \log \left[\frac{Re}{7} \right] \right]^{-2}$	$10^4 < Re < 2.5 \times 10^7$
ESDU 66027(c) ²⁶	$f = \frac{16}{Re}$	Laminar flow
Blasius ¹¹	$f = 0.0791/Re^{0.25}$	$Re \leq 10^5$
Moody ¹⁶	$f = 0.001375 \left[1 + \left(2 \times 10^4 \left(\frac{\varepsilon}{D} + \frac{10^6}{Re} \right)^{1/3} \right) \right]$	$4 \times 10^3 < Re < 10^8$ $0 \leq \varepsilon/D \leq 0.05^*$
Jain ³²	$\frac{1}{\sqrt{f}} = 2.28 - 4 \log \left(\frac{\varepsilon}{D} + \frac{21.25}{Re^{0.9}} \right)$	$5 \times 10^3 < Re < 10^8$ $10^{-6} \leq \varepsilon/D \leq 10^{-2}^*$
Chen ³³	$\frac{1}{\sqrt{f}} = -4 \log \left(\frac{1}{3.7605} \left(\left(\frac{\varepsilon}{D} \right) - \frac{5.0452}{Re} \log \left[\frac{1}{2.8257} \frac{\varepsilon^{1.1098}}{D} + \frac{5.8506}{Re^{0.8981}} \right] \right) \right)$	$Re \geq 4000$
Schlichting ³⁴	Experimental data for smooth pipes	$10^3 < Re < 10^5$
Haaland ³⁷	$\frac{1}{\sqrt{f}} = -1.2 \log \left(\left[\frac{6.9}{Re} \right]^3 + \left[\frac{\varepsilon}{3.75D} \right]^{3.33} \right)$	Turbulent flow
Sonnad and Goudar ⁶²	$\frac{1}{\sqrt{f}} = -1.7372 \ln \left(\frac{Re}{1.964 \ln(Re) - 3.8215} \right)$	$4000 < Re < 10^8$
McKeon <i>et al.</i> ⁵⁶	Experimental data for smooth pipes Tabulated data	$10 < Re < 3.6 \times 10^7$
McKeon <i>et al.</i> ⁶¹	$\frac{1}{\sqrt{f}} = 0.96 \log(Re \sqrt{f}) - 0.2375 - \frac{3.52}{(Re \sqrt{f})^{0.55}}$	$10^4 < Re < 3 \times 10^7$
Draad <i>et al.</i> ⁴⁵	Experimental data for smooth pipes. Plots	$10^3 < Re < 6.6 \times 10^4$

* ε is the effective roughness of the pipe internal surface.

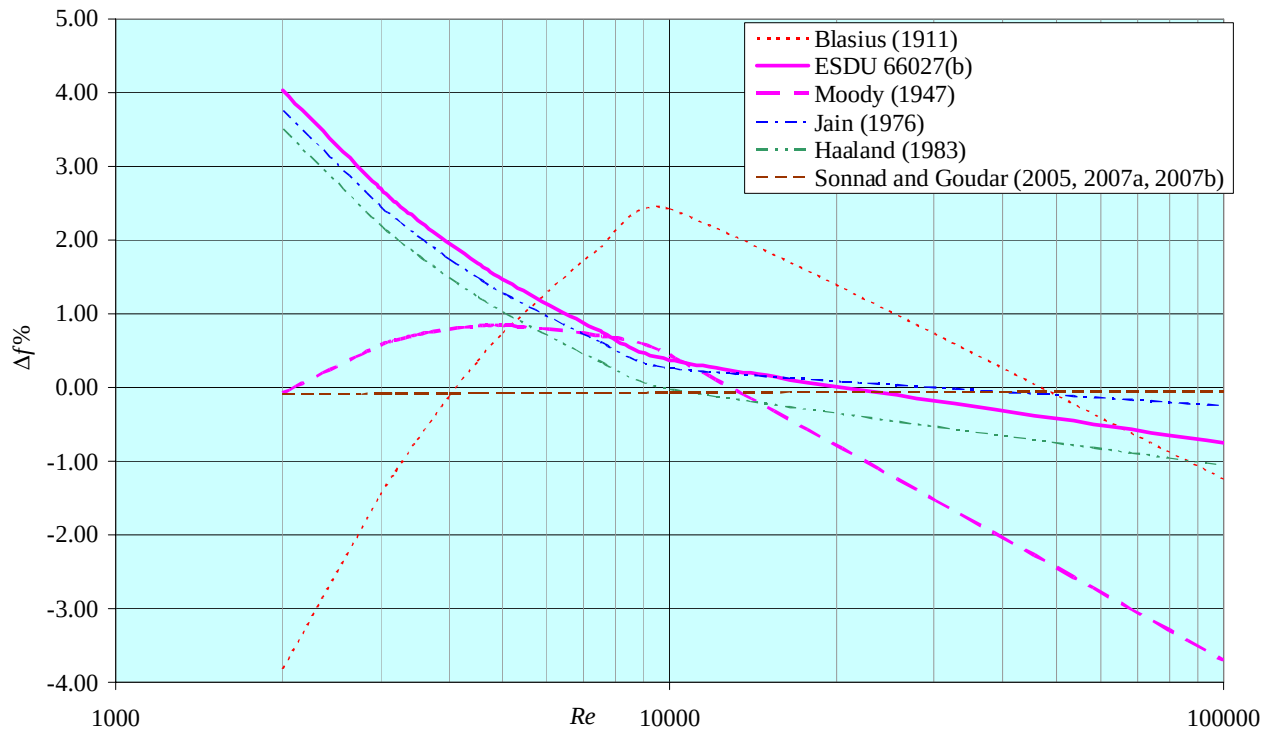


FIGURE 3.4 Friction factor measurements and ESDU 66027(a) and (c) correlations²⁶

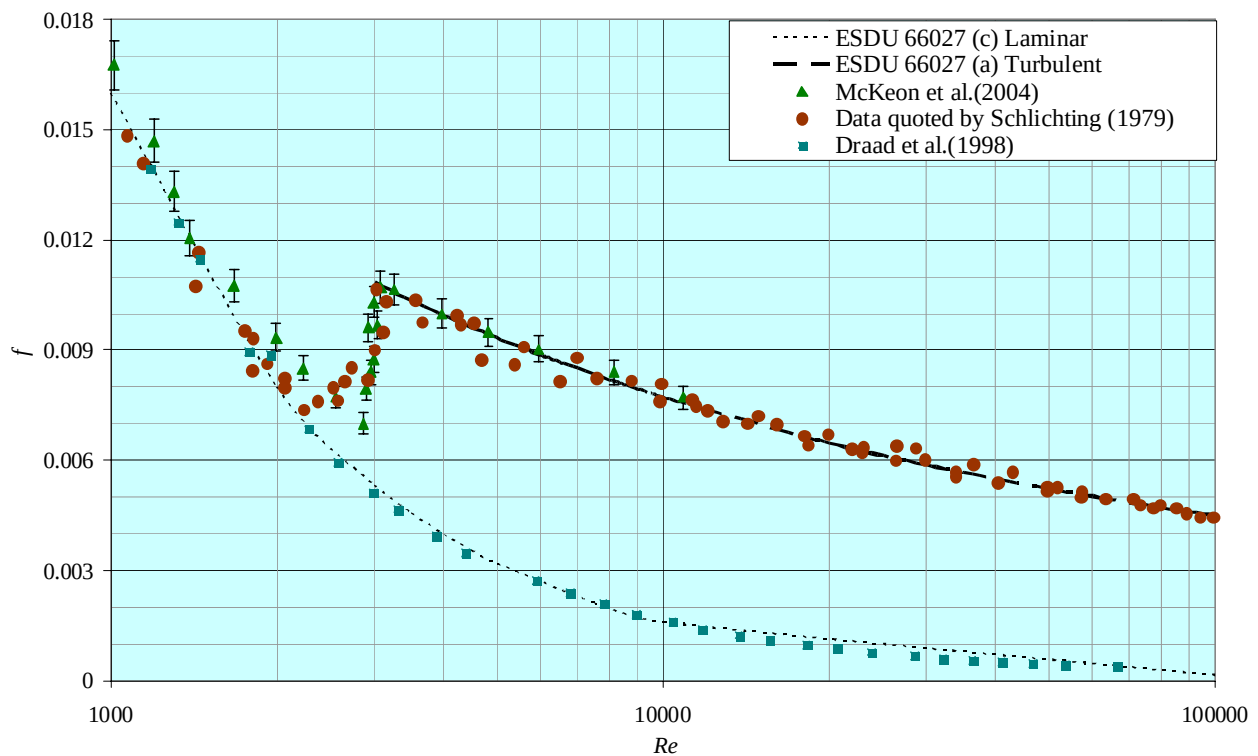


FIGURE 3.5 Comparison of different friction factor correlations with ESDU 66027(a)²⁶