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Revision Memo

12/02 Corrections and minor updates.

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SCOPE

This section describes the basic properties of fluid beds and particles. It provides the background information needed in the design of fluid solids systems. The information is of a general nature and is applicable to all fluid solids processes used within Exxon. However, when dealing with a known process, data and practices based on commercial plant experience should be used when available.

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FLUIDIZATION

DESCRIPTION OF FLUIDIZATION

When a fluid flows upward through a bed of solid particles, the pressure difference across the bed increases approximately linearly with velocity. At a sufficiently high velocity (minimum fluidization velocity, U_{mf}) the drag force on the particles becomes equal to their weight and the particles become neutrally buoyant. The particles are said to be fluidized and the particle bed takes on characteristics of a liquid.

As velocity is increased beyond the minimum fluidization velocity, the bed expands, and under ideal conditions, the pressure difference across the bed remains nearly constant and equal to the weight of the bed per unit cross-sectional area. Above the minimum fluidization velocity, the bed can expand in two ways. In **particulate** fluidization, beds expand uniformly, with the particles maintaining a somewhat regular spacing as they move apart. Fixed bed laws for pressure drop (e.g., the Carman-Kozeny or the Ergun equation) are applicable, with the provision that bed voidage, ϵ increases with fluid velocity in order to maintain constant pressure drop, rather than being independent of velocity as for a fixed bed.

In **aggregative or bubbling bed** fluidization, the bed is non-uniform. Most of the excess gas passes through the bed in fast-moving bubbles (bubble phase) which are nearly empty of particles, while the rest of the bed, called the emulsion phase, remains close to minimum fluidization conditions. Aggregative fluidization may become quite violent at high fluid velocities, and the bed structure and fluid flow patterns are much more complex than in particulate fluidization. Liquid fluidized systems generally display particulate behavior while gas fluidized systems are aggregative. Only gas-fluidized systems have achieved commercial significance, and particulate systems are of mainly academic interest.

The major feature of an aggregatively fluidized bed is the existence of much of the fluidizing gas in the form of large bubbles. Properties of these bubbles help define different types of gas-solid fluidization.



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SUPERFICIAL GAS VELOCITY

The performance of a fluidized bed is determined to a large degree by the velocity of the upward moving gas phase. The measure of the gas velocity normally used is the superficial gas velocity. This is the velocity the gas would have if it were rising up the fluidized bed vessel, empty of solids, but containing internals such as diplegs, overflow wells, etc., at operating temperature and pressure. Corrections for internals may not have been included in previous definitions of superficial gas velocity.

Minimum Fluidization Velocity

Minimum fluidization velocity, U_{mf} , is the superficial velocity at which particles first become suspended by the flowing fluid. U_{mf} can be calculated theoretically by selecting one of the several available expressions for pressure drop through a packed bed and determining the fluid velocity at which pressure drop balances the weight of the particles. Empirical expressions for U_{mf} are more reliable and are more generally used. Minimum fluidization velocity is calculated by PEGASYS cyclone module (see [Section VI-C](#), [Appendix A](#)) or by the following equation which was used to correlate 284 data points over a particle Reynolds Number, Re_p , range of 0.001 to 4000 with a standard deviation of $\pm 34\%$ ([Reference 1](#)).

$$\frac{d_p U_{mf} \rho_g}{\mu} = \left[(33.7)^2 + \frac{0.0408 d_p^3 \rho_g (\rho_p - \rho_g) g}{\mu^2} \right]^{1/2} - 33.7 \quad \text{Eq. (1)}$$

$$Re_p = \frac{d_p U_{mf} \rho_g}{\mu}$$

where: d_p = The volume surface mean particle diameter (ft), (m) [see [Eq. \(36\)](#)]
 U_{mf} = The superficial minimum fluidization gas velocity (ft/s), (m/s)
 ρ_g = The gas density (lbs/ft³), (kg/m³)
 ρ_p = The solid particle density (lbs/ft³), (kg/m³) (see **SOLID PROPERTIES**)
 μ = The gas viscosity (lbs (m)/ft/s), (kg/m/s)
 g = The gravitational acceleration (32.2 ft/s²), (9.81 m/s²)

For smaller particles, the equation reduces to:

$$U_{mf} = \frac{d_p^2 (\rho_p - \rho_g) g}{1650 \mu} \quad \text{for } Re_p < 20 \quad (\text{ft/s}), (\text{m/s}) \quad \text{Eq. (2)}$$

and for large particles:

$$U_{mf} = \left[\frac{d_p^2 (\rho_p - \rho_g) g}{24.5 \rho_g} \right]^{1/2} \quad \text{for } Re_p > 1000 \quad (\text{ft/s}), (\text{m/s}) \quad \text{Eq. (3)}$$

A bed with mixed particle size cannot be operated stably for long at minimum fluidization velocity, because the largest particles may settle out to form a packed, unfluidized zone at the bottom of the bed. If such settling is unacceptable, then the bed must be operated at the velocity above the minimum fluidization velocity for the largest particles.

Particle Terminal Velocity

At relatively low gas velocities, some solids are ejected from the surface of a bubbling bed, but readily fall back into the bed. At higher velocities, some of the solids are entrained and transported out of the fluid bed vessel with the gas. Under these conditions, two phases are present: the dense bubbling bed and the dilute entrained flow phase above it. As the gas velocity is increased, the character of the bubbling bed changes, solids motion becomes more agitated and the interface between the dense and dilute phases becomes less distinct. At some higher velocity, no dense phase is discernible and solids are transported rapidly through the vessel in a dilute phase (vertical pneumatic transport). A steady state dense phase fluidized bed can be operated at a gas velocity in excess of the terminal or free fall velocity of bed particles only if the entrained particles are captured and returned to the bed.

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The single spherical particle terminal (settling) velocity, U_{to} , can be determined from Stoke's law:

$$U_{to} = \frac{d_p^2 (\rho_p - \rho_g) g}{18\mu} \quad (\text{ft/s}), (\text{m/s}) \quad \text{Eq. (4)}$$

at values of the particle Reynolds number, $\frac{d_p U_{to} \rho_g}{\mu}$, less than 1.0. The particle density, ρ_p is defined under **SOLID**

PROPERTIES. The particle diameter d_p , is the weight median size (see **SOLID PROPERTIES**).

At higher Reynolds numbers, U_{to} can be obtained by using Figure 1. For numerical calculations, Figure 1 is represented as:

$$\frac{1}{Y} = \frac{24}{X^2} + \frac{0.69}{\sqrt{X}} \quad \text{Eq. (5)}$$

where: X and Y refer to the axes in Figure 1.

Fine particles at appreciable concentration have settling velocities somewhat higher than the settling velocities for single particles. This is due to forces which cause the particles to behave as clusters or aggregates. The effective diameter of these clusters, expressed as multiples of the single particle diameter, is related to the single particle terminal velocity in Figure 2. The settling velocity of a cluster is calculated as the terminal velocity of a particle with a diameter equal to the effective cluster diameter. For large particles, this cluster settling velocity approximates that for a single particle. The cluster terminal velocity is obtained from Figure 1 using values of n , the cluster index, obtained from Figure 2. Data given in Figure 2 were obtained under vertical pneumatic conveying conditions in a 3 in. (75 mm) pipe (Reference 2). The effect of vessel diameter on settling velocity appears to be small and is not included in these calculations.

If the terminal velocity, U_t , is exceeded, particles will be carried out of the bed, and the bed will gradually disappear if the entrained particles are not returned to be bed or new particles added. If particles are returned or added, however, the bed can retain its identity far above U_t . Thus, although U_t may place a practical limit on simple batch laboratory units, many commercial units operate at much higher velocities. For units which are fitted with two stages of cyclones housed within the top of the reactor vessel, maximum design superficial bed velocity is limited by the cyclone capacity which can be installed in the vessels and not by entrainment rate.

If superficial velocities are increased further, the fluid bed will pass through a transition zone between the dense phase and dilute phase regimes. The transition zone is characterized by pressure surges and unsteady solids flow. Operation in the transition zone should be avoided or minimized to the extent possible. The velocity at the transition point is called the choking velocity. At gas velocities above the choking velocity, a dense bed will not be present and operation will be entirely in the dilute phase flow regime. The choking velocity can be estimated by first calculating the entrainment for a series of gas superficial velocities and for the height of the vessel or riser in question [Eq. (15)]. The gas velocity which yields an entrainment rate equal to the solids circulation rate is the choking velocity.

Commercial beds tend to be designed for gas velocities limited by some upper limit that is characteristic of the particular process of interest. If internal cyclones are used, the limit is as noted above. If entrained particles are excessively sticky or fouling in nature, low velocity may be necessary in order to reduce entrainment and particle deposition above the bed. If no cyclones are to be used, then the velocity and entrainment rate must be selected so as to give tolerable particle losses.

PROPERTIES OF BUBBLES

Fluidized bed properties and performance are, in many cases, related to the size and rise velocity of the bubbles in a bubbling dense bed. Correlations have been developed relating bubble properties to operating conditions. These correlations are then used to predict fluidized bed properties and performance.

Gas bubbles in fluid beds have many of the properties of gas bubbles in low viscosity liquids. Fluid bed bubbles have the same spherical cap shape and flattened or concave bottom as those in liquids.

Bubble Velocities

- **Single Bubble Rise Velocity** - Rise velocity, U_B , of single bubbles in incipiently fluidized beds is given by the following relationship:

$$U_B = 0.71 \sqrt{gD_B} \quad (\text{ft/s}), (\text{m/s}) \quad \text{Eq. (6)}$$

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When the frontal diameter of the bubble D_B becomes greater than 0.3 times the bed diameter D , the bubble is called a slug, and the relationship becomes:

$$U_B = 0.35 \sqrt{gD} \quad (\text{ft/s}), (\text{m/s}) \quad \text{Eq. (7)}$$

This is also applicable to liquid systems and is theoretically derivable. A slug, instead of being centered (axisymmetric), may rise along one side of the bed, behaving as if in a bed of twice the actual diameter. This equation then becomes:

$$U_B = 0.49 \sqrt{gD} \quad (\text{ft/s}), (\text{m/s}) \quad \text{Eq. (8)}$$

- Bubble Velocity During Continuous Fluidization** - When a bed is fluidized by a substantial flow of gas, bubbles are generated continuously, and the bubble rise velocity is greater than when only a single bubble is present. The bubble rise velocity relative to a stationary observer, U_A , is given by:

$$U_A = U_B + U - U_{mf} \quad (\text{ft/s}), (\text{m/s}) \quad \text{Eq. (9)}$$

where: U = The superficial gas velocity

The increase in bubble rise velocity over the case of a single bubble is $U - U_{mf}$ which is the velocity at which the bed of fluidized solids itself is being moved upward by the incoming gas.

This is for an idealized case in which the time average gas flow is uniform across the bed cross section, in which there are no persistent solids circulation patterns, and in which negligible bubble coalescence occurs. In many large fluid beds, coalescence occurs, and gulf stream flow exists in which gas flow is channeled through the center of the bed and solids circulate down at the walls and up in the center. In the case of slug flow, however, the expression for U_A has been proven to be generally applicable.

Bubble Size

Bubbles formed at the grid will coalesce and grow as they rise up through the fluidized bed until a stable size is reached. A stable size is reached when the larger bubbles begin to break into smaller bubbles. One theory of bubble breakage is based on the tendency of bubbles to entrain solids as the bubbles move up the bed. As bubbles grow, their velocities also increase until the bubble velocities approach terminal velocity of the particles. At that point, solids will accumulate in the larger bubbles causing the bubbles to split into smaller bubbles. An equation giving the mean stable bubble size is based on this theory ([Reference 3](#)).

The mean stable bubble size is calculated by PEGASYS cyclone module (see [Section VI-C, Appendix A](#)) or by the following equations:

$$D_B = 0.03 \left[\frac{(\rho_p - \rho_g)^2}{\rho_g \mu} \right]^{0.666} (d_p)^{1.46} \quad (\text{ft}) \quad \text{Eq. (10c)}$$

$$D_B = 0.011 \left[\frac{(\rho_p - \rho_g)^2}{\rho_g \mu} \right]^{0.666} (d_p)^{1.46} \quad (\text{m}) \quad \text{Eq. (10m)}$$

where: D_B = The mean stable bubble size (ft), (m)
 d_p = The volume surface particle diameter [[Eq. \(36\)](#)] (ft), (m)]
 ρ_p = The particle density (lbs/ft³), (kg/m³)
 ρ_g = The gas density (lbs/ft³), (kg/m³)
 μ = The gas viscosity (lbs/ft/s), (kg/m/s)

This correlation is applicable over a cluster Reynolds Number, Re_{cl} , between 0.4 and 500, where Re_{cl} is defined as:

$$Re_{cl} = \frac{d_{cl} U_{tcl} \rho_g}{\mu} \quad \text{Eq. (11)}$$

where: d_{cl} = The cluster diameter calculated for [Eq. \(11\)](#) only as:
 d_{cl} = $0.264 (d_p)^{0.73}$ (ft)
 d_{cl} = $0.192 (d_p)^{0.73}$ (m)
 U_{tcl} = The cluster terminal velocity from [Figure 1](#) where n for this application is the ratio d_c / d_p from [Figure 2](#) (ft/s), (m/s)

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Bubbles achieve the mean stable size fairly close to the grid. In those cases where deep beds are used, the bubble growth region is not important and the mean stable size can be used for fluidization correlations. In shallow bed reactors, or in those cases where rapid reactions occur in the vicinity of the grid, the rate at which the bubbles grow may be important. The bubble size can be calculated at various heights above the grid from the empirical [Eq. \(4\)](#).

$$D_B = 2.8 \times 10^{-2} (1 + 8.29(U - U_{mf}))^{1/3} (1 + 2.08 h)^{1.21} \quad (\text{ft}) \quad \text{Eq. (12c)}$$

$$D_B = 8.5 \times 10^{-3} (1 + 27.2(U - U_{mf}))^{1/3} (1 + 6.82 h)^{1.21} \quad (\text{m}) \quad \text{Eq. (12m)}$$

where: D_B = The bubble size (ft), (m)
 U = Superficial gas velocity (ft/s), (m/s)
 U_{mf} = Superficial gas velocity at minimum fluidization conditions (ft/s), (m/s)
 h = The height above the grid (ft), (m)

The height at which the stable bubble size is reached can be estimated by equating [Eqs. \(10\)](#) and [\(12\)](#) and solving for h .

ENTRAINMENT

Solids entrainment is defined as the weight of solids per volume of gas flowing past a point in the dilute phase region above a dense fluidized bed. The entrainment is usually calculated at the vessel outlet or at the inlet to internal cyclones. Entrainment differs from dilute phase density, discussed later, which is the concentration of solids in a given region above the dense bed, expressed as weight of solids per volume of dilute phase. The entrainment decreases with height above the dense bed, also referred to as outage, up to a point above which the entrainment becomes constant. The value of the outage at which the entrainment becomes constant is called the Transport Disengaging Height or TDH. TDH is used in entrainment and dilute phase holdup calculations.

Transport Disengaging Height (TDH)

A bubble breaking at the surface of a fluidized bed creates a high velocity jet which ejects solids into the dilute phase. Jets formed in this fashion expand and merge with other jets until they reach the vessel wall and are dissipated. Below the point of jet dissipation, solids are carried upward by the jets at effective velocities which decay inversely proportional to the square root of the distance above the bed. As the jets expand and the jet velocities decrease, some solids drop back into the dense bed. Above the point of jet dissipation, gas velocity becomes fairly uniform and the solids entrainment becomes constant with increasing vessel height. At this point, only those solids whose terminal velocities are equal to or less than the gas superficial velocity continue to be entrained. The point at which the entrainment becomes constant is defined as the Transport Disengaging Height or TDH.

For a slugging bed:

$$TDH_S = 1.7 (U_B + U - U_{mf}) \sqrt{U_{BS}} \quad (\text{ft}) \quad \text{Eq. (13c)}$$

$$TDH_S = 3.1 (U_B + U - U_{mf}) \sqrt{U_{BS}} \quad (\text{m}) \quad \text{Eq. (13m)}$$

where: TDH_S = The transport disengaging height for a slugging bed (ft), (m)
 U_{BS} = The slug rise velocity [[Eq. \(7\)](#)] (ft/s), (m/s)
 U = The superficial gas velocity (ft/s), (m/s)
 U_B = The bubble rise velocity [[Eq. \(6\)](#)] (ft/s), (m/s)
 U_{mf} = The superficial velocity at minimum fluidization (ft/s), (m/s)

For a bubbling bed:

$$TDH_B = (1.5) 1.7 (U_B + U - U_{mf}) \sqrt{U_B} \quad (\text{ft}) \quad \text{Eq. (14c)}$$

$$TDH_B = (1.5) 3.1 (U_B + U - U_{mf}) \sqrt{U_B} \quad (\text{m}) \quad \text{Eq. (14m)}$$

where: TDH_B = The transport disengaging height for a bubbling bed (ft), (m)

The factor 1.5 was included in [Eq. \(14\)](#) based on a comparison with commercial FCC data.

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Based on very limited comparisons with measured results, Eqs. (13) and (14) predict TDH to about $\pm 30\%$.

For fluid bed systems with a non-uniform cross-section caused by diameter changes (swaged vessels) or internals such as baffles or cyclones, the concept of a critical outage is used instead of TDH. The critical outage is defined as the vessel height above which entrainment is constant. It can differ from the calculated TDH if changes in effective vessel cross-section occur which affect gas velocity and entrainment.

- TDH, or critical outage, can be calculated with The PEGASYS cyclone module, a general purpose fluid solids program described in [Section VI-C](#), **Appendix A**. PEGASYS cyclone module first calculates bubble size and determines if bubbling or slugging conditions control. The program then selects the proper equations to calculate TDH or the critical outage.

Entrainment Correlation

The calculation of entrainment is based on a correlation involving the square of the ratio of gas velocity to terminal velocities ([Reference 5](#)). The correlation is:

$$\frac{E}{\rho_g} = 14.5 \sum_{i=1}^n x_i \exp \left[\frac{-42.2}{\frac{V^2}{U_{ti}^2} - 1} \right] \quad \text{Eq. (15)}$$

- where:**
- E = The entrainment (lbs solids/ft³ gas), (kg solids/m³ gas)
 - ρ_g = The gas density (lbs/ft³), (kg/m³)
 - x_i = The weight fraction solids in particle size increment i
 - n = The number of particle size increments
 - V = The effective gas velocity at the point at which the entrainment is calculated (ft/s), (m/s).
See further description below.
 - U_{ti} = The terminal velocity for particles in increment i (ft/s), (m/s)

- At levels above TDH (or critical outage), V is the superficial gas velocity, U . Below TDH, V is an effective velocity, computed by the following procedure:

- Determine $U_s(X)$, the superficial gas velocity as a function of height X above the dense bed level. U_s changes because of changes in vessel cross-section area and presence of bed internals that also lead to an effective decrease in available area for gas flow.
- Determine $V_e(X)$

$$V_e(X) = U \left[\frac{\text{TDH}}{X} \right]^{1/2} \quad (\text{ft/s}), (\text{m/s})$$

- where:**
- V_e = The effective velocity (ft/s), (m/s)
 - U = The superficial gas velocity at the top of the dense bed (ft/s), (m/s)
 - X = The height above the dense bed (outage) (ft), (m)
 - TDH = Transport disengaging height (ft), (m)

- Determine critical outage, X_c , which is the vessel height above which entrainment is constant: It is determined by the following expression: $U_s(X_c) = V_e(X_c)$.
- V is then given by the following expression

$$V = \begin{cases} V_e(X), & X < X_c \\ V_e(X_c), & X \geq X_c \end{cases} \quad (\text{ft/s}), (\text{m/s}) \quad \text{Eq. (16)}$$

- The entrainment correlation is included in PEGASYS cyclone module. For larger particles, the program uses [Figure 1](#) to calculate U_{ti} with the inclusion of the Heywood shape factor z . The user can specify the shape factor, or use the program default of 0.52 for spherical particles. For small particles, the program uses the particle clustering concept to determine U_{ti} .

The entrainment correlation predicts measured results to $\pm 30\%$. The correlation should not be used at superficial velocities

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- below 1 ft/s (0.3 m/s). For applications below 1 ft/s (0.3 m/s), consult the Process Engineering Department. A new correlation has been developed to address entrainment at high bed superficial gas velocities, for details see document EE.701E.2002.

FLUID BED PROPERTIES, PERFORMANCE

FLUID BED DENSITY

The density of a bubbling fluidized bed can be derived from the equation for the absolute bubble rise velocity U_A and the two-phase (bubble and emulsion phases) theory of fluidization. The resulting equation is:

$$\frac{H}{H_{mf}} = \frac{\rho_{mf}}{\rho} = 1 + \frac{U - U_{mf}}{U_B} \quad \text{Eq. (17)}$$

where H is the height of the fluidized bed and ρ its density. Subscript $_{mf}$ refers to minimum fluidization conditions. A more convenient form of the equation is:

$$\rho = \rho_{mf} \frac{U_B}{U_B + U - U_{mf}} \quad (\text{lbs / ft}^3), (\text{kg / m}^3) \quad \text{Eq. (18)}$$

The equation may also be used in the absence of experimental data to predict the density of a non-slugging bed. In such a case, the bubble velocity will be given by $U_B = 0.7 \sqrt{gD_B}$ and bubble size can be estimated according to [Eq. \(10\)](#). Bed density at minimum fluidization, ρ_{mf} , is usually known for the solids in question, or can be assumed to be equal to 90 to 95% of the loose bulk density. Fluid bed density can also be expressed in terms of the bed voidage, ε , as given in [Eqs. \(19\), \(21\), and \(22\)](#). Non-ideal behavior, such as gulf stream flow, will often cause actual bed density to be somewhat greater than predicted by the equation, but no method exists to quantify the effect.

BED VOIDAGE

Another property of a fluidized bed related to density is the interstitial voidage, ε . It is related to density by the expression:

$$\varepsilon = 1 - \frac{\rho}{\rho_p} \quad \text{Eq. (19)}$$

where ρ_p is the particle density defined under **SOLID PROPERTIES**. Voidage at minimum fluidization conditions, ε_{mf} , can be obtained from [Eq. \(19\)](#) using ρ_{mf} in place of ρ . In this case:

$$\varepsilon_{mf} = 1 - \frac{\rho_{mf}}{\rho_p} \quad \text{Eq. (20)}$$

[Eq. \(18\)](#) can also be used to give ρ_{mf} in terms of ε_{mf} . This gives another version of the equation for bed density:

$$\rho = \rho_p (1 - \varepsilon_{mf}) \frac{U_B}{U_B + U - U_{mf}} \quad (\text{lbs / ft}^3), (\text{kg / m}^3) \quad \text{Eq. (21)}$$

Voidage at minimum fluidization, ε_{mf} , is usually in the range of 0.45 to 0.5. Voidage can also be obtained from the expression:

$$\varepsilon = 1 - \frac{\rho}{\rho_{mf}} (1 - \varepsilon_{mf}) \quad \text{Eq. (22)}$$

Dilute Phase Density

Dilute phase density is the concentration of solids in the dilute phase region of a fluidized bed vessel expressed as weight of solids per volume of dilute phase. It is used to determine the holdup of solids in the dilute phase for pressure balance calculations. Dilute phase density decreases with outage up to the critical outage. It is constant above the critical outage. Dilute phase density is higher than entrainment because of slip, which is the tendency of the entrained solids to move through the dilute phase region at a velocity less than the rising gas. The relationship between dilute phase density and entrainment is given by [\(Reference 6\)](#):

$$\rho_d = E (\text{S.F.}) = \rho_g \quad (\text{lbs / ft}^3), (\text{kg / m}^3) \quad \text{Eq. (23)}$$

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where: ρ_d = The dilute phase density (lbs/ft³), (kg/m³)
 E = Entrainment (lbs/ft³ gas), (kg/m³ gas)
 $S.F.$ = A slip factor
 ρ_g = The gas density (lbs/ft³), (kg/m³)

The slip factor is defined as:

$$S.F. = U_g / U_s \quad \text{Eq. (24)}$$

where: U_g = The gas velocity (ft/s), (m/s)
 U_s = The solids velocity (ft/s), (m/s)

The slip factor is available from correlations of the slip factor with the ratio E / ρ_p , where ρ_p is the particle density (Reference 6). Dilute phase density is calculated by PEGASYS cyclone module.

Dilute Phase Holdup

The dilute phase holdup is the total weight of solids (lbs or kg) contained in the dilute phase and is obtained by integrating the dilute phase density over the outage. The calculation of dilute phase density and the integration to give dilute phase holdup are made by the cyclone module in PEGASYS.. The integral equation involved in this calculation is:

$$\text{Holdup} = \int_0^{x_f} \frac{U}{V_j} (E) (A) \left[\int_0^x (S.F.) f(d_p) dX \right] dX \quad (\text{lbs}), (\text{kg}) \quad \text{Eq. (25)}$$

where: X = The outage (ft), (m)
 X_f = The maximum outage for the calculation (ft), (m)
 U = The superficial gas velocity (ft/s), (m/s)
 V_j = The gas jet velocity (ft/s), (m/s)
 E = The entrainment (lbs/ft³), (kg/m³)
 A = The vessel area in the dilute phase region (ft²), (m²)
 $S.F.$ = The slip factor
 f = A frequency function (1/ft), (1/m)

The integration is carried out in two regions, below and above the TDH (or critical outage). PEGASYS cyclone module determines the TDH and integrates Eq. (25) accordingly.

A comparison of calculated and measured holdups based on a single test in a commercial FCC plant (Montreal) indicated the calculated values to be about 30% low.

GAS / SOLID CONTACTING

Contacting between gas and solids in a fluidized bed reactor can affect important performance criteria such as conversion, yield product quality, catalyst activity, temperature distribution and may possibly affect unit operability. Contacting is usually described by the two-phase theory which states that gas moves through the bed in two phases, the bubble phase and the emulsion phase (Reference 7).

Bubble Phase

Bubbles are gas voids relatively free of solids. Reactions which require the presence of a solid do not occur to a significant degree in the bubbles. However, vapor phase reactions can occur in the bubbles. Since bubble velocity is related to bubble size [Eq. (6)], the larger the bubbles the higher the upward velocity. Bubble velocities are usually greater than the overall superficial

velocity or velocity through the emulsion phase. The maximum bubble velocity is the terminal velocity of the solids. When bubbles reach this velocity, they break into smaller bubbles with lower velocities.

The fraction of the bed occupied by bubbles, δ , can be approximated by:

$$\delta \cong \frac{U - U_{mf}}{U - U_{mf} + U_B} \quad \text{Eq. (26)}$$

Bubbles are also thought to be surrounded by a gas/solid cloud which is a region with properties intermediate between the bubble and emulsion phases. Large, rapidly rising bubbles have thin clouds, while smaller, slower bubbles have thicker clouds.

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If the gas velocity is greater than twice the velocity at minimum fluidization, or $U > 2 U_{mf}$, the bed is bubbling vigorously with large, fast-rising bubbles. In this case, the bubble gas flow is close to plug flow.

Although bubbles are relatively free of solids, bubbles create a wake as they rise which pulls in solids, carrying them upward with the bubbles to the top of the fluidized bed. This is the mechanism which is responsible for axial mixing of solids in a fluidized bed. Lateral or radial mixing of solids is also caused by the bubbles. In this case, solids adjacent to the bubbles' solids are pushed aside by the rising bubbles, then pulled into the bubble wake where they are mixed with other solids.

In summary, bubble gas flow is characterized by relatively short residence times, minimum contact with solids and is largely in plug flow.

Emulsion Phase

The emulsion phase consists of a bubble-free, gas/solids suspension. Voidage in the emulsion phase, ϵ_e , is usually assumed to be the voidage of minimum fluidization, ϵ_{mf} or about 0.5. Since solids concentration is high, reactions which require the presence of solids occur primarily in the emulsion phase.

Solids which are carried upward in the wake of the bubbles to the top of the fluidized bed, flow downward through the emulsion phase. The solids velocity is given by:

$$U_S = \frac{\delta \alpha U_A}{1 - \delta - \alpha \delta} \quad (\text{ft/s}), (\text{m/s}) \quad \text{Eq. (27)}$$

where: U_S = The solids downward velocity (ft/s), (m/s)
 δ = The bubble fraction in the bed [Eq. (26)]
 α = The fraction of the bubble volume occupied by the bubble wake
 U_A = The absolute bubble velocity (ft/s), (m/s) [Eq. (9)]

α can be assumed to be in the range of 0.2 to 0.4.

Emulsion gas velocity is given as:

$$U_e = \frac{U_{mf}}{\epsilon_{mf}} - U_S \quad (\text{ft/s}), (\text{m/s}) \quad \text{Eq. (28)}$$

where: U_e = The emulsion gas velocity (ft/s), (m/s)

If bed solids are stationary, e.g., at minimum fluidization, $U_S' = 0$ and emulsion velocity equals U_{mf} / ϵ_{mf} . As solids flow is initiated in a bubbling bed, U_e decreases and will become negative when U_S' exceeds U_{mf} / ϵ_{mf} . Therefore, in vigorously bubbling beds, rapidly downward moving solids can be thought to pull emulsion gas downward, causing emulsion gas backmixing. Thus, gas flow through the emulsion phase can range from plug flow at low gas velocities to backmixed flow at high gas velocities. This reversal in the emulsion gas flow direction occurs when the solids downward velocity is greater than U_{mf} / ϵ_{mf} or when:

$$\frac{U}{U_{mf}} > (1 - \delta - \alpha \delta) \left(1 + \frac{1}{\alpha \epsilon_{mf}}\right) \quad \text{Eq. (29)}$$

In summary, in comparison to bubble gas flow, emulsion gas flow is characterized by longer residence times in closer contact with the solids and can range from plug flow to backmixed flow depending on the gas velocity.

Interchange Between Bubble and Emulsion Gas

Gas flowing through a fluidized bed is not restricted to the bubble or emulsion phase, but can move between the two phases. The interchange rate can be expressed in terms of interchange coefficients, defined as volume exchanged/volume of bed/time or crossflow ratios, defined as the number of times bubble gas is replaced as a bubble passes through the bed. Methods to calculate interchange coefficients and crossflow ratios are given in [Reference 7](#).

Gas Dispersion Coefficients

The flow of gas through a fluidized bed can also be described as a diffusion or dispersion process. Dispersion coefficients defined by standard diffusion equations have been measured and correlations developed. Equations based on the two-phase theory have also been developed ([Reference 7](#)). The axial dispersion coefficient, D_a , is dependent on α , δ , U , U_{mf} , ϵ_{mf} and a coefficient describing interchange between the bubble and emulsion phase. The radial dispersion coefficient, D_r , is dependent on D_B , and the bubble to emulsion interchange coefficient. Higher values of the dispersion coefficients, or more correctly the dispersion numbers, indicate more complete gas mixing. The dispersion number is defined as D_e / UH where H is the fluidized bed height. This approach combines mixing occurring in bubble and emulsion phases and mixing due to interchange between the two phases.

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SOLIDS MIXING AND SEGREGATION

Solids in a freely bubbling bed are usually well mixed. This results in a number of process effects. For example, well-mixed solids result in uniform bed temperature. However, solids mixing also results in backmixing of the emulsion gas as well as the solids and this can have a negative effect on conversion and yield. Insufficient solids mixing can cause segregation in the fluid bed with smaller, less dense particles collecting at the top of the bed and larger, denser particles at the bottom. This can affect entrainment rates, solids residence time, and fluidization, all of which can affect process performance.

Solids Mixing

Solids mixing in a fluidized bed is caused by bubble action as previously described. Solids are mixed axially by being pulled upward in bubble wakes and by moving downward in the emulsion phase. Lateral or radial mixing occurs when solids displaced laterally by a bubble are pulled into the bubble wake and mixed with other solids. Axial mixing occurs much more readily than radial mixing. The rate solids flow across a horizontal plane in a freely bubbling bed is given by:

$$J \cong \alpha \rho_p (1 - \varepsilon_{mf}) (U - U_{mf}) \quad (\text{lbs} / \text{ft}^2\text{s}), (\text{kg} / \text{m}^2\text{s}) \quad \text{Eq. (30)}$$

- where:**
- J = The solids flow rate across a horizontal plane (lbs/ft²s), (kg/m²s)
 - α = The fraction of the bubble volume occupied by the bubble wake (usually 0.2 to 0.4)
 - ρ_p = The particle density (lbs/ft³), (kg/m³)
 - ε_{mf} = The bed voidage at minimum fluidization
 - U = The gas superficial velocity (ft/s), (m/s)
 - U_{mf} = The gas superficial velocity at minimum fluidization (ft/s), (m/s)

The degree of solids mixing can also be described by dispersion coefficients, D_{SA} for axial and D_{Sr} for radial mixing. Larger values of D_S indicate higher degrees of mixing. D_{SA} can be estimated from:

$$D_{SA} \cong \frac{\alpha^2 \varepsilon_{mf}}{3 \delta U_{mf}} D_B (U - U_{mf})^2 \quad (\text{ft}^2 / \text{s}), (\text{m}^2 / \text{s}) \quad \text{Eq. (31)}$$

- where:**
- D_{SA} = The solids axial dispersion coefficient (ft²/s), (m²/s)
 - D_B = The bubble diameter [Eq (10)] (ft), (m)
 - All other terms are as previously defined

D_{Sr} can be estimated from:

$$D_{Sr} \cong \frac{3}{16} \frac{\delta}{(1 - \delta)} \frac{U_{mf} D_B}{\varepsilon_{mf}} \quad (\text{ft}^2 / \text{s}), (\text{m}^2 / \text{s}) \quad \text{Eq. (32)}$$

The above equations apply to freely bubbling beds with $U > 2U_{mf}$. Small beds operating in slug flow or beds with coarse solids operated at velocities closer to the minimum fluidization velocity may not follow the above equations. Departures may also be seen in very large commercial plants where mixing caused by the existence of large flow streams (gulf streaming) may occur. The above equations should be used for general guidance. Consult the Process Engineering Department for specific cases and for those cases where the above equations do not apply.

Gulf Streaming

In larger beds, especially if the open area of the fluidizing grid is not equally distributed, large upward flows in the central regions of the bed can be established. The flow splits at the top, runs down the wall and recombines at the bottom. This pattern is referred to as gulf streaming. Although gulf streaming induces effective solids mixing, it can result in very short gas residence times which can lead to poor yields.

Segregation

Segregation is noted by distinct layers of solids of different sizes and/or densities in the fluidized bed. Segregation can occur if the solids size distribution is bimodal, i.e., large concentrations of small and large particles with little intermediate size particles. Appreciable differences in particle densities can also cause segregation. Larger or denser particles can accumulate on the grid, and may defluidize, resulting in increased pressure drop, interference with the operation of internals, solids feeding and withdrawal. Segregation in beds undergoing exothermic reactions can result in temperature runaways and bed fusion.

Segregation may also result in an accumulation of small particles at the bed surface. This can occur in beds operated close to minimum fluidization with little bubble induced mixing.