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Revision Memo

- 12/02** Editorial Revision. Highlight of this revision are:
1. Corrected miscalculated values in the Appendix A Example Problem



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SCOPE

This section covers the analysis and design of foundations supporting reciprocating and rotary machinery used in refineries and chemical plants. Foundations covered by this section are grade level "blocks" and elevated frame and wall supported "tabletops" made of reinforced concrete. The procedures provided herein are also applicable to foundations supporting turbine-generators of up to 100 MW used for cogeneration.

This section is equally applicable to the design of new foundations or for analysis and troubleshooting of existing foundations. The section provides design procedures and guidelines to be used in conjunction with **GP 4-6-2**, and machinery foundation computer tools (**References 19, 20, and 21**).

Guidelines for inspection and repair of existing foundations have been issued separately (**Reference 5**) and are not covered herein.

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OBJECTIVE

The intent of this section is to provide a methodology for accurately predicting dynamic response either for a new or existing foundation. Together with the design loads and criteria set forth in **GP 4-6-2**, the section should produce a conservative foundation design. A conservative approach to new design is warranted considering uncertainties in assumed design parameters/loads and the potentially large operating and financial losses associated with machinery malfunction or downtime.

In order to account for uncertainties in soil and foundation properties, dynamic loading magnitude and direction, it is imperative that a range of values be analyzed and considered in the design. It is important to understand the effect of variables influencing foundation dynamic response. This section provides the guidance required to make qualitative and quantitative judgements required for design.

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BACKGROUND

Heavy machinery with reciprocating or rotary masses requires a support system which can resist cyclic forces and the vibrations caused by these forces. When excessive, such vibrations are detrimental to the machinery, to its support system and to any operating personnel subjected to them for long periods. **GP 4-6-2** contains design requirements specifically directed at preventing the development of such vibrations. For application, however, information supplemental to that in **GP 4-6-2** must be provided.

Machinery foundation analysis and design has continually evolved over the past four decades. In the 1960s, design was based on crude empirical ratios of foundation mass to machine mass without direct consideration of dynamic response. In the 1970s, these empirical approaches were largely replaced with techniques based on structural vibration theory and soil elasticity concepts. These techniques, suitable for hand calculations, are used to determine structural response for each individual vibration mode. Although these techniques included coupled effects of translation and rotational modes, rigid body vibration response governed by soil properties and flexural modes governed by structural parameters were considered separately. These techniques are still predominantly used today because of their ease of use and reasonable level of accuracy.

Dynamic frame and finite-element computer programs have been used during the 1980s and 1990s to analyze and design large, critical, complex machinery foundations. These programs should be used when: extensive soil data is available, a coupled soil/structural response is necessary, and/or when the simple analysis models are invalid.

DESIGN CONSIDERATIONS

FUNCTIONAL CONSIDERATIONS

The basic requirements for any machine foundation are that:

- The machine can operate efficiently and reliably without excessive wear due to foundation vibration.
- The foundation does not suffer damage or settlement sufficient to cause machine malfunction or realignment.
- Transmitted vibration should not adversely affect personnel or adjacent equipment.
- Above requirements are satisfied at minimum cost.

DYNAMIC CONSIDERATIONS

The purpose of a dynamic analysis/design is to avoid excessive vibration and associated problems. Specific items requiring review are:

- Dynamic Response - Displacement/Velocity/Acceleration.
- Structural Resonance.
- Stresses/Fatigue.
- Settlement.

The dynamic displacement (amplitude) of a machinery foundation subjected to dynamic loading is equal to:

$$\Delta_D = M \Delta_S \quad \text{Eq. (1)}$$

where: Δ_D = Dynamic displacement (amplitude)
 M = Dynamic magnification factor
 Δ_S = Static displacement

Figure 1 shows the dynamic magnification for a harmonic loading. Reviewing the above relationship and Figure 1, dynamic response is dependent on three parameters: (1) foundation and soil stiffness, (2) ratio of loading frequency to foundation natural frequency, and (3) amount of damping present. Also, minimizing eccentricity between dynamic load application and the machine/foundation centroid will reduce rotational excitation.

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DESIGN CRITERIA

AMPLITUDES

In order to satisfy the functional considerations, the dynamic response of the foundation must be controlled. Dynamic response can be expressed as amplitude, velocity, or acceleration. For response to harmonic loading, amplitude Δ_D (zero to peak) is given by:

$$\Delta_D = \frac{V}{\omega} = \frac{A}{\omega^2} \quad \text{Eq. (2)}$$

where: ω = Operating frequency, radians/sec
 V = Velocity
 A = Acceleration

GP 4-6-2 provides different peak-to-peak amplitude limits for reciprocating and rotary machinery. Generally, reciprocating machines are more rugged in design, hence the higher allowable limit. For rotary machines, high vibration amplitudes can cause bearing wear/rotor unbalance, thus increasing the dynamic loading from the rotating mass unbalance. These limits are to be applied to maximum vector sum of the translation and rotational components of amplitude predicted on the foundation. This amplitude is determined for each of the X, Y, Z directions and checked against the **GP 4-6-2** limits. The **GP 4-6-2** values are plotted on **Figure 2** along with recommendations from the literature. These values are based on considerations of maintenance and damage prevention of equipment.

If personnel are expected to work significant periods on or near the foundation, then stricter limits must be imposed to prevent personnel discomfort and decreased work efficiency. In these cases, charts given in **International Standard ISO 2631, Guide to the Evaluation of Human Exposure to Whole Body Vibration (Reference 8)**, should be used.

RESONANCE

Figure 1 shows that as the operating frequency or machine speed approaches the natural frequency of the foundation, magnification of static response occurs. Resonance occurs when the machine speed and the natural frequency of the supporting soil/foundation system coincide. Avoiding resonance is one of the critical factors in any successful design where machine induced vibrations are involved, unless the system is highly damped. In most designs, it is more practical to control the dynamic response by modifying the foundation natural frequencies rather than attempting to modify the damping characteristics.

If the foundation natural frequency is less than the machine speed, resonance may occur during equipment start-up and shutdown. **GP 4-6-2** requires that the natural frequencies of the foundation be outside the range of 0.8 to 1.2 times the machine speed.

STRESSES / FATIGUE

Although the amplitude limits will usually govern the stiffness and strength of individual foundation members, stresses should always be checked against fatigue criteria. **GP 4-6-2** provides fatigue and static strength requirements in **Pars. 3.3** and **3.9**. **ACI 215R (Reference 9)** provides a detailed discussion of fatigue strength for reinforced concrete. Key recommendations from this **ACI** Report for nonprestressed members are:

- The stress range in concrete shall not exceed 40 percent of its compressive strength when the minimum stress is zero, or a linearly reduced stress range as the minimum stress is increased so that the permitted stress range is zero when the minimum stress is 0.75 f_c.
- The stress range in straight deformed reinforcement shall be computed as follows and shall not exceed 20 ksi (138 MPa) and not exceed 10 ksi (69 MPa) for bent or tack welded bars (prohibited by AWS D1.4):

$$S_r = 23.4 - 0.33 S_{\min}, \text{ ksi}$$

$$S_r = 161 - 0.33 S_{\min}, \text{ MPa}$$

where: S_r = Stress range, ksi (MPa)
 $S_{\min}$ = Algebraic minimum stress, tension positive, compression negative, ksi (MPa)

The stress range is the difference between the calculated minimum and maximum stress for specified loading. Typically, the minimum stress is due to dead load and the maximum stress is due to dead plus live load.

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DESIGN CRITERIA (Cont)

SETTLEMENT

Settlement control is critical to maintaining close tolerances on machine alignment. Predicted settlements under the combined static and dynamic loading should not exceed 0.5 in. (12 mm). To avoid dynamically induced settlements, **GP 4-6-2 Par. 3.4** specifies that the maximum soil pressure under static and dynamic loading not exceed one-half of the static allowable bearing pressure.

These requirements may not be sufficient to prevent large dynamically induced settlements of foundations bearing on saturated granular soils with a relative density below 75%. For these special cases, soil improvement by excavation and backfill, vibro-compaction, dynamic compaction, etc. is necessary, or alternatively, a piled foundation may be required.

TRANSMITTED VIBRATIONS

GP 4-6-2 Par. 3.4 states that machinery foundations should be independent of other buildings, structures, or foundations. The **GP** also requires a minimum 0.5 in. (12 mm) separation between the foundation and surrounding paving. The purpose of these requirements is to minimize transmission of vibration to other structures. These isolation requirements, together with the amplitude criteria provided above, should prevent damage to other equipment due to transmitted vibrations. A special case where the above criteria may not be satisfactory is when a high-speed compressor requires siting adjacent to a large heavy reciprocating machine. Prediction of transmitted vibrations through soil can not be accurately determined. Qualitatively, the attenuation of vibrations through soils is proportional to $1/r^{0.5}$, where r is the distance between foundations.

MACHINE INFORMATION

TYPES

Machines can be classified as reciprocating or rotary.

Reciprocating machinery found in refineries and chemical plants are: reciprocating air compressors, reciprocating process gas compressors, positive displacement fluid pumps, diesel engine drivers, etc. Reciprocating machinery is characterized by the rotary motion of a master crankshaft and the linear reciprocating motion of the connected pistons. Connecting rods undergo both translation and rotation.

Rotary machinery in refineries and chemical plants include centrifugal air and process gas compressors, axial flow compressors, centrifugal and turbine pumps, steam and gas turbine drivers, electric motor drivers, centrifuges, blowers, etc. These machines are characterized by the rotary motion of impellers or rotors.

DYNAMIC LOADS

Reciprocating machines require a precise number of pistons and a particular crank arrangement in order to balance forces and couples. The mechanical design that is necessary to satisfy process requirements must, of necessity, govern. This leads, in most cases, to piston/cylinder assemblies and crank arrangements which do not completely counter-oppose, and therefore unbalanced forces and/or couples occur which must be resisted by the foundation. Unbalanced forces/couples occur at multiples of the operating speed.

Generally, only the primary and secondary unbalanced loads are considered for dynamic analysis. The primary loads occur at the machine speed while secondary loads occur at twice the speed. The forces and couples are functions of the mass, stroke, arrangements of the pistons and also to mass and arrangement of counterweights on the crankshaft. **Figures 3A/B/C** and **References 10 and 11** provide information on unbalanced loads for various piston arrangements.

Reciprocating machinery vendors usually only report the magnitudes and orientations of the total unbalanced forces and couples. **Although individual piston forces may be balanced and not affect dynamic response, they should always be considered when checking stresses and fatigue.**

Unbalanced forces in rotary machines are created when the mass centroid of the rotating part does not coincide with the axis of rotation. To minimize the effects of mass eccentricity produced during manufacturing, machinery vendors adjust the mass distribution of the rotating parts by "dynamic balancing." In practice, some minor eccentricity always remains. During operation, the eccentric rotating mass produces centrifugal forces, which are proportional to the square of machine speed.

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MACHINE INFORMATION (Cont)

For cases when the mass unbalance is stated by the vendor, the resulting dynamic force is provided by **Eq. (3)**:

$$P_B = m e \omega^2 \quad \text{Eq. (3)}$$

where: P_B = Dynamic force after balancing (zero to peak)
 m = Rotating mass
 e = Measured eccentricity
 ω = Machine speed (radians/second)

The dynamic force used for design should be $2 P_B$ to account for increasing unbalance during the service life of the machine.

Rotary machinery vendors do not usually report the unbalance which remains after balancing. However, **GP 4-6-2** recognizes that some unbalance remains and allows for increasing unbalance with time due to normal machine wear, rotor play, dirt accumulation, etc. Since balancing has been found to improve with machine speed, **GP 4-6-2, Par. 5.3** defines a linear relationship between design unbalanced forces and machine speed:

$$\text{Dynamic force (zero to peak)} = \frac{(\text{rotor weight}) (\text{rotor speed, rpm})}{6,000} \quad \text{Eq. (4)}$$

Note that this force corresponds to a "shutdown" condition and therefore provides a conservative estimate of normal operating amplitudes.

If a foundation supports multiple rotating weights, the weight of each rotating component should be used at its operating speed to compute unbalanced forces. These forces are considered to act at the center of gravity of the rotating part and are transmitted through the bearings to the foundation. The force is dynamic and varies harmonically in both the horizontal and vertical directions at the speed of the machine. The horizontal and vertical forces are equal and 90° out of phase.

DATA REQUIREMENTS

The following checklist can be used when obtaining information from the machinery vendor:

1. A brief description of the power train and driven machinery including:
 - Operating speed or speed range.
 - Number and arrangement of pistons.
 - Rotor/impeller/gear rotating weights.
 - Maximum rated output (power and torque).
 - Gearbox ratio.
 - Operating temperatures at base of machines.
2. A static loading diagram showing component masses and positions of centroids relative to the three principal axes.
3. A detailed drawing indicating position and size of mounting feet and details of anchor bolts.
4. A detailed drawing with suggested foundation dimensions including ancillary equipment and drainage details as necessary.
5. Magnitude, direction, and position of all dynamic loads with their associated frequencies:
 - External Loads (piping, surge, etc).
 - Individual Piston Forces.
 - Primary Unbalanced Forces and Couples.
 - Secondary Unbalanced Forces and Couples.
 - Rotating Mass Eccentricity (after balancing).
6. Mass moments of inertia about three axes.
7. Data on specific machines:
 - Motors - Dynamic short circuit conditions (up to 10 times full load torque).
 - Power Generators - Faulty synchronizing conditions (15 to 20 times full load torque).

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SOIL INFORMATION

STATIC DESIGN

For static design purposes, **GP 4-6-2** requires that soil bearing pressures for all heavy machinery foundations should not exceed 50% of the net allowable values for static loads. As discussed previously, predicted settlements should also not exceed 0.5 in. (12 mm). The **GP** does not provide values or procedures for determining the allowables. These are unique to each particular site and must be determined via onsite soils investigations. Guidelines for conducting soil investigations are provided in [Section XXIX-B, Soils Investigation, Site Preparation, and Earthwork](#).

If soil conditions are such that piles are required, pertinent pile data supplemental to **GP 4-6-2** must be provided (see [DP Section XXIX-B](#)).

DYNAMIC DESIGN METHOD

The method most commonly used to predict the response of soils to dynamic loading is the "elastic half-space model" originally proposed by Richart, et. al. ([Reference 11](#)). This model presumes that a block type foundation on a circular footing rests upon the surface of an elastic half-space (the soil), homogeneous and isotropic, with infinite dimensions. The stress-strain properties of the soil are defined by two elastic constants: dynamic shear modulus (G) and Poisson's ratio (ν). The model assumes the footing is rigid and provides for dissipation of energy by radiation or "geometric" damping.

The half-space model allows the soil to be represented by linear spring constants and damping constants for single degree of freedom, lumped parameter analyses for vertical, horizontal, twisting and rocking modes. Both the spring and damping constants are assumed to be frequency independent. These constants are then used together with the combined foundation and machine mass to predict foundation response. Soil mass is not used in this model. Rectangular foundations are adapted to the model by use of an effective radius, r_0 , and a shape factor.

The accuracy of the model depends on the validity of the assumptions for the particular site. The effect of layered soils with a stiffer strata below the foundation increases the effective stiffness and decreases the damping present. Embedment of the footing below the ground surface increases both the stiffness and damping. Techniques to adapt the model for these conditions are covered later in this section.

The accuracy is also reduced when two or more footings are situated near each other. This case decreases both stiffness and damping. Techniques for accurately determining the response for these cases are not available. The engineer is left with a qualitative judgement to correlate half-space model predictions to the expected response.

DYNAMIC PROPERTIES

There are three soil properties required to use the elastic half-space method:

- Soil unit weight, γ
- Poisson's Ratio, ν
- Shear Modulus, G

Soil unit weight or density is used to calculate damping constants. Soil density is generally provided in reports documenting soil investigations. [Table 1](#) provides general correlations of unit weights for different soil strata types. Total unit weights rather than buoyant weights should be used because both the solid and fluid phases in the soil mass influence dynamic behavior.

Poisson's ratio is used to calculate both soil stiffness and damping although the calculations are relatively insensitive to the values chosen for ν . Consequently, the values shown on [Table 2](#) generally are adequate for dynamic analyses.

Shear modulus is the most important soil parameter influencing dynamic behavior. It is used to calculate soil stiffness. It can be determined by presumptive values, correlations with other soil properties, or by insitu field measurements.

Presumptive values of G for different soil types are shown on [Table 1](#). This approach is recommended for preliminary analyses where only soil type is known. When other soil data is known, the presumptive values may be substantiated/checked by using published empirical correlations to other soil properties. Values for G may be correlated with standard penetration test blow count, N , using the formula given in [Reference 3](#). Other correlations are given in [Reference 12](#) for sands based on void ratio and effective stress, and for clays based on void ratio, effective stress and over-consolidation ratio. Values of G ranging from 0.5 to 1.5 or 0.75 to 1.25 times the nominal value should be considered in the analyses depending on the method used to establish the nominal value. Analyses based on substantiated G values are acceptable for final design for most foundations and for preliminary design of foundations supporting large critical machinery.

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SOIL INFORMATION (Cont)

For final design of large critical foundations, G should be determined using cross-hole or down-hole field tests or resonant-column laboratory measurements. The cross-hole and down-hole techniques measure shear wave velocities in the soil at different depths. In the cross-hole method, two bore holes a known distance apart are drilled, preferably on each side of the base location. A sensor device is placed in one bore hole and a shock producing device is placed in the other. The shear wave velocity is determined by measuring the time it takes for the induced shear wave to travel between bore holes. For the down-hole method, shear waves are generated at the ground surface and the time required to reach different depths in a nearby bore hole are recorded. It has the advantage of requiring only one bore hole. Shear wave velocity based on field measurements and shear modulus are related as follows:

$$G = \rho v_s^2 \quad \text{Eq. (5)}$$

where: G = Dynamic shear modulus
 ρ = Soil mass density (unit weight/gravity)
 v_s = Shear wave velocity of soil

Dynamic Shear modulus values can be estimated from resonant-column tests using a specially constructed triaxial cell capable of imparting a very small amplitude of vibration to a soil specimen. The strain is of the order of 0.002 to 0.00001 (or less) compared to normal triaxial strains in the range of 0.01. For this reason, static triaxial tests can not be used to compute dynamic shear modulus.

A detailed discussion of the cross-hole, down-hole and resonant-column techniques is provided in [Reference 13](#). Analyses based on values determined by these techniques offer the best accuracy. The analyses should consider a range 0.75 to 1.25 G .

For most soil deposits, the shear modulus varies with depth and therefore an effective value of G averaged over depth is often used. The procedure to determine the effective value is as follows:

1. Determine distribution of vertical pressure caused by foundation and machine weight at the edge of an equivalent circular footing with radius r_0 . For rectangular footings, the effective radius is:

$$r_0 = (\text{Base Area} / \pi)^{0.5} \quad \text{Eq. (6)}$$

2. Determine distribution of effective stress in the soils.
3. Add pressures from Steps 1 and 2 and determine depth where minimum value of soil pressure occurs. This depth is where the effective value of G is calculated.

[Figure 4](#) illustrates this procedure.

STIFFNESS

Using the dynamic shear modulus, spring constants can be obtained from the formulas provided on [Figure 5A](#) for circular and rectangular footings for vertical, horizontal, rocking, pitching, and torsional vibration modes. To determine the spring coefficient β_ϕ for pitching, mode of vibration, enter the graph using B/L ratio instead of L/B .

Foundation embedment has the effect of increasing the spring constants of the soil. Embedment should only be considered if the footing/mat is cast directly against undisturbed soil or if it is formed and backfilled carefully with a high compactive effort with low shrink-swell potential soils. The embedment factors, which are multipliers to the spring constant, are shown on [Figure 5B](#).

If the subgrade consists of a relatively soft soil layer overlying an appreciably stiffer soil or bedrock within a depth of 3 to 4 r_0 , the stiffness will increase. Data is limited to vertical stiffness which is twice the half-space stiffness at a ratio of $d / r_0 = 1$. The effect can be neglected when $d / r_0 > 4.0$.

DAMPING

There are two types of damping involved in soil vibration - internal damping and geometric damping. Internal damping involves the hysteretic loss of energy due to stress reversals in the soil. Geometric damping describes the loss of energy through propagation of elastic waves away from the foundation. Damping is usually expressed as the ratio of actual to critical damping. Critical damping is defined as the amount of damping required to produce non-oscillatory dynamic response. A value of 0.05 for internal damping is reasonable for most soils ([Reference 14](#)). Geometric damping ratios are derived from the half-space method and formulas are provided in [Figure 6](#). The combination of geometric and internal damping is plotted on [Figure 7](#).

For design purposes, **GP 4-6-2 Par. 3.2f** specifies limits on geometric damping ratios such that they cannot exceed 2/3 of the theoretical value nor 0.7 for all modes. This reduction accounts for effects from other foundations, layered strata, and other inherent uncertainties.

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SOIL INFORMATION (Cont)

Embedding a foundation can increase damping. Embedment factors for damping are shown in [Figure 6](#). Embedment must be effective, i.e., foundation cast against undisturbed soil or following carefully controlled backfill measures.

Siting a foundation above a stiff lower strata can significantly decrease the geometric damping, due to reflected vibrations. Data covering damping ratios for these cases is very limited. [Reference 12](#) indicates that for $5 < W / \gamma r_o^3 < 10$ and $1 < d_s / r_o < 2$ damping ratios of 15% of geometric half-space values be used together with material damping. For these cases, ratios of 5 to 8% of critical damping should be used for design.

PRELIMINARY FOUNDATION DESIGN

FOUNDATION TYPES AND SELECTION

The designer begins the analysis/design of the machinery foundation by reviewing the foundation type to be used. Foundations are categorized as; low, rigid blocks and elevated, flexible frames. The selection of the block or elevated frame type is based on requirements set by the process or machinery engineer. These foundations may be as follows:

- Rigid Block
 - Soil Bearing
 - Pile Support
- Flexible Frame
 - Rigid Footing
 - Large Flexible Mat
 - Pile Cap

Rigid block type refers to foundations where openings constitute less than 50% of the overall volume (base area times height) and where the height above grade is less than 10 ft (3 m). Rigid blocks are usually supported on soil. Dynamic response of blocks is predominantly controlled by the soil stiffness and damping. Structural flexibility of the foundation is not considered. Note that **GP 4-6-2** requires that rigid blocks be used to support all reciprocating machinery. Rigid blocks may be analyzed using the **MACHFND** computer program ([Reference 19](#)).

The rigid block foundation acts as an inertial anchor and transfer medium. Under the influence of the unbalanced forces and moments from the rotating or reciprocating machinery, the foundation rocks, and translates horizontally and vertically as a rigid body, typically as depicted in [Figure 8A](#). All vibratory movements which cannot be contained simply by its inertia (and that of the machinery) are transferred to the supporting soils or piles, which provide a damping mechanism. According to their stiffnesses and damping characteristics, these further limit the extent of vibratory movement.

Elevated frames consist of a foundation base with columns or walls, and a heavy, stiff slab and/or beams referred as "tabletop." Elevated frames are used to support rotary machinery when heights of 10 to 30 ft (3 to 9 m) above grade are necessary. The foundation base is usually supported on soil. The arrangement/spacing of columns and thickness of the foundation base dictate whether the base is considered rigid or flexible. Dynamic response of elevated frames on heavy rigid bases is governed by the flexural stiffness of the column or walls. Due to the complex dynamic response and soil-structure interaction, a finite-element analysis is recommended. [Reference 20](#) provides the guidelines for performing such an analysis using **STAAD3**.

[Figure 9A](#) shows a representative elevated flexible foundation and the flexing action typical for this type foundation. [Figure 9B](#) shows this same foundation, but now with the transverse frames almost completely filled in; dashed lines show for comparative purposes the frame columns of the flexible foundation of [Figure 9A](#). The height, h_o and width, w_o of the openings in the transverse frames are now less than one-third of total frame height, h_t , and width, w_t , respectively. With this change in stiffness of the superstructure, the elevated foundation is now prone, under the influence of machinery unbalanced forces, to the rigid-body motion depicted in [Figure 9B](#), with the extent of this motion dependent now principally on the spring and damping characteristics of supporting soils and piles.

MASS CONSIDERATIONS

The empirical mass ratios used for design prior to 1970s can be used as guidelines for preliminary sizing of the foundation. These ratios relate the total mass (M_f) of the block or frame to the mass (M_m) of the machine. The ratios are:

- $M_f / M_m > 5$ for reciprocating machines
- $M_f / M_m > 3$ for rotary machines

For elevated frames, the weight of the tabletop plus the top half of the columns should be greater than the weight of the machine.

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PRELIMINARY FOUNDATION DESIGN (Cont)

DIMENSIONAL GUIDELINES

The following guidelines ([Reference 12](#)) can be used together with the mass ratios to initially size and dimension the foundation.

All Foundations

- The bottom of the foundation should be above the water table when possible. It should not rest on previously filled or dumped material or on loose sands. The bearing depth of the foundation should be based on the recommendations of the geotechnical consultant. When soil conditions are poor, piles may be required.
- The foundation should be sufficiently wide to provide 1 ft (0.3 m) horizontal clearance from the edge of the machine base to the edge of the block/tabletop.
- Sufficient space should be provided considering equipment size and clearances for installation, maintenance and operation, piping and anchor bolts.

Block Foundations

- The top of the block should be 8 to 12 in. (200 to 300 mm) above finished grade to avoid drainage related problems.
- The foundation width and length should be at least 1.25 to 1.5 times the vertical distance from the foundation base to the machine centerline to increase damping for rocking modes.
- Foundation plan dimensions should be adjusted so the center of gravity of the machine/foundation and the center of the base coincide within the 5% eccentricity limit per **GP 4-6-2, Par. 3.5**.
- The vertical thickness should be greater than $2 + L/30$ ft ($0.6 + L/30$ m) per **GP 4-6-2, Par. 3.4**.

Elevated Foundations

- Design should satisfy **GP 4-6-2, Par. 3.6**.
- Mat thickness should exceed $0.07 L_s^{4/3}$, ft ($0.10 L_s^{4/3}$, m), where L_s is the average of two adjacent spans between columns.
- All columns should be stressed equally under vertical load. Therefore, column areas should be proportional to the vertical load supported. Column cross-section dimensions should be set so that the axial stress is within the range of 40 to 300 psi (0.3 to 2 MPa). Column spacing should not exceed 12 ft (3.7 m). Individual column resonance with the machine speed should be checked. Column vertical frequency is given by:

For **Customary Units**:

$$f_n = 44800 f'_c{}^{0.25} / (\sigma_\alpha H_c)^{0.5} \quad \text{Eq. (7)}$$

where: f_n = Frequency, rpm
 f'_c = Strength of concrete, psi
 σ_α = Column axial stress, psi
 H_c = Column height, in.

For **Metric Units**:

$$f_n = 65000 f'_c{}^{0.25} / (\sigma_\alpha H_c)^{0.5} \quad \text{Eq. (8)}$$

with: f'_c and σ_α in MPa, and H_c in mm

- Columns should not deflect more than 0.02 in. (0.5 mm) under the equivalent static loads per **GP 4-6-2**.
- The tabletop beams or slab flexural stiffness should be at least twice that of the columns. The depth of the beams should be greater than 1/5 of the clear span. Beam deflections should not exceed 0.02 in. (0.5 mm) under the equivalent static loads.

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BLOCK FOUNDATION ANALYSIS

The soil/foundation system for a block foundation resists dynamic forces through its inertia and stiffness, and through the dissipation of elastic energy (damping). The dynamic analysis of a block foundation involves determining the natural frequencies and amplitudes of vibration. The block foundation is modeled with simple springs and lumped masses which simulate the actual physical system. It is necessary that a proper selection of system parameters be made so that the idealized spring, mass, and damping elements of the model produce similar displacements at critical locations as the actual foundation. The following sections describe the analysis procedure.

ANALYTICAL MODEL

The analysis of block foundation is based on the general equation of motion for a "lumped parameter" system:

$$m \ddot{x} + c \dot{x} + k x = p(t) \quad \text{Eq. (9)}$$

where, as shown in [Figure 10](#):

m	=	Mass or inertia
c	=	Damping
k	=	Stiffness
$\ddot{x}, \dot{x}, x$	=	Acceleration, velocity, and displacement
$p(t)$	=	Force (as a function of time)

The "lumped parameters" are the mass, stiffness, and damping for each mode of vibration. The natural frequencies and amplitudes are functions of these parameters and the geometry of the foundation. The mass properties are determined from the geometry and weight of the machine and foundation. Stiffness and damping parameters are based on the elastic properties of the soil (or piles) using the half-space method.

VIBRATION MODES

The analysis of a block foundation can be separated into a set of one and two degree-of-freedom dynamic problems. If the center of inertia of the block coincides with the center of resistance, six pure modes of vibration are possible. Using a right hand Cartesian coordinate system, these modes are: translation in the X, Y, Z directions, and rotation about the X, Y, and Z axes as shown in [Figure 8A](#). For this case, the analysis of block as a rigid body can be handled using six separate one degree-of-freedom systems (DOF) (one for each mode).

GP 4-6-2 requires that the center of gravity coincides (within 5%), in the X and Y coordinates, with the center of resistance or base contact area. However, the center of gravity can be significantly above (in the Z direction) the center of resistance located at the base of the block. The number of vibration modes can be reduced to four independent modes as follows:

- Vertical translation.
- Torsion about Z axis.
- Coupled X axis sliding and rocking about Y axis.
- Coupled Y axis sliding and rocking about X axis.

These modes are shown on [Figure 8B](#). Vertical translation and torsion about the Z axis can be analyzed using one DOF. The coupled modes are modeled using two DOF systems.

INERTIA / FREQUENCIES

For translational modes of vibration, the total mass, M , is the sum of the individual masses of the machine and foundation:

$$M = \sum_{i=1}^n m_i \quad \text{Eq. (10a)}$$

where: n = Number of individual masses

The coordinates of the center of gravity of the machine and foundation, X_c , Y_c , Z_c are determined as follows:

$$X_c = \sum m_i x_i / M \quad Y_c = \sum m_i y_i / M \quad Z_c = \sum m_i z_i / M \quad \text{Eq. (10b/c/d)}$$

where: x_i, y_i, z_i are the coordinates of the center of each mass.

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BLOCK FOUNDATION ANALYSIS (Cont)

In computing the mass moments of inertia, the rotational inertias of each mass about its own axes are transferred to the CG of the system and then to the axes of rotation or center of resistance, CR. These mass moments of inertia are:

About own axes (usually assumed negligible for machine elements):

$$I_{xi} = m_i (a_{yi}^2 + a_{zi}^2) / 12 \quad \text{Eq. (10e)}$$

$$I_{yi} = m_i (a_{xi}^2 + a_{zi}^2) / 12 \quad \text{Eq. (10f)}$$

$$I_{zi} = m_i (a_{xi}^2 + a_{yi}^2) / 12 \quad \text{Eq. (10g)}$$

where: a_{xi} , a_{yi} , a_{zi} = The x, y, z dimensions of mass i

About CG:

$$I_{\phi x} = [I_{xi} + m_i(y_{ci}^2 + z_{ci}^2)] \quad \text{Eq. (10h)}$$

$$I_{\phi y} = [I_{yi} + m_i(x_{ci}^2 + z_{ci}^2)] \quad \text{Eq. (10i)}$$

$$I_{\phi z} = [I_{zi} + m_i(z_{ci}^2 + y_{ci}^2)] \quad \text{Eq. (10j)}$$

where: x_{ci} = Distance from center of mass, m_i , to CG of system
 x_{ci} = $X_c - x_i$, and similarly for y_{ci} , z_{ci}

About center of resistance, CR:

$$I_x = I_{\phi x} + m z_o^2 \quad \text{Eq. (10k)}$$

$$I_y = I_{\phi y} + m z_o^2 \quad \text{Eq. (10l)}$$

$$I_z = I_{\phi z} \quad \text{Eq. (10m)}$$

where: z_o = Vertical distance (CG to CR)

The following inertia ratios, called gamma ratios, are sometimes used for natural frequency calculations.

$$R_x = I_{\phi x} / I_x \quad R_y = I_{\phi y} / I_y \quad \text{Eq. (10n/o)}$$

Natural frequencies are required for comparison to the excitation frequency, and for calculation of amplitudes.

Natural Frequencies for vertical, pure horizontal, torsional and rocking vibrations are:

$$f_z = \sqrt{K_z / M} \text{ (radians/sec), vertical} \quad \text{Eq. (11a)}$$

$$f_h = \sqrt{K_h / M} \text{ (radians/sec), pure horizontal} \quad \text{Eq. (11b)}$$

$$f_{\phi x} = \sqrt{(K_{\phi x} - m g z_o) / I_x} \text{ (radians/sec), rocking or pitching} \quad \text{Eq. (11c)}$$

$$f_{\phi y} = \sqrt{(K_{\phi y} - m g z_o) / I_y} \text{ (radians/sec), pitching or rocking} \quad \text{Eq. (11d)}$$

$$f_{\phi z} = \sqrt{K_{\phi z} / I_z} \text{ (radians/sec), torsional} \quad \text{Eq. (11e)}$$

The horizontal and rocking modes of vibration are coupled. The resulting natural frequencies are obtained by solving the following equation for two values of f_n^2 :

$$R f_n^4 - (f_{\phi}^2 + f_h^2) f_n^2 + f_{\phi}^2 f_h^2 = 0, \quad \text{Eq. (12)}$$

where: R = Gamma ratio

The solution to the above equation is:

$$f_{n1,2}^2 = 1/2 R [f_{\phi}^2 + f_h^2 \pm [(f_{\phi}^2 + f_h^2)^2 - 4 R f_{\phi}^2 f_h^2]^{0.5}] \quad \text{Eq. (13)}$$

where the appropriate values of R and f^2 may be substituted for rocking about the x or y axis.

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BLOCK FOUNDATION ANALYSIS (Cont)

AMPLITUDES/PHASE ANGLES

In general, dynamic forces and couples act on the foundation out of phase, in more than one direction, and about more than one axis. Also, due to damping, the amplitude corresponding to each dynamic force will be out of phase with that force by some phase angle, θ , as shown in [Figure 11](#). In order to compute total amplitudes, the amplitude and time phase due to each force or couple must be computed at critical points on the foundation and then the amplitudes added in phase.

PILE-SUPPORTED FOUNDATIONS

GENERAL

Pile-supported foundations are required when bearing capacity or settlement design criteria can not be economically satisfied using footings or mat foundations. Compared to foundations supported on soil, piles affect the dynamic response of the foundation in the following manner:

- Increase natural frequency in vertical and rocking modes.
- Decrease natural frequency in horizontal translation modes.
- Exhibit decreased levels of damping.

For dynamic response calculations, piles are assumed to be replaceable with frequency-independent, damped springs. Therefore, equivalent stiffnesses and damping for both vertical and horizontal vibration modes can be determined and then used to predict dynamic response using the elastic half space methodology discussed previously.

PRELIMINARY DESIGN CONSIDERATIONS

Pile foundations must be laid out before any analysis can be performed. Therefore, it will be necessary to initially establish the number of piles and the dimensions of the pile cap. The following factors should be considered in preliminary sizing of a dynamically loaded pile foundation:

1. The factor of safety relative to the ultimate static axial load capacity of a pile should be at least 3 to limit soil stresses and avoid "resonant driving effects," i.e., the soil around the pile resonating with the pile and losing its ability to carry load. For static load:

$$P_{DES} \leq \frac{P_{ULT}}{3} \quad \text{Eq. (14)}$$

where: P_{DES} = The calculated static axial load per pile
 P_{ULT} = The static ultimate load capacity per pile as determined from the soils investigation, preferably from field tests

2. The pile cap should be embedded in competent soil suitably compacted to mobilize additional damping of the pile cap.
3. The pile cap mass should be at least 1.5 to 2.5 times that of a centrifugal machine and 2.5 to 4.0 times that of a reciprocating machine.
4. The top of the pile cap should be at least 8 in. (0.2 m) above the finished floor or ground elevation to prevent accumulation of water.
5. The thickness of the pile cap should be at least 2 ft (0.6 m) or as dictated by pile head embedment requirements.
6. The overall width of the pile cap should be at least 1 to 1.5 times the vertical distance from the top of the pile cap to the machine centerline in order to increase damping due to overturning loads.
7. Distance from the center of the pile to the edge of the pile cap should be 3 times the pile diameter or 1 times the thickness of the mat, whichever is greater.
8. The length and width of the mat is adjusted so the center of gravity of the superstructure, machine and equipment coincides with the center of gravity of the mat.
9. The piles are arranged so that the center of vertical resistance of the group coincides with the overall centroid of mass.
10. Batter piles, if used, are designed to carry all of the horizontal loads. The transverse stiffness and damping of vertical piles is small compared to their axial stiffness and damping, so that, generally, lateral loads on vertical piles should be minimized.

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PILE SUPPORTED FOUNDATIONS (Cont)

11. When piers are used, enlarged bases may be desirable to increase their overall capacity and to increase damping.
12. If a resonance condition is expected, dimensions should be changed and the foundation re-analyzed.
13. The piles or piers must be connected to the cap to properly represent the assumed conditions; for example, full moment capacity for a rigidly attached pile head or full shear capacity for a pinned pile head condition.

The analysis is performed using one of the standard dynamic packages such as **STAAD3** (see [Reference 2](#)), or similar software. The results of the analysis, such as frequency ratios and the maximum amplitude of motion, are checked with required standards per **GP 4-6-2**. Several degrees of freedom exist at each pile head. The rotational degrees of freedom at the pile heads are usually deleted from the computer analysis, with springs and dashpot coefficients specified in all three translatory directions at each joint or node at the location of piles heads. Input values for lateral translation terms should include the appropriate fixity conditions (pinned or fixed) for the pile heads.

STIFFNESS

The total stiffness of a group of piles can be represented as:

$$K_h = \sum_1^n k_h / \sum_1^n \alpha_L + k_{hf} \quad \text{Eq. (15)}$$

$$K_v = \sum_1^n k_v / \sum_1^n \alpha_A + k_{vf} \quad \text{Eq. (16)}$$

$$K_\psi = \sum_1^n (k_\psi + k_v x_r^2 + k_h z_c^2 - 2 z_c k_{h\psi}) + k_{\psi f} \quad \text{Eq. (17)}$$

where: K_h, K_v, K_ψ = Total horizontal, vertical, and rotational stiffness, respectively
 k_h, k_v, k_ψ = Horizontal, vertical, and rotational stiffness per single pile
 $k_{hf}, k_{vf}, k_{\psi f}$ = Horizontal, vertical, and rotational stiffness of pile cap
 n = Total number of piles in group
 α_A = Vertical interaction factor for group effects (varies for ratios of pile spacing/diameter) See [Figure 12A](#) (friction piles) and [Figure 12B](#) (end bearing piles)
 α_L = Horizontal interaction factor for group effects. See [Figure 12C](#) (fixed-head piles) and [Figure 12D](#) (pinned-headed piles)
 $k_{h\psi}$ = $(E_p I_p / r_o^2) f_{9,1}$
 $f_{9,1}$ = Factor from [Table 3](#)
 x_r, z_c = Horizontal and vertical distances, respectively, from the center of gravity of the system to the vertical and horizontal axes at the pile head

Values of k_h and k_v may be determined by:

- Pile Load Tests
- Dynamic Shaker Tests
- **PILAY** Program ([Reference 15](#))
- Approximate Spring Constants

Using static pile load tests, individual pile stiffness should be based on the elastic deformation measured after two to three cycles of loading. Shaker tests indicate resonances at the natural frequencies of the individual pile. The pile stiffness can then be backcalculated. These shaker tests also provide data on damping available. If test data is not available, either the **PILAY** program developed by M. Novak and F. Aboul-Ella at the **University of Western Ontario, Canada**, or the procedures covered herein may be used. The **PILAY** program is suited for highly layered, stratified soil deposits.

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PILE SUPPORTED FOUNDATIONS (Cont)

SPRING CONSTANTS FOR PILES

Vertical Stiffness

The expression for the effective spring constant for piles undergoing vertical motion, k_v , given by Novak (**Reference 16**) is:

$$k_v = (E_p A_p / r_o) f_{18,1} \quad \text{Eq. (18)}$$

where: E_p = Young's modulus for the pile
 A_p = Cross sectional area of pile
 r_o = Equivalent radius of pile
 $f_{18,1}$ = factor given for concrete and timber piles on [Figure 13A](#) as a function of pile penetration, ℓ , r_o and v_s / v_c (soil shear wave velocity to pile compressive wave velocity)

[Figure 13A](#) provides the factors for end-bearing concrete and timber piles. Factors for friction piles are shown on [Figure 13B](#). For steel piles, Novak recommends $f_{18,1} = 0.030$ (for $v_s / v_c = 0.033$ and $\ell / r_o > 80$).

Horizontal Stiffness

Spring constants for fixed head vertical piles undergoing horizontal (sliding) motion are given by:

$$k_h = (E_p I_p / r_o^3) f_{11,1} \quad \text{Eq. (19)}$$

where: $f_{11,1}$ = Factor from [Table 3](#)

Rotational Stiffness

Rotational springs are present at the pile head due to the fixity developed between the pile and the cap. The effective spring constant given by Novak (**Reference 16**) is:

$$k_\psi = (E_p I_p / r_o) f_{7,1} \quad \text{Eq. (20)}$$

where: $f_{7,1}$ = Factor from [Table 3](#)

SPRING CONSTANTS FOR PILE CAP

A pile (or group of piles) is normally capped with a concrete footing. Although it is normally assumed that vibratory loading and/or the natural processes of soil consolidation will prevent permanent contact between the base of the cap and the soil, side friction due to contact with the exterior backfill is often considered effective.

Vertical Motion

Stiffness (spring) constants for the cap-backfill system can be determined from the following:

$$k_{vf} = 2.7 G_{sb} h \quad \text{Eq. (21)}$$

where: G_{sb} = Dynamic shear modulus of backfill
 h = Effective depth of cap embedment in soil

Horizontal Motion

Stiffness (spring) constants for the cap-backfill system can be determined from the following:

$$k_{hf} = G_{sb} h \bar{S}_{u1} \quad \text{Eq. (22)}$$

where: $\bar{S}_{u1}$ = Factors evaluated from [Table 4](#)

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PILE SUPPORTED FOUNDATIONS (Cont)

Rotational Motion

Stiffness (spring) constants for the cap-backfill system can be determined from the following:

$$k_{\psi f} = G_s r_o h^2 \bar{S}_{\psi 1} + G_s h \left[(h^2/3) + z_c^2 - h z_c \right] \bar{S}_{u1} \quad \text{Eq. (23)}$$

where: $\bar{S}_{\psi 1}$ and $\bar{S}_{u1}$ = Factors from [Table 4](#)
 z_c = Vertical distance from the center of gravity of the system to the horizontal axis at the pile head

DAMPING

Vertical Motion

The effective damping constant for vertical motion in a single pile per Novak is given by:

$$c_v = (E_p A_p / v_s) f_{18,2} \quad \text{Eq. (24)}$$

where: c_v = Geometric damping constant
 E_p = Young's modulus of pile
 A_p = Cross sectional area of pile
 v_s = Shear wave velocity of soil
 $f_{18,2}$ = Factor from [Figures 12A](#) and [12B](#)

The effective damping constant for the cap is as follows:

$$c_{vf} = 6.7 h r_o \sqrt{G_{sb} \gamma_{sb} / g} \quad \text{Eq. (25)}$$

where: h = Depth of embedment
 r_o = Equivalent radius of cap $(B L / \pi)^{0.5}$
 γ_{sb} = Unit weight of backfill

Horizontal Motion

The effective damping constant for horizontal motion in a single pile per Novak is given by:

$$c_h = (E_p I_p / r_o^2 v_s^*) f_{11,2} \quad \text{Eq. (26)}$$

where: I_p = Moment of inertia of the pile cross section
 $f_{11,2}$ = Factor from [Table 3](#)
 v_s^* = $(G_s^* g / \gamma_s)^{0.5}$
 G_s^* = $G_s / 4$ to account for the fact that pile-to-soil separations and soft soil near the surface control the load-deformation behavior of laterally-loaded piles

The effective damping constant for the cap is as follows:

$$c_{hf} = h r_o (G_{sb} \gamma_{sb} / g)^{0.5} \bar{S}_{u2} \quad \text{Eq. (27)}$$

where: $\bar{S}_{u2}$ = Factor from [Table 4](#)