
Guidelines for implementation of statistical process control (SPC) —

Part 3: Reference data sets for SPC software validation

*Lignes directrices pour la mise en œuvre de la maîtrise statistique des
processus (MSP) —*

*Partie 3: Jeux de données de référence pour la validation de logiciels
pour MSP*



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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular, the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

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For an explanation of the voluntary nature of standards, the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the World Trade Organization (WTO) principles in the Technical Barriers to Trade (TBT), see www.iso.org/iso/foreword.html.

This document was prepared by Technical Committee ISO/TC 69, *Applications of statistical methods*, Subcommittee SC 4, *Applications of statistical methods in process management*.

A list of all parts in the ISO 11462 series can be found on the ISO website.

Any feedback or questions on this document should be directed to the user's national standards body. A complete listing of these bodies can be found at www.iso.org/members.html.

Introduction

The test examples given in this document were developed for the assessment of statistical process control (SPC) systems. They allow SPC software developers to evaluate their systems. Thus, the end user of those systems can be sure that the data sets are evaluated correctly with a high level of reliability. In order to cover the widest possible spectrum, suitable data sets were prepared individually for various constellations. The evaluation results of those data sets are documented and commented on the following pages.

The results were verified multiple times using different computer programs. This turns the data sets and the results into references for validation of the software. The data sets are listed in Annex A. An electronic version is available at <https://standards.iso.org/iso/tr/11462/-3/ed-1/en/>.

Guidelines for implementation of statistical process control (SPC) —

Part 3: Reference data sets for SPC software validation

1 Scope

This document describes examples for software validation for SPC software implementing the standards of the ISO 7870 series on control charts and the ISO 22514 series on capability and performance. In detail ISO 7870-2, ISO 22514-2 and ISO 22514-8 are covered.

It provides data sets and test results for testing the implementation of the evaluation methods described in these standards. This includes the detection of out of control situations as well as the calculation of sample statistics and process capability indices.

The test examples cover the following situations:

- a) General:
 - different sample and subgroup sizes, accuracy of calculation for large/small numbers;
- b) ISO 22514 series:
 - calculation of sample statistics for location and dispersion;
 - different distribution models;
- c) ISO 7870-2:
 - calculation of control limits;
 - visualization of data (histogram, control charts);
 - detection of out of control situations.

2 Normative references

There are no normative references in this document.

3 Terms and definitions, and symbols and abbreviated terms

3.1 Terms and definitions

No terms and definitions are listed in this document.

ISO and IEC maintain terminological databases for use in standardization at the following addresses:

- ISO Online browsing platform: available at <https://www.iso.org/obp>
- IEC Electropedia: available at <http://www.electropedia.org/>

3.2 Symbols and abbreviated terms

Symbols used in this document are identical to the symbols used in ISO 22514-2 and ISO 7870-2:

C_p	process capability index
C_{pk}	minimum process capability index
C_{pkU}	upper process capability index
C_{pkL}	lower process capability index
U_{CL}	upper control limit
L_{CL}	lower control limit
m	number of subgroups
n	sample size of each subgroup
P_m	machine performance index
P_{mk}	minimum machine performance index
P_{mkU}	upper machine performance index
P_{mkL}	lower machine performance index
P_p	process performance index
P_{pk}	minimum process performance index
P_{pkU}	upper process performance index
P_{pkL}	lower process performance index
T	Centreline (target value) for the respective characteristic in the control charts
U	upper specification limit
L	lower specification limit
U_{tr}	upper specification limit, transformed values
L_{tr}	lower specification limit, transformed values

Abbreviated terms:

SPC	statistical process control
-----	-----------------------------

4 Overview of the test examples

See [Table 1](#) for an overview of the test examples.

Table 1 — Overview of test examples

Test data set number	Subclause	Distribution model	Resulting distribution	Decimal points	Total sample size	Subgroup sample size	Description of data set
1	5.1	A1	normal	4	125	5	Data follows a normal distribution with no outliers
2	5.2	A2	Weibull	3	600	3	Correct calculations for a sample following a Weibull distribution
3	5.3	B	non-normal	2	1 000	5	Normal distribution with time dependent shift of mean
4	5.4	C1	normal	2	1 000	5	Location: random normally distributed; dispersion: constant
5	5.5	C2	non-normal	2	1 000	5	Location: random non-normally distributed; dispersion: constant; resulting distribution: non-normal
6	5.6	C3	non-normal	2	600	6	Capability indices for trend production (tool wear)
7	5.7	C4	non-normal	2	500	5	Capability indices for fix tooling (tool change)
8	5.8	D	non-normal	3	500	5	Systematic and random changes in location and dispersion - non-normal
9	5.9	A2	Rayleigh	3	500	5	Correct calculations for a sample following a Rayleigh distribution
10	5.10	A2	Weibull	0	200	5	Correct calculations for a sample following a Weibull distribution
11	5.11	C2	normal	1	30	(10)	ISO 22514-8 capability of multi-state production processes

NOTE

- The decision which distribution model fits the data and its use is up to the statistical expert. Within this document distribution models for each example are therefore assumed. The procedure to select a statistical distribution is not part of this document.
- According to ISO 22514-2 the calculation method $d = 5$ for the dispersion is not suited for non-normal distributions. It is calculated and given in this document even for non-normal distributions for the purpose of validating the implementation of the calculation method of $d = 5$.
- The resulting distributions for test 2, 9 and 10 are pre-selected models and with no means the "best natural" resulting distribution models for the same reason no goodness of fit statistics are given, knowing they will lead to reject H_0 for the pre-selected distribution models.
- Two more decimal places for sample statistics and control limits than digits of the input values. Resulting values are rounded (ISO/IEC/IEEE 60559, rounded-to-the-nearest).
- Capability index are always given with 2 digits (rounded).

- Control limits were calculated using all values in the test data set.
- All control limits were calculated using tabulated correction factors in ISO 7870-2. Using the exact quantiles of the corresponding distribution functions instead can lead to a deviation in the results <0,1 %.
- The centrelines for the control charts were chosen according to the respective statistics given by ISO 7870-2:2013, 6.1 to 6.3
- The histogram plot is not part of the validation, since the procedure is not specified in ISO 7870 series nor in ISO 22514 series. Histograms are for visualization purposes only.
- No units are given, no unit conversion has to be done.
- Time-dependent distribution models A-D are specified in ISO 22514-2.
- Internally a precision of 15 digits is used, when already calculated values are used again in further calculations.
- If the data are not normal distributed an individual chart and a MR chart cannot be used.

5 Reference data sets description and evaluation

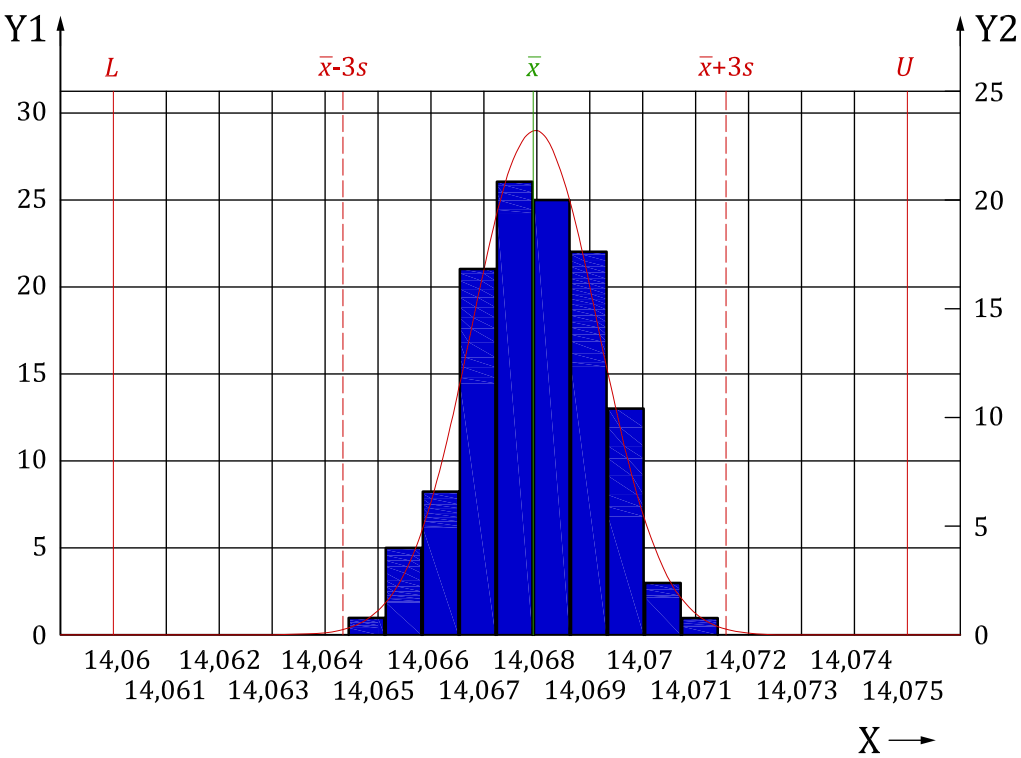
5.1 Test data set 1

5.1.1 Test data set 1 information

This set of test data is taken from a process following a normal distribution and is for checking the accuracy of calculation for control limits, sample statistics and process capabilities. A description of test data set 1 is given in [Table 2](#). [Figure 1](#) shows a histogram and [Figure 2](#) a probability plot of the test data set 1 with the purpose of data visualization.

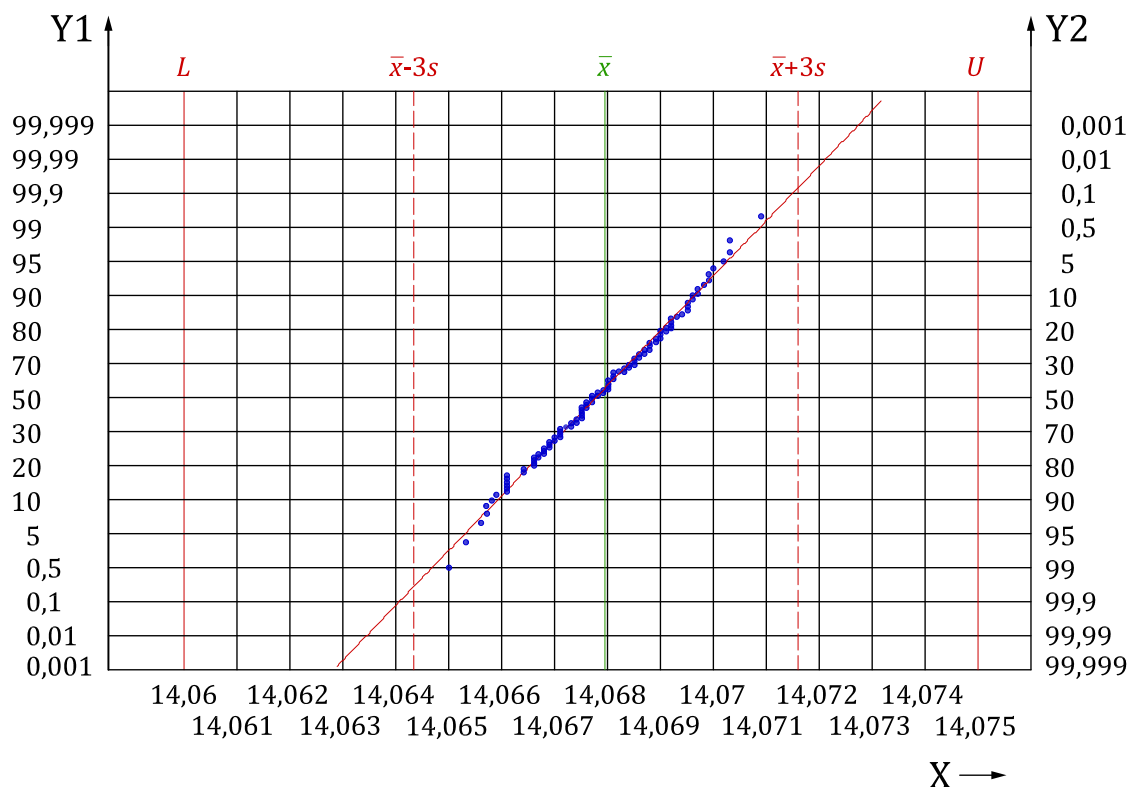
Table 2 — Description of test data set 1

Description of data input			
Distribution model	A1	Resulting distribution	Normal
Data set	Annex A, Table A.1	decimal points	4
Total sample size	125	<i>U</i>	14,075
Size of subgroups	5	<i>L</i>	14,060



Key
X value
Y1 absolute frequency
Y2 relative frequency in %

Figure 1 — Histogram of test data set 1



Key

X value

Y1 probability in %

Y2 1-probability in %

Figure 2 — Probability plot of test data set 1

NOTE The width of the class intervals is 0,000 7.

The class interval with the highest frequency of values is from 14,067 95 to 14,068 02.

The density plot is based on the assumption of normality and estimated parameters $l = 1$, $d = 5$.

5.1.2 Test data set 1 results

5.1.2.1 List of sample statistics

Table 3 lists all sample statistics which are necessary to calculate the target values and control limits for the control charts described in ISO 7870-2 as well as the estimators for location and dispersion given in ISO 22514-2 for the calculation of the process capability indices.

Table 3 — List of sample statistics for test data set 1

Statistic	Value	Reference
Location		
$\bar{x}$ ($l = 1$)	14,067 958	ISO 22514-2:2017, Formula (11)
$\tilde{x}$ ($l = 2$)	14,068 000	ISO 22514-2:2017, Formula (12)
$\bar{\bar{x}}$ ($l = 3$)	14,067 958	ISO 22514-2:2017, Formula (13)
$\bar{\bar{\bar{x}}}$ ($l = 4$)	14,067 828	ISO 22514-2:2017, Formula (14)

Table 3 (continued)

Statistic	Value	Reference
Dispersion		
$\hat{\Delta} (d = 1)$	0,007 240 (normal distribution)	ISO 22514-2:2017: Formula (15)
$\hat{\sigma} (d = 2)$	0,001 187	ISO 22514-2:2017, Formula (16)
$\hat{\sigma} (d = 3)$	0,001 200	ISO 22514-2:2017, Formula (17)
$\hat{\sigma} (d = 4)$	0,001 180	ISO 22514-2:2017, Formula (18)
$\hat{\sigma} (d = 5)$	0,001 207	ISO 22514-2:2017, Formula (19)
$\bar{s}$	0,001 128	ISO 7870-2:2013, 3.2
R	0,005 900	ISO 7870-2:2013, 3.2
$\bar{R}$	0,002 744	ISO 7870-2:2013, 3.2
$\bar{R}_m$	0,001 379	ISO 7870-2:2013, 3.2

5.1.2.2 $\bar{x}$ chart

The $\bar{x}$ control chart calculated according to ISO 7870-2:2013, 6.1, is shown in Figure 3.

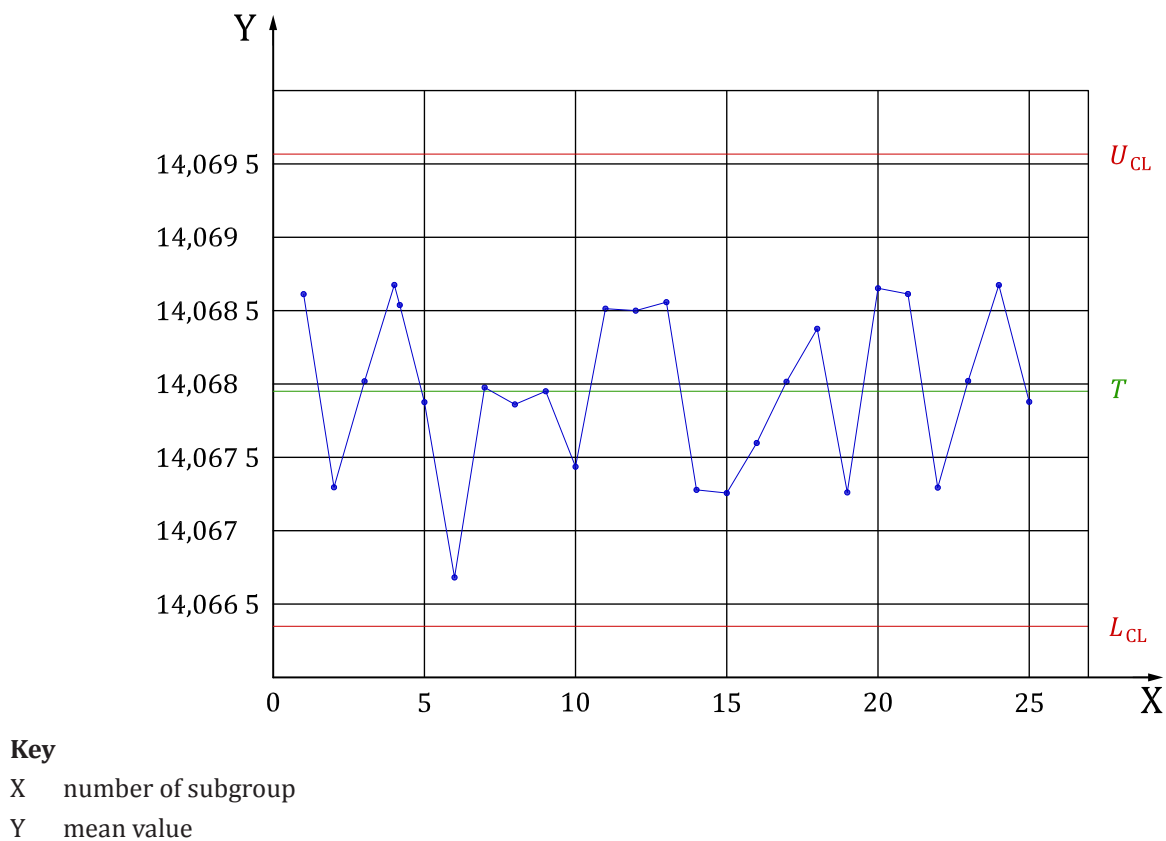


Figure 3 — Mean control chart for test data set 1

The control limits are (see ISO 7870-2:2013, Table 1, using $\bar{s}$):

$$L_{CL} = 14,066\ 349$$

$$U_{CL} = 14,069\ 568$$

5.1.2.3 X chart

The individuals control chart calculated according to ISO 7870-2:2013, 6.2, is shown in [Figure 4](#).

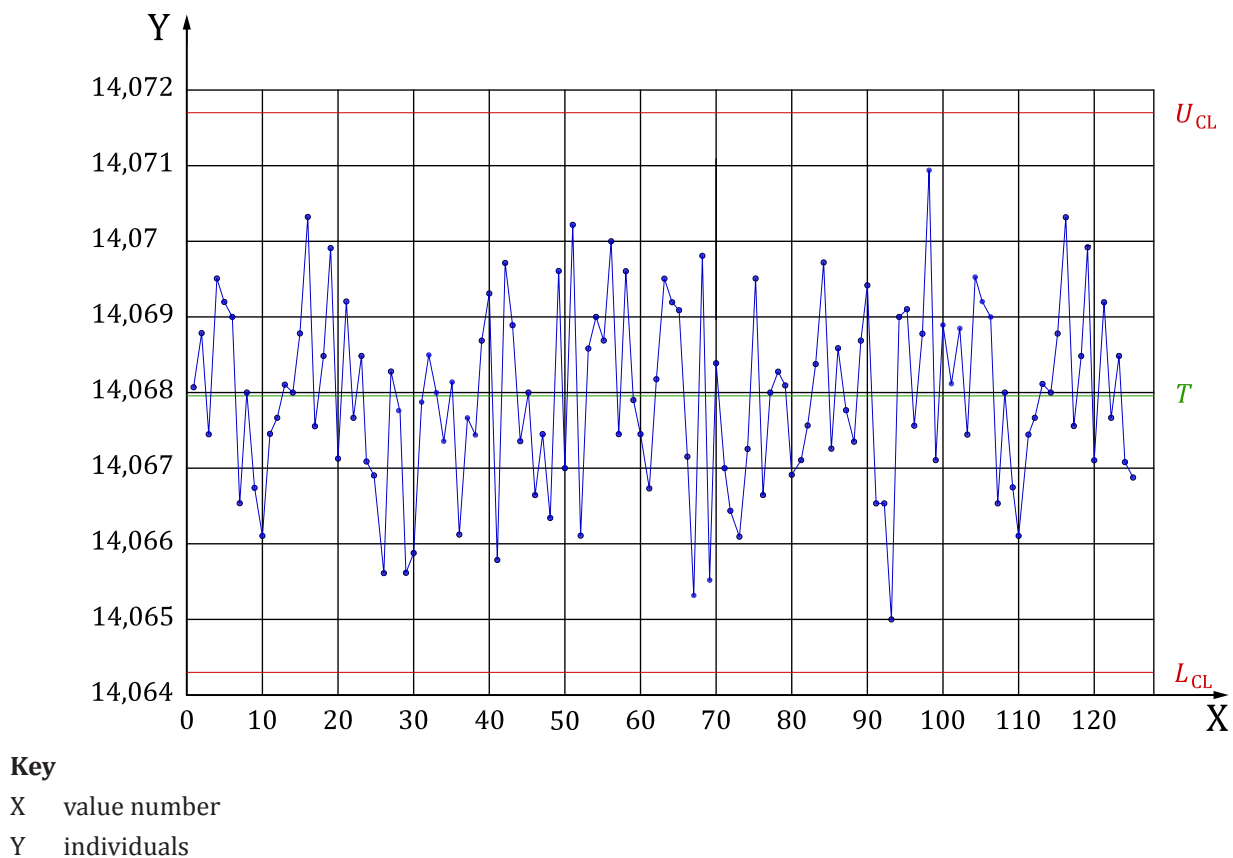


Figure 4 — Individuals control chart for test data set 1

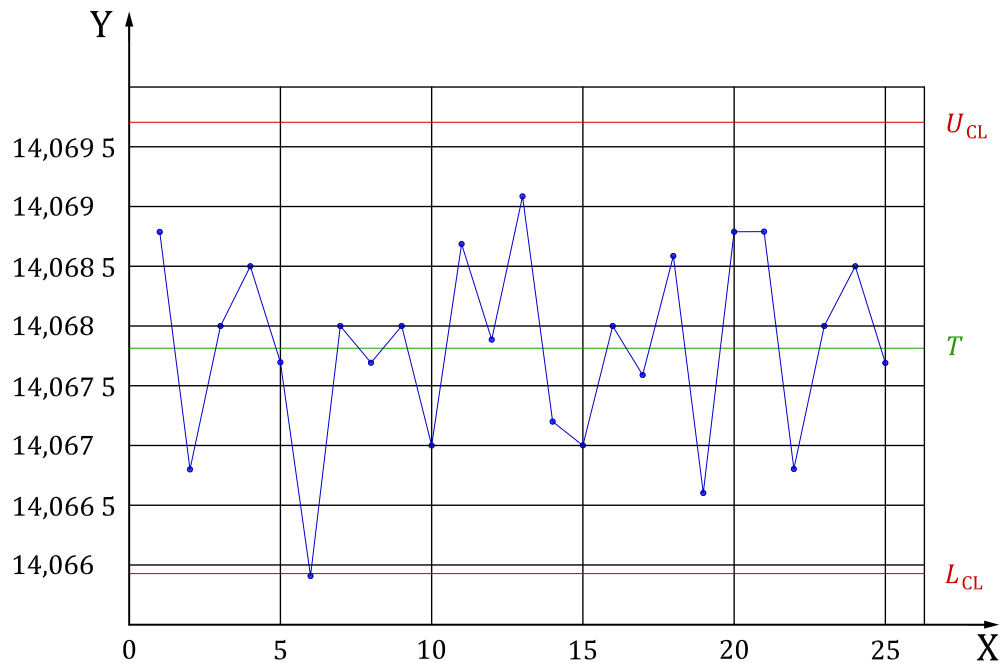
The control limits are (see ISO 7870-2:2013, Table 3):

$$L_{CL} = 14,064\ 29$$

$$U_{CL} = 14,071\ 63$$

5.1.2.4 $\tilde{x}$ chart

The median control chart calculated according to ISO 7870-2:2013, 6.3, is shown in [Figure 5](#).



Key

X subgroup number
Y median value

Figure 5 — Median control chart for test data set 1

The control limits are (see ISO 7870-2:2013, Table 4):

$$L_{CL} = 14,065\ 93$$

$$U_{CL} = 14,069\ 72$$

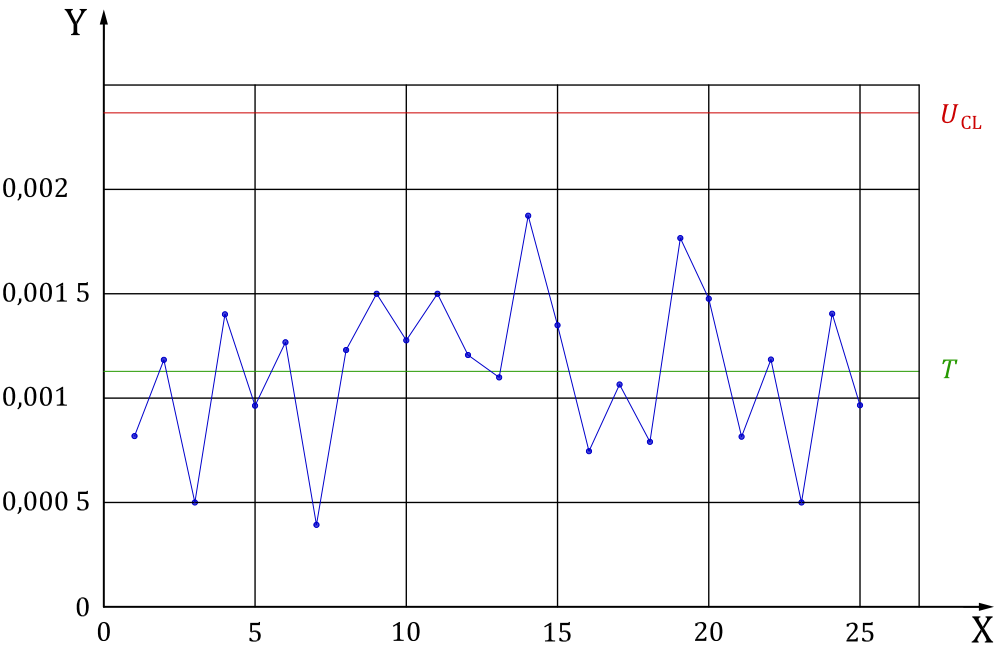
Out of control situations are given in [Table 4](#).

Table 4 — Results of pattern tests for the median control chart and test data set 1

Out of control situations			
Subgroup	Result/violation	Subgroup	Result/violation
6	Violation of L_{CL}		

5.1.2.5 s chart

The s control chart calculated according to ISO 7870-2:2013, 6.1, is shown in [Figure 6](#).



Key
X subgroup number
Y empirical standard deviation s value

Figure 6 — s control chart for test data set 1

The control limits are (see ISO 7870-2:2013, Table 1):

$L_{CL} = 0$
 $U_{CL} = 0,002\ 356$

5.1.2.6 R chart

The R control chart calculated according to ISO 7870-2:2013, 6.3, is shown in [Figure 7](#).

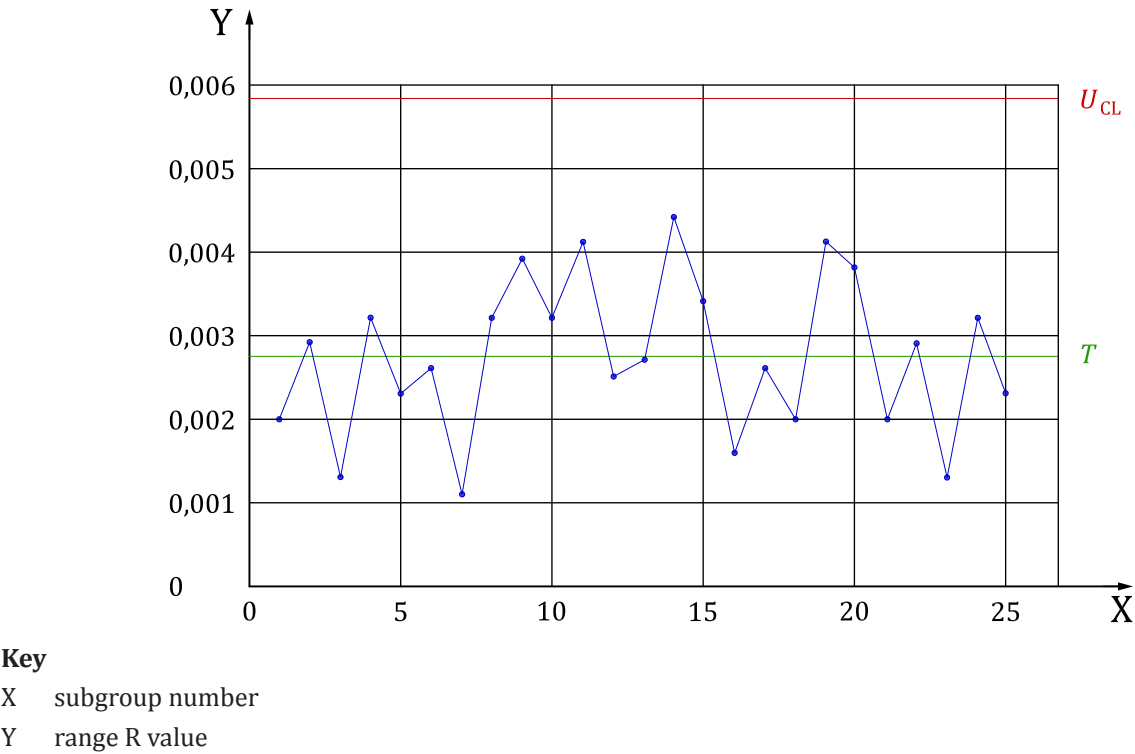


Figure 7 — R control chart for test data set 1

The control limits are (see ISO 7870-2:2013, Table 1):

$$L_{CL} = 0$$

$$U_{CL} = 0,005\ 801$$

5.1.2.7 Moving range chart

The moving range control chart calculated according to ISO 7870-2:2013, 6.3, is shown in [Figure 8](#).

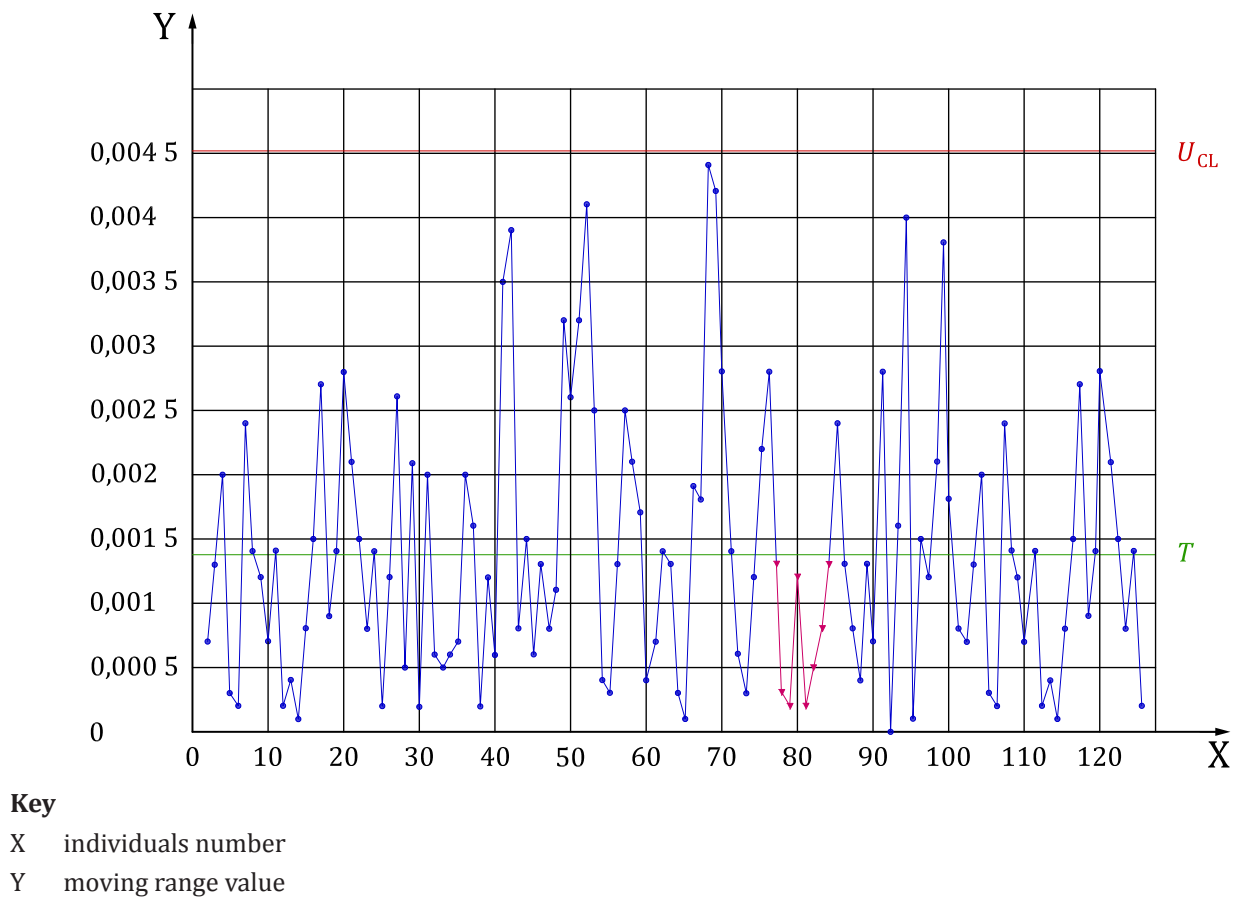


Figure 8 — Moving range control chart for test data set 1

The control limits are (see ISO 7870-2:2013, Table 3):

$$L_{CL} = 0$$

$$U_{CL} = 0,004\ 51$$

Out of control situations are given in [Table 5](#).

Table 5 — Results of pattern tests for the moving range control chart and test data set 1

Out of control situations			
Value	Result/violation	Value	Result/violation
77 to 84	run below centreline		

5.1.2.8 Process capability

The process capability is calculated according to ISO 22514-2:2017, Clause 6, with calculation method $l = 3$ for the location estimator and calculation method $d = 5$ for the dispersion estimator.

Capability indices (calculation method $M_{3,5}$)			
Process is stable (mean, R - chart) - C_p/C_{pk} is used			
C_p	2,07	C_{pk}	1,95
C_{pkL}	2,20	C_{pkU}	1,95

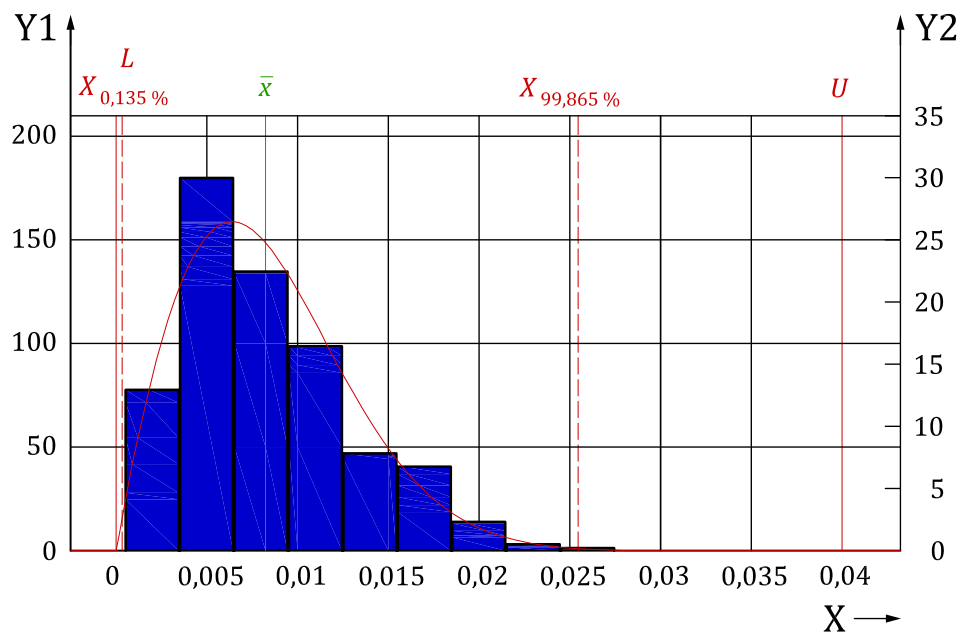
5.2 Test data set 2

5.2.1 Test data set 2 information

This set of test data is taken from a process following a non-normal distribution and is for checking the accuracy of calculation for control limits, sample statistics and process capabilities. A description of test data set 2 is given in [Table 6](#). [Figure 9](#) shows a histogram and [Figure 10](#) a probability plot of the test data set 2 with the purpose of data visualization.

Table 6 — Description of test data set 2

Description of data input			
Distribution model	A2	Resulting distribution	Weibull
Data set	Annex A, Table A.1	decimal points	3
Total sample size	600	U	0,04
Size of subgroups	3	L	0



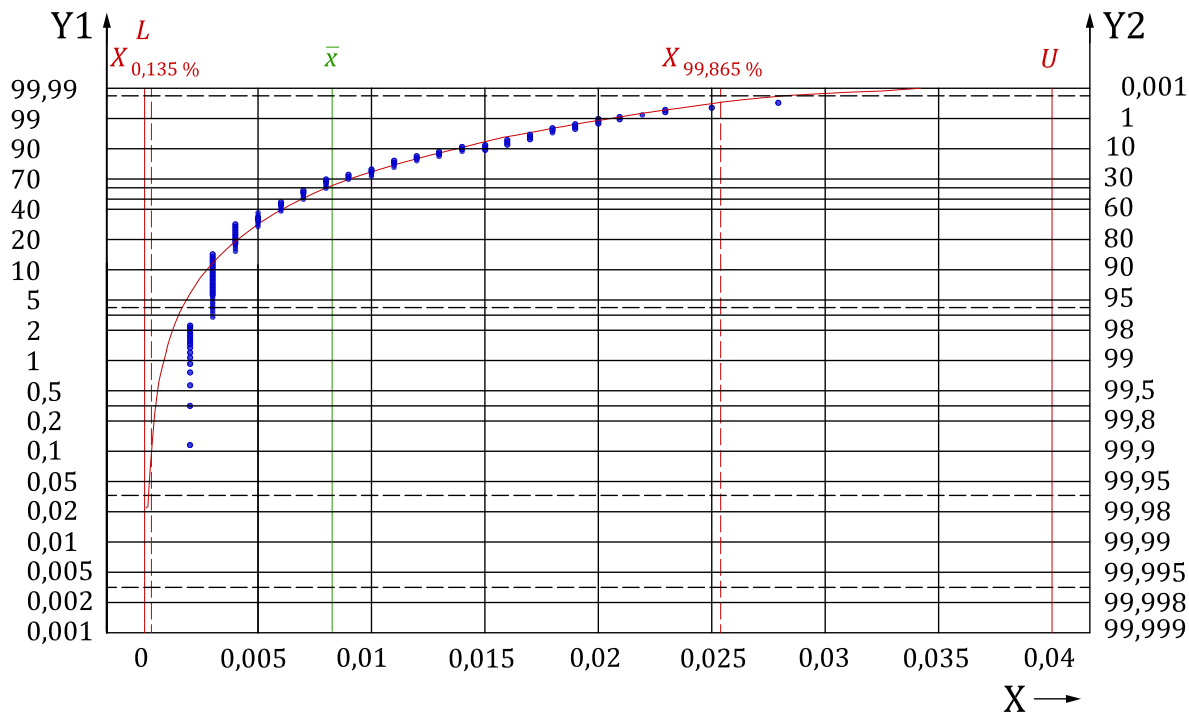
Key

X value

Y1 absolute frequency

Y2 relative frequency in %

Figure 9 — Histogram of test data set 2



Key

- X value
 Y1 probability in %
 Y2 1-probability in %

Figure 10 — Probability plot of test data set 2

NOTE The width of the class intervals is 0,003.

The class interval with the highest frequency of values is from 0,003 5 to 0,006 5.

The density plot is based on the assumption of a two parametric Weibull distribution with estimated parameters scale $a = 0,009\ 39$ and shape $b = 1,897\ 82$. Both estimators have been calculated using the method of Maximum Likelihood.

The 0,135 %- and the 99,865 %-percentiles are calculated using the inverse distribution function of the Weibull distribution

$$F^{-1}(p|a,b) = a \times [-\ln(1-p)]^{\left(\frac{1}{b}\right)}$$

$$X_{0,135\%} = 0,000\ 29$$

$$X_{99,865\%} = 0,025\ 39$$

5.2.2 Test data set 2 results

5.2.2.1 List of sample statistics

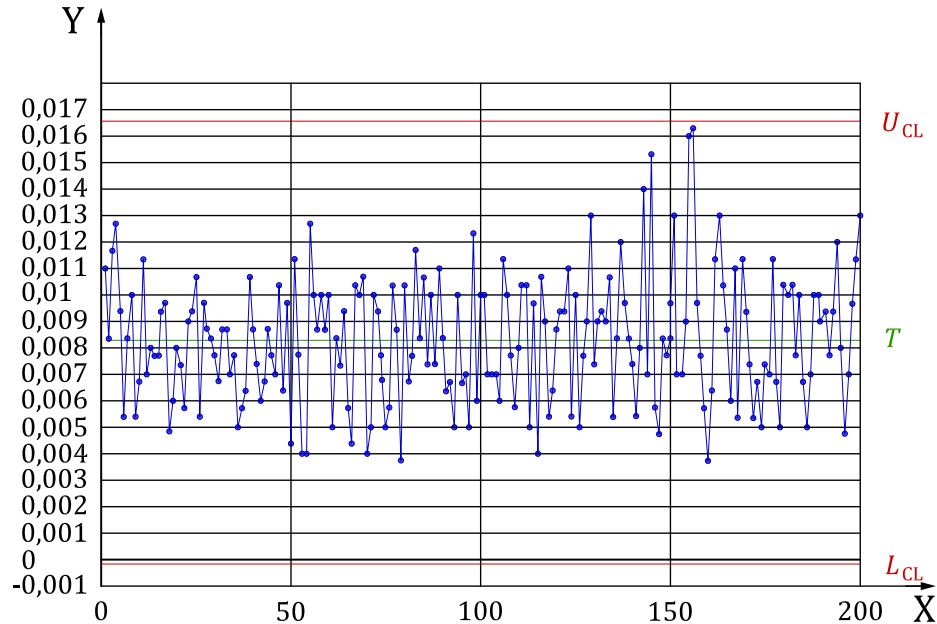
[Table 7](#) lists all sample statistics which are necessary to calculate the target values and control limits for the control charts described in ISO 7870-2 as well as the estimators for location and dispersion given in ISO 22514-2 for the calculation of the process capability indices.

Table 7 — List of sample statistics for test data set 2

Statistic	Value	Reference
Location		
$\bar{x}$ ($l = 1$)	0,008 29	ISO 22514-2:2017, Formula (11)
$\tilde{x}$ ($l = 2$)	0,007 00	ISO 22514-2:2017, Formula (12)
$\bar{\bar{x}}$ ($l = 3$)	0,008 29	ISO 22514-2:2017, Formula (13)
$\tilde{\tilde{x}}$ ($l = 4$)	0,007 51	ISO 22514-2:2017, Formula (14)
Dispersion		
$\hat{\Delta}$ ($d = 1$)	0,025 11 (Weibull using parameters in 5.2.1)	ISO 22514-2:2017, Formula (15)
$\hat{\sigma}$ ($d = 2$)	0,004 88	ISO 22514-2:2017, Formula (16)
$\hat{\sigma}$ ($d = 3$)	0,004 87	ISO 22514-2:2017, Formula (17)
$\hat{\sigma}$ ($d = 4$)	0,004 81	ISO 22514-2:2017, Formula (18)
$\hat{\sigma}$ ($d = 5$)	0,004 67	ISO 22514-2:2017, Formula (19)
$\bar{s}$	0,004 32	ISO 7870-2:2013, 3.2
R	0,026 00	ISO 7870-2:2013, 3.2
$\bar{R}$	0,008 13	ISO 7870-2:2013, 3.2
$\bar{R}_m$	0,005 14	ISO 7870-2:2013, 3.2

5.2.2.2 $\bar{x}$ chart

The $\bar{x}$ control chart calculated according to ISO 7870-2:2013, 6.1, is shown in Figure 11.



Key

- X number of subgroup
- Y mean value

Figure 11 — Mean control chart for test data set 2

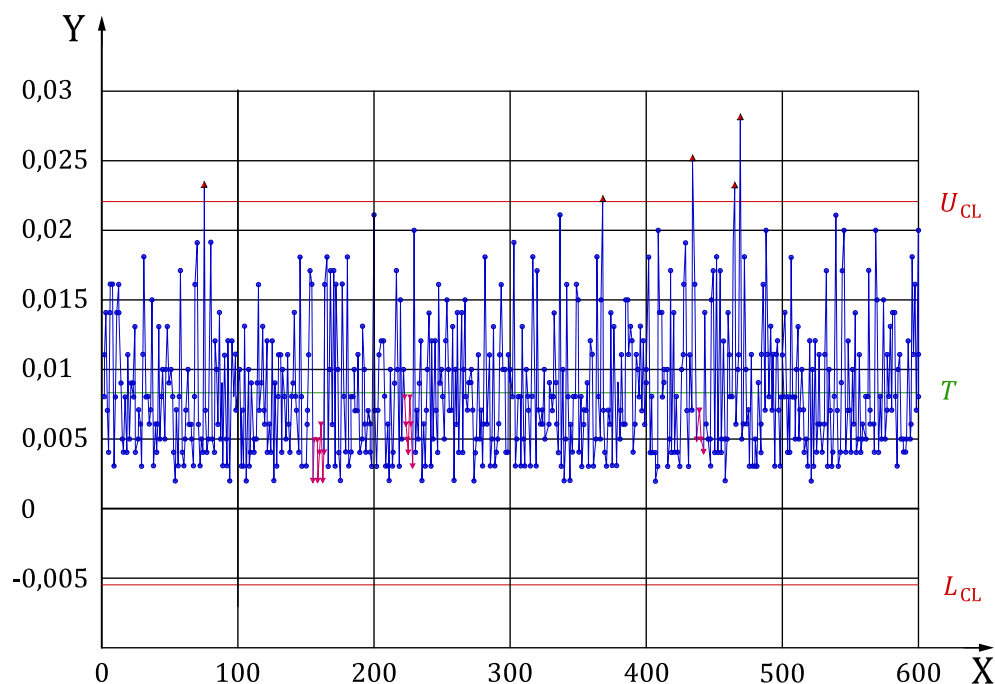
The control limits are (see ISO 7870-2:2013, Table 1, using $\bar{s}$):

$L_{CL} = -0,000\ 15$ (The calculated lower control limit is negative and the dataset is one-sided against 0 limited characteristic. Therefore, the lower control limit is not shown, $L_{CL} = ----$)

$$U_{CL} = 0,016\ 72$$

5.2.2.3 X chart

The individuals control chart calculated according to ISO 7870-2:2013, 6.2, is shown in [Figure 12](#).



Key

X value number
Y individuals

Figure 12 — Individuals control chart for test data set 2

The control limits are (see ISO 7870-2:2013, Table 3):

$$L_{CL} = -0,005\ 39$$

$$U_{CL} = 0,021\ 96$$

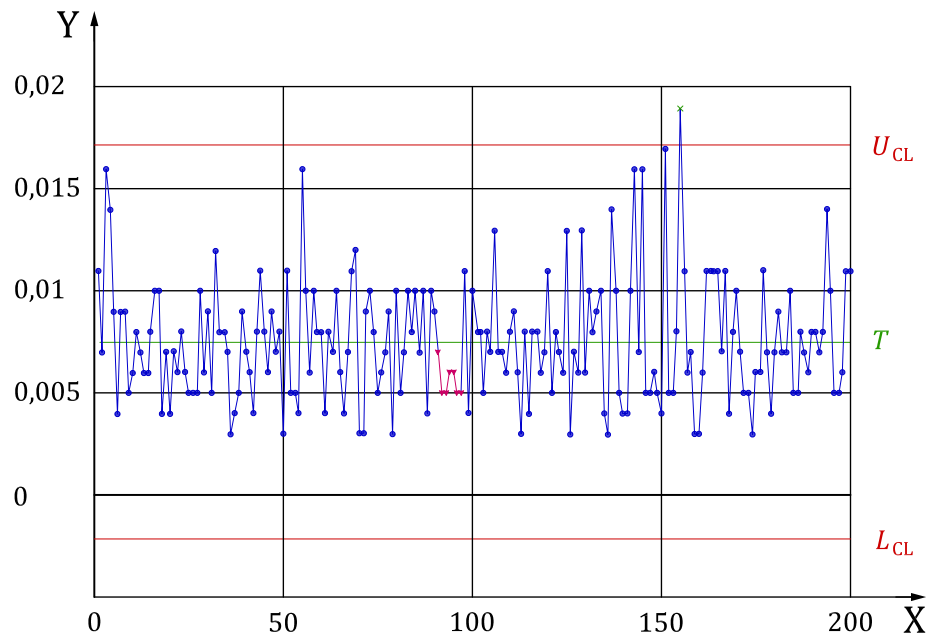
Out of control situations are given in [Table 8](#).

Table 8 — Results of pattern tests for the individuals control chart and test data set 2

Out of control situations			
Value	Result/violation	Value	Result/violation
75	violation of U_{CL}	433	violation of U_{CL}
155 to 163	run below centreline	435 to 441	run below centreline
222 to 228	run below centreline	464	violation of U_{CL}
367	violation of U_{CL}	468	violation of U_{CL}

5.2.2.4 $\tilde{x}$ chart

The median control chart calculated according to ISO 7870-2:2013, 6.3, is shown in [Figure 13](#).



Key

X subgroup number
Y median value

Figure 13 — Median control chart for test data set 2

The control limits are (see ISO 7870-2:2013, Table 4):

$$L_{CL} = -0,002\ 15$$

$$U_{CL} = 0,017\ 17$$

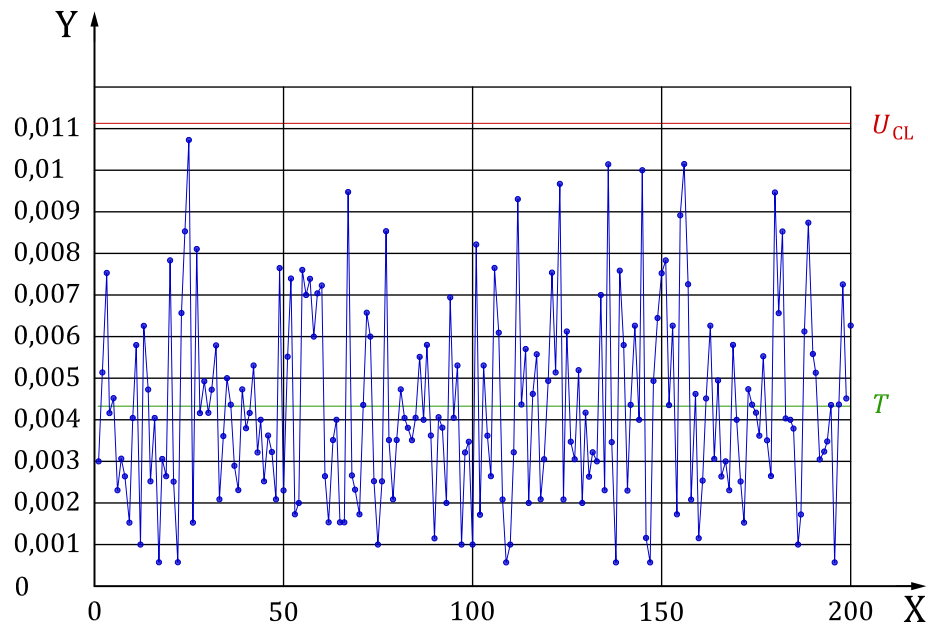
Out of control situations are given in [Table 9](#).

Table 9 — Results of pattern tests for the median control chart and test data set 2

Out of control situations			
Subgroup	Result/violation	Subgroup	Result/violation
91 to 97	run below centreline	155	violation of U_{CL}

5.2.2.5 s chart

The s control chart calculated according to ISO 7870-2:2013, 6.1, is shown in [Figure 14](#).



Key

X subgroup number

Y empirical standard deviation s value

Figure 14 — s control chart for test data set 2

The control limits are (see ISO 7870-2:2013, Table 1):

$$L_{CL} = 0$$

$$U_{CL} = 0,011\ 083$$

5.2.2.6 R chart

The R control chart calculated according to ISO 7870-2:2013, 6.3, is shown in [Figure 15](#).

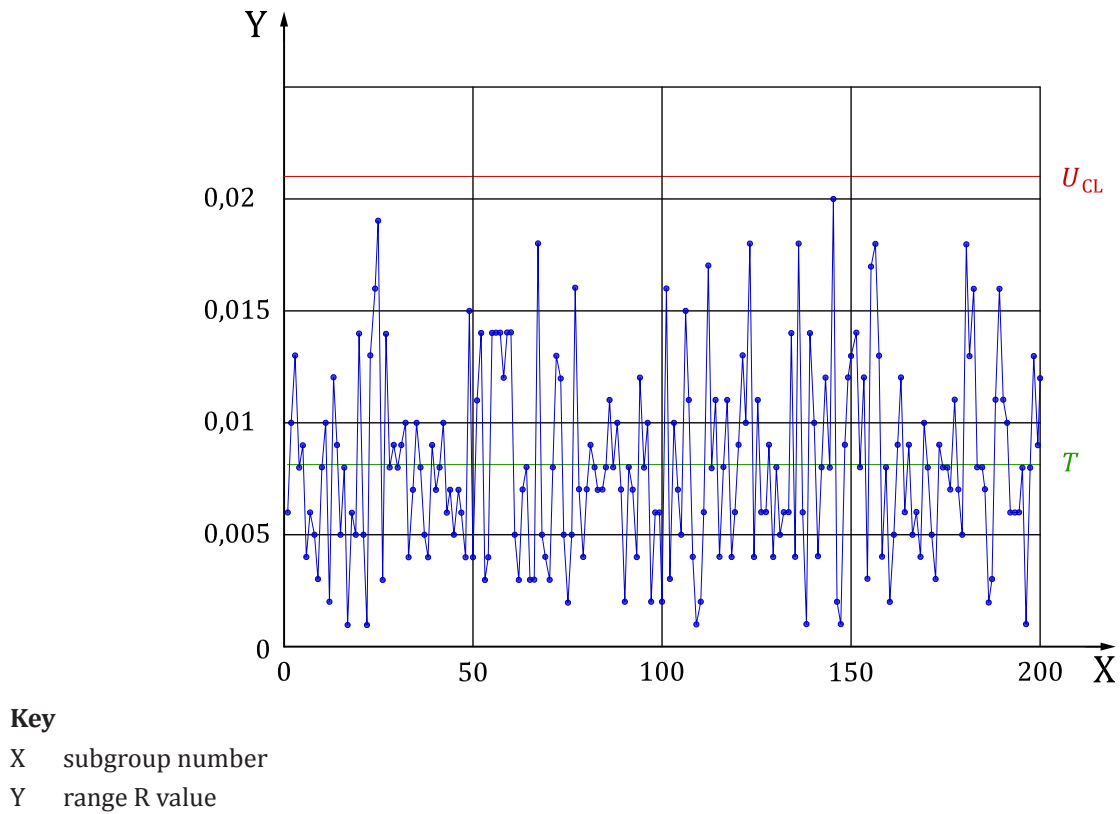


Figure 15 — R control chart for test data set 2

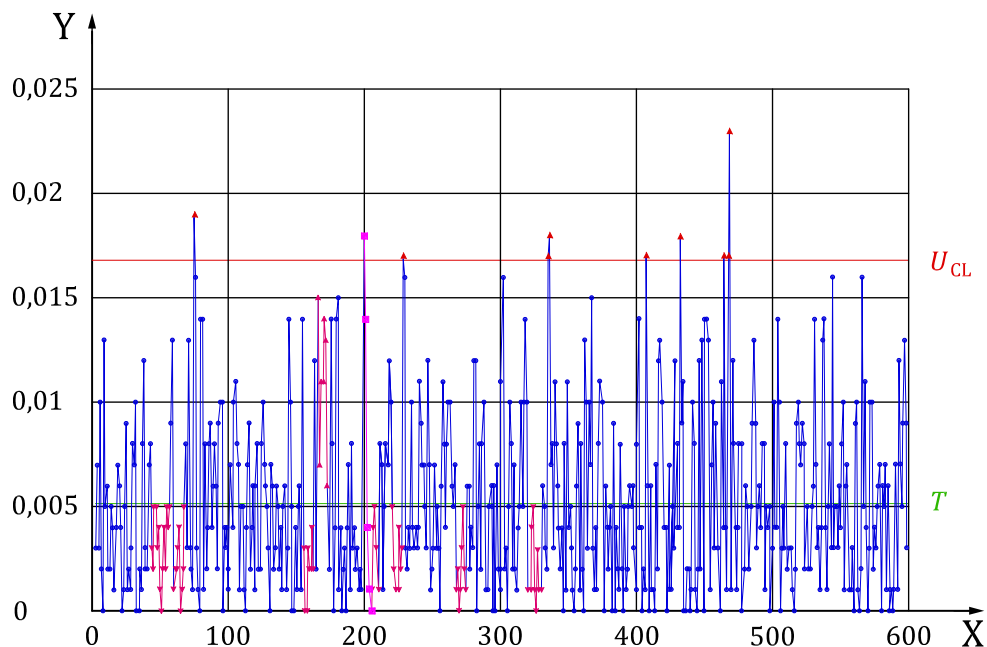
The control limits are (see ISO 7870-2:2013, Table 1):

$$L_{CL} = 0$$

$$U_{CL} = 0,020\ 94$$

5.2.2.7 Moving range chart

The moving range control chart calculated according to ISO 7870-2:2013, 6.3, is shown in [Figure 16](#).



Key

X individuals number

Y moving range value

Figure 16 — Moving range control chart for test data set 2

The control limits are (see ISO 7870-2:2013, Table 3):

$$L_{CL} = 0$$

$$U_{CL} = 0,016\ 80$$

Out of control situations are given in [Table 10](#).

Table 10 — Results of pattern tests for the moving range control chart and test data set 2

Out of control situations			
Value	Result/violation	Value	Result/violation
44 to 57	run below centreline	268 to 275	run below centreline
60 to 68	run below centreline	321 to 331	run below centreline
75	violation of U_{CL}	336	violation of U_{CL}
156 to 163	run below centreline	337	violation of U_{CL}
166 to 174	run below centreline	408	violation of U_{CL}
199 to 206	trend decreasing	433	violation of U_{CL}
199	violation of U_{CL}	465	violation of U_{CL}
201 to 211	run below centreline	468	violation of U_{CL}
221 to 228	run below centreline	469	violation of U_{CL}
229	violation of U_{CL}		

5.2.2.8 Process capability

The process capability is calculated according to ISO 22514-2:2017, Clause 6, with calculation method $l = 3$ for the location estimator and calculation method $d = 5$ for the dispersion estimator.

Capability indices (calculation method $M_{3,5}$)			
Process is stable (mean, R - chart) - C_p/C_{pk} is used			
C_p	1,43	C_{pk}	0,59
C_{pkL}	0,59	C_{pkU}	2,26

Because of non-normal distributed data (Weibull) $M_{3,5}$ cannot be used – instead $M_{3,1}$ is used.

Capability indices (calculation method $M_{3,1}$)			
Process is stable (mean, R - chart) - C_p/C_{pk} is used			
C_p	(1,59) theoretical limit	C_{pk}	1,83
C_{pkL}	---	C_{pkU}	1,83

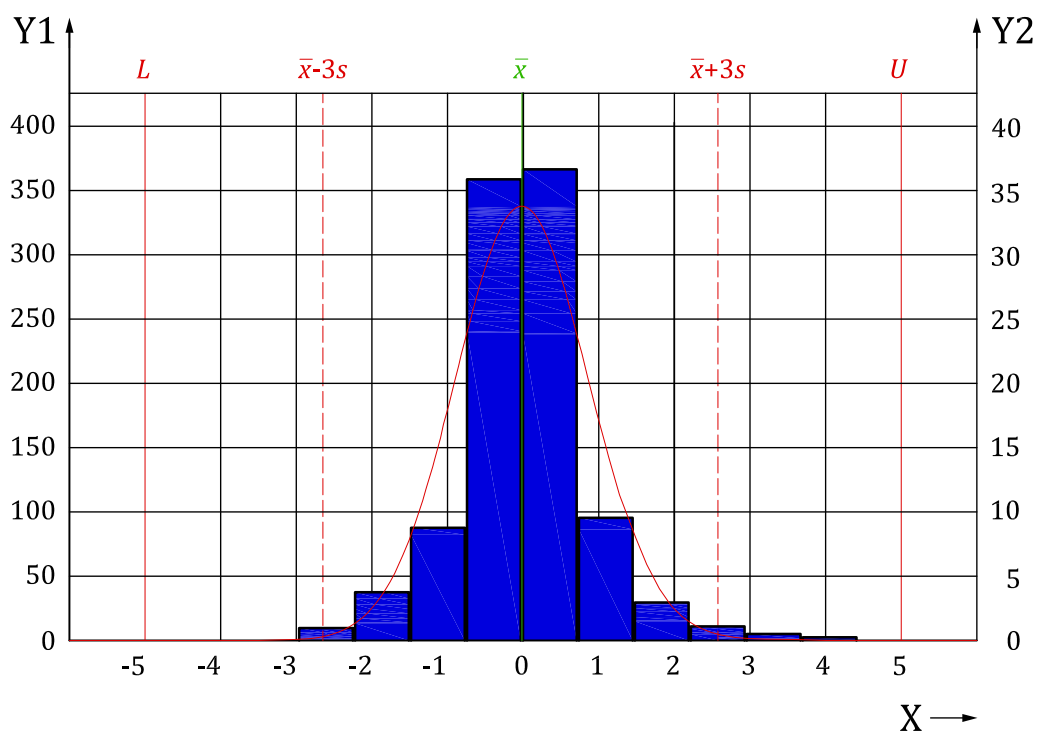
5.3 Test data set 3

5.3.1 Test data set 3 information

This set of test data is taken from a process following a non- normal distribution with a varying dispersion parameter σ and is for checking the accuracy of calculation for control limits, sample statistics and process capabilities. A description of test data set 3 is given in [Table 11](#). [Figure 17](#) shows a histogram and [Figure 18](#) a probability plot of the data set with the purpose of data visualization.

Table 11 — Description of test data set 3

Description of data input			
Distribution model	B	Resulting distribution	non-normal, unimodal
Data set	Annex A, Table A.1	decimal points	2
Total sample size	1 000	U	5
Size of subgroups	5	L	-5

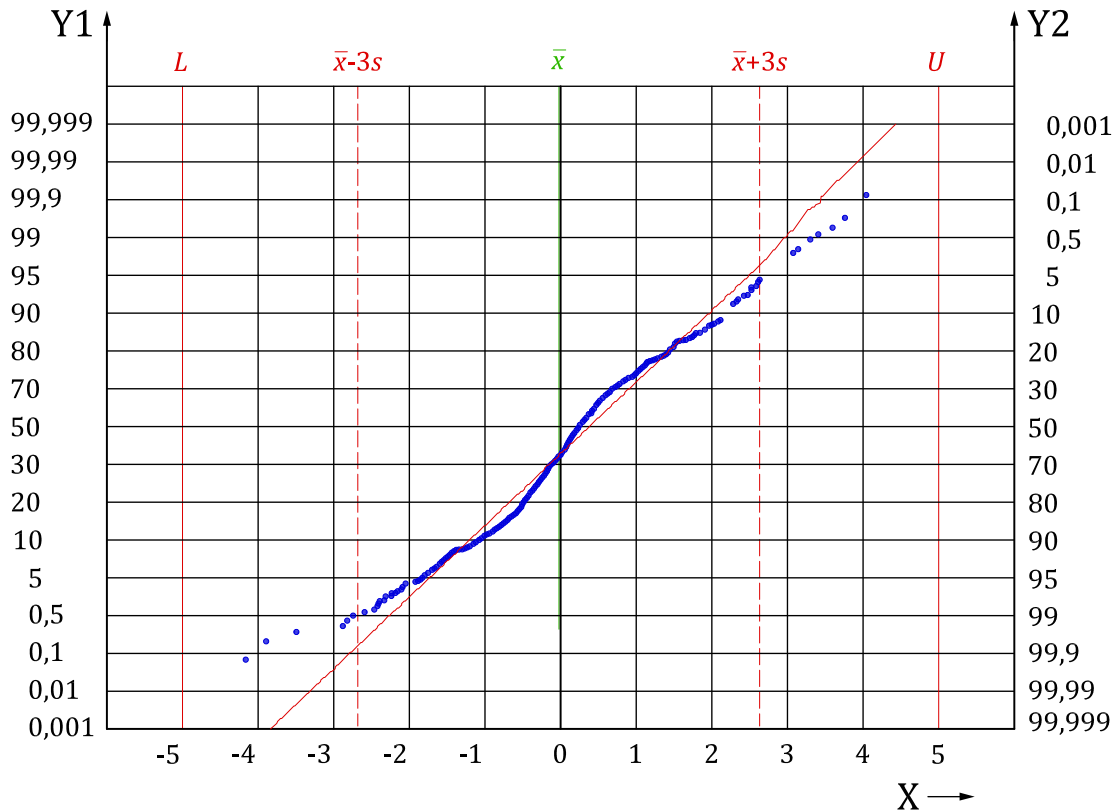


Key

Y1 absolute frequency

Y2 relative frequency in %

Figure 17 — Histogram of test data set 3

**Key**

X value

Y1 probability in %

Y2 1-probability in %

Figure 18 — Probability plot of test data set 3

NOTE The width of the class intervals is 0,74.

The class interval with the highest frequency of values is from $-0,005$ to $0,735$.

The density plot is based on the assumption of normality and estimated parameters $l = 1$, $d = 5$.

5.3.2 Test data set 3 results**5.3.2.1 List of sample statistics**

Table 12 lists all sample statistics which are necessary to calculate the target values and control limits for the control charts described in ISO 7870-2 as well as the estimators for location and dispersion given in ISO 22514-2 for the calculation of the process capability indices.

Table 12 — List of sample statistics for test data set 3

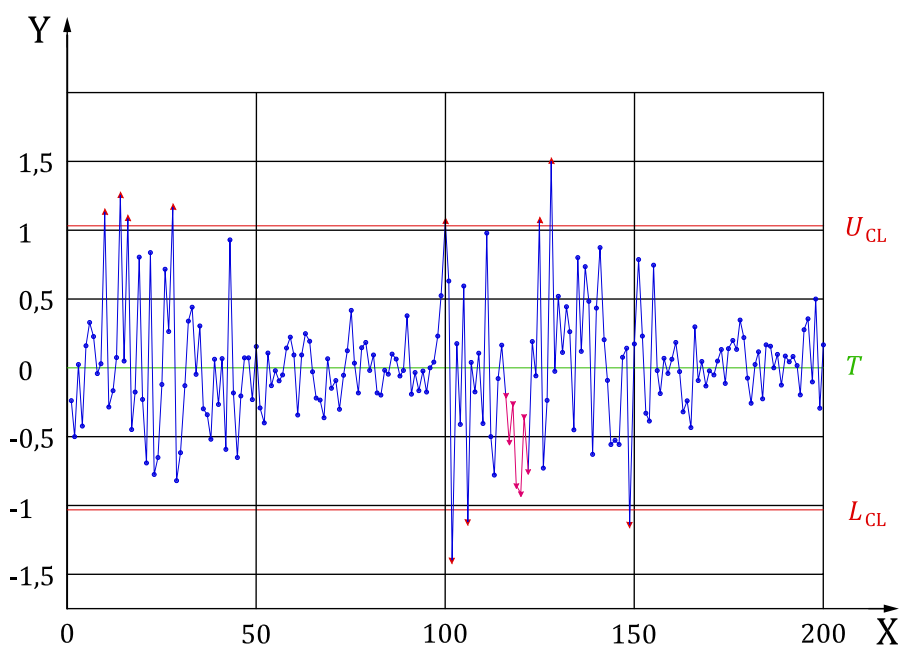
Statistic	Value	Reference
Location		
$\bar{X}$ ($l = 1$)	$-0,002\ 5$	ISO 22514-2:2017, Formula (11)
$\tilde{X}$ ($l = 2$)	$0,000\ 0$	ISO 22514-2:2017, Formula (12)
$\overline{\bar{X}}$ ($l = 3$)	$-0,002\ 5$	ISO 22514-2:2017, Formula (13)
$\tilde{\tilde{X}}$ ($l = 4$)	$0,000\ 6$	ISO 22514-2:2017, Formula (14)

Table 12 (continued)

Statistic	Value	Reference
Dispersion		
$\hat{\Delta} (d = 1)$	5,292 3 (normal distribution)	ISO 22514-2:2017, Formula (15)
$\hat{\sigma} (d = 2)$	0,856 1	ISO 22514-2:2017, Formula (16)
$\hat{\sigma} (d = 3)$	0,759 9	ISO 22514-2:2017, Formula (17)
$\hat{\sigma} (d = 4)$	0,767 5	ISO 22514-2:2017, Formula (18)
$\hat{\sigma} (d = 5)$	0,882 1	ISO 22514-2:2017, Formula (19)
$\bar{s}$	0,714 3	ISO 7870-2:2013, 3.2
R	8,150 0	ISO 7870-2:2013, 3.2
$\bar{R}$	1,785 2	ISO 7870-2:2013, 3.2
$\bar{R}_m$	0,859 0	ISO 7870-2:2013, 3.2

5.3.2.2 $\bar{x}$ - chart

The $\bar{x}$ - control chart calculated according to ISO 7870-2:2013, 6.1, is shown in [Figure 19](#).



Key

X number of subgroup

Y mean value

Figure 19 — Mean control chart for test data set 3

The control limits are (see ISO 7870-2:2013, Table 1, using $\bar{s}$)

$$L_{CL} = -1,021\ 8$$

$$U_{CL} = 1,016\ 8$$

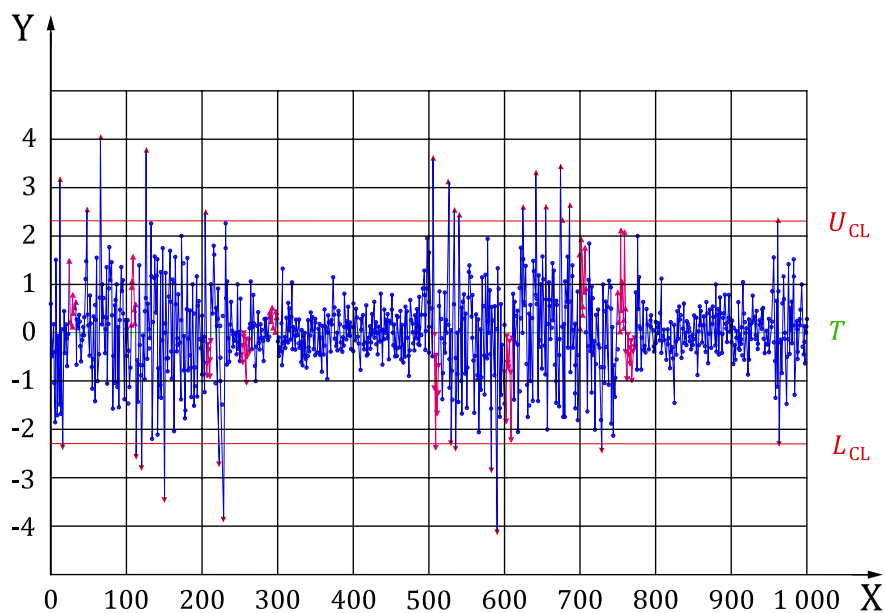
Out of control situations are given in [Table 13](#).

Table 13 — Results of pattern tests for the mean control chart and test data set 3

Out of control situations			
Subgroup	Result/violation	Subgroup	Result/violation
10	violation of U_{CL}	106	violation of L_{CL}
14	violation of U_{CL}	116 to 122	run below centreline
16	violation of U_{CL}	125	violation of U_{CL}
28	violation of U_{CL}	128	violation of U_{CL}
100	violation of U_{CL}	149	violation of L_{CL}
102	violation of L_{CL}		

5.3.2.3 X chart

The individuals control chart calculated according to ISO 7870-2:2013, 6.2, is shown in [Figure 20](#).

**Key**

X value number
Y individuals

Figure 20 — Individuals control chart for test data set 3

The control limits are (see ISO 7870-2:2013, Table 3):

$$L_{CL} = -2,287\ 4$$

$$U_{CL} = 2,282\ 5$$

Out of control situations are given in [Table 14](#).

Table 14 — Results of pattern tests for the individuals control chart and test data set 3

Out of control situations			
Value	Result/violation	Value	Result/violation
12	violation of U_{CL}	523	violation of U_{CL}
16	violation of L_{CL}	528	violation of L_{CL}
24 to 33	run above centreline	532	violation of U_{CL}
48	violation of U_{CL}	534	violation of L_{CL}
66	violation of U_{CL}	538	violation of U_{CL}
106 to 112	run above centreline	581	violation of L_{CL}
113	violation of L_{CL}	588	violation of L_{CL}
120	violation of L_{CL}	599 to 608	run below centreline
126	violation of U_{CL}	622	violation of U_{CL}
150	violation of L_{CL}	639	violation of U_{CL}
204	violation of U_{CL}	652	violation of U_{CL}
205 to 211	run below centreline	672	violation of U_{CL}
222	violation of L_{CL}	673	violation of U_{CL}
228	violation of L_{CL}	684	violation of U_{CL}
251 to 261	run below centreline	697 to 705	run above centreline
288 to 298	run above centreline	726	violation of L_{CL}
504	violation of U_{CL}	747 to 770	run above centreline
505 to 511	run below centreline	958	violation of U_{CL}
507	violation of L_{CL}	960	violation of U_{CL}

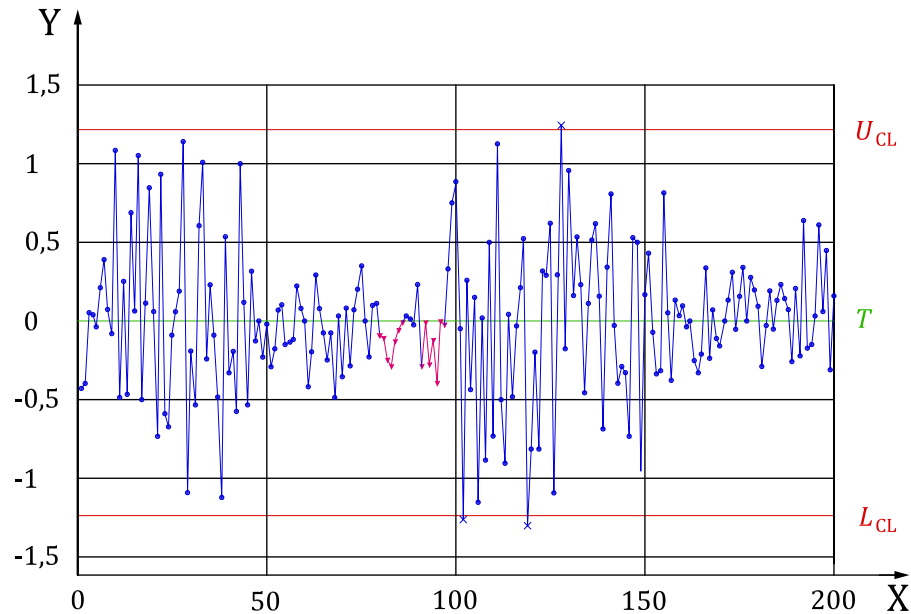
Because of the variation in dispersion another set of control limits can be calculated e.g. based on a Johnson transformation.

$$L_{CL} = -2,964\ 6$$

$$U_{CL} = 2,959\ 7$$

5.3.2.4 $\tilde{x}$ chart

The median control chart calculated according to ISO 7870-2:2013, 6.3, is shown in [Figure 21](#).



Key

X subgroup number

Y median value

Figure 21 — Median control chart for test data set 3

The control limits are (see ISO 7870-2:2013, Table 4):

$$L_{CL} = -1,233\ 0$$

$$U_{CL} = 1,234\ 2$$

Out of control situations are given in [Table 15](#).

Table 15 — Results of pattern tests for the median control chart and test data set 3

Out of control situations			
Subgroup	Result/violation	Subgroup	Result/violation
80 to 86	run below centreline	119	violation of L_{CL}
91 to 97	run below centreline	128	violation of U_{CL}
102	violation of L_{CL}		

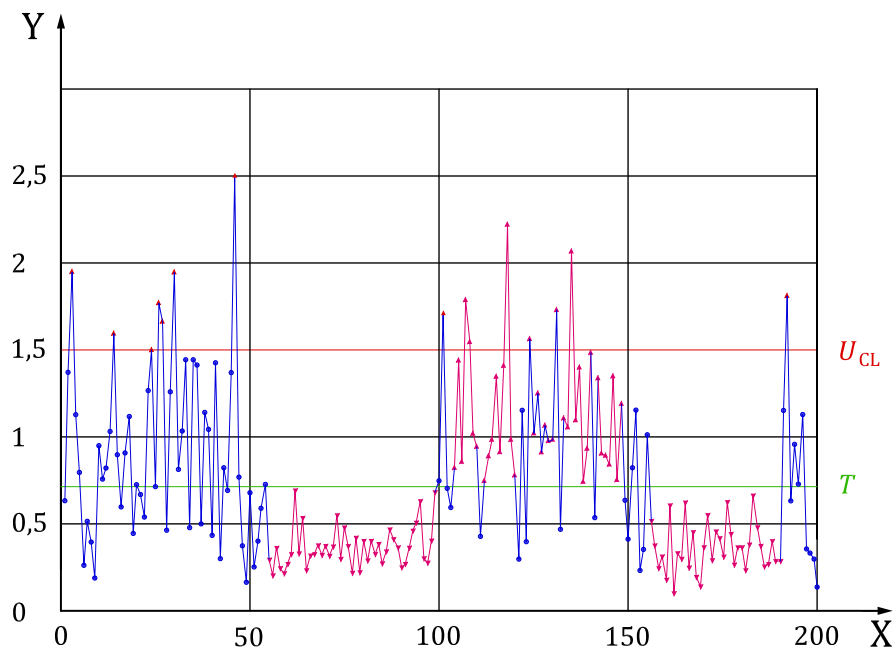
Because of the variation in dispersion another set of control limits can be calculated based on a Johnson Transformation.

$$L_{CL} = -1,377\ 4$$

$$U_{CL} = 1,372\ 4$$

5.3.2.5 s chart

The s control chart calculated according to ISO 7870-2:2013, 6.1, is shown in [Figure 22](#).



Key

X subgroup number

Y empirical standard deviation s value

Figure 22 — s control chart for test data set 3

The control limits are (see ISO 7870-2:2013, Table 1):

$$L_{CL} = 0$$

$$U_{CL} = 1,492\ 1$$

Out of control situations are given in [Table 16](#).

Table 16 — Results of pattern tests for the s control chart and test data set 3

Out of control situations			
Subgroup	Result/violation	Subgroup	Result/violation
3	violation of U_{CL}	108	violation of U_{CL}
14	violation of U_{CL}	112 to 120	run above centreline
24	violation of U_{CL}	118	violation of U_{CL}
26	violation of U_{CL}	124	violation of U_{CL}
27	violation of U_{CL}	124 to 131	run above centreline
30	violation of U_{CL}	131	violation of U_{CL}
46	violation of U_{CL}	133 to 140	run above centreline
55 to 99	run below centreline	135	violation of U_{CL}
101	violation of U_{CL}	142 to 148	run above centreline
104 to 110	run above centreline	156 to 190	run below centreline

Table 16 (continued)

Out of control situations			
Subgroup	Result/violation	Subgroup	Result/violation
107	violation of U_{CL}	192	violation of U_{CL}

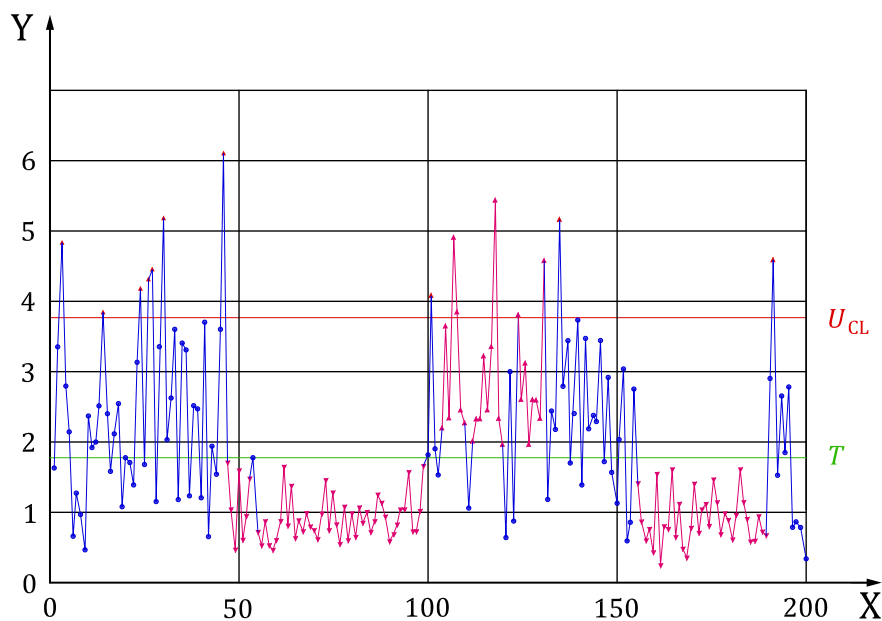
Because of the variation in dispersion another set of control limits can be calculated (Johnson transformation)

$$L_{CL} = 0,139\ 22$$

$$U_{CL} = 1,806\ 07$$

5.3.2.6 R chart

The R control chart calculated according to ISO 7870-2:2013, 6.3, is shown in [Figure 23](#).



Key

X subgroup number

Y range R value

Figure 23 — R control chart for test data set 3

The control limits are (see ISO 7870-2:2013, Table 1):

$$L_{CL} = 0$$

$$U_{CL} = 3,773\ 9$$

Out of control situations are given in [Table 17](#).

Table 17 — Results of pattern tests for the R control chart and test data set 3

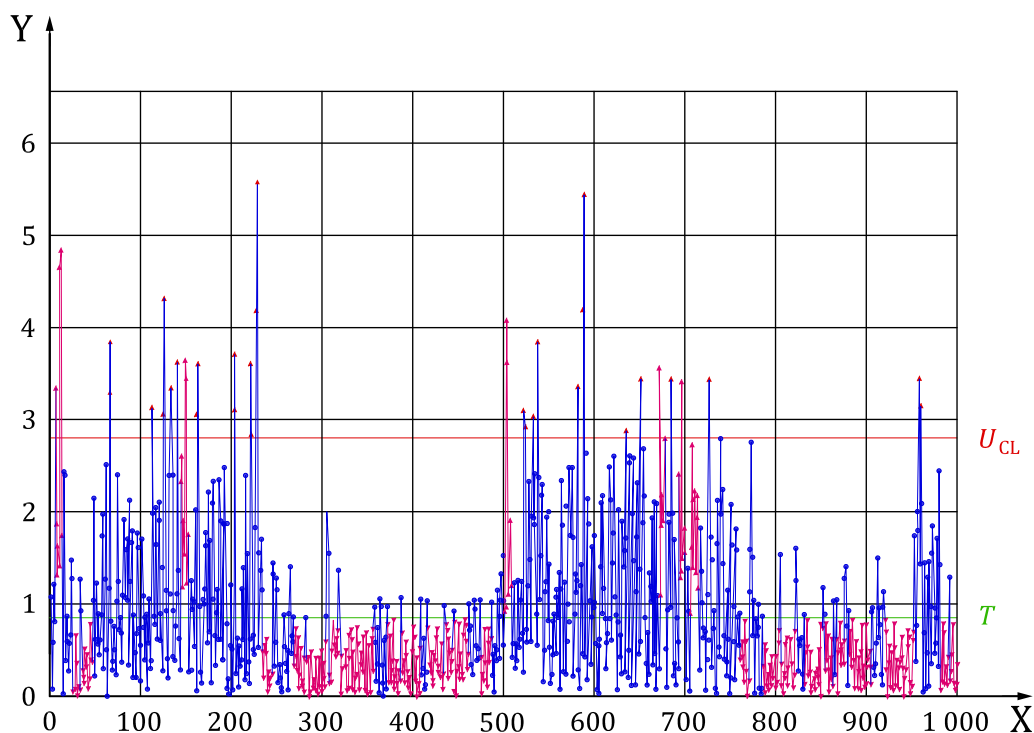
Out of control situations			
Subgroup	Result/violation	Subgroup	Result/violation
3	violation of U_{CL}	107	violation of U_{CL}
14	violation of U_{CL}	108	violation of U_{CL}

Table 17 (continued)

Out of control situations			
Subgroup	Result/violation	Subgroup	Result/violation
24	violation of U_{CL}	112 to 120	run above centreline
26	violation of U_{CL}	118	violation of U_{CL}
27	violation of U_{CL}	124 to 131	run above centreline
29	violation of U_{CL}	124	violation of U_{CL}
46	violation of U_{CL}	131	violation of U_{CL}
47 to 53	run below centreline	135	violation of U_{CL}
55 to 99	run below centreline	156 to 190	run below centreline
101	violation of U_{CL}	192	violation of U_{CL}
104 to 110	run above centreline		

5.3.2.7 Moving range chart

The moving range control chart calculated according to ISO 7870-2:2013, 6.3, is shown in [Figure 24](#).



Key

X individuals number
Y moving range value

Figure 24 — Moving range control chart for test data set 3

The control limits are (see ISO 7870-2:2013, Table 3):

$$L_{CL} = 0$$

$$U_{CL} = 2,806\ 3$$

Out of control situations are given in [Table 18](#).

Table 18 — Results of pattern tests for the moving range control chart and test data set 3

Out of control situations			
Value	Result/violation	Value	Result/violation
7 to 14	run above centreline	446 to 461	run below centreline
7	violation of U_{CL}	475 to 486	run below centreline
12	violation of U_{CL}	502 to 509	run above centreline
13	violation of U_{CL}	504	violation of U_{CL}
26 to 33	run below centreline	505	violation of U_{CL}
36 to 47	run below centreline	523	violation of U_{CL}
66	violation of U_{CL}	524	violation of U_{CL}
67	violation of U_{CL}	533	violation of U_{CL}
113	violation of U_{CL}	538	violation of U_{CL}
126	violation of U_{CL}	582	violation of U_{CL}
127	violation of U_{CL}	588	violation of U_{CL}
134	violation of U_{CL}	589	violation of U_{CL}
141	violation of U_{CL}	636	violation of U_{CL}
145 to 153	run above centreline	652	violation of U_{CL}
150	violation of U_{CL}	672 to 678	run above centreline
151	violation of U_{CL}	672	violation of U_{CL}
163	violation of U_{CL}	685	violation of U_{CL}
164	violation of U_{CL}	694 to 700	run above centreline
205	violation of U_{CL}	697	violation of U_{CL}
205	violation of U_{CL}	706 to 715	run above centreline
222	violation of U_{CL}	727	violation of U_{CL}
223	violation of U_{CL}	761 to 770	run below centreline
228	violation of U_{CL}	786 to 804	run below centreline
229	violation of U_{CL}	806 to 821	run below centreline
235 to 245	run below centreline	832 to 851	run below centreline
270 to 281	run below centreline	854 to 863	run below centreline
283 to 304	run below centreline	867 to 875	run below centreline
308 to 318	run below centreline	881 to 905	run below centreline
320 to 357	run below centreline	920 to 952	run below centreline
373 to 386	run below centreline	958	violation of U_{CL}
388 to 408	run below centreline	960	violation of U_{CL}
417 to 434	run below centreline	982 to 990	run below centreline
436 to 444	run below centreline	992 to 1 000	run below centreline

Because of the variation in dispersion another set of control limits can be calculated.

5.3.2.8 Process capability

The process capability is calculated according to ISO 22514-2:2017, Clause 6, with calculation method $l = 3$ for the location estimator and calculation method $d = 5$ for the dispersion estimator.

Capability indices (calculation method $M_{3,5}$)			
Process is not stable (mean, R - chart) - P_p/P_{pk} is used			
P_p	1,89	P_{pk}	1,89
P_{pkL}	1,89	P_{pkU}	1,89

Because of non-normal distributed data (dispersion not stable) $M_{3,5}$ cannot be used – instead $M_{3,1}$ and a Pearson or Johnson transformation is used.

Capability indices (calculation method $M_{3,1}$)			
Process is not stable (mean, R - chart) - P_p/P_{pk} is used. Calculation based on:			
Pearson transformation			
P_p	1,42	P_{pk}	1,39
P_{pkL}	1,89	P_{pkU}	1,46
Johnson Transformation			
P_p	1,02	P_{pk}	1,02
P_{pkL}	1,03	P_{pkU}	1,02

5.4 Test data set 4

5.4.1 Test data set 4 information

This set of test data is taken from a process following a normal distribution (having changes in the values of parameters μ and σ over time) and is for checking the accuracy of calculation for control limits, sample statistics and process capabilities. A description of test data set 4 is given in [Table 19](#). [Figure 25](#) shows a histogram and [Figure 26](#) a probability plot of the data set with the purpose of data visualization.

Table 19 — Description of test data set 4

Description of data input			
Distribution model	C1	Resulting distribution	normal
Data set	Annex A, Table A.1	decimal points	2
Total sample size	1 000	U	10
Size of subgroups	5	L	-10

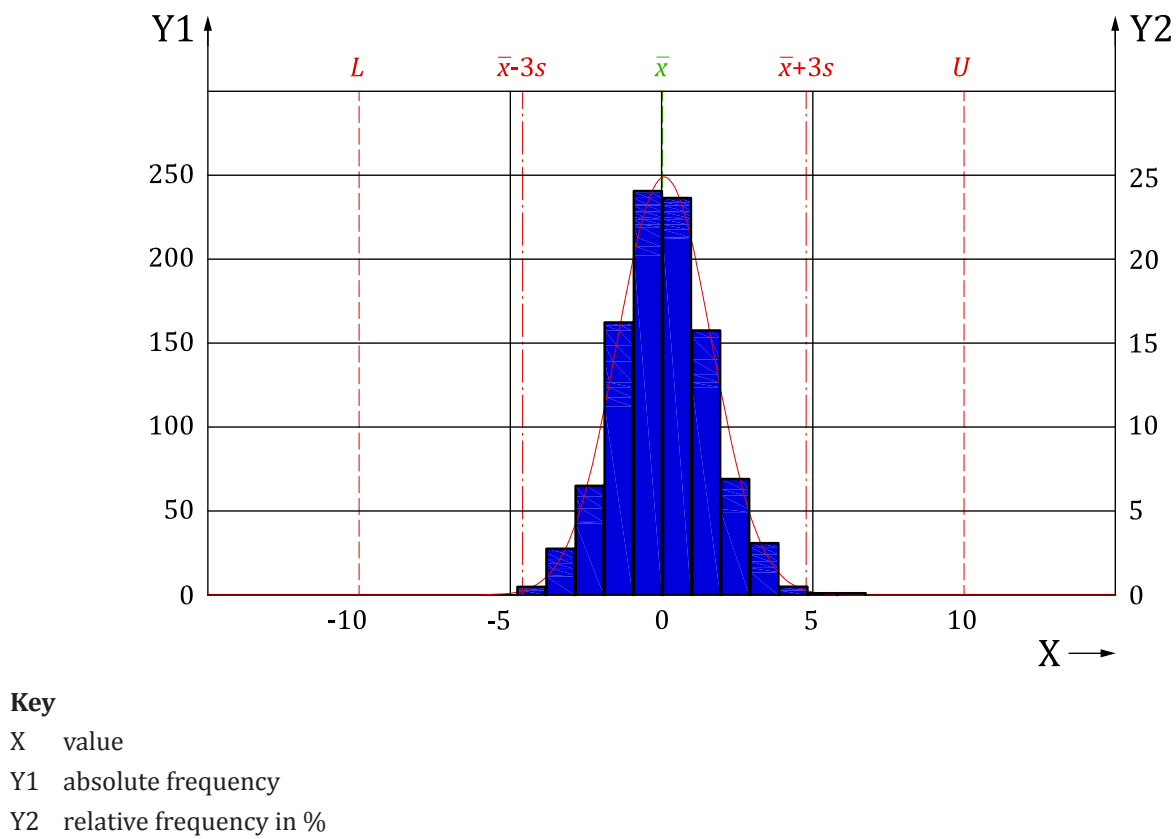
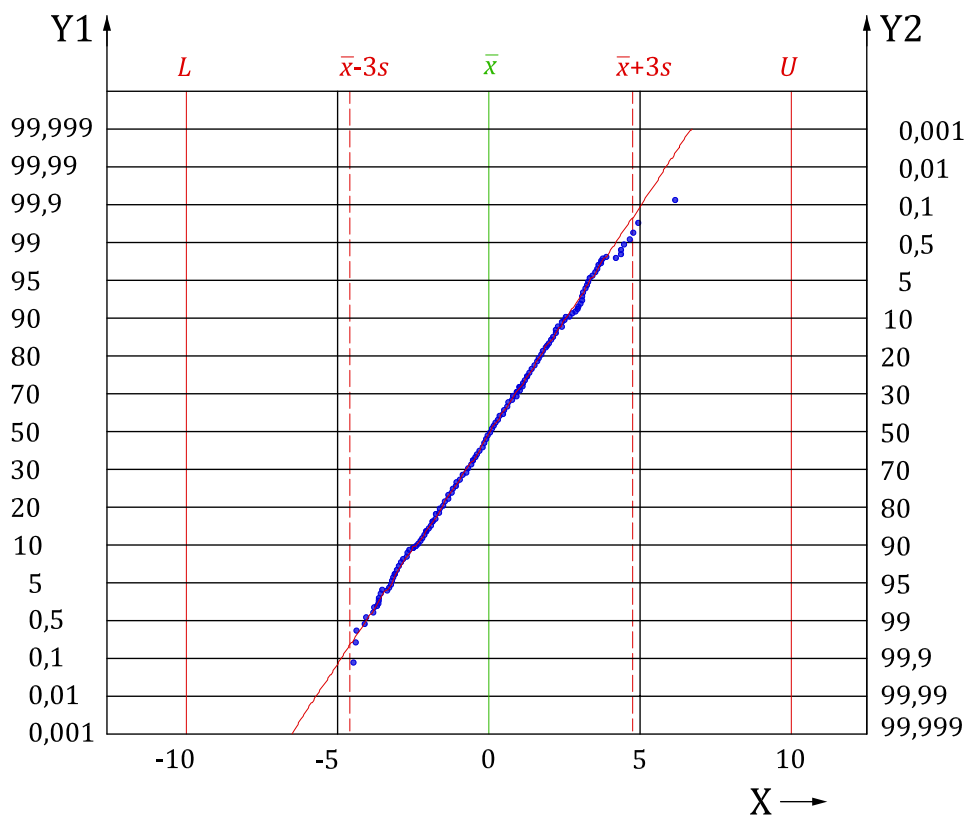


Figure 25 — Histogram of test data set 4



Key
X value
Y1 probability in %
Y2 1-probability in %

Figure 26 — Probability plot of test data set 4

NOTE The width of the class intervals is 0,97.
The class interval with the highest frequency of values is from -0,91 to 0,06.
The density plot is based on the assumption of normality and estimated parameters $l = 1, d = 5$.

5.4.2 Test data set 4 results

5.4.2.1 List of sample statistics

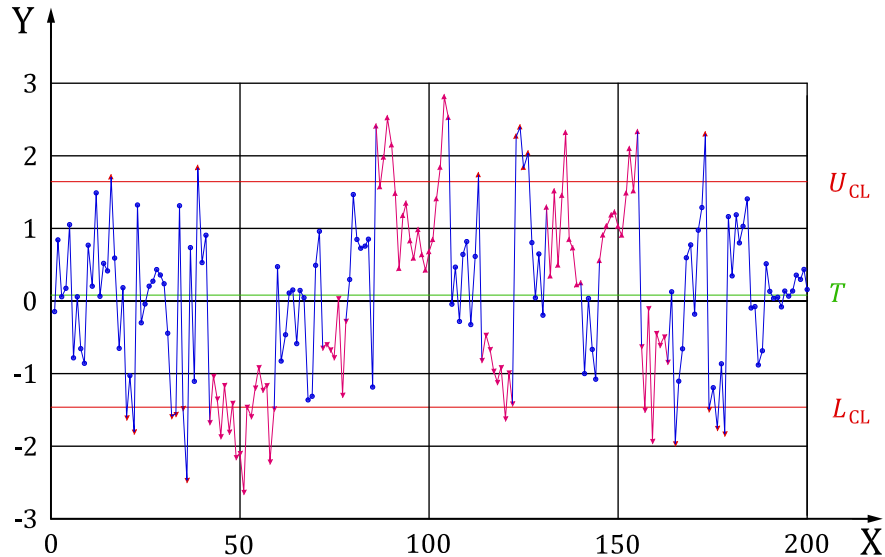
Table 20 lists all sample statistics which are necessary to calculate the target values and control limits for the control charts described in ISO 7870-2 as well as the estimators for location and dispersion given in ISO 22514-2 for the calculation of the process capability indices.

Table 20 — List of sample statistics for test data set 4

Statistic	Value	Reference
Location		
$\bar{x}$ ($l = 1$)	0,088 5	ISO 22514-2:2017, Formula (11)
$\tilde{x}$ ($l = 2$)	0,065 0	ISO 22514-2:2017, Formula (12)
$\bar{\bar{x}}$ ($l = 3$)	0,088 5	ISO 22514-2:2017, Formula (13)
$\tilde{\tilde{x}}$ ($l = 4$)	0,114 3	ISO 22514-2:2017, Formula (14)
Dispersion		
$\hat{\Delta}$ ($d = 1$)	9,349 0 (normal distribution)	ISO 22514-2:2017, Formula (15)
$\hat{\sigma}$ ($d = 2$)	1,157 9	ISO 22514-2:2017, Formula (16)
$\hat{\sigma}$ ($d = 3$)	1,158 3	ISO 22514-2:2017, Formula (17)
$\hat{\sigma}$ ($d = 4$)	1,165 0	ISO 22514-2:2017, Formula (18)
$\hat{\sigma}$ ($d = 5$)	1,558 2	ISO 22514-2:2017, Formula (19)
$\bar{s}$	1,088 8	ISO 7870-2:2013, 3.2
R	10,640 0	ISO 7870-2:2013, 3.2
$\bar{R}$	2,709 7	ISO 7870-2:2013, 3.2
$\bar{R}_m$	1,344 9	ISO 7870-2:2013, 3.2

5.4.2.2 $\bar{x}$ chart

The $\bar{x}$ control chart calculated according to ISO 7870-2:2013, 6.1, is shown in [Figure 27](#).



Key

- X number of subgroup
- Y mean value

Figure 27 — Mean control chart for test data set 4

The control limits are (see ISO 7870-2:2013, Table 1, using $\bar{s}$):

$$L_{CL} = -1,465\ 2$$

$$U_{CL} = 1,642\ 2$$