

INTERNATIONAL STANDARD



Electrical insulating materials – Thermal endurance properties – Part 3: Instructions for calculating thermal endurance characteristics



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INTERNATIONAL ELECTROTECHNICAL COMMISSION

**ELECTRICAL INSULATING MATERIALS –
THERMAL ENDURANCE PROPERTIES –****Part 3: Instructions for calculating
thermal endurance characteristics**

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This redline version of the official IEC Standard allows the user to identify the changes made to the previous edition IEC 60216-3:2006. A vertical bar appears in the margin wherever a change has been made. Additions are in green text, deletions are in strikethrough red text.

IEC 60216-3 has been prepared by IEC technical committee 112: Evaluation and qualification of electrical insulating materials and systems. It is an International Standard.

This third edition cancels and replaces the second edition published in 2006. This edition constitutes a technical revision.

This edition includes the following significant technical changes with respect to the previous edition:

- a) a new computer program has been included;
- b) Annex E " has been completely reworked.

The text of this International Standard is based on the following documents:

Draft	Report on voting
112/475/CDV	112/495/RVC

Full information on the voting for its approval can be found in the report on voting indicated in the above table.

The language used for the development of this International Standard is English.

This document was drafted in accordance with ISO/IEC Directives, Part 2, and developed in accordance with ISO/IEC Directives, Part 1 and ISO/IEC Directives, IEC Supplement, available at www.iec.ch/members_experts/refdocs. The main document types developed by IEC are described in greater detail at www.iec.ch/standardsdev/publications.

A list of all parts in the IEC 60216 series, published under the general title *Electrical insulating materials – Thermal endurance properties*, can be found on the IEC website.

The committee has decided that the contents of this document will remain unchanged until the stability date indicated on the IEC website under webstore.iec.ch in the data related to the specific document. At this date, the document will be

- reconfirmed,
- withdrawn,
- replaced by a revised edition, or
- amended.

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ELECTRICAL INSULATING MATERIALS – THERMAL ENDURANCE PROPERTIES –

Part 3: Instructions for calculating thermal endurance characteristics

1 Scope

This part of IEC 60216 specifies the calculation procedures ~~to be~~ used for deriving thermal endurance characteristics from experimental data obtained in accordance with the instructions of IEC 60216-1 and IEC 60216-2 [1]¹, using fixed ageing temperatures and variable ageing times.

The experimental data ~~may~~ can be obtained using non-destructive, destructive or proof tests. Data obtained from non-destructive or proof tests ~~may~~ can be incomplete, in that it is possible that measurement of times taken to reach the end-point ~~may~~ will have been terminated at some point after the median time but before all specimens have reached end-point.

The procedures are illustrated by worked examples, and suitable computer programs are recommended to facilitate the calculations.

2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

IEC 60216-1:2004/2013, *Electrical insulating materials – Thermal endurance properties – Part 1: Ageing procedures and evaluation of test results*

~~IEC 60216-2:2005, *Electrical insulating materials – Properties of thermal endurance – Part 2: Determination of thermal endurance properties of electrical insulating materials – Choice of test criteria*~~

~~IEC 60493-1:1974, *Guide for the statistical analysis of ageing test data – Part 1: Methods based on mean values of normally distributed test results*~~

3 Terms, definitions, symbols and abbreviated terms

3.1 Terms and definitions

For the purposes of this document, the following definitions apply.

ISO and IEC maintain terminological databases for use in standardization at the following addresses:

- IEC Electropedia: available at <http://www.electropedia.org/>
- ISO Online browsing platform: available at <https://www.iso.org/obp>

¹ Numbers in square brackets refer to the bibliography.

3.1.1

ordered data

group of data arranged in sequence so that in the appropriate direction through the sequence each member is greater than, or equal to, its predecessor

Note 1 to entry: In this document, ascending order implies that the data is ordered in this way, the first being the smallest.

Note 2 to entry: It has been established that the term "group" is used in the theoretical statistics literature to represent a subset of the whole data set. The group comprises those data having the same value of one of the parameters of the set (e.g. ageing temperature). A group may itself comprise a number of sub-groups characterized by another parameter (e.g. time in the case of destructive tests).

3.1.2

order-statistic

~~each individual value in a group of ordered data is referred to as an order statistic identified by its numerical position in the sequence~~

assigned numerical position in the sequence of individual values in a group of ordered data

3.1.3

incomplete data

ordered data, where the values above and/or below defined points are not known

3.1.4

censored data

incomplete data, where the number of unknown values is known

Note 1 to entry: If the censoring is begun above/below a specified numerical value, the censoring is Type I. If above/below a specified order-statistic it is Type II. This document is concerned only with Type II.

3.1.5

degrees of freedom

number of data values minus the number of parameter values

3.1.6

variance of a data group

sum of the squares of the deviations of the data from a reference level

Note 1 to entry: The reference level may be defined by one or more parameters, for example a mean value (one parameter) or a line (two parameters, slope and intercept), divided by the number of degrees of freedom.

3.1.7

central second moment of a data group

sum of the squares of the differences between the data values and the value of the group mean, divided by the number of data in the group

3.1.8

covariance of data groups

for two groups of data with equal numbers of elements where each element in one group corresponds to one in the other, the sum of the products of the deviations of the corresponding members from their group means, divided by the number of degrees of freedom

3.1.9

regression analysis

process of deducing the best-fit line expressing the relation of corresponding members of two data groups by minimizing the sum of squares of deviations of members of one of the groups from the line

Note 1 to entry: The parameters are referred to as the regression coefficients.

3.1.10**correlation coefficient**

number expressing the completeness of the relation between members of two data groups, equal to the covariance divided by the square root of the product of the variances of the groups

Note 1 to entry: The value of its square is between 0 (no correlation) and 1 (complete correlation).

3.1.11**end-point line**

line parallel to the time axis intercepting the property axis at the end-point value

Note 1 to entry: For guidance on the choice of end-point value, refer to IEC 60216-2.

3.2 Symbols and abbreviated terms

		Subclause
a	Regression coefficient (y -intercept)	4.3, 6.2
a_p	Regression coefficient for destructive test calculations	6.1
b	Regression coefficient (slope)	4.3, 6.2
b_p	Regression coefficient for destructive test calculations	6.1
b_r	Intermediate constant (calculation of $\hat{X}_c$)	6.3
c	Intermediate constant (calculation of χ^2)	6.3
f	Number of degrees of freedom	Table C.2 to Table C.5
F	Fisher distributed stochastic variable	4.2, 6.1, 6.3
F_0	Tabulated value of F (linearity of thermal endurance graph)	4.4, 6.3
F_1	Tabulated value of F (linearity of property graph – significance 0,05)	6.1
F_2	Tabulated value of F (linearity of property graph – significance 0,005)	6.1
g	Order number of ageing time for destructive tests	6.1
h	Order number of property value for destructive tests	6.1
HIC	Halving interval at temperature equal to T_I	4.3, Clause 7
HIC_g	Halving interval corresponding to T_{I_g}	7.3
i	Order number of exposure temperature	4.1, 6.2
j	Order number of time to end-point	4.1, 6.2
k	Number of ageing temperatures	4.1, 6.2
m_i	Number of specimens aged at temperature ϑ_i	4.1, 6.1
N	Total number of times to end-point	6.2
n_g	Number of property values in group aged for time τ_g	6.1
n_i	Number of values of y at temperature ϑ_i	4.1, 6.1
$\bar{p}$	Mean value of property values in selected groups	6.1
p	Value of diagnostic property	6.1
P	Significance level of χ^2 distribution	4.4, 6.3.1
p_e	Value of diagnostic property at end-point for destructive tests	6.1
$\bar{p}_g$	Mean of property values in group aged for time τ_g	6.1
p_{gh}	Individual property value	

		Subclause
q	Base of logarithms	6.3
r	Number of ageing times selected for inclusion in calculation (destructive tests)	6.1
r^2	Square of correlation coefficient	6.2.3
s^2	Weighted mean of s_1^2 and s_2^2	6.3
s_1^2	Weighted mean of s_{1i}^2 , pooled variance within selected groups	4.3, 6.1 to 6.3
$(s_1^2)_a$	Adjusted value of s_1^2	4.4, 6.3
s_{1g}^2	Variance of property values in group aged for time τ_g	6.1
s_{1i}^2	Variance of y_{ij} values at temperature ϑ_i	4.3, 6.2
s_2^2	Variance about regression line	6.1 to 6.3
s_a^2	Adjusted value of s^2	6.3
s_r^2	Intermediate constant	6.3
s_Y^2	Variance of Y	6.3
t	Student distributed stochastic variable	6.3
t_c	Adjusted value of t (incomplete data)	6.3
TC	Lower 95 % confidence limit of TI	4.4, 7
TC _a	Adjusted value of TC	7.1
TI	Temperature index	4.3, Clause 7
TI ₁₀	Temperature index at 10 kh	7.1
TI _a	Adjusted value of TI	7.3
TI _g	Temperature index obtained by graphical means or without defined confidence limits	7.3
x	Independent variable: reciprocal of thermodynamic temperature	
$\bar{x}$	Weighted mean value of x	6.2
X	Specified value of x for estimation of y	6.3
$\hat{X}$	Estimated value of x at specified value of y	6.3
$\hat{X}_c$	Upper 95 % confidence limit of $\hat{X}$	6.3
x_i	Reciprocal of thermodynamic temperature corresponding to ϑ_i	4.1, 6.1
$\bar{y}$	Weighted mean value of y	6.2
y	Dependent variable: logarithm of time to end-point	
$\hat{Y}$	Estimated value of y at specified value of x	6.3
Y	Specified value of y for estimation of x	6.3
$\hat{Y}_c$	Lower 95 % confidence limit of $\hat{Y}$	6.3
$\bar{y}_i$	Mean values of y_{ij} at temperature ϑ_i	4.3, 6.2
y_{ij}	Value of y corresponding to τ_{ij}	4.1, 6.1

		Subclause
$\bar{z}$	Mean value of z_g	6.1
z_g	Logarithm of ageing time for destructive tests – group g	6.1
α	Censored data coefficient for variance	4.3, 6.2
β	Censored data coefficient for variance	4.3, 6.2
ε	Censored data coefficient for variance of mean	4.3, 6.2
θ_0	Temperature 0 °C on the thermodynamic scale (273,15 K)	4.1, 6.1
$\hat{\vartheta}$	Estimate of temperature for temperature index	6.3.3
$\hat{\vartheta}_c$	Confidence limit of $\hat{\vartheta}$	6.3.3
ϑ_i	Ageing temperature for group i	4.1, 6.1
μ	Censored data coefficient for mean	4.3, 6.2
$\mu_2(x)$	Central second moment of x values	6.2, 6.3
v	Total number of property values selected at one ageing temperature	6.1
τ_f	Time selected for estimate of temperature	6.3
τ_g	Time of ageing for selected group g	6.1
τ_{ij}	Times to end-point	6.4
χ^2	χ^2 -distributed stochastic variable	6.3

4 Principles of calculations

4.1 General principles

The general calculation procedures and instructions given in Clause 6 are based on the principles set out in IEC 60493-1 [2]. These may be simplified as follows ~~(see 3.7.1 of IEC 60493-1:1974)~~:

- the relation between the mean of the logarithms of the times taken to reach the specified end-point (times to end-point) and the reciprocal of the thermodynamic (absolute) temperature is linear;
- the values of the deviations of the logarithms of the times to end-point from the linear relation are normally distributed with a variance which is independent of the ageing temperature.

The data used in the general calculation procedures are obtained from the experimental data by a preliminary calculation. The details of this calculation are dependent on the character of the diagnostic test: non-destructive, proof or destructive (see 4.2). In all cases the data comprise values of x , y , m , n and k

where

- $x_i = 1/(\vartheta_i + \theta_0)$ is the reciprocal of thermodynamic value of ageing temperature ϑ_i in °C;
- $y_{ij} = \log \tau_{ij}$ is the logarithm of the value of time (j) to end-point at temperature ϑ_i ;
- n_i is the number of y values in group number i aged at temperature ϑ_i ;
- m_i is the number of samples in group number i aged at temperature ϑ_i (different from n_i for censored data);
- k is the number of ageing temperatures or groups of y values.

NOTE Any number ~~may~~ can be used as the base for logarithms, provided consistency is observed throughout calculations. The use of natural logarithms (base e) is ~~recommended~~ beneficial, since most computer programming languages and scientific calculators have this facility.

4.2 Preliminary calculations

4.2.1 General

In all cases, the reciprocals of the thermodynamic values of the ageing temperatures are calculated as the values of x_i .

The values of y_{ij} are calculated as the values of the logarithms of the individual times to end-point τ_{ij} obtained as described below.

In many cases of non-destructive and proof tests, it is advisable for economic reasons, (for example, when the scatter of the data is high) to stop ageing before all specimens have reached the end-point, at least for some temperature groups. In such cases, the procedure for calculation on censored data (see 6.2.1.3) shall be carried out on the (x, y) data available.

Groups of complete and incomplete data or groups censored at a different point for each ageing temperature may be used together in one calculation in 6.2.1.3.

4.2.2 Non-destructive tests

Non-destructive tests (for example, loss of mass on ageing) give directly the value of the diagnostic property of each specimen each time it is measured, at the end of an ageing period. The time to end-point τ_{ij} is therefore available, either directly or by linear interpolation between consecutive measurements.

4.2.3 Proof tests

The time to end-point τ_{ij} for an individual specimen is taken as the mid-point of the ageing period immediately prior to reaching the end-point ~~(6.3.2 of IEC 60216-1:2001)~~.

4.2.4 Destructive tests

When destructive test criteria are employed, each test specimen is destroyed in obtaining a property value and its time to end-point cannot therefore be measured directly.

To enable estimates of the times to end-point to be obtained, the assumptions are made that in the vicinity of the end-point:

- a) the relation between the mean property values and the logarithm of the ageing time is approximately linear;
- b) the values of the deviations of the individual property values from this linear relation are normally distributed with a variance which is independent of the ageing time;
- c) the curves of property versus logarithm of time for the individual test specimens are straight lines parallel to the line representing the relation of a) above.

For the application of these assumptions, an ageing curve is drawn for the data obtained at each of the ageing times. The curve is obtained by plotting the mean value of property for each specimen group against the logarithm of its ageing time. If possible, ageing is continued at each temperature until at least one group mean is beyond the end-point level. An approximately linear region of this curve is drawn in the vicinity of the end-point line (see Figure D.2).

A statistical test (F -test) is carried out to decide whether deviations from linearity of the selected region are acceptable (see 6.1.4, step 4). If acceptable, then, on the same graph, points representing the properties of the individual specimens are drawn. A line parallel to the ageing line is drawn through each individual specimen data point. The estimate of the logarithm of the time to end-point for that specimen (y_{ij}) is then the value of the logarithm of time corresponding to the intersection of the line with the end-point line (Figure D.2).

With some limitations, an extrapolation of the linear mean value graph to the end-point level is permitted.

The above operations are executed numerically in the calculations detailed in 6.1.4.

4.3 Variance calculations

Commencing with the values of x and y obtained in 4.2, the following calculations are made:

For each group of y_{ij} values, the mean $\bar{y}_i$ and variance s_{ii}^2 are calculated, and from the latter the pooled variance within the groups, s_1^2 , is derived, weighting the groups according to size.

For incomplete data, the calculations have been developed from those originated by Saw [3] and given in 6.2.1.3. For the coefficients required (μ for mean, α , β for variance and ε for deriving the variance of mean from the group variance) see Annex C, Table C.1. For multiple groups, the variances are pooled, weighting according to the group size. The mean value of the group values of ε is obtained without weighting, and multiplied by the pooled variance.

NOTE The weighting according to the group size is implicit in the definition of ε , which here is equal to that originally proposed by Saw, multiplied by the group size. This makes for simpler representation in equations.

From the means $\bar{y}_i$ and the values of x_i , the coefficients a and b (the coefficients of the best fit linear representation of the relationship between x and y) are calculated by linear regression analysis.

From the regression coefficients, the values of TI and HIC are calculated. The variance of the deviations from the regression line is calculated from the regression coefficients and the group means.

4.4 Statistical tests

The following statistical tests are made:

- Fisher test for linearity (Fisher test, F -test) on destructive test data prior to the calculation of estimated times to end-point (see 4.2.4);
- variance equality (Bartlett's χ^2 -test) to establish whether the variances within the groups of y values differ significantly;
- F -test to establish whether the ratio of the deviations from the regression line to the pooled variance within the data groups is greater than the reference value F_0 , i.e. to test the validity of the Arrhenius hypothesis as applied to the test data.

In the case of data of very small dispersion, it is possible for a non-linearity to be detected as statistically significant which is of little practical importance.

In order that a result may be obtained even where the requirements of the F -test are not met ~~for this reason~~, a procedure is included as follows:

- increase the value of the pooled variance within the groups $\left(s_1^2\right)$ by the factor F/F_0 so that the F -test gives a result which is just acceptable (see 6.3.2);
- use this adjusted value $\left(s_1^2\right)_a$ to calculate the lower confidence limit TC_a of the result;
- if the lower confidence interval $(TI - TC_a)$ is found acceptable, the non-linearity is deemed to be of no practical importance (see 6.3.2);

- 4) from the components of the data dispersion, $\left(s_1^2\right)$ and $\left(s_2^2\right)$ the confidence interval of an estimate is calculated using the regression equation.

When the temperature index (TI), its lower confidence limit (TC) and the halving interval (HIC) have been calculated, (see 7.1), the result is considered acceptable if

$$TI - TC \leq 0,6 \text{ HIC} \quad (1)$$

When the lower confidence interval (TI – TC) exceeds 0,6 HIC by a small margin, a usable result may still be obtained, provided $F \leq F_0$, by substituting (TC + 0,6 HIC) for the value of TI (see Clause 7).

4.5 Results

The temperature index (TI), its halving interval (HIC) and its lower 95 % confidence limit (TC) are calculated from the regression equation, making allowance as described above for minor deviations from the ~~prescribed~~ specified results of the statistical tests.

The mode of reporting of the temperature index and halving interval is determined by the results of the statistical tests (see 7.2).

It is necessary to emphasize the need to present the thermal endurance graph as part of the report, since a single numerical result, TI (HIC), cannot present an overall qualitative view of the test data, and appraisal of the data cannot be complete without this.

5 Requirements and recommendations for valid calculations

5.1 Requirements for experimental data

5.1.1 General

The data submitted to the procedures of this document shall conform to the requirements of IEC 60216-1:~~2004~~.

5.1.2 Non-destructive tests

For most diagnostic properties in this category, groups of five specimens will be adequate. However, if the data dispersion (confidence interval, see 6.3.3) is found to be too great, more satisfactory results are likely to be obtained by using a greater number of specimens. This is particularly true if it is necessary to terminate ageing before all specimens have reached end-point.

5.1.3 Proof tests

Not more than one specimen in any group shall reach end-point during the first ageing period: if more than one group contains such a specimen, the experimental procedure should be carefully examined (see 6.1.3) and the occurrence included in the test report.

The number of specimens in each group shall be at least five, and for practical reasons the maximum number treatable is restricted to 31 (Table C.1). The recommended number for most purposes is 21.

5.1.4 Destructive tests

At each temperature, ageing should be continued until the property value mean of at least one group is above and at least one below the end-point level. In some circumstances and with appropriate limitations, a small extrapolation of the property value mean past the end-point

level may be permitted (see 6.1.4, step 4). This shall not be permitted for more than one temperature group.

5.2 Precision of calculations

Many of the calculation steps involve summing of the differences of numbers or the squares of these differences, where the differences may be small by comparison with the numbers. In these circumstances it is necessary that the calculations be made with an internal precision of at least six significant digits, and preferably more, to achieve a result precision of three significant digits. In view of the repetitive and tedious nature of the calculations, it is strongly recommended that they be performed using a programmable calculator or microcomputer, in which case internal precision of ten or more significant digits is easily available.

6 Calculation procedures

6.1 Preliminary calculations

6.1.1 Temperatures and x -values

For all types of test, express each ageing temperature in K on the thermodynamic temperature scale, and calculate its reciprocal for use as x_i :

$$x_i = 1/(\vartheta_i + \Theta_0) \quad (2)$$

where $\Theta_0 = 273,15$ K.

6.1.2 Non-destructive tests

For specimen number j of group number i , a property value after each ageing period is obtained. From these values, if necessary by linear interpolation, obtain the time to end-point and calculate its logarithm as y_{ij} .

6.1.3 Proof tests

For specimen number j of group number i , calculate the mid-point of the ageing period immediately prior to reaching the end-point and take the logarithm of this time as y_{ij} .

A time to end-point within the first ageing period shall be treated as invalid. Either:

- a) start again with a new group of specimens, or
- b) ignore the specimen and reduce the value ascribed to the number of specimens in the group (m_i) by one in the calculation for group means and variances (see 6.2.1.3).

If the end-point is reached for more than one specimen during the first period, discard the group and test a further group, paying particular attention to any critical points of experimental procedure.

6.1.4 Destructive tests

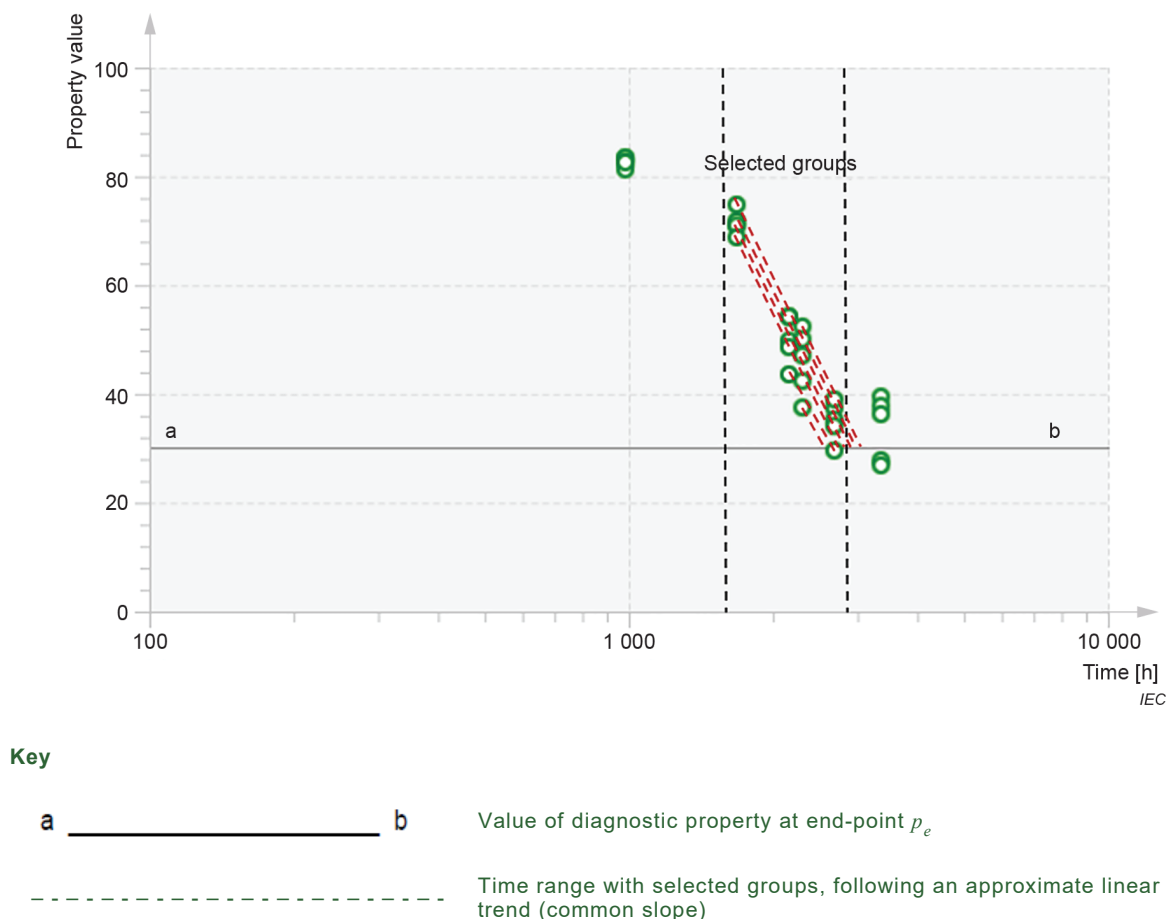
Within the groups of specimens aged at each temperature ϑ_i , carry out the following procedure in five steps:

NOTE The subscript i is omitted from the expressions in step 2 to step 4 in order to avoid confusing multiple subscript combinations in print. The calculations of ~~these subclauses shall be~~ step 2 to step 4 are carried out separately on the data from each ageing temperature.

Step 1 Calculate the mean property value for the data group obtained at each ageing time and the logarithm of the ageing time. Plot these values on a graph with the property value η

as ordinate and the logarithm of the ageing time z as abscissa (see Figure D.2). Fit by visual means a smooth curve through the mean property points (see Figure 1).

Step 2 Select a time range within which the curve so fitted is approximately linear (see step 4). Ensure that this time range includes at least three mean property values with at least one point on each side of the end-point line $p = p_e$. If this is not the case, and further measurements at greater times cannot be made (for example, because no specimens remain), a small extrapolation is permitted, subject to the conditions of step 4.



NOTE Example destructive test data N3 (arbitrary units) from Clause E.3, temperature 150 °C with small extrapolation.

Figure 1 – Example of groups selection

Let the number of selected mean values (and corresponding value groups) be r , the logarithms of the individual ageing times be z_g and the individual property values be p_{gh} , where

$g = 1 \dots r$ is the order number of the selected group tested at time τ_g ;

$h = 1 \dots n_g$ is the order number of the property value within group number g ;

n_g is the number of property values in group number g .

In most cases, the number n_g of specimens tested at each test time is identical, but this is not a necessary condition, and the calculation can be carried out with different values of n_g for different groups.

Calculate the mean value $\bar{p}_g$ and the variance s_{1g}^2 for each selected property value group.

$$\bar{p}_g = \sum_{h=1}^{n_g} p_{gh} / n_g \quad (3)$$

$$s_{1g}^2 = \left(\sum_{h=1}^{n_g} p_{gh}^2 - n_g \bar{p}_g^2 \right) / (n_g - 1) \quad (4)$$

Calculate the logarithms of τ_g :

$$z_g = \log \tau_g \quad (5)$$

Step 3 Calculate the values

$$v = \sum_{g=1}^r n_g \quad (6)$$

$$\bar{z} = \sum_{g=1}^r z_g n_g / v \quad (7)$$

$$\bar{p} = \sum_{g=1}^r \bar{p}_g n_g / v \quad (8)$$

Calculate the coefficients of the regression equation $p = a_p + b_p z$

$$b_p = \frac{\left(\sum_{g=1}^r n_g z_g \bar{p}_g - v \bar{z} \bar{p} \right)}{\left(\sum_{g=1}^r n_g z_g^2 - v \bar{z}^2 \right)} \quad (9)$$

$$a_p = \bar{p} - b_p \bar{z} \quad (10)$$

Calculate the pooled variance within the property groups

$$s_1^2 = \sum_{g=1}^r (n_g - 1) s_{1g}^2 / (v - r) \quad (11)$$

Calculate the weighted variance of the deviations of the property group means from the regression line

$$s_2^2 = \sum_{g=1}^r n_g (\bar{p}_g - \hat{p}_g)^2 / (r - 2)$$

where

$$\hat{p}_g = a_p + b_p z_g \quad (13)$$

This may also be expressed as

$$s_2^2 = \left[\left(\sum_{g=1}^r n_g \bar{p}_g^2 - v \bar{p}^2 \right) - b_p \left(\sum_{g=1}^r n_g z_g \bar{p}_g - v \bar{z} \bar{p} \right) \right] / (r-2) \quad (14)$$

Step 4 Make the F -test for non-linearity at significance level 0,05 by calculating

$$F = s_2^2 / s_1^2 \quad (15)$$

If the calculated value of F exceeds the tabulated value F_1 with $f_n = r - 2$ and $f_d = v - r$ degrees of freedom (see Table C.2)

$$F_1 = F(0,95, r - 2, v - r)$$

change the selection in step 2 and repeat the calculations.

If it is not possible to satisfy the F -test on the significance level 0,05 with $r \geq 3$, make the F -test at a significance level 0,005 by comparing the calculated value of F with the tabulated value F_2 with $f_n = r - 2$ and $f_d = v - r$ degrees of freedom (see Table C.3).

$$F_2 = F(0,995, r - 2, v - r)$$

If the test is satisfied at this level, the calculations may be continued, but the adjustment of TI according to 7.3.2 is not permitted.

If the F -test on significance level 0,005 (i.e. $F \leq F_2$) cannot be satisfied, or the property points plotted according to step 1 are all on the same side of the end-point line, an extrapolation may be permitted, subject to the following condition.

If the F -test on significance level 0,05 can be met for a range of values (with $r \geq 3$) where all mean values $\bar{p}_g$ are on the same side of the end-point value p_e , an extrapolation may be made, provided that the absolute value of the difference between the end-point value p_e and the mean value $\bar{p}_g$ closest to the end-point (usually $\bar{p}_r$) is less than 0,25 of the absolute value of the difference $(\bar{p}_1 - \bar{p}_r)$.

In this case, calculations can be continued, but again it is not permitted to carry out the adjustment of TI according to 7.3.2.

Step 5 For each value of property in each of the selected groups, calculate the logarithm of the estimated time to end-point:

$$y_{ij} = z_g - (p_{gh} - p_e) / b_p \quad (16)$$

$$n_i = v \quad (17)$$

where

$j = 1 \dots n_i$ is the order number of the y -value in the group of estimated y -values at temperature ϑ_i , and the logarithm of the ageing time z_g .

The n_i values of y_{ij} are the log (time) values to be used in the calculations of 6.2.1.

6.1.5 Incomplete data

In the case of incomplete data, arrange each group of y values in ascending order (see 3.1.1).

6.2 Main calculations

6.2.1 Calculation of group means and variances

6.2.1.1 General

Calculate the mean and variance of the group of y -values, y_{ij} , obtained at each temperature ϑ_i .

6.2.1.2 Complete data

For tests where the data are complete (i.e. not censored) the conventional Equations (18) and (19) may be used:

$$\bar{y}_i = \sum_{j=1}^{n_i} y_{ij} / n_i \quad (18)$$

$$s_{ii}^2 = \left(\sum_{j=1}^{n_i} y_{ij}^2 - n_i \bar{y}_i^2 \right) / (n_i - 1) \quad (19)$$

Alternatively, Equations (23) and (24) for incomplete data (6.2.1.3) may be used, although they are much less convenient for this purpose. The coefficients are then given by the following values:

$$\alpha_i = 1 / (n_i - 1) \quad (20)$$

$$\beta_i = \frac{-1}{n_i(n_i - 1)} \quad (21)$$

$$\mu_i = 1 - 1 / n_i \quad (22)$$

NOTE These expressions are derived by simple algebra. If the expression for mean or variance (see Equations (18) and (19)) is equated to that obtained using Equations (23) and (24), the single unknown in each resulting equation can be made the subject of the equation, resulting in the expressions of Equations (20) to (22). The value of ε is obviously 1.

For a worked example see Annex D, Table D.2.

6.2.1.3 Censored data

Instead of Equations (18) and (19), the following equations shall be used:

$$\bar{y}_i = (1 - \mu_i) y_{in_i} + \mu_i \sum_{j=1}^{n_i-1} \frac{y_{ij}}{(n_i - 1)} \quad (23)$$

$$s_{1i}^2 = \alpha_i \sum_{j=1}^{n_i-1} (y_{in_i} - y_{ij})^2 + \beta_i \left[\sum_{j=1}^{n_i-1} (y_{in_i} - y_{ij}) \right]^2 \quad (24)$$

The values of μ_i , α_i , and β_i shall be read from the appropriate lines of Table C.1. Where data are partially censored (i.e. one or more temperature groups is complete and one or more censored) the values shall be derived using Equations (20) to (22).

For a worked example see Annex D, Table D.1.

6.2.2 General means and variances

Calculate the total number of y_{ij} values, N , the weighted mean value of x , $(\bar{x})$, and the weighted mean value of y , $(\bar{y})$:

$$N = \sum_{i=1}^k n_i \quad (25)$$

$$\bar{x} = \sum_{i=1}^k n_i x_i / N \quad (26)$$

$$\bar{y} = \sum_{i=1}^k n_i \bar{y}_i / N \quad (27)$$

For censored data, calculate the total number of test specimens:

$$M = \sum_{i=1}^k m_i \quad (28)$$

For complete data, $M = N$.

For censored data, read the values of ε_i from Table C.1. For complete data, or if $n_i = m_i$ in partially censored data, the value of ε_i shall be 1.

Calculate the general mean variance factor:

$$\varepsilon = \sum_{i=1}^k \varepsilon_i / k \quad (29)$$

Calculate the pooled variance within the data groups:

$$s_1^2 = \varepsilon \sum_{i=1}^k (n_i - 1) s_{1i}^2 / (N - k) \quad (30)$$

Calculate the second central moment of the x values:

$$\mu_2(x) = \frac{\left(\sum_{i=1}^k n_i x_i^2 - N \bar{x}^2 \right)}{N} \quad (31)$$

6.2.3 Regression calculations

In the expression for the regression line:

$$y = a + bx \quad (32)$$

Calculate the slope:

$$b = \frac{\left(\sum_{i=1}^k n_i x_i \bar{y}_i - N \bar{x} \bar{y} \right)}{\left(\sum_{i=1}^k n_i x_i^2 - N \bar{x}^2 \right)} \quad (33)$$

the intercept on the y -axis

$$a = \bar{y} - b \bar{x} \quad (34)$$

and the square of the correlation coefficient:

$$r^2 = \frac{\left(\sum_{i=1}^k n_i x_i \bar{y}_i - N \bar{x} \bar{y} \right)^2}{\left(\sum_{i=1}^k n_i x_i^2 - N \bar{x}^2 \right) \left(\sum_{i=1}^k n_i y_i^2 - N \bar{y}^2 \right)} \quad (35)$$

Calculate the variance of the deviations of the y -means from the regression line:

$$s_2^2 = \sum_{i=1}^k \frac{n_i (\bar{y}_i - \hat{Y}_i)^2}{(k-2)}, \quad \hat{Y}_i = a + b x_i \quad (36)$$

or

$$s_2^2 = \frac{(1-r^2)}{k-2} \left(\sum_{i=1}^k n_i \bar{y}_i^2 - N \bar{y}^2 \right) \quad (37)$$

6.3 Statistical tests

6.3.1 Variance equality test

Calculate the value of Bartlett's χ^2 function:

$$\chi^2 = \frac{\ln q}{c} \left[(N-k) \log_q \frac{s_1^2}{\varepsilon} - \sum_{i=1}^k (n_i-1) \log_q s_{1i}^2 \right] \quad (38)$$

where

$$c = 1 + \frac{\left(\sum_{i=1}^k \frac{1}{n_i-1} - \frac{1}{N-k} \right)}{3(k-1)} \quad (39)$$

q is the base of the logarithms used in this equation. It need not be the same as that used in the calculations elsewhere in Clause 6.

If $q = 10$, $\ln q = 2,303$, if $q = e$, $\ln q = 1$.

Compare the value of χ^2 with the tabulated value for $f = (k-1)$ degrees of freedom (Table C.5). If the value of χ^2 is greater than the value tabulated for a significance level of 0,05, report the value of χ^2 and the significance level tabulated for the highest value less than χ^2 . Alternatively, if both χ^2 and its significance level are calculated by a computer program, report these.

6.3.2 Linearity test (F -test)

The variance of the deviations from the regression line s_2^2 is compared with the pooled variance within the k groups of measurements s_1^2 by the F -test at a significance level of 0,05.

Calculate the ratio

$$F = s_2^2 / s_1^2 \quad (40)$$

and compare its value with the tabulated value F_0 with $f_n = k-2$ and $f_d = N-k$ degrees of freedom (Table C.2 and Table C.3).

$$F_0 = F(0,95, k-2, N-k)$$

a) If $F \leq F_0$, calculate the pooled variance estimate

$$s^2 = \frac{(N-k)s_1^2 + (k-2)s_2^2}{(N-2)} \quad (41)$$

b) If $F > F_0$, adjust s_1^2 to $(s_1^2)_a = s_1^2 (F / F_0)$ and calculate an adjusted value of s^2

$$s_a^2 = \frac{(N-k)(s_1^2)_a + (k-2)s_2^2}{(N-2)} \quad (42)$$

6.3.3 Confidence limits of X and Y estimates

Obtain the tabulated value of Student's t with $N - 2$ degrees of freedom at a confidence level of 0,95, $t_{0,95,N-2}$ (Table C.4).

Calculate the value of t (t_c) corrected for the amount of censoring of the data:

$$t_c = \left(\frac{1}{t_{0,95,N-2}} - \frac{(1-N/M)}{(N/8+4,5)} \right)^{-1} \quad (43)$$

a) Y -estimates

Calculate the estimated value of Y corresponding to the given X and its lower 95 % confidence limit:

$$\hat{Y}_c = \hat{Y} - t_c s_Y, \quad \hat{Y} = a + bx \quad (44)$$

$$s_Y^2 = \frac{s^2}{N} \left[1 + \frac{(X - \bar{x})^2}{\mu_2(x)} \right] \quad (45)$$

For the confidence limit curve of the thermal endurance graph (see 6.4), Y_c is calculated for several (X, Y) pairs of values over the range of interest, and the curve drawn through the points (X, Y_c) plotted on the graph.

If $F > F_0$ the value of s^2 shall be replaced by s_a^2 (Equation (42)).

b) X -estimates

Calculate the value of $\hat{X}$ and its upper 95 % confidence limit, corresponding to a time to end-point τ_f :

$$\hat{X}_c = \bar{x} + \frac{(Y - \bar{y})}{b_r} + \frac{t_c s_r}{b_r} \quad (46)$$

$$Y = \log \tau_f : \hat{X} = (Y - a) / b \quad (47)$$

$$b_r = b - \frac{t_c^2 s^2}{Nb \mu_2(x)} \quad (48)$$

$$s_r^2 = \frac{s^2}{N} \left(\frac{b_r}{b} + \frac{(\hat{X} - \bar{x})^2}{\mu_2(x)} \right) \quad (49)$$

The temperature estimate and its lower 95 % confidence limit shall be calculated from the corresponding X estimate and its upper confidence limit:

$$\hat{\vartheta} = \frac{1}{\hat{X}} - \Theta_0, \quad \hat{\vartheta}_c = \frac{1}{\hat{X}_c} - \Theta_0 \quad (50)$$

6.4 Thermal endurance graph

When the regression line has been established, it is drawn on the thermal endurance graph, i.e. a graph with $y = \log(\tau)$ as ordinate and $x = 1/(\vartheta + \Theta_0)$ as abscissa. Usually x is plotted as increasing from right to left and the corresponding values of ϑ in degrees Celsius (°C) are marked on this axis (see Figure D.1a) and Figure D.1b)). Special graph paper is obtainable for this purpose.

Alternatively, a computer program executing this calculation may include a subroutine to plot the graph on the appropriate non-linear scale.

The individual values $y_{ij} = \log(\tau_{ij})$ and the mean values $\bar{y}_i$ obtained as in 6.2.1 are plotted on the graph at the corresponding values of x_i :

$$x_i = 1/(\vartheta_i + \Theta_0) \quad (51)$$

The thermal endurance graph may be completed by drawing the lower 95 % confidence curve (see 6.3.3).

7 Calculation and requirements for results

7.1 Calculation of thermal endurance characteristics

Using the regression equation

$$y = a + bx \quad (52)$$

(the coefficients a and b being calculated according to 6.2.3), calculate the temperature in degrees Celsius (°C) corresponding to a time to end-point of 20 kh. The numerical value of this temperature is the temperature index, TI.

Calculate by the same method the numerical value of the temperature corresponding to a time to end-point of 10 kh, TI_{10} . The halving interval HIC is:

$$HIC = TI_{10} - TI \quad (53)$$

Calculate by the method of 6.3.3 b), with $Y = \log 20\,000$, the lower 95 % confidence limit of TI: TC or TC_a if the adjusted value s_a^2 is used.

Determine the value of $(TI - TC)/HIC$ or $(TI - TC_a)/HIC$.

Plot the thermal endurance graph (see 6.4).

7.2 Summary of statistical tests and reporting

For a summary of statistical tests and reporting see Annex B. In Table B.1, if the condition in the column headed "Test or action" is not met, the action is as indicated in the final column. If the condition is met, the action is as indicated at the next step. The same sequence is indicated in the decision flow chart for thermal endurance calculations, see Figure A.1 in Annex A.

7.3 Reporting of results

7.3.1 If the value of $(TI - TC)/HIC$ is $\leq 0,6$, the test result shall be reported in the format

$$TI (HIC): xxx (xx,x) \quad (54)$$

in accordance with IEC 60216-1:2004.

7.3.2 If $0,6 < (TI - TC)/HIC \leq 1,6$ and at the same time, $F \leq F_0$ (see 6.3.2) the value

$$TI_a = TC + 0,6 HIC \quad (55)$$

together with HIC shall be reported as TI (HIC): xxx (xx,x).

7.3.3 In all other cases the result shall be reported in the format

$$TI_g = \dots, \quad HIC_g = \dots \quad (56)$$

7.3.4 If a time different from 20 000 h has been used for deriving TI, the relevant time expressed in kh shall be stated, followed by "kh". The format of TI is then:

$$TI \text{ xx kh (HIC): } xxx (xx,x) \quad (57)$$

and correspondingly for TI_a and TI_g .

8 Test report

The test report shall include

- a description of the tested material including dimensions and any conditioning of the specimens;
- the property investigated, the chosen end-point, and, if it was required to be determined, the initial value of the property;
- the test method used for the determination of the property (for example, by reference to an IEC publication);
- any relevant information on the test procedure, for example, ageing environment;
- the individual test temperatures, with the appropriate data for the test type;
 - for non-destructive tests, the individual times to end-point;
 - for proof tests, the numbers and durations of the ageing cycles, with the numbers of specimens reaching end-point during the cycles;
 - for destructive tests, the ageing times and individual property values, with the graphs of variation of property with ageing time;
- the thermal endurance graph;
- the temperature index and halving interval reported in the format defined in 7.3;

- h) the values of χ^2 and P if required by 6.3.1;
- i) first-cycle failures in accordance with 5.1.3.

Annex A (normative)

Decision flow chart

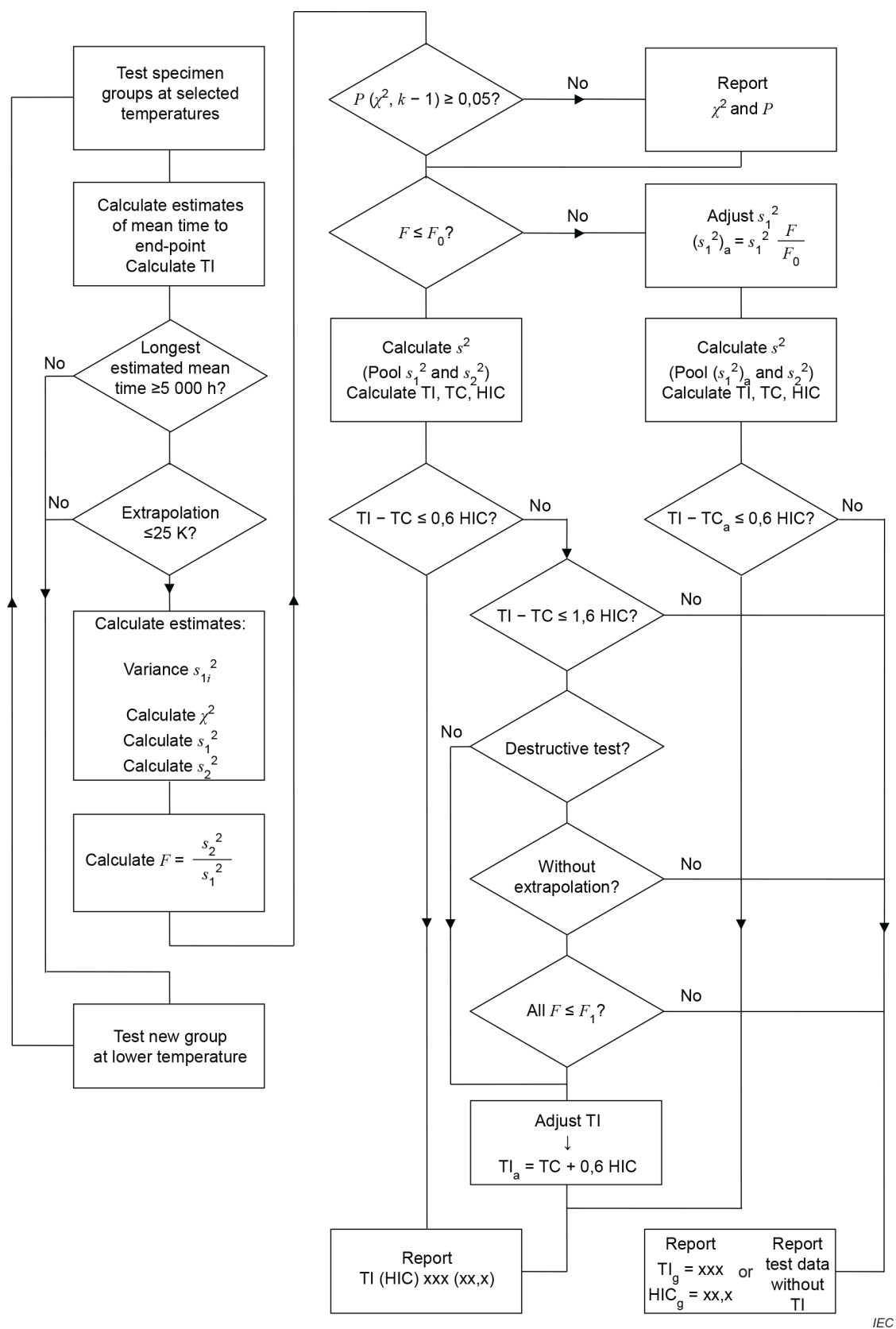


Figure A.1 – Decision flow chart

Annex B (normative)

Decision table

Table B.1 – Decisions and actions according to tests

Step	Test or action ^a	Reference	Action if "NO" in test
1	Longest mean time to end-point $\geq 5\,000$ h	5.5 of IEC 60216-1:2013	Go to step 15
2	Extrapolation ≤ 25 K	5.5 of IEC 60216-1:2013	Go to step 15
3	$P(\chi^2, f) \geq 0,05$	6.3.1	Report χ^2 and P Go to step 4
4	$F \leq F_0$	6.3.2	Go to step 12
5	TI – TC $\leq 0,6$ HIC	7.3	Go to step 7
6	Report TI (HIC): xxx (xx,x)	7.3	
7	TI – TC $\leq 1,6$ HIC	7.3	Go to step 14
8	Destructive test criteria used	6.1.4, step 4	Go to step 11
9	Were data processed without extrapolation?	6.1.4, step 4	Go to step 14
10	Were all values of $F \leq F_1$?	6.1.4, step 4	Go to step 14
11	Report TI_a = TC + 0,6 HIC as TI (HIC): ... (..)	7.3	
12	TI – TC _a $\leq 0,6$ HIC	6.3.2	Go to step 14
13	Report TI (HIC): xxx (xx,x)	7.3	
14	Report TI_g = xxx, HIC_g = xx,x	7.3	
15	Test new group at a lower temperature		

^a An action is indicated in bold type.

Annex C (informative)

Statistical tables

Table C.1 – Coefficients for censored data calculations

m	n	α	β	μ	ε
5	3	614,470 506 172 8	–100,380 198 559 7	0,000 000 000 0	860,448 288 888 9
5	4	369,315 310 001 2	–70,671 293 489 9	472,493 715 084 2	874,074 589 444 7
6	4	395,414 213 960 5	–58,270 118 352 3	222,691 521 846 8	835,765 030 646 5
6	5	272,528 723 805 2	–44,098 885 093 6	573,512 612 381 5	887,106 668 142 6
7	4	415,588 035 156 3	–46,540 155 273 4	0,000 000 000 0	841,774 673 437 5
7	5	289,191 447 008 9	–38,006 043 810 7	364,264 215 381 5	837,368 126 781 9
7	6	215,514 679 687 5	–30,136 366 210 9	642,234 560 615 2	898,799 440 429 7
8	5	302,255 954 330 4	–32,045 551 009 5	173,745 192 558 9	823,132 502 297 0
8	6	227,132 033 490 0	–26,714 924 272 0	462,394 689 655 8	845,589 167 341 7
8	7	178,019 204 785 1	–21,890 905 564 9	692,008 291 149 8	908,717 523 176 5
9	5	312,981 200 000 0	–26,384 270 000 0	0,000 000 000 0	830,502 200 000 0
9	6	236,385 800 000 0	–23,298 610 000 0	296,052 630 000 0	821,317 260 000 0
9	7	186,640 100 000 0	–19,789 890 000 0	534,460 180 000 0	855,209 670 000 0
9	8	151,512 000 000 0	–16,614 080 000 0	729,711 990 000 0	917,058 320 000 0
10	6	244,119 156 089 0	–20,004 774 072 9	142,373 900 284 7	815,821 088 682 6
10	7	193,620 588 004 7	–17,666 360 481 4	386,952 601 761 8	825,759 043 775 3
10	8	158,230 060 832 0	–15,243 793 158 2	589,634 132 230 7	864,621 929 488 4
10	9	131,803 038 236 3	–13,034 762 797 6	759,253 366 384 2	924,098 919 253 1
11	6	250,685 932 098 8	–16,853 035 429 5	0,000 000 000 0	822,972 912 731 5
11	7	199,469 546 848 7	–15,583 654 537 4	249,259 995 307 9	812,630 898 625 4
11	8	163,699 612 133 7	–13,837 118 255 7	457,209 096 574 3	832,548 816 179 9
11	9	137,229 924 382 7	–12,100 190 779 3	633,229 292 467 8	873,335 541 088 0
11	10	116,591 321 046 4	–10,496 956 971 8	783,017 794 944 4	930,088 037 299 4
12	7	204,534 992 422 9	–13,576 711 024 4	120,574 855 492 1	810,980 305 184 0
12	8	168,329 219 660 0	–12,443 988 079 5	332,551 955 767 4	814,726 902 133 0
12	9	141,642 522 967 4	–11,121 946 667 6	513,149 341 538 3	840,062 504 581 7
12	10	121,088 479 244 8	–9,835 950 775 4	668,539 265 126 9	881,240 032 296 2
12	11	104,506 080 037 5	–8,633 379 584 8	802,544 129 235 6	935,228 223 004 9
13	7	208,940 611 828 4	–11,645 614 282 7	0,000 000 000 0	817,592 186 339 0
13	8	172,346 425 140 0	–11,086 526 420 1	215,202 335 515 1	807,269 942 297 3
13	9	145,417 868 782 7	–10,147 234 899 2	399,323 652 033 8	819,318 009 509 0
13	10	124,737 192 422 5	–9,130 008 532 8	558,746 158 905 5	847,590 859 692 6
13	11	108,301 805 863 3	–8,151 081 966 3	697,715 856 087 3	888,359 118 118 9
13	12	94,679 614 970 6	–7,225 211 787 4	818,869 702 877 8	939,679 419 663 9
NOTE α , β , μ and ε are all in units of 1×10^{-3} .					

m	n	α	β	μ	ε
14	8	175,901 842 209 0	–9,774 682 609 8	104,554 351 6980	807,510 679 332 7
14	9	148,706 654 321 0	–9,189 143 374 5	291,514 076 5844	807,927 394 074 1
14	10	127,881 689 678 0	–8,422 450 692 9	454,060 900 2065	825,039 882 806 3
14	11	111,381 769 972 9	–7,626 697 130 2	596,623 583 2604	854,823 830 446 3
14	12	97,927 824 691 4	–6,863 605 925 9	722,224 918 8477	894,761 415 308 6
14	13	86,536 307 523 1	–6,135 526 882 2	832,719 252 4487	943,566 894 197 6
15	8	179,051 340 576 2	–8,507 153 076 2	0,000 000 000 0	813,556 818 212 9
15	9	151,627 445 154 0	–8,256 692 317 2	189,315 731 952 4	803,657 234 619 6
15	10	130,638 736 267 4	–7,722 878 628 9	354,390 697 378 5	810,944 133 571 3
15	11	114,045 779 796 6	–7,097 395 186 3	499,752 662 880 0	831,192 011 019 8
15	12	100,571 888 183 6	–6,464 822 448 7	628,585 928 820 5	861,635 264 831 5
15	13	89,346 612 386 1	–5,857 855 430 9	743,099 738 270 9	900,526 205 166 5
15	14	79,679 695 687 0	–5,275 139 366 7	844,614 393 863 7	946,988 901 484 6
16	9	154,251 868 908 5	–7,352 734 812 9	92,286 597 662 4	804,890 154 565 0
16	10	133,092 655 230 3	–7,037 490 348 3	259,470 300 502 6	803,417 948 946 8
16	11	116,397 190 014 4	–6,571 880 798 3	407,107 444 694 2	815,225 911 951 0
16	12	102,862 022 796 0	–6,059 026 278 1	538,470 351 887 8	837,405 616 491 7
16	13	91,647 511 041 4	–5,548 523 480 8	655,915 300 372 3	867,986 413 358 9
16	14	82,133 483 929 8	–5,057 399 050 1	761,089 730 468 5	905,730 213 237 4
16	15	73,828 121 853 0	–4,583 976 609 5	854,940 091 579 0	950,022 975 937 6
17	9	156,610 475 842 1	–6,476 460 274 5	0,000 000 000 0	810,419 011 339 7
17	10	135,306 977 099 1	–6,369 862 523 4	168,979 564 112 2	801,066 074 880 2
17	11	118,497 493 348 7	–6,054 318 734 9	318,520 886 724 6	805,318 062 739 4
17	12	104,894 493 937 6	–5,654 673 321 1	451,948 602 041 3	820,151 369 194 9
17	13	93,641 407 943 0	–5,231 044 716 6	571,696 183 063 2	843,486 177 866 0
17	14	84,157 807 920 1	–4,813 301 797 2	679,548 045 681 0	873,880 335 131 3
17	15	75,987 691 268 4	–4,410 061 254 4	776,751 703 284 6	910,442 891 855 0
17	16	68,776 185 039 1	–4,020 399 239 0	863,986 627 489 9	952,730 802 137 3
18	10	137,319 690 100 1	–5,720 840 122 8	82,592 591 372 5	802,835 654 113 7
18	11	120,396 550 341 6	–5,547 705 212 4	233,762 521 677 5	800,258 419 848 3
18	12	106,717 957 142 0	–5,254 869 270 6	368,923 773 992 3	808,487 834 862 6
18	13	95,417 915 235 3	–4,913 521 939 3	490,558 207 272 5	825,357 995 890 6
18	14	85,912 982 279 7	–4,560 657 091 3	600,519 390 056 5	849,333 989 100 0
18	15	77,784 669 734 1	–4,214 502 545 1	700,184 082 553 0	879,339 504 407 5
18	16	70,690 282 324 6	–3,879 229 298 2	790,508 013 638 6	914,725 238 932 5
18	17	64,370 690 391 9	–3,554 819 683 0	871,976 998 724 4	955,161 899 362 0
19	10	139,149 625 000 0	–5,090 018 125 0	0,000 000 000 0	807,909 618 750 0
19	11	122,130 237 500 0	–5,053 480 937 5	152,583 847 187 5	799,119 842 812 5
19	12	108,370 456 250 0	–4,861 845 937 5	289,231 816 562 5	801,360 262 500 0
19	13	97,018 825 000 0	–4,598 604 062 5	412,455 355 937 5	812,396 743 437 5
19	14	87,480 900 000 0	–4,306 963 437 5	524,150 835 000 0	830,631 200 000 0
19	15	79,344 375 000 0	–4,010 505 625 0	625,758 673 437 5	854,902 153 125 0
19	16	72,297 331 250 0	–3,720 423 750 0	718,357 160 937 5	884,394 535 000 0
19	17	66,078 087 307 1	–3,438 596 529 0	802,684 840 281 0	918,630 065 987 3
19	18	60,495 123 456 8	–3,165 752 232 4	879,085 324 747 8	957,356 388 289 5

NOTE α , β , μ and ε are all in units of 1×10^{-3} .

m	n	α	β	μ	ε
20	11	123,720 724 690 7	–4,571 903 849 4	74,739 952 689 8	801,179 011 626 4
20	12	109,882 247 113 5	–4,477 035 548 8	212,683 662 366 2	797,948 281 173 8
20	13	98,473 823 238 1	–4,287 939 233 2	337,273 238 927 2	803,677 721 219 6
20	14	88,899 384 983 5	–4,054 686 452 3	450,433 824 821 7	816,713 086 237 3
20	15	80,740 119 043 3	–3,804 781 413 9	553,643 825 389 0	835,843 732 032 9
20	16	73,694 598 203 3	–3,553 609 281 2	648,041 435 461 8	860,173 538 768 6
20	17	67,524 357 313 6	–3,308 057 336 8	734,481 449 050 2	889,078 451 004 8
20	18	62,027 051 120 2	–3,068 869 206 8	813,538 080 764 9	922,202 807 269 0
20	19	57,059 331 163 4	–2,837 292 341 8	885,449 527 637 9	959,347 069 438 1
21	11	125,180 504 268 8	–4,102 787 081 4	0,000 000 000 0	805,857 221 187 1
21	12	111,274 858 447 6	–4,101 040 726 7	139,085 614 417 5	797,605 437 620 2
21	13	99,807 327 895 4	–3,982 732 403 3	264,868 574 231 4	798,472 591 530 8
21	14	90,192 703 419 5	–3,805 109 379 9	379,291 522 952 8	806,763 785 482 7
21	15	82,006 895 840 0	–3,599 696 102 2	483,858 887 792 2	821,225 961 821 7
21	16	74,946 575 450 5	–3,384 632 353 4	579,743 276 288 7	840,921 205 171 3
21	17	68,784 814 683 3	–3,170 115 019 5	667,856 652 288 5	865,147 773 729 6
21	18	63,335 741 064 5	–2,960 377 331 2	748,883 265 049 3	893,423 810 538 9
21	19	58,441 207 543 7	–2,755 639 453 1	823,271 305 249 0	925,482 471 420 9
21	20	53,992 487 284 4	–2,557 464 289 7	891,180 261 676 2	961,160 991 780 3
22	12	112,562 249 376 3	–3,733 942 654 3	68,249 899 230 9	799,813 456 437 8
22	13	101,038 358 565 9	–3,683 676 456 5	195,088 306 477 2	796,200 753 033 8
22	14	91,380 460 456 0	–3,559 186 854 7	310,616 120 426 8	800,134 506 430 3
22	15	83,165 536 713 6	–3,396 328 281 6	416,355 757 299 6	810,324 221 550 3
22	16	76,085 740 665 3	–3,215 758 572 9	513,502 991 079 9	825,800 027 083 5
22	17	69,915 447 090 2	–3,029 759 742 9	603,001 704 223 8	845,821 800 161 5
22	18	64,479 595 568 7	–2,845 164 997 2	685,591 100 846 1	869,833 743 632 6
22	19	59,631 110 650 6	–2,664 566 007 3	761,823 205 764 4	897,461 327 883 1
22	20	55,245 182 109 6	–2,487 974 468 3	832,048 472 778 3	928,502 565 642 9
22	21	51,238 188 548 3	–2,317 111 964 4	896,367 325 561 6	962,820 644 707 6
23	12	113,753 114 824 5	–3,375 661 462 4	0,000 000 000 0	804,147 498 958 3
23	13	102,180 592 915 5	–3,391 035 253 9	127,779 779 922 2	796,393 802 656 5
23	14	92,478 714 378 2	–3,317 568 453 9	244,286 853 730 7	796,302 286 039 9
23	15	84,232 065 039 4	–3,195 430 410 7	351,054 316 620 9	802,558 394 510 6
23	16	77,130 671 601 8	–3,047 956 733 6	449,294 709 215 6	814,162 196 954 9
23	17	70,946 228 319 6	–2,889 128 739 5	539,972 715 916 5	830,348 307 946 7
23	18	65,506 773 051 7	–2,727 427 071 3	623,857 738 790 5	850,523 990 198 2
23	19	60,674 543 903 7	–2,567 467 256 7	701,554 759 605 1	874,245 203 107 7
23	20	56,331 747 664 1	–2,410 824 975 7	773,511 902 625 0	901,219 290 370 3
23	21	52,378 971 244 4	–2,257 458 807 1	840,003 110 782 9	931,291 926 725 9
23	22	48,750 967 330 6	–2,109 138 220 4	901,084 347 837 2	964,344 871 037 5
NOTE α , β , μ and ε are all in units of 1×10^{-3} .					

m	n	α	β	μ	ε
24	13	103,243 347 881 9	–3,104 836 192 9	62,796 296 393 4	798,667 677 335 2
24	14	93,499 899 161 3	–3,080 597 976 9	180,179 665 703 1	794,841 653 545 8
24	15	85,219 822 400 0	–2,997 560 243 2	287,859 544 598 4	797,456 360 640 0
24	16	78,094 848 041 1	–2,881 836 788 2	387,060 117 227 4	805,486 809 924 8
24	17	71,894 122 886 0	–2,749 128 344 1	478,761 192 082 8	818,144 554 412 7
24	18	66,444 704 304 8	–2,609 144 801 8	563,751 195 249 6	834,815 363 694 5
24	19	61,612 691 563 3	–2,467 829 940 8	642,664 080 518 4	855,018 951 419 5
24	20	57,287 908 800 0	–2,328 319 155 2	715,998 983 808 0	878,398 189 184 0
24	21	53,375 054 194 3	–2,191 560 665 7	784,121 449 345 2	904,723 395 218 0
24	22	49,794 229 859 7	–2,057 530 704 7	847,245 055 046 6	933,875 440 847 1
24	23	46,493 767 000 5	–1,927 973 165 6	905,392 264 548 8	965,749 572 297 4
25	13	104,232 885 613 2	–2,825 050 151 1	0,000 000 000 0	802,701 301 544 1
25	14	94,453 143 892 0	–2,848 396 832 3	118,172 687 883 0	795,402 438 793 7
25	15	86,139 657 001 5	–2,803 071 458 2	226,670 678 353 7	794,630 540 737 9
25	16	78,989 382 024 2	–2,717 860 940 0	326,724 816 668 1	799,346 648 224 3
25	17	72,770 823 174 3	–2,610 275 202 5	419,326 141 499 8	808,740 684 495 8
25	18	67,309 061 167 5	–2,491 180 827 8	505,276 666 008 1	822,180 644 398 6
25	19	62,470 447 683 2	–2,367 461 361 3	585,228 093 541 2	839,166 598 102 9
25	20	58,148 780 157 1	–2,243 330 943 3	659,707 591 544 9	859,305 787 476 1
25	21	54,254 772 141 2	–2,120 927 932 5	729,129 747 248 1	882,309 499 023 6
25	22	50,710 634 469 5	–2,000 815 184 9	793,793 828 693 6	907,996 803 098 9
25	23	47,451 582 466 4	–1,883 013 652 0	853,865 474 685 9	936,274 654 862 4
25	24	44,436 084 435 5	–1,769 195 964 9	909,341 937 225 5	967,048 258 250 8
26	14	95,344 951 652 9	–2,620 976 346 5	58,149 346 185 6	797,692 105 701 4
26	15	87,000 011 060 1	–2,612 140 016 6	167,386 495 331 3	793,760 045 585 9
26	16	79,823 556 347 0	–2,556 356 239 1	268,207 052 434 6	795,387 979 746 5
26	17	73,585 712 493 3	–2,472 950 569 9	361,609 787 669 1	801,749 088 924 2
26	18	68,110 255 082 3	–2,373 992 685 8	448,407 042 544 8	812,194 055 876 7
26	19	63,262 342 617 2	–2,267 184 422 5	529,262 848 347 4	826,208 688 490 2
26	20	58,936 792 778 9	–2,157 486 675 1	604,721 224 122 4	843,381 329 569 6
26	21	55,048 042 936 4	–2,047 914 524 5	675,223 991 890 1	863,388 484 003 6
26	22	51,522 935 221 6	–1,939 929 951 9	741,117 446 777 6	885,995 805 567 7
26	23	48,297 466 480 8	–1,833 861 504 0	802,647 219 753 4	911,060 297 192 9
26	24	45,318 643 413 4	–1,729 780 273 4	859,940 670 651 4	938,508 290 093 4
26	25	42,552 583 209 7	–1,629 261 554 1	912,976 149 171 2	968,252 478 710 8
NOTE α , β , μ and ε are all in units of 1×10^{-3} .					

m	n	α	β	μ	ε
27	14	96,179 952 415 7	–2,398 330 795 0	0,000 000 000 0	801,462 097 378 7
27	15	87,807 233 951 0	–2,424 824 879 2	109,908 447 002 3	794,576 240 291 9
27	16	80,604 908 585 4	–2,397 518 691 3	211,422 762 924 1	793,315 879 921 6
27	17	74,346 695 559 0	–2,337 441 207 8	305,545 934 609 8	796,846 398 872 9
27	18	68,856 363 573 0	–2,257 899 609 6	393,098 036 470 7	804,502 877 056 5
27	19	63,997 827 886 1	–2,167 418 784 4	474,752 453 008 4	815,757 617 370 5
27	20	59,665 408 004 9	–2,071 567 175 3	551,064 568 013 3	830,186 793 990 7
27	21	55,774 966 628 5	–1,973 967 843 6	622,492 414 801 3	847,448 219 212 0
27	22	52,256 650 509 1	–1,876 793 609 6	689,408 781 849 6	867,273 947 639 7
27	23	49,049 953 893 3	–1,781 045 141 6	752,104 268 197 1	889,473 159 384 9
27	24	46,101 825 203 4	–1,686 910 857 4	810,780 782 969 1	913,932 486 779 3
27	25	43,368 537 622 7	–1,594 507 505 2	865,534 983 389 9	940,592 671 976 3
27	26	40,822 044 246 6	–1,505 300 292 0	916,331 145 646 2	969,372 165 667 9
28	15	88,566 025 912 5	–2,241 132 818 2	54,142 478 801 1	796,851 152 756 7
28	16	81,339 370 105 7	–2,241 441 397 0	156,288 646 095 9	792,882 432 527 5
28	17	75,060 373 981 7	–2,203 940 607 0	251,064 797 130 7	793,761 059 837 3
28	18	69,554 150 197 3	–2,143 147 145 2	339,296 241 903 9	798,810 986 602 2
28	19	64,683 999 121 8	–2,068 438 756 0	421,663 641 341 7	807,491 370 564 5
28	20	60,343 313 763 4	–1,985 966 124 9	498,730 858 604 3	819,367 766 769 3
28	21	56,447 978 854 4	–1,899 822 629 5	570,966 583 043 2	834,086 917 498 6
28	22	52,929 720 988 9	–1,812 683 033 9	638,759 337 098 7	851,362 210 800 1
28	23	49,730 866 708 1	–1,726 122 211 5	702,425 476 425 6	870,970 736 294 0
28	24	46,800 965 426 3	–1,640 824 982 1	762,209 793 536 6	892,756 725 488 4
28	25	44,095 734 093 7	–1,556 898 149 9	818,278 335 252 4	916,630 022 383 7
28	26	41,578 780 488 7	–1,474 495 826 6	870,703 044 247 0	942,542 088 687 8
28	27	39,226 562 034 9	–1,394 969 127 3	919,437 834 977 3	970,415 906 516 9
29	15	89,279 883 950 6	–2,061 068 353 9	0,000 000 000 0	800,388 437 037 0
29	16	82,031 509 834 9	–2,088 154 781 9	102,724 036 530 3	793,877 036 548 2
29	17	75,732 129 652 0	–2,072 559 754 0	198,095 960 802 7	792,263 582 227 8
29	18	70,209 350 617 3	–2,029 914 279 8	286,943 773 786 0	794,868 014 074 1
29	19	65,326 705 651 2	–1,970 453 580 3	369,953 573 812 1	801,138 249 032 6
29	20	60,976 857 817 2	–1,900 926 199 3	447,695 899 552 2	810,629 267 057 7
29	21	57,075 122 222 2	–1,825 852 469 1	520,647 203 374 5	822,979 169 012 3
29	22	53,553 594 979 1	–1,748 283 440 2	589,206 152 039 4	837,890 741 721 5
29	23	50,356 178 834 6	–1,670 211 381 3	653,704 451 693 1	855,122 458 451 0
29	24	47,434 795 061 7	–1,592 782 963 0	714,411 894 115 2	874,488 236 049 4
29	25	44,747 071 219 0	–1,516 466 229 5	771,535 321 180 3	895,860 662 948 3
29	26	42,255 794 376 7	–1,441 322 471 7	825,211 204 490 1	919,167 805 180 3
29	27	39,930 419 412 0	–1,367 534 108 1	875,491 537 135 3	944,369 090 532 7
29	28	37,750 921 973 1	–1,296 339 683 3	922,322 734 544 5	971,391 163 917 5
NOTE α , β , μ and ε are all in units of 1×10^{-3} .					

m	n	α	β	μ	ε
30	16	82,684 820 885 4	–1,937 664 400 8	50,652 014 573 0	796,118 572 279 5
30	17	76,366 256 465 8	–1,943 349 624 5	146,570 247 501 8	792,158 476 319 7
30	18	70,826 762 953 8	–1,918 317 017 4	235,981 114 359 5	792,461 131 587 5
30	19	65,930 930 098 6	–1,873 623 032 8	319,574 313 579 2	796,467 129 928 9
30	20	61,571 167 864 8	–1,816 627 096 1	397,925 628 146 2	803,721 454 578 9
30	21	57,662 159 653 0	–1,752 277 838 5	471,517 592 025 1	813,853 822 773 7
30	22	54,135 742 160 1	–1,683 950 682 5	540,756 077 379 4	826,559 615 528 3
30	23	50,936 394 609 4	–1,613 946 311 8	605,982 564 971 8	841,586 818 964 2
30	24	48,017 520 085 7	–1,543 759 560 3	667,481 860 128 2	858,730 910 242 0
30	25	45,338 701 707 0	–1,474 228 249 3	725,485 016 653 2	877,836 129 828 8
30	26	42,864 116 365 2	–1,405 671 510 2	780,167 231 081 2	898,798 090 502 7
30	27	40,562 288 770 8	–1,338 127 121 8	831,640 469 649 9	921,559 182 169 8
30	28	38,407 368 531 3	–1,271 797 397 1	879,940 590 381 4	946,084 740 241 1
30	29	36,382 112 999 3	–1,207 813 151 8	925,008 722 656 2	972,304 453 992 4
31	16	83,301 992 538 5	–1,789 978 738 8	0,000 000 000 0	799,449 240 316 8
31	17	76,966 136 087 7	–1,816 327 048 7	96,420 883 672 2	793,277 568 074 3
31	18	71,410 327 810 5	–1,808 419 671 8	186,348 933 495 6	791,408 319 114 9
31	19	66,500 911 757 5	–1,778 059 7559	270,475 566 988 6	793,280 391 567 8
31	20	62,130 594 837 7	–1,733 208 979 2	349,380 454 238 2	798,430 692 473 8
31	21	58,213 650 031 6	–1,679 255 477 3	423,551 017 869 7	806,480 582 844 0
31	22	54,681 451 819 7	–1,619 889 288 3	493,398 689 540 3	817,118 733 927 4
31	23	51,478 457 936 1	–1,557 665 638 7	559,271 735 187 4	830,086 415 779 9
31	24	48,558 751 556 4	–1,494 336 391 4	621,464 461 258 2	845,168 568 248 7
31	25	45,883 258 032 6	–1,431 029 978 5	680,222 614 151 8	862,191 333 503 9
31	26	43,417 750 283 1	–1,368 360 139 7	735,744 785 101 6	881,024 058 266 6
31	27	41,131 756 949 6	–1,306 543 790 7	788,179 632 726 8	901,581 102 905 3
31	28	38,998 487 430 7	–1,245 608 341 8	837,618 735 482 7	923,816 123 588 4
31	29	36,995 887 902 7	–1,185 768 788 8	884,084 886 238 3	947,698 822 700 0
31	30	35,108 942 438 4	–1,128 054 899 5	927,515 641 210 7	973,161 491 746 6
NOTE α , β , μ and ε are all in units of 1×10^{-3} .					

Table C.2 – Fractiles of the F -distribution, $F(0,95, f_n, f_d)$

f	f_n								
f_d	1	2	3	4	5	6	7	8	9
10	4,965	4,103	3,708	3,478	3,326	3,217	3,135	3,072	3,020
11	4,844	3,982	3,587	3,357	3,204	3,095	3,012	2,948	2,896
12	4,747	3,885	3,490	3,259	3,106	2,996	2,913	2,849	2,796
13	4,667	3,806	3,411	3,179	3,025	2,915	2,832	2,767	2,714
14	4,600	3,739	3,344	3,112	2,958	2,848	2,764	2,699	2,646
15	4,543	3,682	3,287	3,056	2,901	2,790	2,707	2,641	2,588
16	4,494	3,634	3,239	3,007	2,852	2,741	2,657	2,591	2,538
17	4,451	3,592	3,197	2,965	2,810	2,699	2,614	2,548	2,494
18	4,414	3,555	3,160	2,928	2,773	2,661	2,577	2,510	2,456
19	4,381	3,522	3,127	2,895	2,740	2,628	2,544	2,477	2,423
20	4,351	3,493	3,098	2,866	2,711	2,599	2,514	2,447	2,393
25	4,242	3,385	2,991	2,759	2,603	2,490	2,405	2,337	2,282
30	4,171	3,316	2,922	2,690	2,534	2,421	2,334	2,266	2,211
40	4,085	3,232	2,839	2,606	2,449	2,336	2,249	2,180	2,124
50	4,034	3,183	2,790	2,557	2,400	2,286	2,199	2,130	2,073
100	3,936	3,087	2,696	2,463	2,305	2,191	2,103	2,032	1,975
500	3,860	3,014	2,623	2,390	2,232	2,117	2,028	1,957	1,899

f	f_n								
f_d	10	11	12	13	14	15	16	17	18
10	2,978	2,943	2,913	2,887	2,865	2,845	2,828	2,812	2,798
11	2,854	2,818	2,788	2,761	2,739	2,719	2,701	2,685	2,671
12	2,753	2,717	2,687	2,660	2,637	2,617	2,599	2,583	2,568
13	2,671	2,635	2,604	2,577	2,554	2,533	2,515	2,499	2,484
14	2,602	2,565	2,534	2,507	2,484	2,463	2,445	2,428	2,413
15	2,544	2,507	2,475	2,448	2,424	2,403	2,385	2,368	2,353
16	2,494	2,456	2,425	2,397	2,373	2,352	2,333	2,317	2,302
17	2,450	2,413	2,381	2,353	2,329	2,308	2,289	2,272	2,257
18	2,412	2,374	2,342	2,314	2,290	2,269	2,250	2,233	2,217
19	2,378	2,340	2,308	2,280	2,256	2,234	2,215	2,198	2,182
20	2,348	2,310	2,278	2,250	2,225	2,203	2,184	2,167	2,151
25	2,236	2,198	2,165	2,136	2,111	2,089	2,069	2,051	2,035
30	2,165	2,126	2,092	2,063	2,037	2,015	1,995	1,976	1,960
40	2,077	2,038	2,003	1,974	1,948	1,924	1,904	1,885	1,868
50	2,026	1,986	1,952	1,921	1,895	1,871	1,850	1,831	1,814
100	1,927	1,886	1,850	1,819	1,792	1,768	1,746	1,726	1,708
500	1,850	1,808	1,772	1,740	1,712	1,686	1,664	1,643	1,625

	19	20	25	30	40	50	100	500
10	2,785	2,774	2,730	2,700	2,661	2,637	2,588	2,548
11	2,658	2,646	2,601	2,570	2,531	2,507	2,457	2,415
12	2,555	2,544	2,498	2,466	2,426	2,401	2,350	2,307
13	2,471	2,459	2,412	2,380	2,339	2,314	2,261	2,218
14	2,400	2,388	2,341	2,308	2,266	2,241	2,187	2,142
15	2,340	2,328	2,280	2,247	2,204	2,178	2,123	2,078
16	2,288	2,276	2,227	2,194	2,151	2,124	2,068	2,022
17	2,243	2,230	2,181	2,148	2,104	2,077	2,020	1,973
18	2,203	2,191	2,141	2,107	2,063	2,035	1,978	1,929
19	2,168	2,155	2,106	2,071	2,026	1,999	1,940	1,891
20	2,137	2,124	2,074	2,039	1,994	1,966	1,907	1,856
25	2,021	2,007	1,955	1,919	1,872	1,842	1,779	1,725
30	1,945	1,932	1,878	1,841	1,792	1,761	1,695	1,637
40	1,853	1,839	1,783	1,744	1,693	1,660	1,589	1,526
50	1,798	1,784	1,727	1,687	1,634	1,599	1,525	1,457
100	1,691	1,676	1,616	1,573	1,515	1,477	1,392	1,308
500	1,607	1,592	1,528	1,482	1,419	1,376	1,275	1,159

Table C.3 – Fractiles of the F -distribution, $F(0,995, f_n, f_d)$

f	f_n								
f_d	1	2	3	4	5	6	7	8	9
10	12,826	9,427	8,081	7,343	6,872	6,545	6,302	6,116	5,968
11	12,226	8,912	7,600	6,881	6,422	6,102	5,865	5,682	5,537
12	11,754	8,510	7,226	6,521	6,071	5,757	5,525	5,345	5,202
13	11,374	8,186	6,926	6,233	5,791	5,482	5,253	5,076	4,935
14	11,060	7,922	6,680	5,998	5,562	5,257	5,031	4,857	4,717
15	10,798	7,701	6,476	5,803	5,372	5,071	4,847	4,674	4,536
16	10,575	7,514	6,303	5,638	5,212	4,913	4,692	4,521	4,384
17	10,384	7,354	6,156	5,497	5,075	4,779	4,559	4,389	4,254
18	10,218	7,215	6,028	5,375	4,956	4,663	4,445	4,276	4,141
19	10,073	7,093	5,916	5,268	4,853	4,561	4,345	4,177	4,043
20	9,944	6,986	5,818	5,174	4,762	4,472	4,257	4,090	3,956
25	9,475	6,598	5,462	4,835	4,433	4,150	3,939	3,776	3,645
30	9,180	6,355	5,239	4,623	4,228	3,949	3,742	3,580	3,450
40	8,828	6,066	4,976	4,374	3,986	3,713	3,509	3,350	3,222
50	8,626	5,902	4,826	4,232	3,849	3,579	3,376	3,219	3,092
100	8,241	5,589	4,542	3,963	3,589	3,325	3,127	2,972	2,847
500	7,950	5,355	4,330	3,763	3,396	3,137	2,941	2,789	2,665

f	f_n								
f_d	10	11	12	13	14	15	16	17	18
10	5,847	5,746	5,661	5,589	5,526	5,471	5,422	5,379	5,340
11	5,418	5,320	5,236	5,165	5,103	5,049	5,001	4,959	4,921
12	5,085	4,988	4,906	4,836	4,775	4,721	4,674	4,632	4,595
13	4,820	4,724	4,643	4,573	4,513	4,460	4,413	4,372	4,334
14	4,603	4,508	4,428	4,359	4,299	4,247	4,200	4,159	4,122
15	4,424	4,329	4,250	4,181	4,122	4,070	4,024	3,983	3,946
16	4,272	4,179	4,099	4,031	3,972	3,920	3,875	3,834	3,797
17	4,142	4,050	3,971	3,903	3,844	3,793	3,747	3,707	3,670
18	4,030	3,938	3,860	3,793	3,734	3,683	3,637	3,597	3,560
19	3,933	3,841	3,763	3,696	3,638	3,587	3,541	3,501	3,465
20	3,847	3,756	3,678	3,611	3,553	3,502	3,457	3,416	3,380
25	3,537	3,447	3,370	3,304	3,247	3,196	3,151	3,111	3,075
30	3,344	3,255	3,179	3,113	3,056	3,006	2,961	2,921	2,885
40	3,117	3,028	2,953	2,888	2,831	2,781	2,737	2,697	2,661
50	2,988	2,900	2,825	2,760	2,703	2,653	2,609	2,569	2,533
100	2,744	2,657	2,583	2,518	2,461	2,411	2,367	2,326	2,290
500	2,562	2,476	2,402	2,337	2,281	2,230	2,185	2,145	2,108

f	f_n							
f_d	19	20	25	30	40	50	100	500
10	5,305	5,274	5,153	5,071	4,966	4,902	4,772	4,666
11	4,886	4,855	4,736	4,654	4,551	4,488	4,359	4,252
12	4,561	4,530	4,412	4,331	4,228	4,165	4,037	3,931
13	4,301	4,270	4,153	4,073	3,970	3,908	3,780	3,674
14	4,089	4,059	3,942	3,862	3,760	3,698	3,569	3,463
15	3,913	3,883	3,766	3,687	3,585	3,523	3,394	3,287
16	3,764	3,734	3,618	3,539	3,437	3,375	3,246	3,139
17	3,637	3,607	3,492	3,412	3,311	3,248	3,119	3,012
18	3,527	3,498	3,382	3,303	3,201	3,139	3,009	2,901
19	3,432	3,402	3,287	3,208	3,106	3,043	2,913	2,804
20	3,347	3,318	3,203	3,123	3,022	2,959	2,828	2,719
25	3,043	3,013	2,898	2,819	2,716	2,652	2,519	2,406
30	2,853	2,823	2,708	2,628	2,524	2,459	2,323	2,207
40	2,628	2,598	2,482	2,401	2,296	2,230	2,088	1,965
50	2,500	2,470	2,353	2,272	2,164	2,097	1,951	1,821
100	2,257	2,227	2,108	2,024	1,912	1,840	1,681	1,529
500	2,075	2,044	1,922	1,835	1,717	1,640	1,460	1,260

Table C.4 – Fractiles of the t -distribution, $t_{0,95}$

f	t
1	6,314
2	2,920
3	2,353
4	2,132
5	2,015
6	1,943
7	1,895
8	1,860
9	1,833
10	1,812
11	1,796
12	1,782
13	1,771
14	1,761
15	1,753
16	1,746
17	1,740
18	1,734
19	1,729
20	1,725
25	1,708
30	1,697
40	1,684
50	1,676
100	1,660
500	1,640

Table C.5 – Fractiles of the χ^2 -distribution

f	$p = 0,95$	$p = 0,99$	$p = 0,995$
1	3,8	6,6	7,9
2	6,0	9,2	10,6
3	7,8	11,3	12,8
4	9,5	13,3	14,9
5	11,1	15,1	16,7
6	12,6	16,8	18,5

NOTE The significance level P is equal to $(1 - p)$, for example significance 0,05 corresponds to $p = 0,95$.

Annex D (informative)

Worked examples

Table D.1 – Worked example 1 – Censored data (proof tests: file CENEX3.DTA)

g_i	240	260	280
x_i	0,001 948 747 929	0,001 875 644 753	0,001 807 827 895
j	τ_{ij} y_{ij}	τ_{ij} y_{ij}	τ_{ij} y_{ij}
1	1 764 7,475 339 237	756 6,628 041 376	108 4,682 131 227
2	2 772 7,927 324 360	924 6,828 712 072	252 5,529 429 088
3	2 772 7,927 324 360	924 6,828 712 072	324 5,780 743 516
4	3 780 8,237 479 289	1 176 7,069 874 128	324 5,780 743 516
5	4 284 8,362 642 432	1 176 7,069 874 128	468 6,148 468 296
6	4 284 8,362 642 432	2 184 7,688 913 337	612 6,416 732 283
7	4 284 8,362 642 432	2 520 7,832 014 181	684 6,527 957 918
8	5 292 8,573 951 525	2 856 7,957 177 323	756 6,628 041 376
9	7 308 8,896 724 917	2 856 7,957 177 323	756 6,628 041 376
10	7 812 8,963 416 292	3 192 8,068 402 959	828 6,719 013 154
11	7 812 8,963 416 292	3 192 8,068 402 959	828 6,719 013 154
12		3 864 8,259 458 195	972 6,879 355 804
13		4 872 8,491 259 809	1 428 7,264 030 143
14		5 208 8,557 951 184	1 596 7,375 255 778
15		5 544 8,620 471 541	1 932 7,566 311 015
16		5 880 8,679 312 041	1 932 7,566 311 015
17		5 880 8,679 312 041	2 100 7,649 692 624
18		5 880 8,679 312 041	2 268 7,726 653 665
19			2 604 7,864 804 003
20			2 772 7,927 324 360
m_i	21	21	21
n_i	11	18	20
α_i	0,125 180 504 27	0,063 335 741 06	0,053 992 487 28
β_i	–0,004 102 787 08	–0,002 960 377 33	–0,002 557 464 29
μ_i	0	0,748 883 265 05	0,891 180 261 68
ε_i	0,805 857 221 19	0,893 423 810 54	0,961 160 991 78
$\sum_{j=1}^{n_i-1} y_{ij}$	83,089 487 275 2	133,285 066 669	127,452 728 95
$\sum_{j=1}^{n_i-1} (y_{in_i} - y_{ij})^2$	6,127 249 075 70	19,955 744 346 8	41,422 442 313 8
$\bar{y}_i$	8,963 416 292	8,050 988 496	6,840 720 748 66
s_{1i}^2	0,591 278 355 53	0,661 652 813 85	0,863 951 396 023

Term	Value	Equation #
$\sum_{i=1}^k \varepsilon_i / k$	0,886 814 007 835	(29)
$\sum_{i=1}^k n_i x_i^2$	0,000 170 463 415 664	
$\sum_{i=1}^k n_i \bar{y}_i^2$	2 986,411 838 81	
$\sum_{i=1}^k n_i x_i \bar{y}_i$	0,711 293 042 041	
$M = \sum_{i=1}^k m_i$	63	(28)
$N = \sum_{i=1}^k n_i$	49	(25)
$\sum_{i=1}^k n_i x_i / N$	0,001 864 375 319 83	(26)
$\sum_{i=1}^k n_i \bar{y}_i / N$	7,761 832 390 07	(27)
b	15 327,985 78	(33)
a	–20,815 286 004 4	(34)
s_1^2	0,647 296 300 122	(30)
s_2^2	0,395 498 398 826	(36)
F	0,611 000 555 311	(40)
F_0	4,051 748 692 214	
χ^2	0,554 692 947 413	(38)
c	1,031 619 329 65	(39)
$t_{0,95, N-2}$	1,677 926 722	(43)
t_c	1,738 953 340 31	(43)
$\mu_2(x)$	2,949 884 440 3 × 10 ^{–9}	(31)
s^2	0,641 938 897 967	(41)
$TI = \hat{g}$	225,827 791 333	(50)
$TC = \hat{g}_c$	214,550 619 764	(50)
HIC	11,518 995 303 8	(53)
$(TI - TC)/HIC$	0,979 006 525 432	
TI_a	221,462 017 221	(55)
Result	TI (HIC): 221,5 (11,5)	

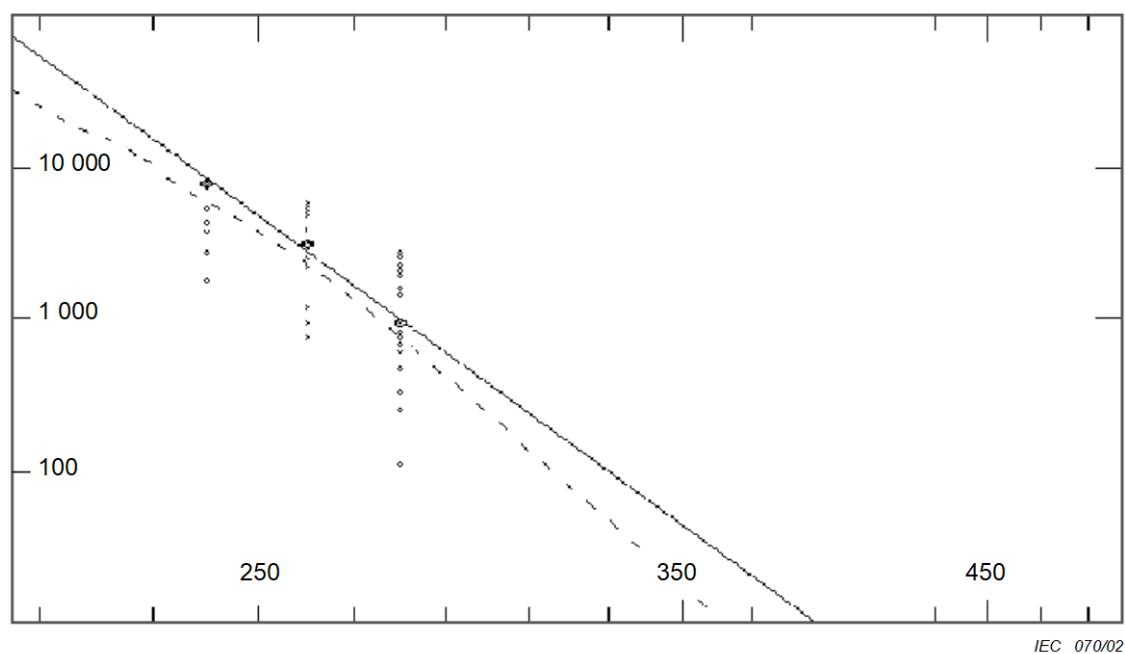
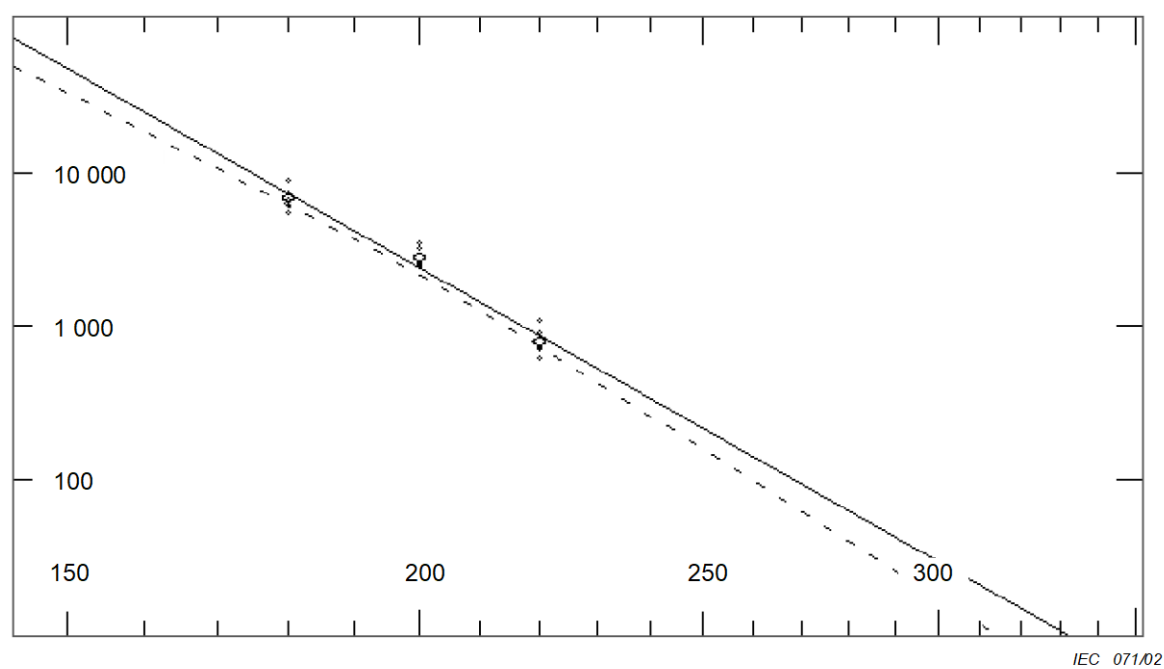
**Table D.2 – Worked example 2 – Complete data
(non-destructive tests: file TEST2.DTA)**

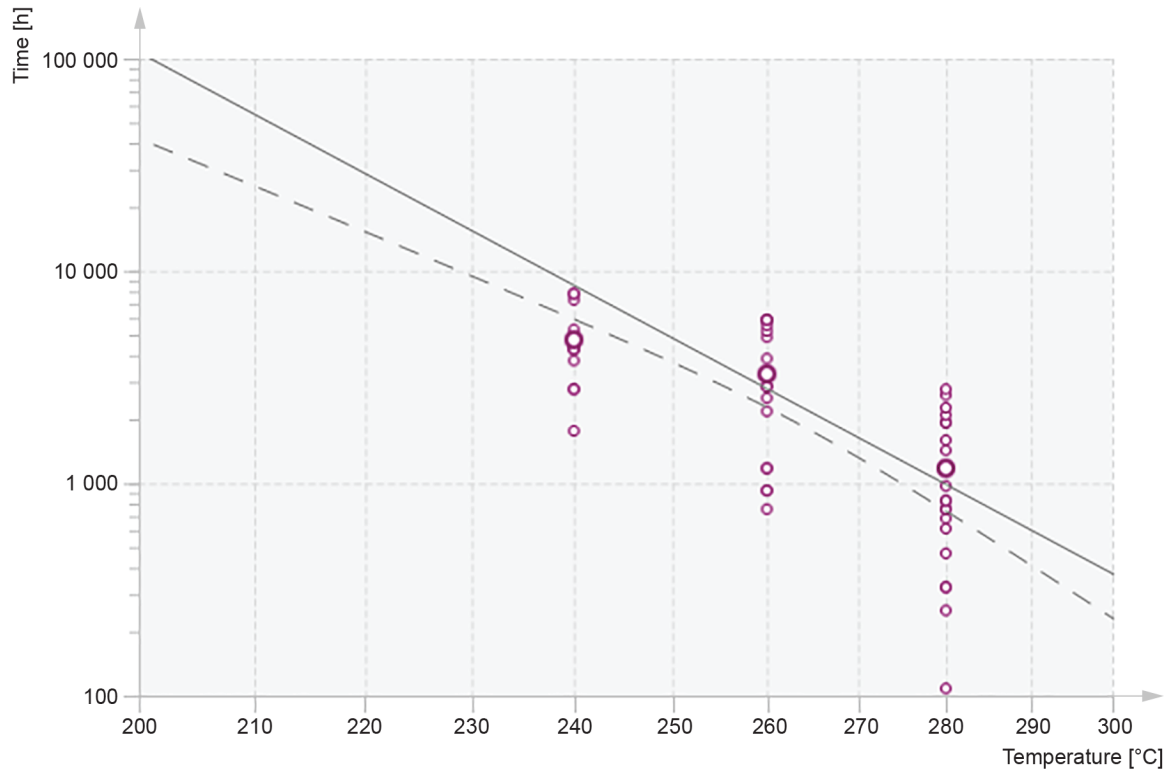
g_i	180	200	220
x_i	0,002 206 774 799	0,002 113 494 663	0,002 027 780 594
j	τ_{ij} y_{ij}	τ_{ij} y_{ij}	τ_{ij} y_{ij}
1	7 410 8,910 585 718	3 200 8,070 906 089	1 100 7,003 065 459
2	6 610 8,796 338 933	2 620 7,870 929 597	740 6,606 650 186
3	6 170 8,727 454 117	2 460 7,807 916 629	720 6,579 251 212
4	5 500 8,612 503 371	2 540 7,839 919 360	620 6,429 719 478
5	8 910 9,094 929 520	3 500 8,160 518 247	910 6,813 444 599

m_i	5	5	5
n_i	5	5	5
ε_i	1	1	1
$\sum_{j=1}^{n_i} y_{ij}$	44,141 811 66	39,750 189 92	33,432 130 93
$\sum_{j=1}^{n_i} y_{ij}^2$	389,835 529 1	316,113 013 5	223,741 618
$\bar{y}_i$	8,828 362 332	7,950 037 984	6,686 426 187
s_{1i}^2	0,033 905 452 03	0,0024 373 442	0,0050 035 781 4

Term	Value	Equation #
$\sum_{i=1}^k \varepsilon_i / k$	1	(29)
$\sum_{i=1}^k n_i x_i^2$	6,724 304 421 1 $\times 10^{-5}$	
$\sum_{i=1}^k n_i \bar{y}_i^2$	929,256 902 85	
$\sum_{i=1}^k n_i x_i \bar{y}_i$	0,249 215 878 14	
$M = \sum_{i=1}^k m_i$	15	(28)
$N = \sum_{i=1}^k n_i$	15	(25)
$\sum_{i=1}^k n_i x_i / N$	2,116 016 685 4 $\times 10^{-3}$	(26)
$\sum_{i=1}^k n_i \bar{y}_i / N$	7,821 608 834 4	(27)
b	11 929,077 582	(33)
a	–17,420 518 37	(34)
s_1^2	0,036 104 891 8	(30)

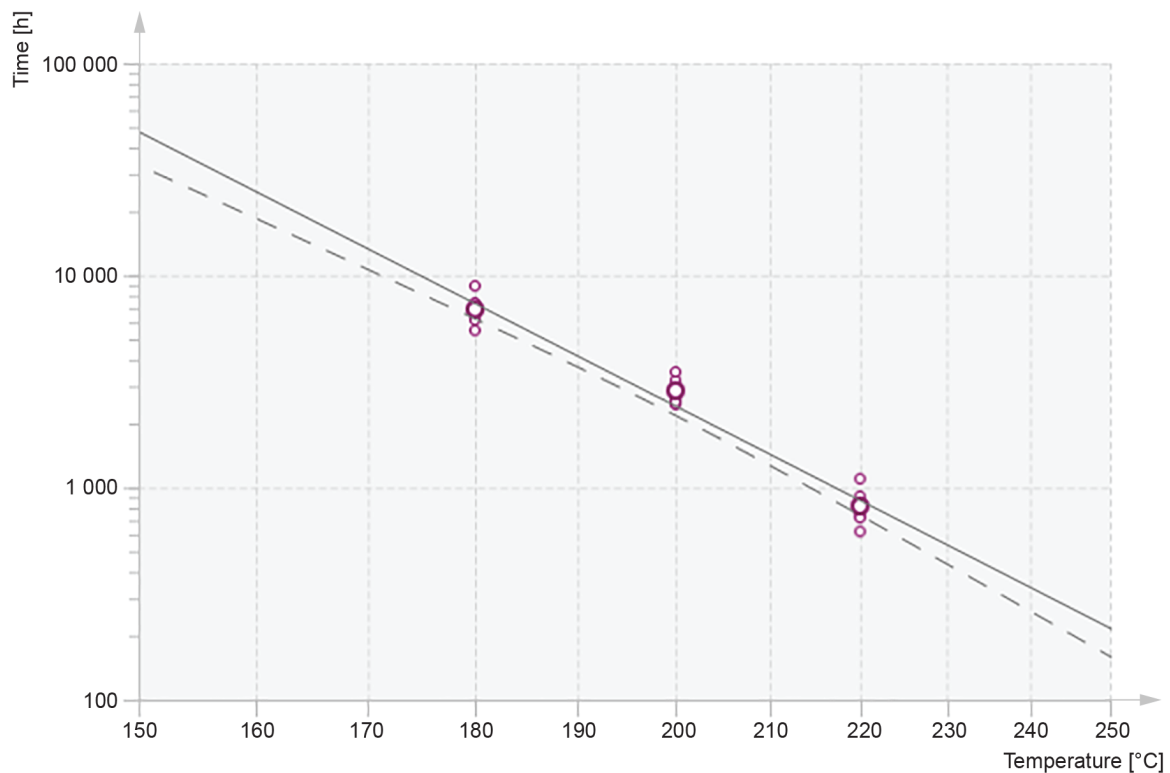
Term	Value	Equation #
s_2^2	0,188 563 697 29	(36)
F	5,222 663 409	(40)
F_0	4,747 225 347	
χ^2	0,466 116 435 248	(38)
c	1,111 111 111 1	(39)
$t_{0,95, N-2}$	1,770 933 396 2	(43)
t_c	1,770 933 396 2	(43)
$\mu_2(x)$	$5,343\ 001\ 171\ 0 \times 10^{-9}$	(31)
s_a^2	0,05119958608 0,051 170 274 78	(42)
$TI = \hat{g}$	163,428 648 665	(50)
$TC_a = \hat{g}_c$	158,670330155 158,671 846 470	(50)
HIC	11,363 255 775 6	(53)
$(TI - TC_a)/HIC$	0,41874605344 0,418 612 613 23	
Result	TI (HIC): 163 (11,4)	(54)

**Figure D.1a – Example 1****Figure D.1b – Example 2**



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a) Example 1



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b) Example 2

NOTE In the above figures, the full line represents the regression equation, and the dotted line the lower 95 % confidence limit of a temperature estimate. The figures are as drawn by the program given in Annex E.

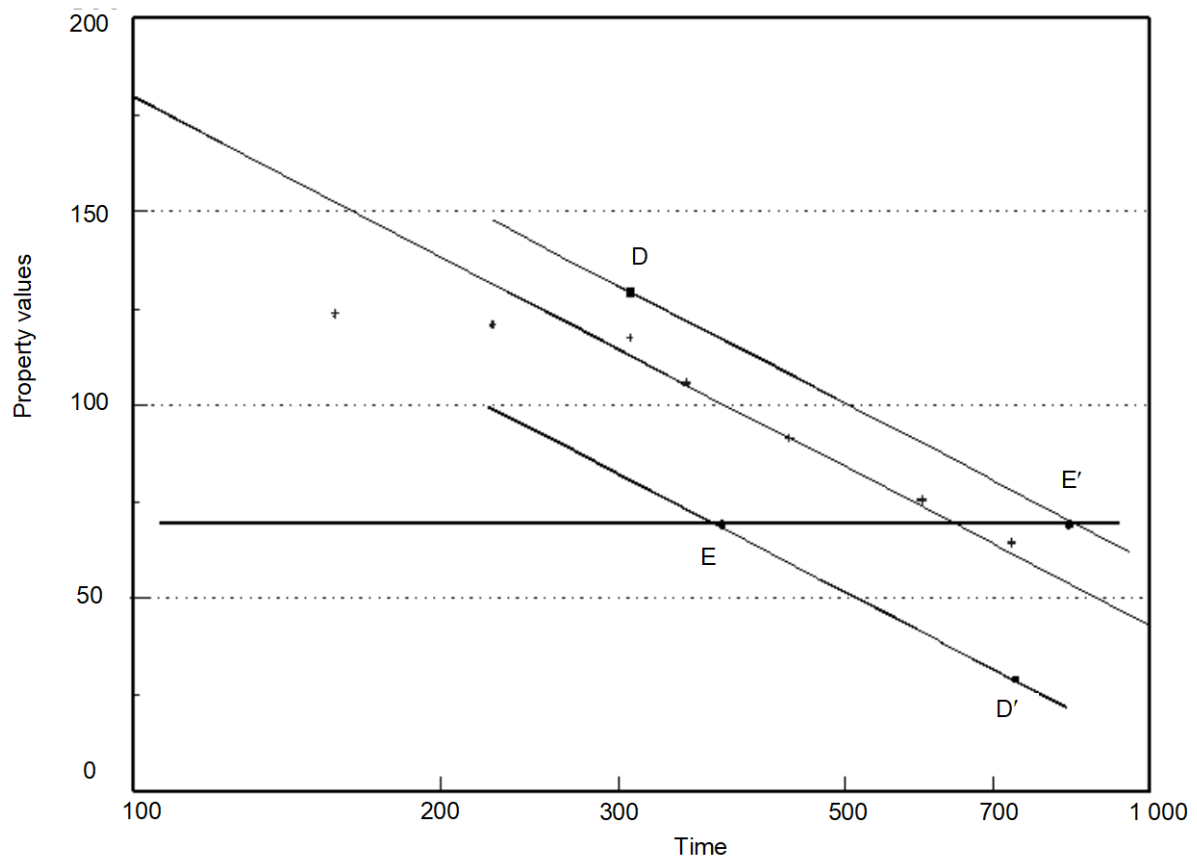
Figure D.1 – Thermal endurance graph

Table D.3 – Worked example 3 – Destructive tests

End-point p_e : 70,0					
τ_g	288	336	432	624	720
p_{gh}	139,5	121,9	101,2	77,8	69,6
	125,0	109,3	99,5	74,6	69,4
	120,8	98,3	98,4	71,4	67,2
	112,7	96,5	92,4	68,2	60,4
	112,0	93,0	78,1	60,5	59,4
n_g	5	5	5	5	5
$\bar{p}_g$	122,00	103,80	93,92	70,50	65,20
s_{1g}^2	125,795	139,510	89,197	44,050	24,420
$\log \tau_g$	5,662 960	5,817 111	6,068 426	6,436 150	6,579 251
n_i			25		
$\bar{z}$			6,112 8		
$\bar{p}$			91,084		
b_p			–59,493 7		
a_p			454,756		
s_1^2			84,594		
s_2^2			77,266		
F			0,913		
F_1			3,098		
z_{gh}	6,831 151	6,689 472	6,592 851	6,567 257	6,572 528
	6,587 428	6,477 685	6,564 276	6,513 469	6,569 166
	6,516 832	6,292 792	6,545 787	6,459 682	6,532 187
	6,380 683	6,262 536	6,444 936	6,405 895	6,417 890
	6,368 917	6,203 707	6,204 574	6,276 470	6,401 081

This worked example is given to illustrate the calculations specific for destructive test data, and relates to a single test temperature. The data from this calculation and further test temperatures would be entered into a calculation similar to that exemplified in worked example 2 (Table D.2).

In the graph below (Figure D.2), displaying the data of example 3, the line passing through the data points marked E and E' indicates the chosen end-point criteria. The points marked D and D' are two randomly selected data points, with lines parallel to the regression line intersecting the end-point line at E and E'. The other points marked on the graph are the means of the property value groups. The red circles represent individual data points grouped to $r = 5$ ageing times.



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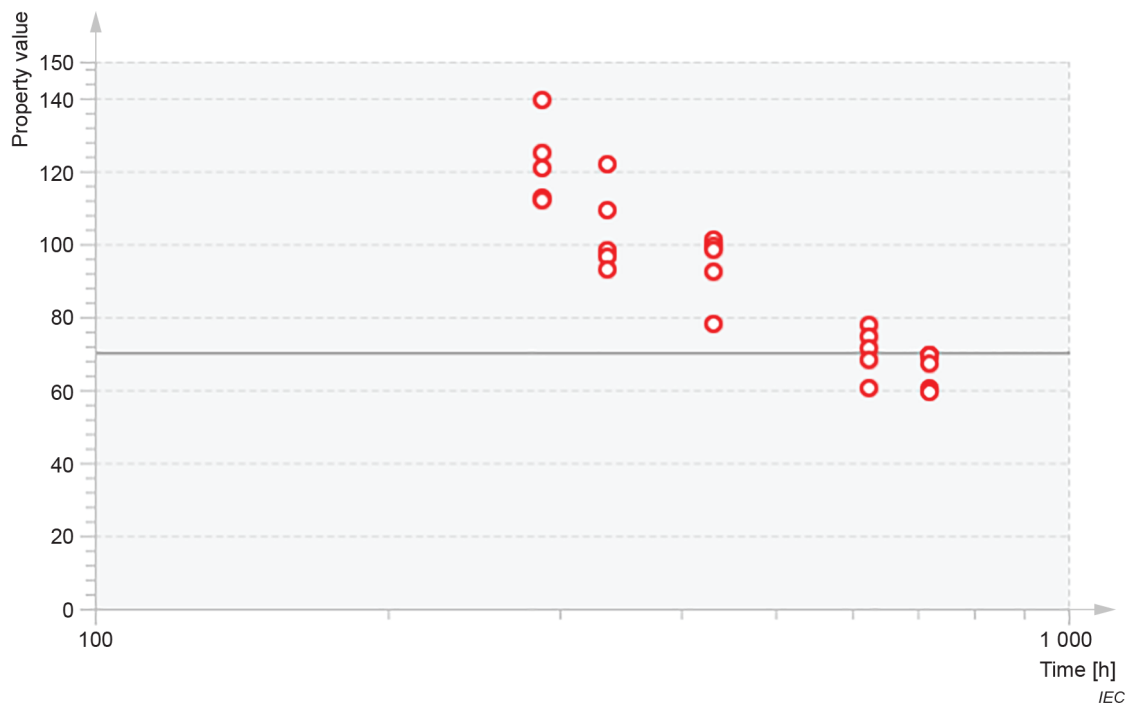


Figure D.2 – Example 3: Property-time graph
(destructive test data)

Table D.4 – Worked example 3 – Selection of groups

τ_g	288	336	432	624	720	F	F_1	$F_1 - F$
a	x	x	x	x	x	0,913	3,098	2,185
b		x	x	x	x	0,325	3,634	3,309
c			x	x	x	0,449	4,747	4,298
d		x	x	x		0,480	4,747	4,267
e	x	x	x	x		0,967	3,634	2,667

Table D.4 shows all valid group selections, see 6.1.4. At least three groups have to be selected from one range in time ($r \geq 3$: step 2). The F -test is satisfied, if the F value is smaller than the criteria F_1 or F_2 . The F value tends to get lower, if the pooled variance within the property groups (s_1) is large compared to the weighted variance of the deviations of the property group means from the regression line (s_2). In this example, the lowest F value is found for selection b). On the other hand, the values of the F -test criteria F_1 and F_2 tend to get higher, if the number of selected groups r and data points v is kept lowest. This is the case for selections c) and d). Even though all selections a) to e) fulfil the F -test criterion $F \leq F_1$ and therefore are permitted, in total the highest degree of linearity is found for selection c), where the expression $F_1 - F$ is maximized.

Extrapolation

If, in the above data set, only the data up to an ageing time of 624 h were available, the ageing curve would not have crossed the end-point line, since $70,5 > 70,0$. In this case, the extrapolation required would be

$$(70,5 - 70,0) / (122,0 - 70,5) = 0,009\ 7$$

This would be permitted (criterion: $< 0,25$), subject to the other restrictions of 6.1.4, step 4. However, if only the data up to an ageing time of 432 h were available, such an extrapolation is no longer permitted, because

$$(93,9 - 70,0) / (122,0 - 93,2) = 0,829\ 9$$

Annex E (informative)

Computer program

E.1 General

The programs on the accompanying CD-ROM for making calculations in this standard are for use in conjunction with the disk operating system MSDOS or equivalent. The preferred mode of operation is in a Windows 95 (or later) system in a “DOS” window.

CD-ROM content

Annex E.doc
Entry 3.bas, Entry 3.exe
216-3.bas, 216-3.exe
Test2.dta
Genex3.dta
N3.dst

The text based program files (*.bas) are written in a dialect of the Basic language known as “Quick Basic”, chosen mainly for historical reasons. They may be edited if necessary using either a text editor or the “Quick Basic” programs QB45 or QBX and saved as ASCII files with the filename ending in “.bas”. They may then be executed using QB45 or QBX or compiled for stand-alone execution. The resultant executable files will have the ending “.exe” and the same file stem as the “.bas” files.

These executable files are suitable for direct execution in DOS, and may be executed in Windows 95 or later (98 etc.) by creating an icon for the file and double-clicking that icon. The “short-cut” options should be set so that the program is run in a normal (full screen) window.

To create the icon, in Windows Explorer, right-click the file name and select “Create Shortcut”, or drag the program into the desktop.

NOTE It is not possible, without substantial modification, to run the text based programs in a “Visual Basic” environment.

The program code is in two separate files, one being the actual calculation processor and the other for entry of the data to a file in a format suitable for retrieval and processing by the first. The actual format or structure of these files is described in the next section, and the actual content of the files used in the worked examples is given in the subsequent section.

E.1.1 216-3.bas (or .exe)

The program requires data to be entered in the form of an ASCII text file obtained using the associated program Entry 3.bas described below.

Data analysis and statistical tests in accordance with this standard are carried out and the results reported in the appropriate format. The report is recorded in a text file with the ending “.rep”, which may be edited in a word processor program.

Result parameters as required for calculation of RTE are automatically entered into a file having the same name as the experimental data file and with the ending “.int”.