

INTERNATIONAL STANDARD

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Reliability testing – Compliance tests for constant failure rate and constant failure intensity

Essais de fiabilité – Plans d'essai de conformité pour un taux de défaillance constant et une intensité de défaillance constante



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Reliability testing – Compliance tests for constant failure rate and constant failure intensity

Essais de fiabilité – Plans d’essai de conformité pour un taux de défaillance constant et une intensité de défaillance constante

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CONTENTS

FOREWORD.....	6
INTRODUCTION.....	8
1 Scope.....	10
2 Normative references	10
3 Terms, definitions, abbreviated terms and symbols.....	11
3.1 Terms and definitions.....	11
3.2 Abbreviated terms and symbols	11
3.2.1 Abbreviated terms	11
3.2.2 Symbols	11
4 General requirements and area of application.....	13
4.1 Requirements and characteristics	13
4.2 Applicability to replaced and repaired items	13
4.3 Types of test plans.....	14
4.3.1 General	14
4.3.2 Advantages and disadvantages of the different test plan types	14
5 General test procedure	16
5.1 Test conditions	16
5.2 General characteristics of the test plans	17
5.3 Data to be recorded	17
5.4 Calculation of accumulated test time, T^*	17
5.5 Number of failures	18
6 Truncated sequential probability ratio test (SPRT) plans.....	18
6.1 General.....	18
6.2 Common test procedure.....	19
6.3 Decision criteria	19
6.4 Operating characteristic (OC) curve	20
6.5 Expected accumulated test time to decision (ETT)	21
6.6 Overview of test plans.....	22
7 Fixed time/failure terminated test plans – Fixed duration (to acceptance) test plans	25
7.1 General.....	25
7.2 Common test procedure.....	26
7.3 Decision criteria	26
7.4 Test plans	26
8 Design of alternative time/failure terminated test plans (FTFT)	27
8.1 General.....	27
8.2 Design procedures	27
8.3 Common FTFT procedure	28
8.4 Decision criteria	28
9 Calendar time/failure terminated test plans (FTFT) for non-replaced items	28
9.1 General.....	28
9.2 Common test procedure.....	29
9.3 Decision criteria	29
9.4 Use of IEC 61123:2019, Table 5 for fixed calendar time tests	29
9.4.1 General	29
9.4.2 Procedure when the test time is given	30

9.4.3	Procedure when the number of items is given	30
10	Combined test plans	30
10.1	General	30
10.2	Common test procedure	30
10.3	Decision criteria	31
10.4	Test plans	31
11	Performing the test and presenting the results	32
Annex A	(normative) Tables for border lines of SPRT plans (types A and C)	33
A.1	Symbols	33
A.2	Border lines	33
A.3	Example of the SPRT plan from Clause 6	37
Annex B	(normative) Tables and graphs for combined test plans (type D)	39
B.1	General	39
B.2	Test plans D.3 and C.3 ($\alpha = \beta = 10\%$, $D = 1,7$)	41
Annex C	(informative) Extension of the set of SPRTs type A	44
C.1	Symbols	44
C.2	Extension of the set of type A tests (through interpolation by α and β)	44
Annex D	(informative) Approximation of operating characteristic for type A SPRTs by Wald's formula	47
D.1	Symbols	47
D.2	Approximations of OC in this document	47
D.3	Approximation of OC for type A SPRT by Wald's formula	47
D.4	Construction of the approximated OC curve using a spreadsheet	49
Annex E	(informative) Mathematical references and examples for fixed time/failure terminated test (FTFT) plans	51
E.1	Symbols	51
E.2	Mathematical references	51
E.2.1	General	51
E.2.2	Mathematical references	51
E.2.3	Design procedure {a}	54
E.2.4	Design procedure {b}	55
E.2.5	Design procedure {c}	55
E.2.6	Design procedure {d}	56
E.3	Examples of FTFT design using test plans B	56
E.3.1	Example 1	56
E.3.2	Example 2	57
E.4	Test OC approximation using formula for FTFT	58
Annex F	(informative) Examples of FTFT design using a spreadsheet program	59
F.1	General	59
F.2	Finding the test border lines using optimization on the example of the design procedure {b}	61
F.3	ETT and OC curves	63
F.4	Example of FTFT design by procedure {a}	65
F.5	Example of FTFT design by procedure {c}	67
F.6	Example of FTFT design by procedure {d}	69
F.7	Example of a test with replacement of failed items	72
F.8	Evaluation of an approximate OC for non-FTFT plans using a spreadsheet	73
Annex G	(informative) Examples and mathematical references for the calendar time terminated test plans	78

G.1	Examples	78
G.1.1	Example 1	78
G.1.2	Example 2	78
G.2	Mathematical background	79
Annex H (informative) Derivation and mathematical reference for the optimized test plans of GOST R 27 402 [12]		80
H.1	Symbols	80
H.2	Test plan types and terminology	81
H.3	Introductory remarks	81
H.4	Procedure used for developing the optimized test plans	82
Bibliography		89

Figure 1 – Relative ETT (T_e^*/m_0) and MaxTT (T_t^*/m_0) of various tests with the same risks	16
Figure 2 – SPRT diagram and test example	20
Figure 3 – OC curve, P_a	21
Figure 4 – SPRT – Curve of expected accumulated test time to decision (ETT)	22
Figure 5 – Example of a decision graph for combined test plan (type D) and for SPRT type C	31
Figure A.1 – Decision graph of SPRT plan	34
Figure B.1 – Expected accumulated test time to acceptance decision, $T_e^*(+)$ for D.3 and C.3 test plans	43
Figure B.2 – Operating characteristic P_a for D.3 and C.3 test plans	43
Figure D.1 – Approximation of OC for type A SPRT using Wald's formula	48
Figure E.1 – Example 1 – Expected accumulated test time to decision (ETT) of tests B.2 and A.25	57
Figure E.2 – Example 1 – Operating characteristic of tests B.2 and A.25	58
Figure F.1 – Using Solver to find T_t^*/m_0 – Accumulated test truncation time in terms of m_0	63
Figure F.2 – ETT plotted from the spreadsheet calculations	64
Figure F.3 – OC curve plotted from the spreadsheet calculations	64
Figure F.4 – Using Solver to find T_t^*/m_0 and c in Step {a1}	66
Figure F.5 – Using Solver to find T_t^*/m_0 in Step {a2}	67
Figure F.6 – Using Solver to find T_t^*/m_0 in Step {c2}	69
Figure F.7 – Using Solver to find D and c in Step {d1}	71
Figure F.8 – Using Solver to find D and T_t^* in Step {d2}	72
Figure F.9 – Using Solver to find c and T_t^*/m_0 from Clause F.8	75
Figure F.10 – OC approximated by formula for FTFT (example from Clause F.8)	77
Figure H.1 – Test plan types and terminology	81
Figure H.2 – Principle of test plans	83
Figure H.3 – Partitioning of the test plan graph	83
Figure H.4 – Interior nodes and border nodes	83
Figure H.5 – Paths to the accept line	84

Figure H.6 – Paths to the reject line	84
Figure H.7 – Probabilities of paths transfer between nodes	85
Figure H.8 – Recurrent element – Two cases	88
Table 1 – Advantages and disadvantages for the different test plan types	15
Table 2 – OC curve	20
Table 3 – Relative ETT versus m/m_0	21
Table 4 – Overview of type A SPRT plans	23
Table 5 – Overview of type C SPRT plans	25
Table 6 – Type B FTFT plans	27
Table 7 – Overview of type D combined plans	32
Table A.1 – Constants for border line formulae and their coordinates for type A SPRT plans	35
Table A.2 – Constants for border line formulae and their coordinates for type C SPRT plans	36
Table A.3 – Example for SPRT using test plan A.41 (with example data)	38
Table B.1 – Combined test plans in Annex B	39
Table B.2 – Type D test plans – Accept and reject lines	40
Table B.3 – Expected accumulated test time to acceptance decision, $T_e^*(+)$, for D and C test plans	41
Table B.4 – Accept and reject lines for D.3 and C.3 test plans	42
Table C.1 – Example for interpolation by α and β	46
Table D.1 – Spreadsheet set-up for construction of the OC curve by Wald	50
Table D.2 – Formulae embedded in the spreadsheet	50
Table E.1 – List of the typical FTFT design procedures	54
Table F.1 – Set-up of the spreadsheet with embedded formulae	60
Table F.2 – Formulae embedded in the spreadsheet	61
Table F.3 – Fragment from Table 6	62
Table F.4 – Set-up 1 of the spreadsheet for example by procedure {a}	65
Table F.5 – Set-up 2 of the spreadsheet for example by procedure {a}	66
Table F.6 – Set-up 3 (final solution) for example by procedure {a}	67
Table F.7 – Set-up 2 for example by procedure {c}	68
Table F.8 – Set-up 3 (final solution) for example by procedure {c}	69
Table F.9 – Set-up 1 of the spreadsheet for example by procedure {d}	70
Table F.10 – Set-up 2 of the spreadsheet for example by procedure {d}	71
Table F.11 – Set-up 3 (final solution) for example by procedure {d}	72
Table F.12 – Set-up of the spreadsheet with embedded formulae from Clause F.8	74
Table F.13 – Set-up 1 of the spreadsheet from Clause F.8	75
Table F.14 – Set-up 2 of the spreadsheet for example from Clause F.8	76

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COMPLIANCE TESTS FOR CONSTANT FAILURE RATE
AND CONSTANT FAILURE INTENSITY****FOREWORD**

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This fourth edition cancels and replaces the third edition published in 2012. This edition constitutes a technical revision.

This edition includes the following significant technical changes with respect to the previous edition:

- a) The truncated sequential probability ratio test (SPRT) [1], [2], [3]¹ has been significantly developed in recent years [4], [5], [6]. In this edition, type A test plans (optimally truncated SPRT) have been significantly changed, as follows:

¹ Numbers in square brackets refer to the Bibliography.

- the tests are significantly truncated (the maximal test time is low) without substantially increasing the expected accumulated test time to decision (ETT);
 - the true producer's and consumer's risks (α' , β') are given and are very close to the nominal values;
 - the range of the test parameters is wide (risks and discrimination ratio);
 - the test plans include various risk ratios (not restricted to equal risks only);
 - the values of the ETT are accurate and given in the relevant region (for practical use);
 - guidelines for extension of the tests set (using accurate interpolation) are included.
- b) Other ready-to-use test plans (types B, C, D) are not changed, only the form of presentation of the data on their border lines and the characteristics has been changed. This form is made unified for all types of test plans, which helps the comparison of different plans and, accordingly, to facilitate the selection of the most appropriate.
- c) FTFT design procedures, to extend the set of test plans B, are significantly changed and make the design accurate and simple. The implementation of this design is given on a spreadsheet program. A unified approach to the calculation of the operational characteristics of all types of test plans is introduced.

The text of this International Standard is based on the following documents:

Draft	Report on voting
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Full information on the voting for its approval can be found in the report on voting indicated in the above table.

The language used for the development of this International Standard is English.

This document was drafted in accordance with ISO/IEC Directives, Part 2, and developed in accordance with ISO/IEC Directives, Part 1 and ISO/IEC Directives, IEC Supplement, available at www.iec.ch/members_experts/refdocs. The main document types developed by IEC are described in greater detail at www.iec.ch/standardsdev/publications.

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INTRODUCTION

A compliance test is an essential part of the reliability assurance system [7], [8], [9]. Reliability is affected by many random factors, so its prediction is not accurate. The direct way to check if the item meets its reliability specifications is to perform a compliance test.

The tests described in this document can be applied to items that have a failure rate or failure intensity (denoted by λ) which can be considered as a constant. The procedures are based on the assumption that trials of the test are statistically independent. If it is necessary to test the constant failure rate and constant failure intensity assumption, the procedures given in IEC 60605-6 should be used.

The test serves to verify the compliance with a specified λ_0 , that is, to verify that $\lambda \leq \lambda_0$.

The probability of making the correct decision in the test depends on the test duration and on the sample size (number of failures). The tests usually require a large sample size and, accordingly, a large consumption of time and funds. The consumptions are especially high for reliability testing. For this reason, sampling plans of the tests should be carefully planned in order to reduce the consumption.

This document is dedicated to sampling plans for the tests.

The tests are characterized by the operating characteristic (OC) and test duration until the test stops with the accept/reject decision on the compliance.

OC is the probability of accepting an item as meeting the requirements. In this document, the OC is represented by the coordinates of its two points (see ISO 3534-2 [10]):

- $(\lambda_0, 1 - \alpha)$ are the coordinates of the producer's risk point (PRP);
- (λ_1, β) are the coordinates of the consumer's risk point (CRP);

where α and β are producer's and consumer's risks, and $\lambda_1 > \lambda_0$.

The test duration (test time) is a random value and in this document is usually characterized by its expected (ETT) and maximum (MaxTT) values.

This document contains the following types of tests:

- optimally truncated sequential probability ratio test (SPRT, type A);
- maximally truncated SPRT (type C);
- fixed time/failure terminated test (FTFT, type B);
- FTFT – calendar time terminated test without replacement;
- combined test plan (type D).

The tests can be used for testing equipment (repaired or non-repaired) as well as for components (replaced or not replaced when failing).

All the plans in this document are sequential, that is, every time an event occurs during the test, a decision is made to continue or stop the test. An event occurs in two cases: when a failure occurs, or when the acceptance boundary is crossed, which means that there is compliance with the requirements. The decision can be one of three types:

- accept the compliance and stop the test;
- reject the compliance and stop the test;
- continue the test, because there is not enough information to stop it.

The difference between the types of tests is in the shape of border lines.

The FTFT is characterized by decision rules for accepting or rejecting compliance when the MaxTT has been reached, or the acceptable number of failures has been exceeded. This test has the smallest MaxTT among all tests with specified PRP and CRP. If, for a tested item $\lambda \leq \lambda_0$, then ETT is close to MaxTT; otherwise, if $\lambda > \lambda_0$, then ETT decreases. In fact, the only advantage of the FTFT over the SPRT is the simplicity of designing new test plans. A detailed procedure for the design is provided in this document.

The optimally truncated SPRT (type A) has a MaxTT of 1,1 to 1,2 times greater than the FTFT with the same PRP and CRP. However, the ETT of the SPRT is significantly smaller than that of the corresponding FTFT, and for $\lambda \leq \lambda_0$ it can be 1,4 to 1,8 times smaller. This is a great advantage of the SPRT. This document contains an extensive set of ready-to-use type A plans. The set also allows the design of additional tests by simple interpolation according to the procedure provided in this document.

The maximally truncated SPRT (type C) has a MaxTT, like the FTFT; however, its ETT is less than that of the FTFT, but greater than that of the type A SPRT.

In the combined test plan (type D), test items with early failures will not be rejected in the initial stages of the test.

Some of the ready-to-use tests listed in this document have a very large maximal acceptable number of failures, which is why they are likely to be rarely used. However, the data allows the user of this document to assess the economic benefit of the OC test requirements and, in general, to assess the advisability of performing the test.

Accumulated test time can be reduced by accelerated testing (see IEC 62506 [11]).

An example of objects covered by this document can be electronic equipment and its components, which usually have a failure rate or failure intensity that can be considered constant.

Clause 4 presents the requirements and area of application of the tests and recommendations for their selection. Clause 5 explains the general elements of the test procedure. Clause 6 explains the characteristics of the ready-to-use SPRT and the parameters of the border lines (their values are given in Annex A). Extension of the set of SPRT tests are given in Annex C. Clause 7 is devoted to the ready-to-use FTFT. Clause 8 presents the design of FTFT plans that are not covered in the tables of this document. Mathematical references and procedures of the design of FTFT plans are given in Annex E and in Annex F. Clause 9 is devoted to the calendar FTFT for non-replaced items (examples and mathematical references of their design are given in Annex G). Clause 10 is devoted to the combined test plans (parameters of their border lines are given in Annex B). Clause 11 explains how to perform the test and presentation of results. Annex D presents the approximation of OC by Wald's formula. Annex H is devoted to the mathematical reference for the test plans of GOST R 27.402 [12].

RELIABILITY TESTING – COMPLIANCE TESTS FOR CONSTANT FAILURE RATE AND CONSTANT FAILURE INTENSITY

1 Scope

This document gives a number of optimized test plans, the corresponding border lines and characteristics. In addition, the algorithms for designing test plans using a spreadsheet program are also given, together with guidance on how to choose test plans.

This document specifies procedures to test whether an observed value of

- failure rate,
- failure intensity,
- mean operating time to failure (MTTF),
- mean operating time between failures (MTBF),

conforms to a given requirement.

It is assumed, except where otherwise stated, that during the accumulated test time, the times to failure or the operating times between failures are independent and identically exponentially distributed. This assumption implies that the failure rate or failure intensity is assumed to be constant.

Four types of test plans are described as follows:

- truncated sequential probability ratio test (SPRT);
- fixed time/failure terminated test (FTFT);
- fixed calendar time terminated test without replacement;
- combined test.

This document does not cover guidance on how to plan, perform, analyse and report a test. This information can be found in IEC 60300-3-5.

This document does not describe test conditions. This information can be found in IEC 60605-2 and in IEC 60300-3-5.

2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

IEC 60050-192, *International Electrotechnical Vocabulary (IEV) – Part 192: Dependability*, available at <http://www.electropedia.org>

IEC 60300-3-5:2001, *Dependability management – Part 3-5: Application guide – Reliability test conditions and statistical test principles*

IEC 60605-2, *Equipment reliability testing – Part 2: Design of test cycles*

IEC 60605-4:2001, *Equipment reliability testing – Part 4: Statistical procedures for exponential distribution – Point estimates, confidence intervals, prediction intervals and tolerance intervals*

IEC 60605-6, *Equipment reliability testing – Part 6: Tests for the validity and estimation of the constant failure rate and constant failure intensity*

IEC 61123:2019, *Reliability testing – Compliance test plans for success ratio*

3 Terms, definitions, abbreviated terms and symbols

3.1 Terms and definitions

For the purposes of this document the terms and definitions given in IEC 60050-192 apply.

ISO and IEC maintain terminology databases for use in standardization at the following addresses:

- IEC Electropedia: available at <https://www.electropedia.org/>
- ISO Online browsing platform: available at <https://www.iso.org/obp>

The terms "failure rate" and "failure intensity" are used as meaning constant failure rate and constant failure intensity.

3.2 Abbreviated terms and symbols

3.2.1 Abbreviated terms

ADT	accept decision time – time that the test was terminated with accept decision
CDF	cumulative distribution function
CRP	consumer's risk point
ETT	expected accumulated test time to accept/reject decision
FTFT	fixed time/failure terminated test
MaxTT	maximal accumulated test time and accumulated truncation time of the test
MTBF	mean operating time between failures
MTTF	mean operating time to failure
OC	operating characteristic
PDF	probability density function
PRP	producer's risk point
RDS	reject decision segment – segment of time/failure line when the test was terminated with reject (see Figure 2)
SPRT	truncated sequential probability ratio test (in some literature called probability ratio sequential test (PRST)).

3.2.2 Symbols

The generic symbol λ is used in this document for failure rate and failure intensity.

The symbol m is used to denote both the following reliability measures:

- mean operating time between failures, MTBF;
- mean operating time to failure, MTTF.

When used, the relationship between the above quantities, under the given assumptions, is:

$$\lambda = 1/m$$

Test plans are based on m as a reliability measure; thus in these cases:

$$m = 1/\lambda$$

c	acceptable number of failures during FTFT
D	discrimination ratio; $D = m_0/m_1$ or $D = \lambda_1/\lambda_0$
m	true MTBF or MTTF
m_j	indexed m , where $j = L, 1, M, 0, H$ (as in Figure 1 and in Table 3)
m_0	specified MTTF or MTBF, $m_0 = 1/\lambda_0$
m_1	lower limit for MTTF or MTBF, $m_1 = 1/\lambda_1$
n	number of test items at the beginning of the test
P_a	probability of acceptance
p_0	acceptable failure ratio
q_0	acceptable success ratio, $q_0 = 1 - p_0$
$R(t)$	reliability function
r	observed number of failures during the test
r_0	maximum number of failures where an "accept" decision is possible (see Figure 2)
r_e	expected number of failures to reach a decision
T^*	accumulated test time
T_a^*	accumulated test time stated as accept criterion (ADT)
$T_{a,min}^*$	minimum test time for $r = 0$ stated as accept criterion
T_e^*	expected accumulated test time to decision
$T_{e,j}^*$	indexed T_e^* , where $j = L, 1, M, 0, H$ (which are related to the five values of m_j as in Figure 1)
$T_e^*(+)$	expected accumulated test time to acceptance
T_r^*	accumulated test time stated as reject criterion
T_t^*	accumulated test time stated as termination criterion for FTFT and MaxTT for A, C and D tests
t	test time
t^*	test time for each tested item
t_e^*	expected test time to decision
t_t^*	test truncation time
t_i	test time of failed item i
$t_{cal,t}^*$	calendar test time stated as termination criterion
R_D	optimization criterion, Formula (E.11)
R_R	optimization criterion, Formula (E.12)

$P(r)$	probability of r failures
α	producer's risk (type I risk)
α'	true producer's risk (type I risk)
β	consumer's risk (type II risk)
β'	true consumer's risk (type II risk)
λ	true failure rate per item
λ_0	specified expected failure rate per item (design goal)
λ_1	upper limit for constant failure rate per item

4 General requirements and area of application

4.1 Requirements and characteristics

It is assumed, except where otherwise stated, that during the accumulated test time, the times to failure or the operating times between failures are independent, and identically exponentially distributed. This assumption implies that the failure rate or failure intensity is assumed to be constant. Under this assumption, there is no difference between failure rate and failure intensity. Therefore, they are both called λ and referred to in this document as failure rate.

It is assumed that the requirement is specified in one of the following terms: the acceptable constant failure rate or the acceptable mean number of failures per time unit, λ_0 , or the acceptable mean operating time to failure or mean operating time between failures, m_0 .

The tests are characterized by an operating characteristic (OC) and test time to decision.

The OC of a test is the probability of accepting an item as meeting the requirements (see example in Figure 3). The OC is a function of the true value of m , or of λ . In this document, the OC is represented by the coordinates of its two points (see ISO 3534-2):

- $(m_0, 1 - \alpha)$ are the coordinates of the producer's risk point (PRP), or $(\lambda_0, 1 - \alpha)$;
- (m_1, β) are the coordinates of the consumer's risk point (CRP), or (λ_1, β) .

The test time to reaching a decision regarding the test is a random value and in this document is usually characterized by its expected (ETT) and maximum (MaxTT) values (see example in Figure 1). ETT is a function of the true value of m , or λ .

It is possible that special treatments will be required to ensure the constant failure rate or failure intensity, for example, screening for the elimination of early life failure period.

If it is necessary to test the constant failure rate/constant failure intensity assumption, the procedures given in IEC 60605-6 shall be used.

4.2 Applicability to replaced and repaired items

The truncated sequential probability ratio test plans (see Clause 6), the time/failure terminated test plans (see Clause 7) and the combined test plans (see Clause 10) are applicable to the following:

- replacement of failed items;
- without replacement of failed items;

under the assumption that

- an item can be replaced by repair of the item itself;
- the accumulated test time is calculated as elapsed operating item-time, in accordance with 5.4;
- replaced items belong to the same population as the original items;
- repaired items can be considered to have the same failure intensity after repair as they had before they failed.

The calendar time/failure terminated test plans in Clause 9, however, are applicable to cases where failed items are not replaced and where a fixed number of items are placed on test for a fixed calendar time. This means that the test is running, even though the number of items under test will not remain constant because some items will not survive.

4.3 Types of test plans

4.3.1 General

Test plans are given for the following test types:

- optimally truncated SPRT (type A ready-to-use plans, see Clause 6);
- maximally truncated SPRT (type C ready-to-use plans, see Clause 6);
- FTFT (type B ready-to-use plans see Clause 7, and Clause 8 for design of alternative FTFT);
- FTFT – calendar time terminated tests without replacement (see Clause 9);
- combined test plans (see Clause 10).

All plans in this document are sequential, that is, every time an event occurs during the test, a decision is made to continue or stop the test. The event is: failure or crossing the acceptance boundary of the hypothesis compliance. The decision can be one of three:

- accept the compliance hypothesis and stop the test;
- reject the compliance hypothesis and stop the test;
- continue the test, because there is not enough information to stop it.

The difference between the test types is the shape of border lines, as shown in Table 1.

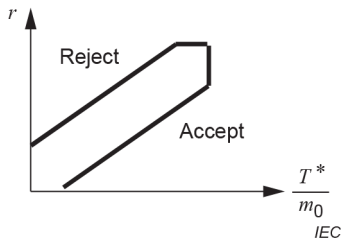
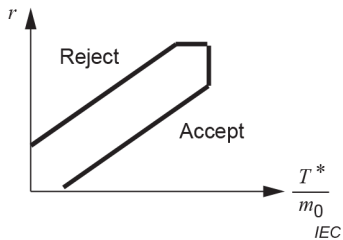
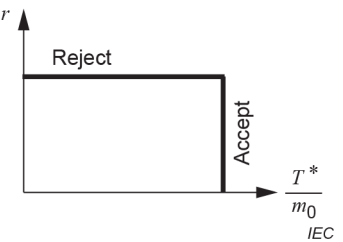
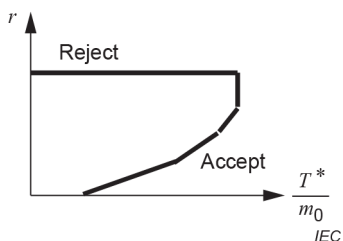
The duration of the tests is a bounded continuous random variable, which means that the test stops when MaxTT is reached, if it has not already been stopped. This random variable is conveniently characterized by ETT and MaxTT. ETT is a function of the true value of m (Figure 1).

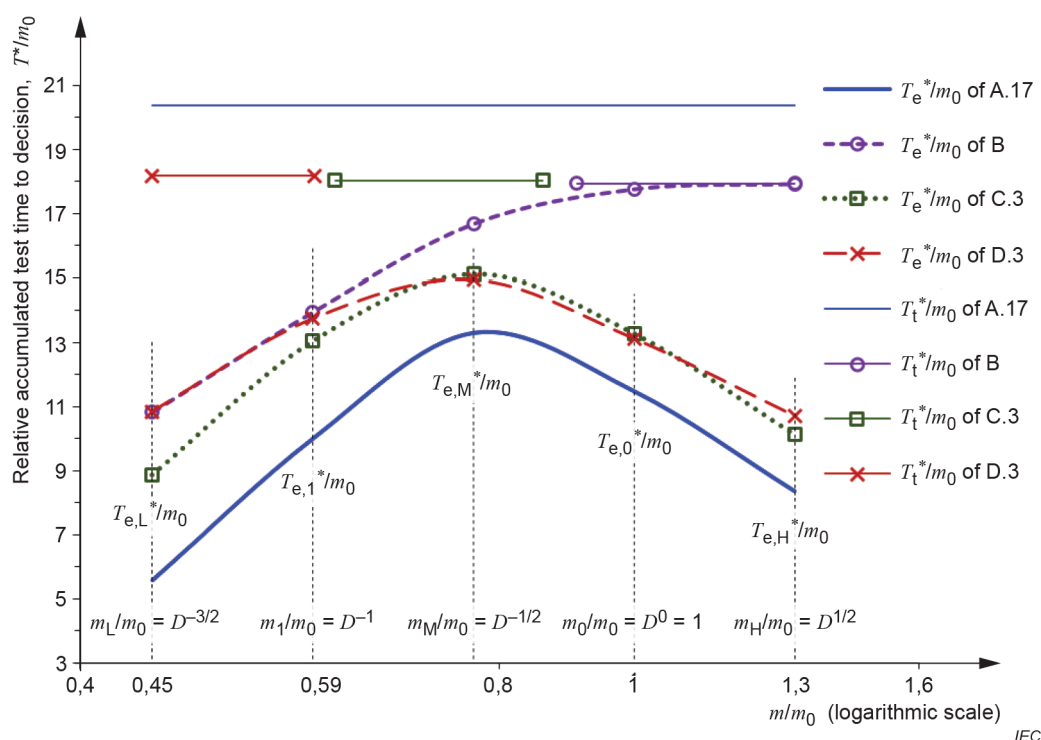
NOTE Ready-to-use plans are test plans for which this document specifies: border lines of the test, D , α' , β' , ETT and MaxTT.

4.3.2 Advantages and disadvantages of the different test plan types

The comparison between the different types of test plans are shown in Table 1. Figure 1 shows examples of MaxTT and function ETT of the different types of plans. For all plans in Figure 1, the OS curves are very close and coincide at two points (PRP and CRP) with the coordinates: $(m_0, 1 - \alpha')$ and (m_1, β') , respectively. See [5].

Table 1 – Advantages and disadvantages for the different test plan types

Test plan type	Advantages	Disadvantages
Optimally truncated SPRT (type A plans, see Clause 6) 	Ensures highest test efficiency (lowest ETT without significant increment of the MaxTT). This document contains a wide set of these test plans. The user can plan additional test plans using interpolation (Annex C). The ready-to-use A test plans includes true α, β (given in Table 4) which are very close to the nominal.	MaxTT is higher than in the other tests listed here (5 % to 20 %). This can affect the test (with a low probability) only when $m_1 < m < m_0$. For m outside this interval, MaxTT is practically not reached and, accordingly, does not have an effect.
Maximally truncated SPRT (type C plans, see Clause 6) 	MaxTT of C tests is lower than that of the A tests and is almost equal to the MaxTT of the FTFT. The ready-to-use C test plans include true α, β (given in Table 5) which are very close to the nominal.	ETT of the C tests is much longer than that of the A tests. A very limited set of ready-to-use C test plans (only for $D = 1,7$). For additional plan development, a time-consuming iterative process is required.
FTFT - ready-to-use test plans B in Clause 7, and additional tests that can be planned according to Clause 8. - calendar time terminated tests without replacement that can be planned according to Clause 9. 	The user can develop additional test plans using a spreadsheet program. MaxTT of FTFTs is lower than that of the A tests. Easy administration and known accumulated test time to accept decision.	ETT is much longer than that of the other tests listed here.
Combined Test plans in Clause 10. 	Very reliable test items will be accepted in less time than in C plans, while test items with early failures will not be rejected early in the test. MaxTT of type D tests is equal to that of type C tests.	A very limited set of ready-to-use D test plans (only for $D = 1,7$). For additional plan development, a time-consuming and complex iterative process is required.



NOTE 1 This Figure 1 is drawn for $D = 1,7$, $\alpha' = \beta' = 10\%$.

NOTE 2 A.17, C.3 and D.3 are tests from Table 4, Table 5 and Table 7, respectively.

NOTE 3 B is the virtual FTFT according to Clause 8 with characteristics given in NOTE 1. Its T_e^*/m_0 is calculated according to Annex F. The test is virtual because its acceptable number of failures is not an integer.

Figure 1 – Relative ETT (T_e^*/m_0) and MaxTT (T_t^*/m_0) of various tests with the same risks

5 General test procedure

5.1 Test conditions

A guide on how to plan, perform, analyse and report a test can be found in IEC 60300-3-5. Test conditions can be found in IEC 60605-2 and IEC 60300-3-5.

All of these test types can be accelerated to accommodate high reliability requirements and enable shorter time to market (see IEC 62506 [11]). Mission and life-cycle environment profiles and test conditions are provided as follows.

- The mission profiles should be used to determine the environmental specifications and should be derived from the operational life profile defined by the equipment or system requirements. If this information is not provided in the original contractual documentation, provision should be made to derive the mission profiles and the equipment environmental specifications. Here, the term "equipment" is used in place of the more appropriate general term "item" to match the terminology of the referenced standards.
- This derivation should make use of historical data on similar equipment applications and mounting platforms and the effect of equipment location in the platform should be accounted for. Each significant life-cycle event shall be considered including transportation, handling, installation and testing as well as for platform category and operational conditions.
- Environmental test conditions (for reliability growth, qualification, acceptance tests) should be used. All of the test types should be performed under the influence of the appropriate test conditions which are expected to be combined in the used environments.

- The test level for these test conditions should be derived from the equipment's mission and environment profiles:
 - When the equipment is designed for one application with a single mission or one type of repetitive mission, there is a simple relationship between the test profile, mission profile and life-cycle environmental profile. The test conditions should simulate the stress level during the mission.
 - If the equipment is designed for multi-missions and environmental conditions, the test profile should represent a composite of those missions, with the test levels and durations being rated according to the percentage of each mission type expected during the equipment's life-cycle. In order to derive realistic test conditions and levels, the actual environments (especially temperature and vibration) should be measured at the location where the equipment is to be mounted during the actual operation.
 - If design changes are made during the test, the accumulated test time shall start from 0 after the changes have been incorporated in the test items, unless specifically agreed between the parties and justified in the test report (see IEC 60300-3-5 and IEC 61014 [13]).

5.2 General characteristics of the test plans

All test plans and decision criteria are based on either of the two following characteristics:

- an acceptable number of failures in a specified test time;
- an acceptable test time for a specified number of failures.

The detailed criteria for test plans, with variants, are given in 6.3, 7.3, 8.4 and 9.3.

5.3 Data to be recorded

In order to apply the statistical procedures defined in this document, the following data shall be recorded:

- the observed number of failures;
- the accumulated test time.

If it is necessary to test the assumption stated in 4.1, the following data shall also be recorded:

- the time of occurrence of each failure.

5.4 Calculation of accumulated test time, T^*

Whenever the term "time" is used in this document it can be replaced by other similar measures of the duration of the operation for the item(s) under test, for example distance driven, operating cycles, start and stop cycles (see IEC 60605-2).

The accumulated test time T^* is calculated as the sum of the test time(s) accumulated by each item in the test. For more than one repaired item under test, the accumulated test time is the sum of all elapsed operational times (excluding all repair times and other down times).

When n items are put on test at the same time, the accumulated test time can be determined using the following formulae:

$$T^* = nt_t^* \quad \text{for tests with replacement of failed items;}$$

$$T^* = (n-r)t_t^* + \sum_{i=1}^r t_i \quad \text{for tests without replacement of failed items;}$$

where

n is the number of items at the beginning of the test;

t_t^* is the test truncation time;

t_i is the test time of failed items;

r is the number of failed items.

It is necessary to monitor the test items during the test in order to replace failed items, register accumulated test time and, where required, record time to failures (see 5.3).

Several items can be tested simultaneously or sequentially. The recommended sample size is $n \geq 1$ with replacement and $n \geq r_0 + 1$ (Clauses 6 and 10) and $n \geq c + 1$ (Clauses 7, 8 and 9) without replacement.

Calendar time is used to state the test time when more than one item is on test simultaneously (see Clause 9).

Calculation of accumulated test time when items enter the test after the test has started, or are removed from the test before the test ends (censoring) shall comply with IEC 60300-3-5 and IEC 60605-4:2001, Annex C. The accumulated test time is normally counted as the operating time. If the parties to the test wish to include other operational modes such as "idle" or "stand by" in the test time, for example in a test cycle, this shall be specified and agreed before the test is started (shall comply with IEC 60605-2).

5.5 Number of failures

In this document the test result is determined by the test time and the number of failures. However, in practice, failures can have very different consequences. Therefore, the user should consider if there are failures that are so critical that the test has failed even though the total number of failures are not exceeded. Alternatively, there can be failures where the consequences are so insignificant that they should not be considered as relevant failures. All failures are counted except when they can be classified as non-relevant, for example caused by factors not relevant for the reliability of the item in the field. Such classification rules should be agreed between the parties before the start of the test (see IEC 60300-3-5). Re-classification of failures requires agreement between the parties and shall be justified in the test report.

6 Truncated sequential probability ratio test (SPRT) plans

6.1 General

SPRTs are characterized by decision rules for accepting or rejecting compliance or continuing the test at any test time. The rules are determined by selected values of risks and discrimination ratio. To implement a test plan, m_0 shall be specified. An overview of test plans is given in 6.6. The test plan's border lines and example are stated in Annex A. SPRTs type A (optimally truncated) are derived using a methodology based on [5], [6], [7].

The SPRT type C (maximally truncated) is derived by a different methodology based on GOST R 27.402 [12]. Descriptions of the iterative procedure for creating those test plans can be found in Annex H. An overview of test plans, type C, is given in 6.6. Comparison of C and D tests is given in Annex B.

6.2 Common test procedure

The test procedure should include the following steps.

- Decide the combined environmental conditions to be applied during the test based on analysis to determine appropriate test conditions (see 5.1 and IEC 60300-3-5).
- Specify m_0 , according to requirement or agreement.
- Choose risks, α and β , and discrimination ratio, D , considering that low values of α , β and D make a more powerful test, but require a longer test time or more items or both.
- Select an appropriate test plan from Table 4, Table 5 and Annex A.
- Note the test times, T_a^* for accept, and T_r^* for reject (Annex A).
- Perform the test and apply the decision criteria in accordance with 6.3.

6.3 Decision criteria

The accumulated test time and the number of failures shall be compared with the accept and reject criteria continuously during the test (Figure 2), or at intervals not greater than the length of the test cycle or the monitoring interval (see 5.1 and IEC 60300-3-5:2001, 7.1.3). The following criteria shall apply.

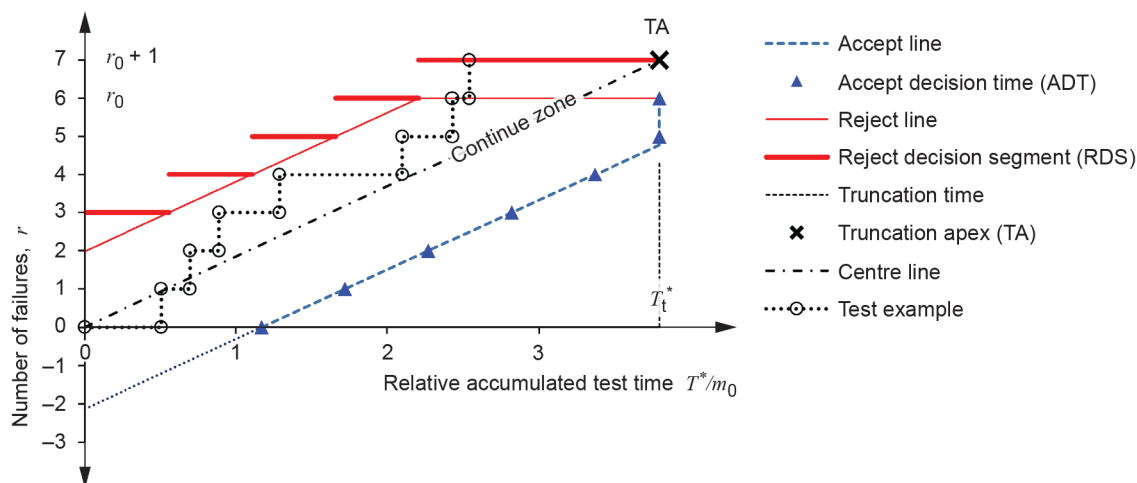
Accept the compliance hypothesis if the elapsed $T^* \geq T_a^*$ for the observed $r \leq r_0$.
The specified requirement is regarded as being complied with.

Continue testing if the elapsed T^* is between the limits $T_r^* < T^* < T_a^*$ for the observed $r \leq r_0$.
No decision can be taken.

Reject the compliance hypothesis if the elapsed $T^* \leq T_r^*$ for the observed $r \leq r_0$, or if $r \geq r_0 + 1$.
The specified requirement is regarded as not being complied with.

Below are explanations for the SPRT diagram and the test example as shown in Figure 2:

- accept line is the border line for acceptance;
- accept decision time (ADT) are points on the accept line for the test termination;
- reject line is the border line for reject when passing;
- reject decision segment (RDS) is the segment when the test was terminated with reject;
- centre line is a line parallel to accept and reject lines and starts at the origin;
- truncation time (T_t^*);
- test example is an example for a test course.



NOTE This Figure 2 is drawn for A.41 test plan (Table 4, $D = 3$, $\alpha' = \beta' = 10,0\%$).

Figure 2 – SPRT diagram and test example

6.4 Operating characteristic (OC) curve

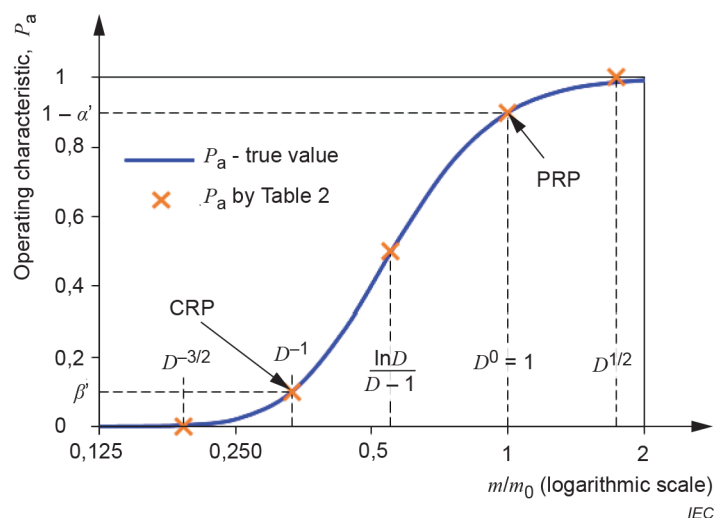
For all the tests that are described in this document (including the non-SPRTs), the following approximate points on the OC curve are given in Table 2 and Figure 3. The approximation is based on two given OC points: CRP and PRP. This approximation has sufficient accuracy for practical purposes.

Table 2 – OC curve

m		P_a
Designation	Value	
m_H	$m_0 D^{0,5}$	~ 1
m_0	m_0	$1 - \alpha'$
m_S	$m_0 \frac{\ln D}{D - 1}$	$\frac{\ln(1 - \beta') - \ln \alpha'}{\ln(1 - \beta') - \ln \alpha' - \ln \beta' + \ln(1 - \alpha')}$
m_1	$m_1 = m_0 D^{-1}$	β'
m_L	$m_0 D^{-1,5}$	~ 0

NOTE 1 For $\alpha' = \beta'$ case, $P_a(m_S) = 0,5$.

NOTE 2 The table is suitable for all the tests in this document.



NOTE This Figure 3 is drawn for $D = 3$, $\alpha' = \beta' = 10,0\%$ (Test plan A.41).

Figure 3 – OC curve, P_a

NOTE 1 The procedure for the exact calculation of the OC of a test plan at its border lines is given in [2]. Another procedure for the same purpose is given in Annex D. These procedures are quite difficult to implement with a spreadsheet program.

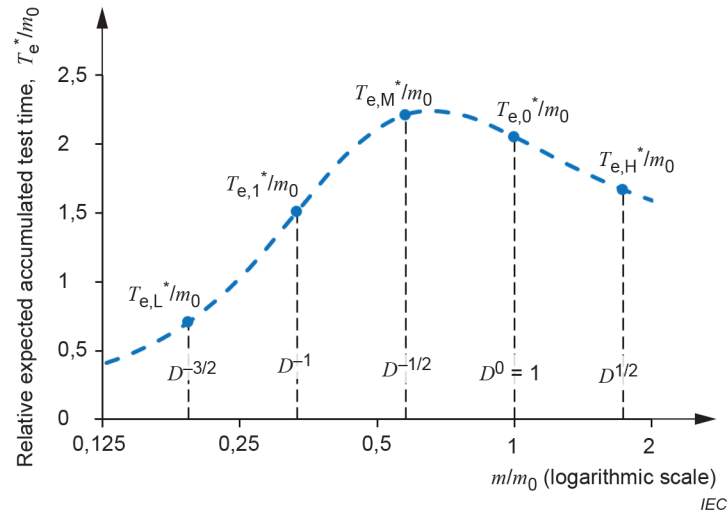
NOTE 2 It is possible to approximate the OC with high accuracy based on Wald's formula [1] or the formula for FTFTs. These approximations are based on the CRP and PRP; they do not require knowledge of the border lines of the test and can be performed using a spreadsheet program. One of the purposes of these approximations is to obtain more OC points than given in Table 2. The procedure for approximation by the Wald's formula (better suited for type A tests) is described in Annex D and the procedure based on the formula for FTFTs (better for type C and D tests) is described in Clauses E.4 and F.8. In addition, the detailed methodology for approximation of the OC is described in [4], Subsection 2.2 and in [5], Appendix A.

6.5 Expected accumulated test time to decision (ETT)

For any of the SPRTs in Table 4, the relative ETT curve is defined via five points $(T_{e,j}^*/m_0)$ versus the true ratio, m_j/m_0 (according to Table 3 and Figure 4).

Table 3 – Relative ETT versus m/m_0

m/m_0	$m_L/m_0 = D^{-3/2}$	$m_1/m_0 = D^{-1}$	$m_M/m_0 = D^{-1/2}$	$m_0/m_0 = D^0$	$m_H/m_0 = D^{1/2}$
$T_{e,j}^*/m_0$	$T_{e,L}^*/m_0$	$T_{e,1}^*/m_0$	$T_{e,M}^*/m_0$	$T_{e,0}^*/m_0$	$T_{e,H}^*/m_0$
NOTE $j = L, 0, M, 1, H$ (see Figure 4).					



NOTE This Figure 4 is drawn for $D = 3$, $\alpha' = \beta' = 10,0\%$.

Figure 4 – SPRT – Curve of expected accumulated test time to decision (ETT)

The expected number of failures to reach a decision is:

$$r_e = T_e^* / m \quad (1)$$

The expected test time to reach a decision in the test with replacement is:

$$t_e^* = T_e^* / n \quad (2)$$

When the test is without replacement, the expected test time to arrive at a decision for $n \geq r_0 + 1$ items under test is [2]:

$$t_e^* = m \ln \left(\frac{n}{n - r_e} \right) \quad (3)$$

where r_e is from Formula (1).

6.6 Overview of test plans

Table 4 and Table 5 show the parameters of the characteristics of SPRT plans of type A and C, respectively. These parameters are sufficient to choose a specific test plan. The parameters of the border lines for each of the tests are given in Annex A. An example of an SPRT plan is given in Clause A.3.

Table 4 – Overview of type A SPRT plans

Test plan No.	Characteristics of the plan			Minimum test time (Acceptance at 0 failures)	ETT in multiples of m_0 for various m					Test truncation time in multiples of m_0
	Discri- mination ratio	True risks			$m = m_L$	$m = m_1$	$m = m_M$	$m = m_0$	$m = m_H$	
		D	$\alpha' \%$							
A.1	1,5	5	5	6,54	13,6	27,0	40,8	30,5	19,3	63,93
A.2	1,5	5	10	5,24	13,8	24,8	31,8	23,7	15,4	48,37
A.3	1,5	5	20	3,61	12,5	19,5	21,5	15,8	10,5	36,22
A.4	1,5	10	5	6,60	11,1	21,3	32,1	27,9	19,2	48,69
A.5	1,5	10	10	5,03	10,4	18,3	24,2	20,6	14,5	37,96
A.6	1,5	10	20	3,63	9,8	14,5	16,4	13,8	10,3	24,97
A.7	1,5	10	40	1,99	6,3	7,6	7,7	6,6	5,3	13,70
A.8	1,5	20	5	6,09	7,6	14,2	21,8	21,9	17,0	37,39
A.9	1,5	20	10	4,71	7,5	12,4	16,4	16,0	12,8	25,92
A.10	1,5	20	20	3,23	6,3	8,8	10,2	9,8	8,3	16,09
A.11	1,5	30	30	1,88	3,0	3,5	3,9	3,9	3,7	6,44
A.12	1,5	40	10	3,97	4,0	6,0	7,9	8,8	8,6	14,26
A.13	1,7	5	5	4,65	7,1	14,6	22,3	16,9	11,1	34,80
A.14	1,7	5	10	3,70	7,1	13,3	17,3	13,1	8,8	26,50
A.15	1,7	5	20	2,55	6,4	10,4	11,7	8,7	6,0	19,87
A.16	1,7	10	5	4,70	5,9	11,6	17,6	15,6	11,1	26,38
A.17	1,7	10	10	3,59	5,6	10,0	13,3	11,5	8,4	20,38
A.18	1,7	10	20	2,54	5,1	7,7	8,9	7,6	5,8	13,70
A.19	1,7	10	40	1,45	3,4	4,2	4,2	3,7	3,1	7,11
A.20	1,7	20	5	4,35	4,1	7,7	12,0	12,2	9,8	20,18
A.21	1,7	20	10	3,41	3,9	6,6	8,9	8,9	7,4	14,10
A.22	1,7	20	20	2,22	3,2	4,7	5,5	5,4	4,6	9,03
A.23	1,7	30	30	1,29	1,5	1,9	2,1	2,1	2,1	3,76
A.24	1,7	40	10	2,81	2,1	3,3	4,4	4,9	4,9	7,75
A.25	2	5	5	3,24	3,6	7,7	11,9	9,3	6,3	18,65
A.26	2	5	10	2,59	3,7	7,1	9,3	7,2	5,0	14,00
A.27	2	5	20	1,75	3,2	5,4	6,2	4,7	3,4	10,82
A.28	2	10	5	3,24	3,0	6,1	9,4	8,5	6,2	14,29
A.29	2	10	10	2,49	2,8	5,2	7,1	6,3	4,7	11,00
A.30	2	10	20	1,69	2,4	3,9	4,6	4,0	3,2	7,81
A.31	2	10	40	0,95	1,7	2,1	2,2	2,0	1,7	3,98
A.32	2	20	5	2,97	2,0	4,0	6,4	6,6	5,5	11,28
A.33	2	20	10	2,29	1,9	3,4	4,7	4,8	4,1	7,95
A.34	2	20	20	1,54	1,7	2,5	3,0	2,9	2,6	4,79
A.35	2	30	30	0,88	0,8	1,0	1,1	1,2	1,2	2,05
A.36	2	40	10	1,85	1,0	1,6	2,2	2,6	2,6	4,82
A.37	3	5	5	1,59	0,9	2,3	3,8	3,1	2,3	6,91

Test plan No.	Characteristics of the plan			Minimum test time (Acceptance at 0 failures)	ETT in multiples of m_0 for various m					Test truncation time in multiples of m_0
	Discri- mination ratio	True risks			$m = m_L$	$m = m_1$	$m = m_M$	$m = m_0$	$m = m_H$	
		D	α' %							
A.38	3	5	10	1,24	0,9	2,1	2,9	2,4	1,8	4,65
A.39	3	5	20	0,87	0,9	1,6	2,0	1,6	1,2	3,34
A.40	3	10	5	1,60	0,8	1,8	3,0	2,8	2,3	4,58
A.41	3	10	10	1,17	0,7	1,5	2,2	2,0	1,7	3,80
A.42	3	10	20	0,82	0,7	1,2	1,5	1,3	1,1	2,48
A.43	3	10	40	0,47	0,5	0,6	0,7	0,7	0,6	1,19
A.44	3	20	5	1,47	0,5	1,2	2,0	2,2	2,0	3,67
A.45	3	20	10	1,17	0,6	1,1	1,6	1,7	1,5	2,32
A.46	3	20	20	0,73	0,4	0,7	0,9	1,0	0,9	1,61
A.47	3	30	30	0,42	0,2	0,3	0,3	0,4	0,4	1,08
A.48	3	40	10	0,95	0,3	0,5	0,7	0,9	1,0	1,36
A.49	5	5	5	0,76	0,2	0,7	1,3	1,1	0,9	2,40
A.50	5	5	10	0,58	0,2	0,6	1,0	0,9	0,7	1,88
A.51	5	5	20	0,40	0,2	0,5	0,7	0,6	0,5	1,39
A.52	5	10	5	0,76	0,2	0,6	1,0	1,0	0,9	1,71
A.53	5	10	10	0,56	0,2	0,5	0,8	0,7	0,7	1,58
A.54	5	10	20	0,38	0,2	0,4	0,5	0,5	0,4	1,09
A.55	5	10	40	0,21	0,1	0,2	0,2	0,2	0,2	0,55
A.56	5	20	5	0,71	0,1	0,4	0,7	0,8	0,8	1,41
A.57	5	20	10	0,54	0,2	0,3	0,5	0,6	0,6	0,87
A.58	5	20	20	0,35	0,1	0,2	0,3	0,4	0,4	0,78
A.59	5	23,5	26,2	0,27	0,1	0,1	0,2	0,2	0,3	0,39
A.60	5	37,2	9,8	0,46	0,1	0,2	0,3	0,4	0,4	0,55

Table 5 – Overview of type C SPRT plans

Test plan No.	Characteristics of the plan			Minimum test time (Acceptance at 0 failures)	ETT in multiples of m_0 for various m					Test truncation time in multiples of m_0
	Discrimination ratio	True risks			$m = m_L$	$m = m_1$	$m = m_M$	$m = m_0$	$m = m_H$	
					$\underline{T_{e,L}^*}$ m_0	$\underline{T_{e,1}^*}$ m_0	$\underline{T_{e,M}^*}$ m_0	$\underline{T_{e,0}^*}$ m_0	$\underline{T_{e,H}^*}$ m_0	
	D	$\alpha \%$	$\beta \%$	$\underline{T_{a,min}^*}$ m_0						$\underline{T_t^*}$ m_0
C.1	1,7	5	5	7,58	10,3	18,9	26,0	23,9	17,3	29,30
C.2	1,7	10	5	5,03	10,1	16,3	19,8	16,6	11,8	24,24
C.3	1,7	10	10	4,43	8,9	13,0	15,1	13,3	10,1	18,03
C.4	1,7	15	5	5,84	9,9	14,2	17,2	16,4	13,1	19,90
C.5	1,7	20	10	4,02	5,5	8,2	10,1	10,0	8,6	12,60
C.6	1,7	20	20	2,34	3,8	5,1	5,8	5,6	4,8	8,30
C.7	1,7	30	20	2,10	2,6	3,3	3,9	4,0	3,7	5,50
C.8	1,7	30	30	1,28	1,5	1,8	2,0	2,1	2,0	3,80

7 Fixed time/failure terminated test plans – Fixed duration (to acceptance) test plans

7.1 General

Fixed time/failure terminated test (FTFT) plans are characterized by decision rules for accepting or rejecting compliance when the test time for termination has been reached, or the acceptable number of failures has been exceeded. The test plans are determined by selected values of risks and discrimination ratio. To implement a test plan, m_0 shall be specified (or derived).

The FTFT plans have the advantage that they are easy to administer, and that the accumulated test time to accept decision is known before the test, which is important for planning and for booking of the test equipment.

There are four types of fixed duration tests:

- time terminated tests with replacement;
- time terminated tests without replacement;
- failure terminated tests with replacement;
- failure terminated tests without replacement.

The plans mentioned in (a) and (c) differ just by the representation on the graphs of the border lines of the test. In case (a), the x-axis is time, and the y-axis is the number of failures. In case (c), the x-axis is the number of failures, and the y-axis is time. This is also true for the pair (b) and (d).

In this document, Clauses 7 to 9 and Annex E, Annex F and Annex G are devoted to FTFTs.

Clause 7 contains 13 ready-to-use FTFTs (B plans, Table 6); no additional calculations are required to use them. Examples of application of the B plans are given in Clause E.3, and mathematical references are given in Annex E.

Clause 8 is devoted to the design of additional (alternative) FTFTs. Mathematical references and algorithms of the design are given in Annex E. Annex F describes the design of FTFTs using a spreadsheet program and illustrates the design on a comprehensive number of examples.

Clause 9 is dedicated to calendar FTFTs for non-replaced items. Examples and mathematical references for the plans are given in Annex G.

7.2 Common test procedure

The test procedure should include the following steps.

- Decide the combined environmental conditions to be applied during the test based on analysis to determine appropriate test conditions (see 5.1 and IEC 60300-3-5).
- Specify m_0 according to requirement or agreement.
- Select an appropriate test plan from Table 6, considering that low values of α , β and D make a more powerful test, but require a longer time or more items or both.
- Obtain from Table 6 the accumulated test time for termination, T_t^* , and the acceptable number of failures, c .
- Perform the test and apply the decision criteria according to 7.3.

7.3 Decision criteria

The following decision criteria shall apply.

- | | |
|-----------------|--|
| Accept the test | if $r \leq c$ at T_t^* .
The specified requirement is regarded as being complied with. |
| Reject the test | if $r > c$ at or before T_t^* .
The specified requirement is regarded as not being complied with. |

7.4 Test plans

Table 6 summarizes the recommended FTFT plans. The test time is accumulated until either a predetermined amount of test time has been reached (accept), or a predetermined number of failures has been exceeded (reject).

For any of the FTFTs in Table 6, the ETT curve is defined via five points ($T_{e,j}^*/m_0$ versus the true m) (also given in Table 6) according to Table 3 and Figure 1.

For any of the FTFTs, the five approximate points on the OC are given in Table 2 and the example is shown in Figure 3. The exact calculation of the OC using the spreadsheet program is given in Annex F.

Table 6 – Type B FTFT plans

Test plan No.	Characteristics of the plan			Test time for termination	Acceptable number of failures	ETT in multiples of m_0 for various m				
	True risks		Discrimination ratio			$m_L = m_0 D^{-1,5}$	$m_1 = m_0 D^{-1}$	$m_M = m_0 D^{-0,5}$	m_0	$m_H = m_0 D^{0,5}$
	$\alpha' \%$	$\beta' \%$	D	T_t^*/m_0	c	$T_{e,L}^*/m_0$	$T_{e,1}^*/m_0$	$T_{e,M}^*/m_0$	$T_{e,0}^*/m_0$	$T_{e,H}^*/m_0$
B.1	4,96	4,84	1,5	54,10	66	36,5	44,5	51,7	54,0	54,1
B.2	4,97	4,99	2	15,71	22	8,1	11,4	14,6	15,6	15,7
B.3	5,35	5,40	3	4,76	8	1,7	3,0	4,3	4,7	4,8
B.4	4,25	4,29	5	1,88	4	0,4	1,0	1,7	1,9	1,9
B.5	10,00	10,20	1,5	32,14	39	21,8	26,4	30,3	31,9	32,1
B.6	10,00	10,07	2	9,47	13	4,9	6,9	8,6	9,3	9,5
B.7	9,40	9,90	3	3,10	5	1,2	1,9	2,7	3,0	3,1
B.8	9,96	9,48	5	1,08	2	0,3	0,6	0,9	1,0	1,1
B.9	19,49	19,94	1,5	14,30	17	9,8	11,6	13,1	13,9	14,2
B.10	20,40	20,44	2	3,93	5	2,1	2,8	3,4	3,7	3,9
B.11	18,37	18,40	3	1,47	2	0,6	0,9	1,2	1,4	1,4
B.12	29,99	29,95	1,5	5,41	6	3,7	4,2	4,7	5,0	5,2
B.13	28,28	28,54	2	1,85	2	1,0	1,3	1,5	1,7	1,8

8 Design of alternative time/failure terminated test plans (FTFT)

8.1 General

The FTFT plans described in this Clause 8 are alternatives to the ready-to-use FTFT plans given in Clause 7.

Alternative FTFT plans are characterized by decision rules for accepting or rejecting compliance when the fixed test time for termination has been reached or the fixed acceptable number of failures has been exceeded. The test plans are determined by selected values of risks and test time. The test time is chosen independently of λ_0 which enables test plans to be designed individually to meet particular needs including, for example, economy, resources or time scheduling.

In order to implement a test plan, λ_0 shall be specified (or derived). The transformation between λ_0 and m_0 is given in 3.2.2.

It is also possible to choose different test plan parameters and determine unknown parameters as described in Annex E and Annex F. Specifically, Annex E contains mathematical references for FTFT plans; Annex F describes the design of FTFTs using a spreadsheet program and illustrates the design on a comprehensive number of examples.

8.2 Design procedures

The procedures given in Annex E and Annex F can be applied directly to tests in which:

- replacement occurs, and replacement or repair times are negligible;
- replacement does not occur, and $\lambda_0 t_t^* < 0,1$ is valid.

In such cases, $T_t^* = n t_t^*$.

However, the procedures can also be applied to tests in which:

- replacement occurs, and replacement times are long (e.g. due to repair);
- replacement does not occur, and $\lambda_0 t_t^* > 0,1$.

In such cases, the formulae involving accumulated test time, T_t^* (see 5.4) shall be used, and in the formulae nt_t^* shall be replaced by T_t^* . This implies that n and t_t^* cannot be separated.

In each case, when applicable, the values of α and β equal to 5 %, 10 % or 20 % shall be considered as preferred values.

8.3 Common FTFT procedure

Input parameters for the test can be test's resources (maximum test time and number of test items available) or OC, or their combination.

The test procedure should include the following steps.

- a) Decide the combined environmental conditions to be applied during the test based on analysis to determine appropriate test conditions (see 5.1 and IEC 60300-3-5).
- b) Specify the acceptable λ_0 (or m_0), according to requirement or agreement.
- c) Choose risks, α and β , test time for each item, t_t^* , and number of test items, n , according to resources.
- d) Design the test plan according to Annex E and Annex F.
- e) Evaluate the test plan especially regarding the resources allocated to the test, and perhaps redesign with modified parameters to satisfy the requirement.
- f) Perform the test with the stated test time, $T_t^* = nt_t^*$ and the derived acceptable number of failures, c , and apply the decision criteria according to 8.4.

8.4 Decision criteria

The following decision criteria shall apply.

- | | |
|-----------------|--|
| Accept the test | if $r \leq c$ at T_t^* .
The specified requirement is regarded as being complied with. |
| Reject the test | if $r > c$ at or before T_t^* .
The specified requirement is regarded as not being complied with. |

9 Calendar time/failure terminated test plans (FTFT) for non-replaced items

9.1 General

Calendar FTFTs for non-replaced items are characterized by decision rules for accepting or rejecting compliance when the test time for termination has been reached, or the acceptable number of failures has been exceeded. The test plans are determined by selected values of risks and discrimination ratios. To implement a test plan, λ_0 shall be specified (or derived). The transformation between λ_0 and m_0 is described in 3.2.2.

Since no replacement takes place, the number of items being tested will generally not remain constant, and the accumulated time of operation is undetermined and, hence, is not relevant. Thus, the compliance test plans for success ratio specified in IEC 61123:2019, Table 5 shall be used. An alternative is to use IEC 61710 (Power law) [14] or IEC 61649 (Weibull analysis) [15].

However, if the number of items is large compared to the number of failures, the FTFT plans (see Clause 7) can be used.

Examples and a mathematical reference are given in Annex G.

9.2 Common test procedure

The test procedure should include the following steps.

- Decide the combined environmental conditions to be applied during the test based on analysis to determine appropriate test conditions (see 5.1 and IEC 60300-3-5).
- Specify λ_0 , according to requirement or agreement.
- Choose risks, α and β , and discrimination ratio, D , considering that low values of α , β and D make a more powerful test but require a longer time or more items or both.
- Choose test termination time, $t_{\text{cal},t}^*$, or number of items, n , and calculate according to 9.4.2 or 9.4.3, respectively.
- Select an appropriate test plan from IEC 61123:2019, Table 5, using the derived q_0 and the found n , to give the acceptable number of failures c .
- Perform the test and apply the decision criteria according to 9.3.

9.3 Decision criteria

The decision criteria include the following.

- Accept the test if $r \leq c$ at $t_{\text{cal},t}^*$
 The specified requirement is regarded as being complied with.
- Reject the test if $r > c$ at or before $t_{\text{cal},t}^*$
 The specified requirement is regarded as not being complied with.

9.4 Use of IEC 61123:2019, Table 5 for fixed calendar time tests

9.4.1 General

Two procedures can be used. The first one, given in 9.4.2, is used when the (maximum available) test time is given. The second one, given in 9.4.3, is used when the (maximum available) number of items is given.

The following formulae apply:

$$p_0 = 1 - q_0 = 1 - e^{-t/m_0} = 1 - e^{-\lambda_0 t} \quad (4)$$

where $t = t_{\text{cal},t}^*$

and the derived

$$t_{\text{cal},t}^* = -m_0 \cdot \ln(1 - p_0) = -m_0 \cdot \ln(q_0) \quad (5)$$

where

p_0 is the acceptable failure ratio;

q_0 is the acceptable success ratio, $q_0 = 1 - p_0$.

NOTE 1 According to the normal use of IEC 61123:2019, p_0 or q_0 is specified, but in this case p_0 is derived from the specified m_0 or λ_0 , according to Formula (4). For further information on p_0 and q_0 , see IEC 61123:2019.

NOTE 2 In this case, p_0 expresses the failure probability $1 - R(t)$ for $t = t_{\text{cal},t}^*$.

NOTE 3 The procedures can only determine the product nt^* .

9.4.2 Procedure when the test time is given

The test procedure should include the following steps.

- a) Choose $t_{\text{cal},t}^* \leq 0,223$ and $m_0 = 0,223/\lambda_0$ (so that p_0 is within the range of IEC 61123:2019, Table 5).
- b) Calculate p_0 using Formula (4).
- c) Adjust $t_{\text{cal},t}^*$ if necessary to fit a $q_0 = 1 - p_0$ stated in IEC 61123:2019, Table 5 using Formula (5).
- d) Select a relevant test plan from IEC 61123:2019, Table 5 fulfilling the required α , β and D , and thereby determine n and c .

9.4.3 Procedure when the number of items is given

The test procedure should include the following steps.

- a) Select a relevant test plan from IEC 61123:2019, Table 5 fulfilling the required α , β and D , and with n close to the indicated n , and thereby determine $q_0 = 1 - p_0$, n and c .
- b) Calculate $t_{\text{cal},t}^*$ using Formula (5).

10 Combined test plans

10.1 General

The combined test plans use a reject line with a constant number of failures, while the accept line is curved (see Figure 5 and Table 1). Therefore, test items with a high reliability will be accepted early while items with many early failures will not be rejected immediately. Test plans are also suitable for a test, analysis and fix program (TAAF) as more information is collected about the failure modes than for SPRTs. The test plans are derived using an iterative procedure that is described in Annex H.

Clause 10 contains characteristics of eight ready-to-use combined test plans (D plans, Table 7). The parameters of the border lines, for each of the tests, are given in Annex B. It also contains an example of the plotting of border lines for these parameters; no additional calculations are required to use them.

10.2 Common test procedure

The test procedure should include the following steps.

- Decide the combined environmental conditions to be applied during the test based on analysis to determine appropriate test conditions (see 5.1 and IEC 60300-3-5).
- Specify m_0 , according to requirement or agreement.
- Choose risks, α and β , and discrimination ratio, D , considering that low values of α , β and D make a more powerful test, but require a longer time or more items or both.
- Select an appropriate test plan from Table 7 and Annex B.
- Note the test times, T_a^* for accept and the number of failures for reject, $r_0 + 1$ (Table B.2).
- Perform the test and apply the decision criteria in accordance with 10.3.

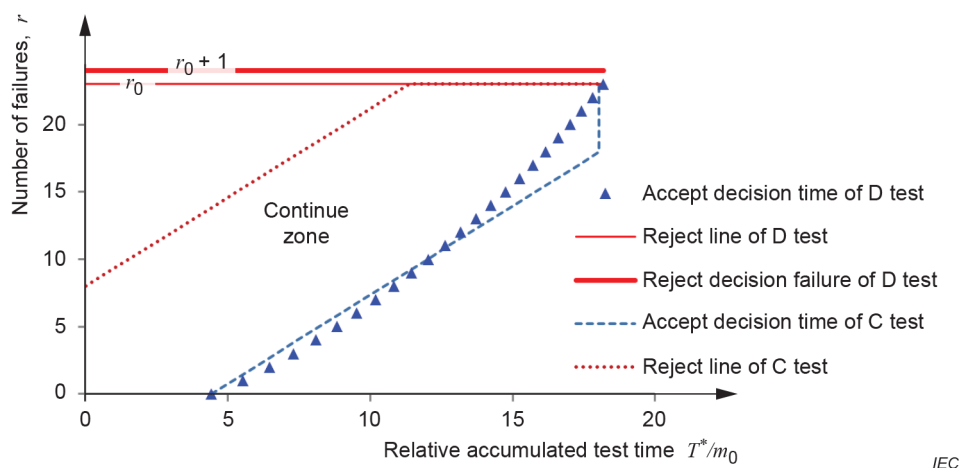
10.3 Decision criteria

The accumulated test time and number of failures shall be compared continuously with the accept and reject criteria during the test, or at intervals short enough to avoid substantially biasing the test results (see 5.1 and IEC 60300-3-5:2001, 7.1.3).

The following criteria shall apply.

- Accept the test if the elapsed $T^* \geq T_a^*$ for the observed $r \leq r_0$.
The specified requirement is regarded as being complied with.
- Continue testing if $T^* < T_a^*$ for the observed $r \leq r_0$.
No decision can be taken.
- Reject the test if the number of failures $r \geq r_0 + 1$ during the test.
The specified requirement is regarded as not being complied with.

Figure 5 shows an example of the border lines of the type D test in accordance with the above criteria. For comparison, the figure shows accept and reject lines for type C SPRT with the same OC. A comparison of the characteristics of these tests is given in 4.3.2.



NOTE 1 This Figure 5 is drawn for D.3 and C.3 test plans (Table 7 and Table 5 respectively, $D = 3$, $\alpha' = \beta' = 10\%$).

NOTE 2 "Reject the test" occurs when the observed test results rise higher than the reject line.

Figure 5 – Example of a decision graph for combined test plan (type D) and for SPRT type C

10.4 Test plans

Table 7 shows the parameters of the characteristics of combined test plans. These parameters are sufficient to choose a specific test plan. The parameters of the border lines, for each of the tests, are given in Annex B. It also contains examples of plotting the border lines and characteristics of these tests.

For any of the combined test plans in Table 7, the ETT curve is defined via five points given in Table 7 ($T_{e,j}^*/m_0$ versus the true m according to Table 3 and Figure 1).

For any of the combined test plans, the five approximate points on the OC are given in Table 2 and the example is shown in Figure 3. The tests in Table 7 have a maximal accumulated test time (MaxTT) close to MaxTT of the FTFT (with the same D , α' and β'). Therefore, the OC of these tests D will exactly coincide with the OC of the corresponding FTFT. Hence, for very accurate OC calculations, the procedure in Clause E.4 and the spreadsheet program in Clause F.8 can be used.

Table 7 – Overview of type D combined plans

Test plan No.	Characteristics of the plan			Minimum test time to acceptance (at 0 failures)	ETT in multiples of m_0 for various m					Maximal accumulated test time
	Discrimination ratio	True risks			$m = m_L$	$m = m_1$	$m = m_M$	$m = m_0$	$m = m_H$	
				D	$\alpha' \%$	$\beta' \%$	$\frac{T_{a,min}^*}{m_0}$	$\frac{T_{e,L}^*}{m_0}$	$\frac{T_{e,1}^*}{m_0}$	$\frac{T_{e,M}^*}{m_0}$
	D.1	1,7	5	5	6,45	17,6	22,7	25,7	22,1	17,2
D.2	1,7	10	5	4,72	14,0	17,9	19,9	16,6	12,5	24,52
D.3	1,7	10	10	4,43	10,8	13,7	15,0	13,1	10,5	18,18
D.4	1,7	15	5	5,73	11,3	14,6	17,0	16,2	13,6	19,99
D.5	1,7	20	10	3,95	7,2	9,1	10,3	9,8	8,4	12,76
D.6	1,7	20	20	2,44	4,9	5,8	6,1	5,6	4,8	8,57
D.7	1,7	30	20	2,14	3,1	3,7	4,0	4,0	3,7	5,71
D.8	1,7	30	30	1,22	2,0	2,3	2,4	2,3	2,1	4,39

11 Performing the test and presenting the results

Guidance on test conditions and practical management of the test can be found in 5.1, in IEC 60300-3-5 and in IEC 60605-2.

When presenting the results, the following information shall be given:

- test conditions (see 5.1 and IEC 60300-3-5);
- identification of test items, number of items on test and possible replacement during test;
- the test plan used;
- specified acceptable λ_0 or m_0 ;
- test plan parameters given or derived, for example risk levels, discrimination ratio, accumulated test time for termination, acceptable number of failures;
- observed number of failures and justification for any failures classified as non-relevant;
- definition of accumulated test time especially if test cycles or several mission modes are used;
- accumulated test time when the test is stopped;
- result(s) of other analyses, if required;
- conclusions.

Annex A (normative)

Tables for border lines of SPRT plans (types A and C)

NOTE 1 See Clause 6.

NOTE 2 Table A.2 contains type C SPRT plans based on GOST R 27.402 [12].

A.1 Symbols

In addition to the symbols given in 3.2.2, the following symbols are used in this Annex A:

- a accept line's intersection with r axis (see Figure A.1)
- b slope of accept and reject lines
- b^* related slope (see Formula (A.3) and Figure A.1)
- c reject line's intersection with the r axis (see Figure A.1)
- r_a ordinate (number of failures) of the point belongs to the accept line
- r_r ordinate (number of failures) of the point belongs to the reject line
- r_0 the maximum number of failures possible for "accept" decision (see Figure A.1)
- T_t^* accumulated test time stated as termination criterion (see Figure A.1)
- t^* test time for each test item
- t_a^* test time for each test item, stated as acceptance criterion
- t_r^* test time for each test item, stated as reject criterion

A.2 Border lines

Table A.1 and Table A.2 shows the data of the border lines of SPRT plans of types A and C, respectively. The data contains the constants and the extreme coordinates of the border lines, including r_0 and T_t^* (see Figure A.1). Following are the border line formulae.

Accept border line:

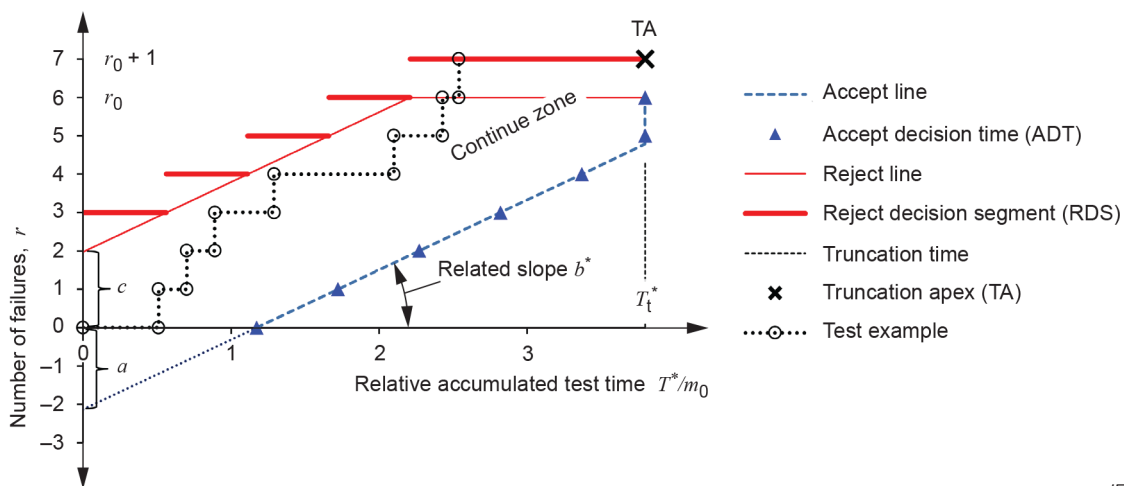
$$r_a = a + b^* \cdot T^* / m_0 \quad (\text{A.1})$$

Reject line:

$$r_r = c + b^* \cdot T^* / m_0 \quad (\text{A.2})$$

where

$$b^* = b \cdot m_0 = (D-1) / (\ln(D)) \quad (\text{A.3})$$



NOTE This Figure A.1 is drawn for A.41 test plan (Table 4, $D = 3$, $\alpha' = \beta' = 10,0\%$).

Figure A.1 – Decision graph of SPRT plan

Instead of plotting a diagram as in Figure A.1, Table A.3 can be used. The data in the three left columns in Table A.3 can be derived from the reject and accept lines, Formulae (A.1) and (A.2), written in the following form:

Accept border line:

$$T^*/m_0 = (r_a - a)/b^* \quad (\text{A.4})$$

Reject line:

$$T^*/m_0 = (r_r - c)/b^* \quad (\text{A.5})$$

Extreme coordinates of the border lines, including r_0 and T_t^* , are given in Table A.3.

Table A.1 – Constants for border line formulae and their coordinates for type A SPRT plans

Test plan	Characteristics of the test plan			Constants for border line formulae			Coordinates for border lines $\{T^*/m_0; r\}$							
							Reject line				Accept line			
	D	α' %	β' %	a	b^*	c	From		To		From		To	
							T	$r = c$	T	r_0	$T_{a,min}^*$	r	T_t^*	r
A.1	1,5	5	5	-8,07	1,233 3	7,90	0	7,90	56,84	78	6,54	0	63,93	70,78
A.2	1,5	5	10	-6,47	1,233 3	8,05	0	8,05	42,12	60	5,24	0	48,37	53,19
A.3	1,5	5	20	-4,45	1,233 3	7,51	0	7,51	31,21	46	3,61	0	36,22	40,22
A.4	1,5	10	5	-8,14	1,233 3	6,40	0	6,40	41,84	58	6,60	0	48,69	51,91
A.5	1,5	10	10	-6,21	1,233 3	6,05	0	6,05	32,39	46	5,03	0	37,96	40,61
A.6	1,5	10	20	-4,48	1,233 3	5,96	0	5,96	20,30	31	3,63	0	24,97	26,32
A.7	1,5	10	40	-2,46	1,233 3	4,59	0	4,59	10,87	18	1,99	0	13,70	14,44
A.8	1,5	20	5	-7,52	1,233 3	4,30	0	4,30	31,38	43	6,09	0	37,39	38,60
A.9	1,5	20	10	-5,81	1,233 3	4,36	0	4,36	20,79	30	4,71	0	25,92	26,16
A.10	1,5	20	20	-3,98	1,233 3	3,92	0	3,92	12,23	19	3,23	0	16,09	15,87
A.11	1,5	30	30	-2,32	1,233 3	2,35	0	2,35	3,77	7	1,88	0	6,44	5,62
A.12	1,5	40	10	-4,90	1,233 3	2,26	0	2,26	10,33	15	3,97	0	14,26	12,69
A.13	1,7	5	5	-6,13	1,319 4	6,04	0	6,04	29,53	45	4,65	0	34,80	39,79
A.14	1,7	5	10	-4,89	1,319 4	6,11	0	6,11	21,90	35	3,70	0	26,50	30,08
A.15	1,7	5	20	-3,36	1,319 4	5,67	0	5,67	16,17	27	2,55	0	19,87	22,85
A.16	1,7	10	5	-6,21	1,319 4	4,94	0	4,94	21,27	33	4,70	0	26,38	28,60
A.17	1,7	10	10	-4,74	1,319 4	4,73	0	4,73	16,12	26	3,59	0	20,38	22,15
A.18	1,7	10	20	-3,36	1,319 4	4,52	0	4,52	10,22	18	2,54	0	13,70	14,71
A.19	1,7	10	40	-1,91	1,319 4	3,59	0	3,59	4,86	10	1,45	0	7,11	7,48
A.20	1,7	20	5	-5,74	1,319 4	3,31	0	3,31	15,68	24	4,35	0	20,18	20,89
A.21	1,7	20	10	-4,50	1,319 4	3,23	0	3,23	10,44	17	3,41	0	14,10	14,10
A.22	1,7	20	20	-2,93	1,319 4	2,91	0	2,91	6,13	11	2,22	0	9,03	8,98
A.23	1,7	30	30	-1,70	1,319 4	1,63	0	1,63	1,80	4	1,29	0	3,76	3,26
A.24	1,7	40	10	-3,70	1,319 4	1,74	0	1,74	4,75	8	2,81	0	7,75	6,52
A.25	2	5	5	-4,68	1,442 9	4,58	0	4,58	14,84	26	3,24	0	18,65	22,23
A.26	2	5	10	-3,74	1,442 9	4,76	0	4,76	10,56	20	2,59	0	14,00	16,46
A.27	2	5	20	-2,53	1,442 9	4,23	0	4,23	8,16	16	1,75	0	10,82	13,08
A.28	2	10	5	-4,67	1,442 9	3,74	0	3,74	10,57	19	3,24	0	14,29	15,95
A.29	2	10	10	-3,59	1,442 9	3,54	0	3,54	7,94	15	2,49	0	11,00	12,29
A.30	2	10	20	-2,44	1,442 9	3,17	0	3,17	5,42	11	1,69	0	7,81	8,83
A.31	2	10	40	-1,38	1,442 9	2,58	0	2,58	2,37	6	0,95	0	3,98	4,36
A.32	2	20	5	-4,28	1,442 9	2,41	0	2,41	8,03	14	2,97	0	11,28	11,99
A.33	2	20	10	-3,30	1,442 9	2,32	0	2,32	5,33	10	2,29	0	7,95	8,17
A.34	2	20	20	-2,23	1,442 9	2,25	0	2,25	2,60	6	1,54	0	4,79	4,68
A.35	2	30	30	-1,27	1,442 9	1,10	0	1,10	0,62	2	0,88	0	2,05	1,69
A.36	2	40	10	-2,67	1,442 9	1,01	0	1,01	2,76	5	1,85	0	4,82	4,29
A.37	3	5	5	-2,89	1,820 8	2,81	0	2,81	3,95	10	1,59	0	6,01	8,05
A.38	3	5	10	-2,25	1,820 8	2,77	0	2,77	2,87	8	1,24	0	4,05	6,22

Test plan	Characteristics of the test plan			Constants for border line formulae			Coordinates for border lines $\{T^*/m_0; r\}$							
							Reject line				Accept line			
	D	α' %	β' %	a	b^*	c	From		To		From		To	
							T	$r = c$	T	r_0	$T_{a,min}^*$	r	T_t^*	r
A.39	3	5	20	-1,58	1,820 8	2,64	0	2,64	1,84	6	0,87	0	3,34	4,51
A.40	3	10	5	-2,90	1,820 8	2,29	0	2,29	2,58	7	1,60	0	4,58	5,43
A.41	3	10	10	-2,14	1,820 8	1,98	0	1,98	2,21	6	1,17	0	3,80	4,78
A.42	3	10	20	-1,49	1,820 8	1,91	0	1,91	1,15	4	0,82	0	2,48	3,02
A.43	3	10	40	-0,85	1,820 8	1,60	0	1,60	0,22	2	0,47	0	1,19	1,32
A.44	3	20	5	-2,68	1,820 8	1,38	0	1,38	1,99	5	1,47	0	3,67	4,00
A.45	3	20	10	-2,14	1,820 8	1,73	0	1,73	0,70	3	1,17	0	2,32	2,09
A.46	3	20	20	-1,33	1,820 8	1,23	0	1,23	0,42	2	0,73	0	1,61	1,61
A.47	3	30	30	-0,77	1,820 8	0,45	0	0,45	0,30	1	0,42	0	1,08	1,19
A.48	3	40	10	-1,73	1,820 8	0,78	0	0,78	0,12	1	0,95	0	1,36	0,75
A.49	5	5	5	-1,88	2,486 1	1,67	0	1,67	1,34	5	0,76	0	2,40	4,07
A.50	5	5	10	-1,44	2,486 1	1,64	0	1,64	0,95	4	0,58	0	1,88	3,23
A.51	5	5	20	-1,00	2,486 1	1,57	0	1,57	0,58	3	0,40	0	1,39	2,45
A.52	5	10	5	-1,88	2,486 1	1,41	0	1,41	0,64	3	0,76	0	1,71	2,37
A.53	5	10	10	-1,39	2,486 1	1,06	0	1,06	0,78	3	0,56	0	1,58	2,53
A.54	5	10	20	-0,95	2,486 1	0,98	0	0,98	0,41	2	0,38	0	1,09	1,75
A.55	5	10	40	-0,52	2,486 1	0,86	0	0,86	0,06	1	0,21	0	0,55	0,84
A.56	5	20	5	-1,76	2,486 1	0,85	0	0,85	0,46	2	0,71	0	1,41	1,76
A.57	5	20	10	-1,34	2,486 1	0,95	0	0,95	0,02	1	0,54	0	0,87	0,83
A.58	5	20	20	-0,86	2,486 1	0,63	0	0,63	0,15	1	0,35	0	0,78	1,08
A.59	5	23,5	26,2	-0,67	2,486 1	0,46	0	0,46	-0,18	0	0,27	0	0,39	0,30
A.60	5	37,2	9,8	-1,16	2,486 1	0,49	0	0,49	-0,20	0	0,46	0	0,55	0,20

Table A.2 – Constants for border line formulae and their coordinates for type C SPRT plans

Test plan	Characteristics of the test plan			Constants for border line formulae			Coordinates for border lines $\{T^*/m_0; r\}$							
							Reject line				Accept line			
	D	α' %	β' %	a	b^*	c	From		To		From		To	
							T	$r = c$	T	r_0	$T_{a,min}^*$	r	T_t^*	r
C.1	1,7	5	5	-9,76	1,319 4	8,96	0	8,96	22,01	38	7,40	0	29,30	28,90
C.2	1,7	10	5	-6,63	1,319 4	8,99	0	8,99	15,92	30	5,02	0	24,24	25,35
C.3	1,7	10	10	-5,83	1,319 4	8,20	0	8,20	11,22	23	4,42	0	18,03	17,96
C.4	1,7	15	5	-7,66	1,319 4	9,56	0	9,56	10,95	24	5,80	0	19,90	18,60
C.5	1,7	20	10	-5,32	1,319 4	5,02	0	5,02	7,56	15	4,03	0	12,60	11,31
C.6	1,7	20	20	-3,08	1,319 4	3,76	0	3,76	4,73	10	2,33	0	8,30	7,87
C.7	1,7	30	20	-2,77	1,319 4	2,68	0	2,68	2,52	6	2,10	0	5,50	4,48
C.8	1,7	30	30	-1,67	1,319 4	1,62	0	1,62	1,81	4	1,26	0	3,80	3,35

A.3 Example of the SPRT plan from Clause 6

Specified: $n = 500$ items that are placed on test with replacement. Producer and consumer agree on $\lambda_0 = 9 \times 10^{-7} \text{ h}^{-1}$, $\alpha = \beta = 10 \%$ and $D = 3$.

To be derived: SPRT plan and simulated examples.

Procedure: $m_0 = 1/\lambda_0 = 1,11 \times 10^6 \text{ h}$ (= 126,7 years).

Since replacement is used, the accumulated test time for 500 components, T^* , is:

$$T^* = nt^* = 500t^*$$

where t^* is the test time per item.

Test plan A.41 satisfies the specified α , β and D , while T_a^* and T_r^* are the accumulated test times for accept and reject lines, respectively; they are stated as multiples of m_0 , that is, T_a^*/m_0 and T_r^*/m_0 .

Each time a failure occurs or accept border lines are reached, the point $(T^*/m_0, r) = (nt^*/m_0, r)$ should be plotted in the diagram and it should be noted whether it falls (see 6.3):

- above the reject border lines, reject;
- on or below the accept border lines, accept;
- otherwise, continue testing.

The border lines have the following formulae (by data from Table A.1).

Reject lines: $r = 1,98 + 1,8208(T^*/m_0)$, $0 \leq (T^*/m_0) < 2,21$ and $r = 6$, $2,21 \leq (T^*/m_0) < 3,80$.

Accept lines: $r = -2,14 + 1,8208(T^*/m_0)$, $1,17 \leq (T^*/m_0) < 3,80$ and $4,78 < r \leq 6$, $(T^*/m_0) = 3,80$.

The border lines are shown in Figure A.1. The OC of the test is shown in Figure 3, and the ETT in Figure 4.

The abscissa T^*/m_0 can be changed to t^* by multiplying by $m_0/n = 1,11 \times 10^6/500 = 2\,222 \text{ h}$.

Instead of plotting a diagram as in Figure A.1, Table A.3 can be used. The data in the three left columns in Table A.3 can be derived from the reject and accept lines formulae as above. The columns of t_a^* and t_r^* (in test hours) calculated by $t_a^* = [T_a^*/m_0] \cdot (m_0/n)$ and $t_r^* = [T_r^*/m_0] \cdot (m_0/n)$ where the quantities in the square brackets are the stated values.

An easy way to perform SPRT is to put the table into a spreadsheet program with graphics.

The results from an example with fictive (simulated) data are shown in Table A.3. In the second right column (t^* in hours), the time from the beginning of the test to the r -th failure is given. The same example is shown in Figure A.1.

Table A.3 – Example for SPRT using test plan A.41 (with example data)

$\alpha' = \beta' = 10 \text{ \%}, D = 3, m_0 = 1,11 \times 10^6 \text{ h}, t^* = 2\,222 \cdot T^*/m_0 \text{ h}$							
Test plan A.41					Test result		
r	T_r^*/m_0 Reject (equal or less)	T_a^*/m_0 Accept (equal or more)	t_r^* in hours Reject (equal or less)	t_a^* in hours Accept (equal or more)	T^*/m_0	t^* in hours	Decision
0	N/A	1,17	N/A	2 610	0	0	Continue
1	N/A	1,72	N/A	3 832	0,51	1 133	Continue
2	0,01	2,27	24	5 052	0,70	1 555	Continue
3	0,56	2,82	1 245	6 273	0,89	1 978	Continue
4	1,11	3,37	2 465	7 493	1,29	2 866	Continue
5	1,66	3,80	3 685	8 444	2,10	4 666	Continue
$r_0 = 6$	2,21	3,80	4 906	8 444	2,43	5 399	Continue
7	3,80	N/A	8 444	N/A	2,54	5 644	Reject

Annex B (normative)

Tables and graphs for combined test plans (type D)

NOTE 1 This annex uses the symbols listed in 3.2.2.

NOTE 2 This Annex B contains combined test plans based on GOST R 27.402 [12].

B.1 General

The combined test plans bring together the advantages of time/failure terminated test plans (test plans B, Clause 7) with the advantages of the sequential test plans (test plans C, Clause 6). They reduce the test time for highly reliable test items, while they do not reject low reliability items early. They require about the same administrative work as the sequential test plans.

More test plans of these types can be found in GOST R 27.402 [12].

This Annex B contains the test plans listed in Table B.1.

Table B.1 – Combined test plans in Annex B

Combined test plans D $D = 1, 7$		α				
		5 %	10 %	15 %	20 %	30 %
β	5 %	D.1	D.2	D.4		
	10 %		D.3		D.5	
	20 %				D.6	D.7
	30 %					D.8

For each combined plan of Table B.1, there is a corresponding SPRT plan in Table 5 (that is, a plan C with the same D , α' and β').

For each test plan, the following data is provided in tables and figures:

- the accept and reject lines as the number of failures r versus the relative accumulated test time T^*/m_0 (Table B.2, the example in Figure 5 and in Table B.4);
- the expected accumulated test time T_e^* as a function of m/m_0 (Table 7 and the example in Figure 1);
- the expected accumulated test time to acceptance $T_e^*(+)$ as a function of m/m_0 (Table B.3 and the example in Figure B.1);
- the OC as a function of m/m_0 . Five approximate points on the OC are given in Table 2. To obtain more OC points, the procedure described in Clause E.4 and its implementation by the spreadsheet program (Clause F.8) can be used. See the example in Figure B.2.

In order to make it easy to compare the test plans of types C and D, it is recommended to plot their characteristics in the same figure or table, as shown in Figure 5, Figure 1, Figure B.1 and in Table B.3.

Table B.2 – Type D test plans – Accept and reject lines

Test plan No.	D.1	D.2	D.3	D.4	D.5	D.6	D.7	D.8
α' %	5	10	10	15	20	20	30	30
β' %	5	5	10	5	10	20	20	30
Reject failure, $r_0 + 1$	39	31	24	25	16	11	7	5
Failures, r	Accumulated test time of accept line, T^*/m_0							
0	6,454	4,722	4,426	5,732	3,95	2,436	2,144	1,219
1	7,769	5,896	5,531	6,946	4,966	3,257	2,888	1,931
2	8,878	6,899	6,466	7,960	5,818	3,973	3,533	2,689
3	9,875	7,813	7,310	8,866	6,580	4,633	4,126	3,509
4	10,799	8,667	8,092	9,697	7,280	5,258	4,682	4,388
5	11,668	9,479	8,829	10,473	7,932	5,854	5,210	N/A
6	12,494	10,256	9,528	11,204	8,545	6,430	5,715	
7	13,285	11,006	10,196	11,897	9,123	6,987	N/A	
8	14,046	11,732	10,837	12,558	9,670	7,528		
9	14,780	12,438	11,455	13,189	10,189	8,055		
10	15,491	13,127	12,050	13,794	10,681	8,568		
11	16,181	13,799	12,626	14,374	11,147	N/A		
12	16,851	14,456	13,182	14,931	11,588			
13	17,504	15,100	13,721	15,466	12,004			
14	18,140	15,732	14,242	15,980	12,394			
15	18,760	16,352	14,746	16,473	12,757			
16	19,365	16,961	15,234	16,946	N/A			
17	19,956	17,559	15,705	17,399				
18	20,534	18,148	16,160	17,832				
19	21,098	18,726	16,599	18,244				
20	21,649	19,296	17,021	18,637				
21	22,188	19,856	17,426	19,008				
22	22,714	20,408	17,814	19,358				
23	23,229	20,951	18,184	19,686				
24	23,731	21,485	N/A	19,99				
25	24,221	22,011		N/A				
26	24,700	22,529						
27	25,167	23,038						
28	25,621	23,539						
29	26,064	24,032						
30	26,494	24,517						
31	26,912	N/A						
32	27,318							
33	27,710							
34	28,090							
35	28,455							
36	28,806							
37	29,142							
38	29,463							
39	N/A							
NOTE For all the tests, the discrimination ratio $D = 1,7$								

Table B.3 – Expected accumulated test time to acceptance decision, $T_e^*(+)$, for D and C test plans

Test plan No.	Characteristics of the plan			Expected accumulated time to acceptance decision in multiples of m_0 for various m				
	Discrimination ratio	True risks						
		$\alpha' \%$	$\beta' \%$	$m = m_L$	$m = m_1$	$m = m_M$	$m = m_0$	$m = m_H$
	D			$\frac{T_{e,L}^*(+)}{m_0}$	$\frac{T_{e,1}^*(+)}{m_0}$	$\frac{T_{e,M}^*(+)}{m_0}$	$\frac{T_{e,0}^*(+)}{m_0}$	$\frac{T_{e,H}^*(+)}{m_0}$
D.1	1,7	5	5	27,2	27,2	25,6	21,8	17,2
C.1				28,5	28,8	27,7	23,8	17,7
D.2	1,7	10	5	16,8	19,7	18,8	16,0	12,4
C.2				15,2	19,8	19,3	16,0	11,8
D.3	1,7	10	10	15,2	15,5	14,6	12,8	10,5
C.3				14,7	16,0	15,2	13,0	10,1
D.4	1,7	15	5	18,5	18,4	17,6	15,9	13,6
C.4				18,3	18,8	18,1	16,2	13,1
D.5	1,7	20	10	10,7	10,9	10,4	9,5	8,3
C.5				11,0	11,3	10,9	9,9	8,5
D.6	1,7	20	20	5,6	5,9	5,7	5,3	4,7
C.6				5,6	6,1	6,0	5,4	4,7
D.7	1,7	30	20	4,0	4,1	4,0	3,8	3,6
C.7				4,2	4,3	4,2	4,0	3,7
D.8	1,7	30	30	1,9	2,1	2,1	2,1	2,0
C.8				2,2	2,3	2,3	2,2	2,1

B.2 Test plans D.3 and C.3 ($\alpha = \beta = 10 \%$, $D = 1,7$)

In this Clause B.2, test plan D.3 is presented as an example of a combined test plan. SPRT plan C.3 is also presented for comparison.

Accept and reject lines of all combined tests are given in Table B.2. In Table B.4, this data is given for tests D.3 and C.3 in a more convenient decision-making form. The decision graphs of the tests are shown in Figure 5.

The expected accumulated test time T_e^* as a function of m/m_0 is given in Table 7 and Figure 1. Graphs clearly indicate that test C.3 has an advantage over D.3 for $m \leq m_0$.

The expected accumulated test time to acceptance $T_e^*(+)$ as a function of m/m_0 is given in Table B.3 and Figure B.1. It is seen that D.3 has no advantage over C.3.

The operating characteristic P_a as a function of m/m_0 is presented in Table 2 and Figure B.2. The graphs in Figure B.2 are the same for tests D.3 and C.3. The graph " P_a – true value" was calculated by the method given in Annex H. The P_a estimates in Table 2 agree well with the true values.

Table B.4 – Accept and reject lines for D.3 and C.3 test plans

Failures r	Accumulated test time T^*/m_0			
	Plan C.3		Plan D.3	
	Reject	Accept	Reject	Accept
0	N/A	4,429	N/A	4,426
1	N/A	5,187	N/A	5,531
2	N/A	5,946	N/A	6,466
3	N/A	6,704	N/A	7,310
4	N/A	7,462	N/A	8,092
5	N/A	8,220	N/A	8,829
6	N/A	8,978	N/A	9,528
7	N/A	9,736	N/A	10,196
8	0,015	10,494	N/A	10,837
9	0,773	11,252	N/A	11,455
10	1,531	12,010	N/A	12,050
11	2,289	12,768	N/A	12,626
12	3,047	13,526	N/A	13,182
13	3,805	14,284	N/A	13,721
14	4,563	15,042	N/A	14,242
15	5,321	15,800	N/A	14,746
16	6,079	16,558	N/A	15,234
17	6,838	17,316	N/A	15,705
18	7,596	18,030	N/A	16,160
19	8,354	18,030	N/A	16,599
20	9,112	18,030	N/A	17,021
21	9,870	18,030	N/A	17,426
22	10,628	18,030	N/A	17,814
$r_0 = 23$	11,386	18,030	N/A	18,184
24	18,030	N/A	Always	N/A
Always reject at 24 failures or more.				

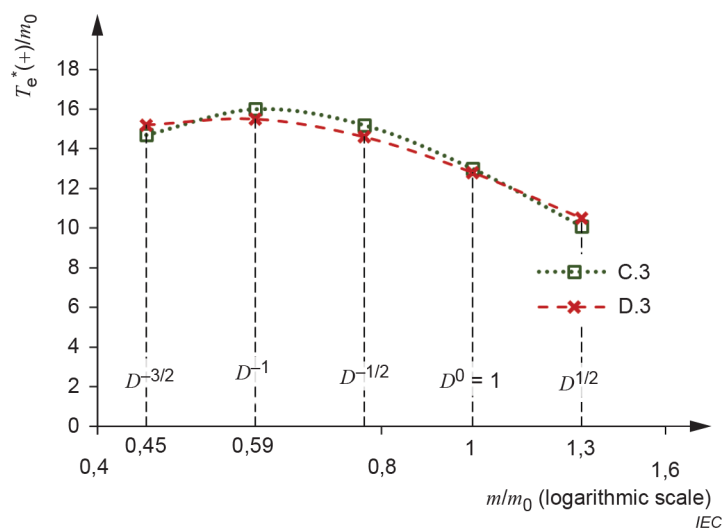
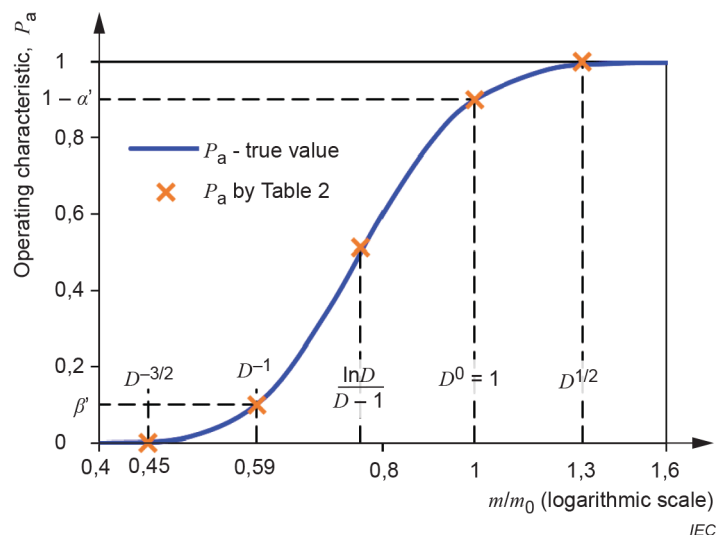


Figure B.1 – Expected accumulated test time to acceptance decision, $T_e^*(+)$ for D.3 and C.3 test plans



NOTE This Figure B.2 is drawn for $D = 1.7$, $\alpha' = \beta' = 10.0\%$.

Figure B.2 – Operating characteristic P_a for D.3 and C.3 test plans

Annex C (informative)

Extension of the set of SPRTs type A

C.1 Symbols

In addition to the symbols given in 3.2.2, the following symbols are used:

a	accept line's intersection with r axis (see Figure A.1)
b	slope of accept and reject lines
b^*	related slope (see Formula (A.3) and Figure A.1)
c	reject line's intersection with the r axis (see Figure A.1)
r_{0x}, r_{01}, r_{02}	maximum number of failures for which it is possible to make a decision "accept" (see Figure A.1) of the designed test and the referenced test pair from Table 4, respectively
y, y_1, y_2	parameter values of the designed test and the referenced test pair, respectively
$\alpha_x, \alpha_1, \alpha_2$	α values (type I risk) of the designed test and the referenced test pair from Table 4, respectively
β_x	β values (type II risk) of the designed test

C.2 Extension of the set of type A tests (through interpolation by α and β)

If a test plan is required, which is not included in, its planning is possible according to the following.

For every D , and α/β ratio available in Table 4 and in Table A.1, it is possible to interpolate the test parameters for any α and β . It is performed by linear interpolation of the required parameters in $\ln \alpha$.

The general formula for linear interpolation is:

$$y = \frac{y_2 - y_1}{\ln \alpha_2 - \ln \alpha_1} (\ln \alpha_x - \ln \alpha_1) + y_1 \quad (\text{C.1})$$

where

α_1, α_2	are the α values (type I risk) of the referenced test pair from Table 4;
α_x	is the α value of the designed test;
y, y_1, y_2	are the parameter values of the designed test and the referenced test pair, respectively.

The parameters $a, c, T_t^*/m_0, r_0$ and $T_{e,j}^*/m_0$ are calculated by Formula (C.1). The value of b^* is unchanged.

Below is the calculation procedure and example for the following parameter values:

$$D = 2, \alpha = 0,15, \alpha/\beta = 2 \text{ (nominal values)} \quad (\text{C.2})$$

Step 1: Calculation and rounding of r_{0x}

$$r_{0x} = \frac{r_{02} - r_{01}}{\ln \alpha_2 - \ln \alpha_1} (\ln \alpha_x - \ln \alpha_1) + r_{01} \quad (\text{C.3})$$

$$r_{0x} = \frac{10 - 19}{\ln 0,2 - \ln 0,1} (\ln 0,15 - \ln 0,1) + 19 = 13,7 \approx 14$$

Step 2: Calculation of α_x and β_x for integer r_{0x}

$$\alpha_x = \exp \left\{ \frac{\ln \alpha_2 - \ln \alpha_1}{r_{02} - r_{01}} (r_{0x} - r_{0,1}) + \ln \alpha_1 \right\} \quad (\text{C.4})$$

$$\alpha_x = \exp \left\{ \frac{\ln 0,2 - \ln 0,1}{10 - 19} (14 - 19) + \ln 0,1 \right\} = 0,147$$

β_x is calculated from α_x and the specified α/β value:

$$\beta_x = \alpha_x / (\alpha/\beta) \quad (\text{C.5})$$

$$\beta_x = 0,147 / 2 \approx 0,074$$

Step 3: Calculation of the parameters a , c , T_t^*/m_0 , and T_{ej}^*/m_0 by Formula (C.1), for example:

$$c_x = \frac{c_2 - c_1}{\ln \alpha_2 - \ln \alpha_1} (\ln \alpha_x - \ln \alpha_1) + c_1 \quad (\text{C.6})$$

$$c_x = \frac{2,32 - 3,74}{\ln 0,2 - \ln 0,1} (\ln 0,147 - \ln 0,1) + 3,74 = 2,95$$

All other test parameters and characteristics should be calculated according to the same method. Table C.1 contains all the tests in question and demonstrates the high accuracy of the method. Construction of five points of the OC curve for the test obtained by the interpolation is given in Table 2. This curve is sufficient for practical purposes. In addition, in Clause D.4 the procedure for calculating more OC points is provided (see Figure D.1).

Table C.1 – Example for interpolation by α and β

D, b^*	α' %	β' %	a	c	r_0	$\frac{T_t^*}{m_0}$	$\frac{T_{e,L}^*}{m_0}$	$\frac{T_{e,1}^*}{m_0}$	$\frac{T_{e,M}^*}{m_0}$	$\frac{T_{e,0}^*}{m_0}$	$\frac{T_{e,H}^*}{m_0}$	
$D = 2,0 \quad b^* = 1,4429$	15	7,5										Target values
	10	5	-4,67	3,74	19	14,29	3,0	6,1	9,4	8,5	6,2	First referenced test: A.28
	20	10	-3,30	2,32	10	7,95	1,9	3,4	4,7	4,8	4,1	Second referenced test: A.33
	14,7	7,4	-3,91	2,95	14	10,77	2,4	4,6	6,8	6,4	4,9	Interpolated designed test
	14,6	7,4	-3,91	2,95	14	10,77	2,4	4,6	6,7	6,4	5,1	Exact value by designed border lines
	-0,7	0	---	---	---	---	0	0	1,4	0	4,1	Error of interpolated test relative to exact value, %

Annex D (informative)

Approximation of operating characteristic for type A SPRTs by Wald's formula

NOTE In this Annex D, probabilities are not given as percentages.

D.1 Symbols

In addition to the symbols given in 3.2.2, the following symbols are used.

A, B constants for the approximation

h independent variable

D.2 Approximations of OC in this document

The OC of all tests in this document is available in Table 2. The table shows the values for only five points of the operating system, which is sufficient from a practical point of view. However, this document provides the following calculation procedures for a larger number of OC points.

- For type A SPRTs, the procedure is given in Clauses D.3 and D.4. This procedure is based on the approximation of the OC by the Wald formula [1]. The formula has a high accuracy for optimally truncated tests, in particular for A tests.
- For FTFTs, an accurate calculation procedure is given in E.2.2 and Clause F.3.
- For type C SPRTs and combined type D tests, the procedure is given in Clauses E.4 and F.8. This procedure is based on the approximation of the OC by Formula (E.6) for FTFTs, which has a high accuracy for highly truncated tests, which in turn include C and D tests.

The approximations by the Wald formula and the formula for FTFTs coincide with the true OC value for all tests at the CRP and PRP points. At other points, they are very close to each other and provide the upper and lower limits for estimating the OC of SPRTs.

D.3 Approximation of OC for type A SPRT by Wald's formula

The OC curve, $P_a(m)$, is approximated by the expression [1] given in a parametric form by Formulae (D.1) and (D.2).

$$P_a(h) = \frac{A^h - 1}{A^h - B^h} \quad (\text{D.1})$$

$$m(h) = m_0 \cdot \frac{D^h - 1}{h \cdot (D - 1)} \quad (\text{D.2})$$

where h is an independent variable.

The constants A and B are

$$A = (1 - \beta) / \alpha \quad (D.3)$$

$$B = \beta / (1 - \alpha) \quad (D.4)$$

Figure D.1 shows the approximated OC calculated using Formula (D.1) and Formula (D.2) for the test obtained by interpolation in Clause C.2. The test parameters are given in Table C.1, in the row "Interpolated designed test". Figure D.1 also shows the true OC of the tests calculated by the procedure described in [5]. Figure D.1 illustrates the high accuracy of the approximation.

The following principles can be applied to determine h in Formula (D.1) and Formula (D.2).

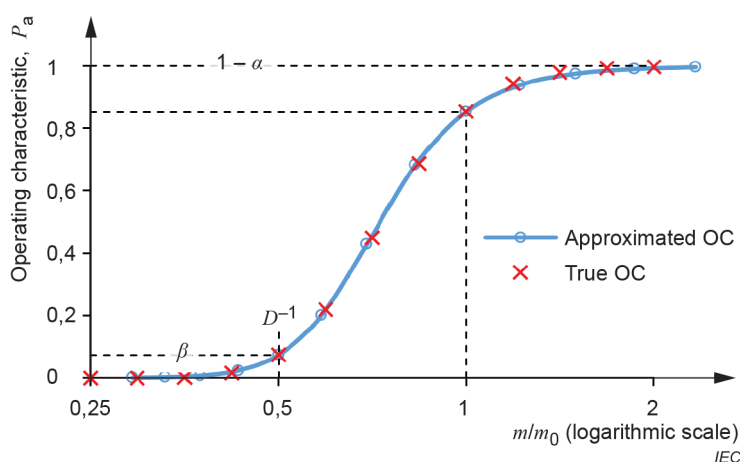
- a) It is recommended to assign h a value ranging from -3 to 3 with a constant step of $0,5$ (see, for example, graph "Approximated OC" in Figure D.1). In this case, $h = 0$ should be replaced by a small number, for example, $h = 10^{-5}$, in order to avoid uncertainty in the calculation. Otherwise, when $h = 0$, the following limits can be used:

$$\lim_{h \rightarrow 0} P_a(h) = \frac{\ln(1 - \beta) - \ln \alpha}{\ln(1 - \beta) - \ln \alpha - \ln \beta + \ln(1 - \alpha)}; \quad (D.5)$$

$$\lim_{h \rightarrow 0} m(h) = m_0 \cdot \frac{\ln D}{(D - 1)}. \quad (D.6)$$

- b) For a predetermined m (for example, as in Table 3) the value of h is derived from Formula (D.2). Another example is the "True OC" graph in Figure D.1, where the values of m form a geometric progression:

$$m_i = m_0 D^{(i-1)/4} \quad \text{for } i = 1 \dots 13 \quad (D.7)$$



NOTE This Figure D.1 is drawn for $D = 2$, $\alpha = 0,147$, $\beta = 0,074$.

Figure D.1 – Approximation of OC for type A SPRT using Wald's formula