

INTERNATIONAL STANDARD



**Superconductivity –
Part 17: Electronic characteristic measurements – Local critical current density
and its distribution in large-area superconducting films**



THIS PUBLICATION IS COPYRIGHT PROTECTED

Copyright © 2021 IEC, Geneva, Switzerland

All rights reserved. Unless otherwise specified, no part of this publication may be reproduced or utilized in any form or by any means, electronic or mechanical, including photocopying and microfilm, without permission in writing from either IEC or IEC's member National Committee in the country of the requester. If you have any questions about IEC copyright or have an enquiry about obtaining additional rights to this publication, please contact the address below or your local IEC member National Committee for further information.

IEC Central Office
3, rue de Varembe
CH-1211 Geneva 20
Switzerland

Tel.: +41 22 919 02 11
info@iec.ch
www.iec.ch

About the IEC

The International Electrotechnical Commission (IEC) is the leading global organization that prepares and publishes International Standards for all electrical, electronic and related technologies.

About IEC publications

The technical content of IEC publications is kept under constant review by the IEC. Please make sure that you have the latest edition, a corrigendum or an amendment might have been published.

IEC publications search - webstore.iec.ch/advsearchform

The advanced search enables to find IEC publications by a variety of criteria (reference number, text, technical committee, ...). It also gives information on projects, replaced and withdrawn publications.

IEC Just Published - webstore.iec.ch/justpublished

Stay up to date on all new IEC publications. Just Published details all new publications released. Available online and once a month by email.

IEC Customer Service Centre - webstore.iec.ch/csc

If you wish to give us your feedback on this publication or need further assistance, please contact the Customer Service Centre: sales@iec.ch.

IEC online collection - oc.iec.ch

Discover our powerful search engine and read freely all the publications previews. With a subscription you will always have access to up to date content tailored to your needs.

Electropedia - www.electropedia.org

The world's leading online dictionary on electrotechnology, containing more than 22 000 terminological entries in English and French, with equivalent terms in 18 additional languages. Also known as the International Electrotechnical Vocabulary (IEV) online.



IEC 61788-17

Edition 2.0 2021-04
REDLINE VERSION

INTERNATIONAL STANDARD



**Superconductivity –
Part 17: Electronic characteristic measurements – Local critical current density
and its distribution in large-area superconducting films**

INTERNATIONAL
ELECTROTECHNICAL
COMMISSION

ICS 17.220.20; 29.050

ISBN 978-2-8322-9765-0

Warning! Make sure that you obtained this publication from an authorized distributor.

CONTENTS

FOREWORD	5
INTRODUCTION	2
1 Scope	9
2 Normative references	9
3 Terms and definitions	9
4 Requirements	10
5 Apparatus	11
5.1 Measurement equipment	11
5.2 Components for inductive measurements	12
6 Measurement procedure	13
6.1 General	13
6.2 Determination of the experimental coil coefficient	13
6.3 Measurement of J_C in sample films	18
6.4 Measurement of J_C with only one frequency	18
6.5 Examples of the theoretical and experimental coil coefficients	19
7 Uncertainty in the test method	20
7.1 Major sources of systematic effects that affect the U_3 measurement	20
7.2 Effect of deviation from the prescribed value in the coil-to-film distance	21
7.3 Uncertainty in the experimental coil coefficient and the obtained J_C	22
7.4 Effects of the film edge	23
7.5 Specimen protection	23
8 Test report	23
8.1 Identification of test specimen	23
8.2 Report of J_C values	23
8.3 Report of test conditions	23
Annex A (informative) Additional information relating to Clauses 1 to 8	24
A.1 Comments on other methods for measuring the local J_C of large-area HTS films	24
A.2 Requirements	24
A.3 Theory of the third-harmonic voltage generation	25
A.4 Calculation of the induced electric fields	26
A.5 Theoretical coil coefficient k and experimental coil coefficient k'	27
A.6 Scaling of the U_3 – I_0 curves and the constant-inductance criterion to determine I_{th}	27
A.7 Effects of reversible flux motion	29
Annex B (informative) Optional measurement systems	30
B.1 Overview	30
B.2 Harmonic noises arising from the power source and their reduction	31
Annex C (informative) Uncertainty considerations	40
Annex C (informative) Evaluation of the uncertainty	40
C.1 Evaluation of the uncertainty in the experimental coil coefficient	40
C.2 Uncertainty in the calculation of induced electric fields	41
C.3 Experimental results on the effect of the deviation of the coil-to-film distance	42

C.4	Examples of the Type-A uncertainties of J_C and n -values, originating from the experimental uncertainty in the U_3 measurement	42
C.5	Evaluation of the uncertainty in the obtained J_C	43
C.6	Experimental results that reveal the effect of the film edge	44
	Bibliography	46

Figure 1	Diagram for an electric circuit used for inductive J_C measurement of HTS films	11
Figure 2	Illustration showing techniques to press the sample coil to HTS films	12
Figure 3	Example of a calibration wafer used to determine the coil coefficient	13
Figure 4	Illustration of the sample coil and the magnetic field during measurement	15
Figure 5	Illustration of the sample coil and its magnetic field generation	15
Figure 6	Example of the normalized third-harmonic voltages (U_3/I_0) measured with various frequencies	15
Figure 6	E - J characteristics measured by a transport method and the U_3 inductive method	17
Figure 7	Illustration of coils 1 and 3 in Table 2	20
Figure 8	The coil-factor function $F(r) = 2H_0/I_0$ calculated for the three coils	20
Figure 9	The coil-to-film distance Z_1 dependence of the theoretical coil coefficient k	22
Figure A.1	Illustration of the sample coil and the magnetic field during measurement	26
Figure A.2	U_3 and U_3/I_0 plotted against I_0 in a YBCO thin film measured in applied DC magnetic fields, and the scaling observed when normalized by I_{th} (insets)	28
Figure A.3	Example of the normalized third-harmonic voltages (U_3/I_0) measured with various frequencies	28
Figure B.1	Schematic diagram for the variable- RL -cancel circuit	31
Figure B.2	Diagram for an electrical circuit used for the two-coil method	31
Figure B.3	Harmonic noises arising from the power source	32
Figure B.4	Noise reduction using a cancel coil with a superconducting film	32
Figure B.5	Normalized harmonic noises (U_3/I_0) arising from the power source	33
Figure B.6	Normalized noise voltages after the reduction using a cancel coil with a superconducting film	33
Figure B.7	Normalized noise voltages after the reduction using a cancel coil without a superconducting film	34
Figure B.8	Normalized noise voltages with the two-coil system shown in Figure B.2	34
Figure C.1	Effect of the coil position against a superconducting thin film on the measured J_C values	45

Table 1	Specifications and theoretical coil coefficients k of sample coils	16
Table 2	Specifications and coil coefficients of typical sample coils	19
Table C.1	Output signals from two nominally identical extensometers	19
Table C.2	Mean values of two output signals	19
Table C.3	Experimental standard deviations of two output signals	19
Table C.4	Standard uncertainties of two output signals	19
Table C.5	Coefficient of variations of two output signals	19

Table C.1 – Uncertainty budget table for the experimental coil coefficient k'	41
Table C.2 – Examples of repeated measurements of J_C and n -values	43

INTERNATIONAL ELECTROTECHNICAL COMMISSION

SUPERCONDUCTIVITY –

**Part 17: Electronic characteristic measurements –
Local critical current density and its distribution
in large-area superconducting films**

FOREWORD

- 1) The International Electrotechnical Commission (IEC) is a worldwide organization for standardization comprising all national electrotechnical committees (IEC National Committees). The object of IEC is to promote international co-operation on all questions concerning standardization in the electrical and electronic fields. To this end and in addition to other activities, IEC publishes International Standards, Technical Specifications, Technical Reports, Publicly Available Specifications (PAS) and Guides (hereafter referred to as “IEC Publication(s)”). Their preparation is entrusted to technical committees; any IEC National Committee interested in the subject dealt with may participate in this preparatory work. International, governmental and non-governmental organizations liaising with the IEC also participate in this preparation. IEC collaborates closely with the International Organization for Standardization (ISO) in accordance with conditions determined by agreement between the two organizations.
- 2) The formal decisions or agreements of IEC on technical matters express, as nearly as possible, an international consensus of opinion on the relevant subjects since each technical committee has representation from all interested IEC National Committees.
- 3) IEC Publications have the form of recommendations for international use and are accepted by IEC National Committees in that sense. While all reasonable efforts are made to ensure that the technical content of IEC Publications is accurate, IEC cannot be held responsible for the way in which they are used or for any misinterpretation by any end user.
- 4) In order to promote international uniformity, IEC National Committees undertake to apply IEC Publications transparently to the maximum extent possible in their national and regional publications. Any divergence between any IEC Publication and the corresponding national or regional publication shall be clearly indicated in the latter.
- 5) IEC itself does not provide any attestation of conformity. Independent certification bodies provide conformity assessment services and, in some areas, access to IEC marks of conformity. IEC is not responsible for any services carried out by independent certification bodies.
- 6) All users should ensure that they have the latest edition of this publication.
- 7) No liability shall attach to IEC or its directors, employees, servants or agents including individual experts and members of its technical committees and IEC National Committees for any personal injury, property damage or other damage of any nature whatsoever, whether direct or indirect, or for costs (including legal fees) and expenses arising out of the publication, use of, or reliance upon, this IEC Publication or any other IEC Publications.
- 8) Attention is drawn to the Normative references cited in this publication. Use of the referenced publications is indispensable for the correct application of this publication.
- 9) Attention is drawn to the possibility that some of the elements of this IEC Publication may be the subject of patent rights. IEC shall not be held responsible for identifying any or all such patent rights.

This redline version of the official IEC Standard allows the user to identify the changes made to the previous edition IEC 61788-17:2013. A vertical bar appears in the margin wherever a change has been made. Additions are in green text, deletions are in strikethrough red text.

IEC 61788-17 has been prepared by IEC technical committee 90: Superconductivity. It is an International Standard.

This second edition cancels and replaces the first edition published in 2013. This edition constitutes a technical revision.

This edition includes the following a significant technical change with respect to the previous edition:

- a) A simple method to calculate theoretical coil coefficient k is described in 6.2.1.

The text of this International Standard is based on the following documents:

FDIS	Report on voting
90/462/FDIS	90/464/RVD

Full information on the voting for the approval of this International Standard can be found in the report on voting indicated in the above table.

The language used for the development of this International Standard is English.

This document was drafted in accordance with ISO/IEC Directives, Part 2, and developed in accordance with ISO/IEC Directives, Part 1 and ISO/IEC Directives, IEC Supplement, available at www.iec.ch/members_experts/refdocs. The main document types developed by IEC are described in greater detail at www.iec.ch/standardsdev/publications.

A list of all the parts of the IEC 61788 series, published under the general title *Superconductivity*, can be found on the IEC website.

The committee has decided that the contents of this document will remain unchanged until the stability date indicated on the IEC website under "<http://webstore.iec.ch>" in the data related to the specific document. At this date, the document will be

- reconfirmed,
- withdrawn,
- replaced by a revised edition, or
- amended.

IMPORTANT – The 'colour inside' logo on the cover page of this publication indicates that it contains colours which are considered to be useful for the correct understanding of its contents. Users should therefore print this document using a colour printer.

INTRODUCTION

Over thirty years after their discovery in 1986, high-temperature superconductors are now finding their way into products and technologies that will revolutionize information transmission, transportation, and energy. Among them, high-temperature superconducting (HTS) microwave filters, which exploit the extremely low surface resistance of superconductors, have already been commercialized. They have two major advantages over conventional non-superconducting filters, namely: low insertion loss (low noise characteristics) and high frequency selectivity (sharp cut) [1]¹. These advantages enable a reduced number of base stations, improved speech quality, more efficient use of frequency bandwidths, and reduced unnecessary radio wave noise.

Large-area superconducting thin films have been developed for use in microwave devices [2]. They are also ~~used~~ **considered** for **use in** emerging superconducting power devices, such as resistive-type superconducting fault-current limiters (SFCLs) [3] [4] [5], superconducting fault detectors used for superconductor-triggered fault current limiters [6] [7] and persistent-current switches used for persistent-current HTS magnets [8] [9]. The critical current density J_c is one of the key parameters that describe the quality of large-area HTS films. Nondestructive, AC inductive methods are widely used to measure J_c and its distribution for large-area HTS films [10] [11] [12] [13], among which the method utilizing third-harmonic voltages $U_3 \cos(3\omega t + \theta)$ is the most popular [10] [11], where ω , t and θ denote the angular frequency, time, and initial phase, respectively. However, these conventional methods are not accurate because they have not considered the electric-field E criterion of the J_c measurement [14] [15] and sometimes use an inappropriate criterion to determine the threshold current I_{th} from which J_c is calculated [16]. A conventional method can obtain J_c values that differ from the accurate values by 10 % to 20 % [15]. It is thus ~~necessary~~ **important** to establish standard test methods to precisely measure the local critical current density and its distribution, to which all involved in the HTS filter industry can refer for quality control of the HTS films. Background knowledge on the inductive J_c measurements of HTS thin films is summarized in Annex A.

In these inductive methods, AC magnetic fields are generated with AC currents $I_0 \cos \omega t$ in a small coil mounted just above the film, and J_c is calculated from the threshold coil current I_{th} , at which full penetration of the magnetic field to the film is achieved [17]. For the inductive method using third-harmonic voltages U_3 , U_3 is measured as a function of I_0 , and the I_{th} is determined as the coil current I_0 at which U_3 starts to emerge. The induced electric fields E in the superconducting film at $I_0 = I_{th}$, which are proportional to the frequency f of the AC current, can be estimated by a simple Bean model [14]. A standard method has been proposed to precisely measure J_c with an electric-field criterion by detecting U_3 and obtaining the n -value (index of the power-law E - J characteristics) by measuring I_{th} precisely at various frequencies [14] [15] [18] [19]. This method not only obtains precise J_c values, but also facilitates the detection of degraded parts in inhomogeneous specimens, because the decline of n -value is more ~~remarkable~~ **noticeable** than the decrease of J_c in such parts [15]. It is noted that this standard method is excellent for assessing homogeneity in large-area HTS films, although the relevant parameter for designing microwave devices is not J_c , but the surface resistance. For application of large-area superconducting thin films to SFCLs, knowledge on J_c distribution is vital, because J_c distribution significantly affects quench distribution in SFCLs during faults.

The International Electrotechnical Commission (IEC) draws attention to the fact that it is claimed that compliance with this document may involve the use of a patent ~~concerning the determination of the E - J characteristics by inductive J_c measurements as a function of frequency, given in the Introduction, Clause 1, Clause 4 and 5.1~~. IEC takes no position concerning the evidence, validity, and scope of this patent right.

¹ Numbers in square brackets refer to the Bibliography.

The holder of this patent right has assured IEC that s/he is willing to negotiate licences ~~free of charge~~ under reasonable and non-discriminatory terms and conditions with applicants throughout the world. In this respect, the statement of the holder of this patent right is registered with IEC. Information may be obtained from the patent database available at <http://patents.iec.ch>.

~~Name of holder of patent right:~~

~~National Institute of Advanced Industrial Science and Technology~~

~~Address:~~

~~Intellectual Property Planning Office, Intellectual Property Department-
1-1-1, Umezono, Tsukuba, Ibaraki Prefecture, Japan~~

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights other than those ~~identified above~~ in the patent database. IEC shall not be held responsible for identifying any or all such patent rights.

~~ISO (www.iso.org/patents) and IEC (<http://patents.iec.ch>) maintain on-line data bases of patents relevant to their standards. Users are encouraged to consult the data bases for the most up-to-date information concerning patents.~~

SUPERCONDUCTIVITY –

Part 17: Electronic characteristic measurements – Local critical current density and its distribution in large-area superconducting films

1 Scope

This part of IEC 61788 ~~describes~~ specifies the measurements of the local critical current density (J_c) and its distribution in large-area high-temperature superconducting (HTS) films by an inductive method using third-harmonic voltages. The most important consideration for precise measurements is to determine J_c at liquid nitrogen temperatures by an electric-field criterion and obtain current-voltage characteristics from its frequency dependence. Although it is possible to measure J_c in applied DC magnetic fields [20] [21], the scope of this document is limited to the measurement without DC magnetic fields.

This technique intrinsically measures the critical sheet current that is the product of J_c and the film thickness d . The range and measurement resolution for $J_c d$ of HTS films are as follows.

- $J_c d$: from 200 A/m to 32 kA/m (based on results, not limitation).
- Measurement resolution: 100 A/m (based on results, not limitation).

2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

~~IEC 60050 – (all parts), International Electrotechnical Vocabulary (available at <http://www.electropedia.org><<http://www.electropedia.org>>)~~

IEC 60050-815, *International Electrotechnical Vocabulary – Part 815: Superconductivity* (available at <<http://www.electropedia.org>>)

3 Terms and definitions

For the purposes of this document, the terms and definitions given in IEC 60050-815:2000 apply, some of which are repeated here for convenience.

ISO and IEC maintain terminological databases for use in standardization at the following addresses:

- IEC Electropedia: available at <http://www.electropedia.org/>
- ISO Online browsing platform: available at <http://www.iso.org/obp>

3.1 critical current

I_c

maximum direct current that can be regarded as flowing without resistance practically

Note 1 to entry: I_c is a function of magnetic field strength, temperature and strain.

[SOURCE: IEC 60050-815:20002015, 815-0312-01]

3.2

critical current criterion

I_c criterion

criterion to determine the critical current, I_c , based on the electric field strength, E , or the resistivity, ρ

Note 1 to entry: $E = 10 \mu\text{V/m}$ or $E = 100 \mu\text{V/m}$ is often used as electric field criterion, and $\rho = 10^{-14} \Omega \cdot \text{m}$ or $\rho = 10^{-13} \Omega \cdot \text{m}$ is often used as resistivity criterion. (~~" $E = 10 \text{ V/m}$ or $E = 100 \text{ V/m}$ " in the current edition is mistaken and is scheduled to be corrected in the second edition).~~

[SOURCE: IEC 60050-815:20002015, 815-0312-02]

3.3

critical current density

J_c

electric current density at the critical current using either the cross-section of the whole conductor (overall) or of the non-stabilizer part of the conductor if there is a stabilizer

Note 1 to entry: The overall current density is called engineering current density (symbol: J_e).

[SOURCE: IEC 60050-815:20002015, 815-0312-03]

3.4

transport critical current density

J_{ct}

critical current density obtained by a resistivity or a voltage measurement

[SOURCE: IEC 60050-815:20002015, 815-0312-04]

3.5

n -value

<superconductor> exponent obtained in a specific range of electric field strength or resistivity when the voltage/current $U(I)$ curve is approximated by the equation $U \propto I^n$

[SOURCE: IEC 60050-815:20002015, 815-0312-10]

4 Requirements

The critical current density J_c is one of the most fundamental parameters that describe the quality of large-area HTS films. In this document, J_c and its distribution are measured non-destructively via an inductive method by detecting third-harmonic voltages $U_3 \cos(3\omega t + \theta)$. A small coil, which is used both to generate AC magnetic fields and detect third-harmonic voltages, is mounted just above the HTS film and used to scan the measuring area. To measure J_c precisely with an electric-field criterion, the threshold coil currents I_{th} , at which U_3 starts to emerge, are measured repeatedly at different frequencies and the E - J characteristics are determined from their frequency dependencies.

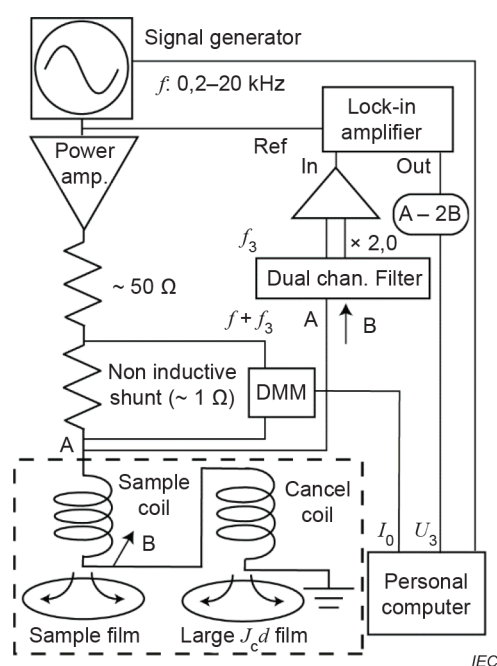
The target relative combined standard uncertainty in the method used to determine the absolute value of J_c is less than 10 %. However, the target uncertainty is less than 5 % for the purpose of evaluating the homogeneity of J_c distribution in large-area superconducting thin films.

5 Apparatus

5.1 Measurement equipment

Figure 1 shows a schematic diagram of a typical electric circuit used for the third-harmonic voltage measurements. This circuit is comprised of a signal generator, power amplifier, digital multimeter (DMM) to measure the coil current, band-ejection filter to reduce the fundamental wave signals and lock-in amplifier to measure the third-harmonic signals. It involves the single-coil approach in which the coil is used to generate an AC magnetic field and detect the inductive voltage. This method can also be applied to double-sided superconducting thin films **without hindrance with no obstacles**. In the methods proposed here, however, there is an additional system to reduce harmonic noise voltages generated from the signal generator and the power amplifier [14]. In an example of Figure 1, a cancel coil of specification being the same as the sample coil is used for cancelling. The sample coil is mounted just above the superconducting film, and a superconducting film with a $J_c d$ sufficiently larger than that of the sample film is placed below the cancel coil to adjust its inductance to that of the sample coil. **Note that the inductance of the sample coil decreases by 20 % to 30 % due to the superconducting shielding current when it is mounted on a superconducting film.** Both coils and superconducting films are immersed in liquid nitrogen (a broken line in Figure 1). Other optional measurement systems are described in Annex B.

NOTE In this circuit, coil currents of about 0,1 A (RMS) and power source voltages of > 6 V (RMS) are needed to measure the superconducting film of $J_c d \approx 10$ kA/m while using coil 1 or 2 of Table 2. A precision power amplifier, **such as NF-HSA4011**, with sufficiently high power is **necessary** used to supply such large currents and voltages.



NOTE The broken line surrounds elements immersed in liquid nitrogen.

Figure 1 – Diagram for an electric circuit used for inductive J_c measurement of HTS films

5.2 Components for inductive measurements

5.2.1 Coils

Currently available large-area HTS films are deposited on areas as large as about 25 cm in diameter, while films about 5 cm in diameter are commercially used to prepare microwave filters [22]. Larger $\text{YBa}_2\text{Cu}_3\text{O}_7$ (YBCO) films, about 10 cm in diameter and 2,7 cm × 20 cm, were used to fabricate fault current limiter modules [3] [4] [5]. For the J_c measurements of such films, the appropriate outer diameter of the sample coils ranges from 2 mm to 5 mm. The requirement for the sample coil is to generate as high a magnetic field as possible at the upper surface of the superconducting film, for which flat coil geometry is suitable. Typical specifications are as follows.

- Inner winding diameter D_1 : 0,9 mm, outer diameter D_2 : 4,2 mm, height h : 1,0 mm, 400 turns of a 50 μm diameter copper wire.
- D_1 : 0,8 mm, D_2 : 2,2 mm, h : 1,0 mm, 200 turns of a 50 μm diameter copper wire.

5.2.2 Spacer film

Typically, a polyimide film with a thickness of 50 μm to 125 μm is used to protect the HTS films. The coil has generally some protection layer below the coil winding, which also insulates the thin film from Joule heat in the coil. The typical thickness is 100 μm to 150 μm , and the coil-to-film distance Z_1 is kept to be 200 μm .

5.2.3 Mechanism for the set-up of the coil

To maintain a prescribed value for the spacing Z_1 between the bottom of the coil winding and the film surface, the sample coil should be pressed to the film with sufficient pressure, typically exceeding about 0,2 MPa [18]. Techniques to achieve this are to use a weight or spring, as shown in Figure 2. The system schematically shown in the figure left is used to scan a wide area of the film. Before the U_3 measurement the coil is initially ~~moved~~ raised up to some distance, moved laterally to the target position, and then ~~moved~~ lowered down and pressed to the film. An appropriate pressure should be determined so that too high pressure does not damage the bobbin, coil, HTS thin film or the substrate. It is reported that the YBCO deposited on biaxially-textured pure Ni substrate was degraded by transverse compressive stress of about 20 MPa [23].

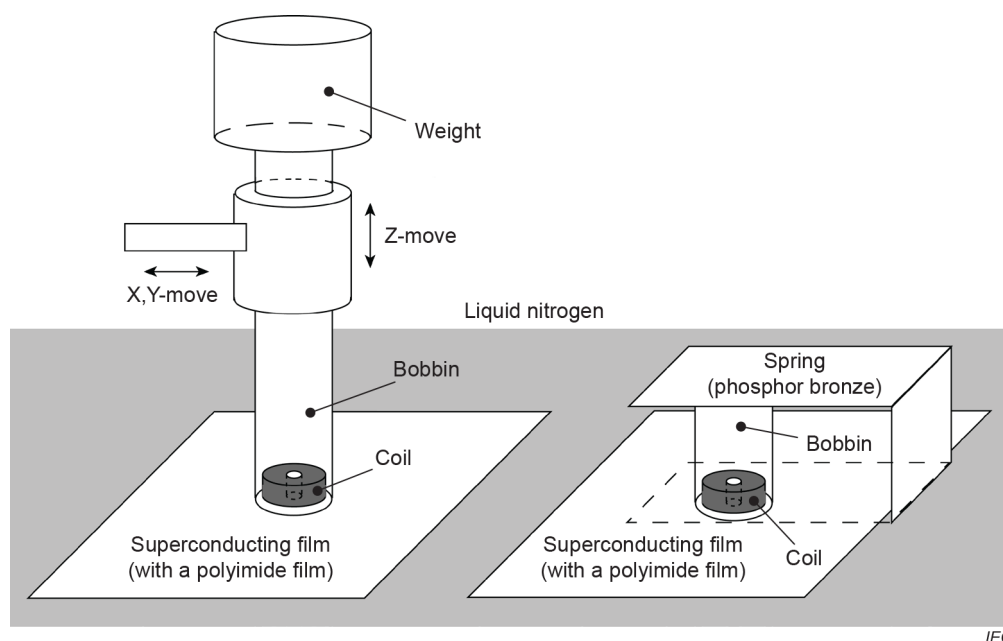


Figure 2 – Illustration showing techniques to press the sample coil to HTS films

5.2.4 Calibration wafer

A calibration wafer is used to determine the experimental coil coefficient k' described in Clause 6. It is made by using a homogeneous large-area (typically about 5 cm diameter) YBCO thin film. It consists of bridges for transport measurement and an inductive measurement area (Figure 3). Typical dimensions of the transport bridges are 20 μm to 70 μm wide and 1 mm to 2 mm long, which were prepared either by UV photolithography technique or by laser etching [24]. In the transport bridge area shown in Figure 3, a transport current can be passed from current terminal 1 to another current terminal 3 through the bridge "a". In this case, terminals 2 and 12 are used as voltage terminals. Similarly, a transport current can be passed from current terminal 1 to another current terminal (5, 7, 9 or 11) through the bridge "b", "c", "d" or "e". In this case, terminals 4, 6, 8 or 10, and 12 are used as voltage terminals.

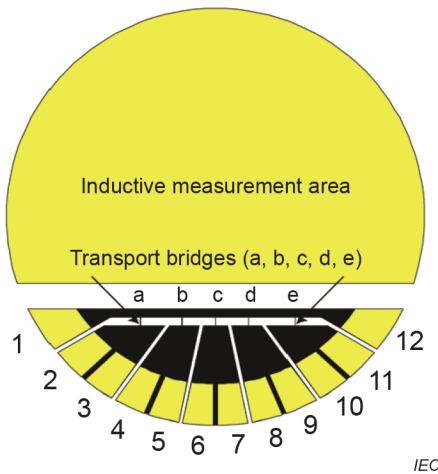


Figure 3 – Example of a calibration wafer used to determine the coil coefficient

6 Measurement procedure

6.1 General

The procedures used to determine the experimental coil coefficient k' and measure the J_c of the films under test are described as follows, with the meaning of k' expressed in Clause A.5.

6.2 Determination of the experimental coil coefficient

6.2.1 Calculation of the theoretical coil coefficient k

Calculate the theoretical coil coefficient $k = J_c d / I_{th}$ from

$$k = F_m, \quad (1)$$

where F_m is the maximum of $F(r)$ that is a function of r , the distance from the central axis of the coil whose inner diameter is D_1 , outer diameter is D_2 and height is h (Figure 4). The coil-factor function $F(r) = -2H_r(r, t)/I_0 \cos \omega t = 2H_0/I_0$ is obtained by

$$F(r) = \frac{N}{2\pi S} \int_{R_1}^{R_2} dr' \int_0^{2\pi} d\theta \int_{Z_1}^{Z_2} dz \frac{r' z \cos \theta}{(z^2 + r^2 + r'^2 - 2rr' \cos \theta)^{3/2}}, \quad (2)$$

where ~~N is the number of windings, $S = (R_2 - R_1)h$ is the cross-sectional area, $R_1 = D_1/2$ is the inner radius, $R_2 = D_2/2$ is the outer radius of the coil, Z_1 is the coil-to-film distance, and $Z_2 = Z_1 + h$ [17]. The derivation of the Equation (2) is described in A.3.~~

$H_r(r, t)$ is the radial component of the magnetic field generated by the sample coil at a upper surface of the superconducting film, N is the number of turns in the sample coil, $R_1 = D_1/2$ is the inner radius, $R_2 = D_2/2$ is the outer radius of the coil, $S = (R_2 - R_1)h$ is the cross-sectional area, Z_1 is the coil-to-film distance, and $Z_2 = Z_1 + h$ [17]. The explanation of Equations (1) and (2) is given in Clause A.3.

A simple method to obtain k is as follows.

- a) Calculate the magnetic-field amplitude $H_0(r) = H_r(r, t = 0)$ as a function of r at a position below the coil with a distance Z_1 when a current of $I_0 = 1$ mA is passed in the sample coil (Figure 5).
- b) Obtain the (local) maximum value of $H_0(r)$ when r is changed near $r \approx (R_1 + R_2)/2$.
- c) The maximum value of $H_0(r)$ should have a unit of A/m, then the doubled value divided by $I_0 (= 1$ mA) becomes k (unit: 1/mm). Note that the magnetic field arising from the image coil (i.e. from the shielding current flowing in the superconducting film) cancels out the perpendicular component H_z , and the parallel component H_r doubles. The image coil and its magnetic field generation are shown by the broken lines in Figure 5.
- d) For the calculation of coil magnetic fields, a free web site may be used; for example, http://www.sc.kyushu-u.ac.jp/~kajikawa/javascript/field_and_potential-e.html (the calculation of this site is based on a paper entitled "Calculation of Magnetic Field Distribution of Solenoid Coil by Computer" [25]).²

Some examples of the theoretical coil coefficient k for typical sample coils are shown in Table 1 with the specifications.

² This information is given for the convenience of users of this document and does not constitute an endorsement by IEC.

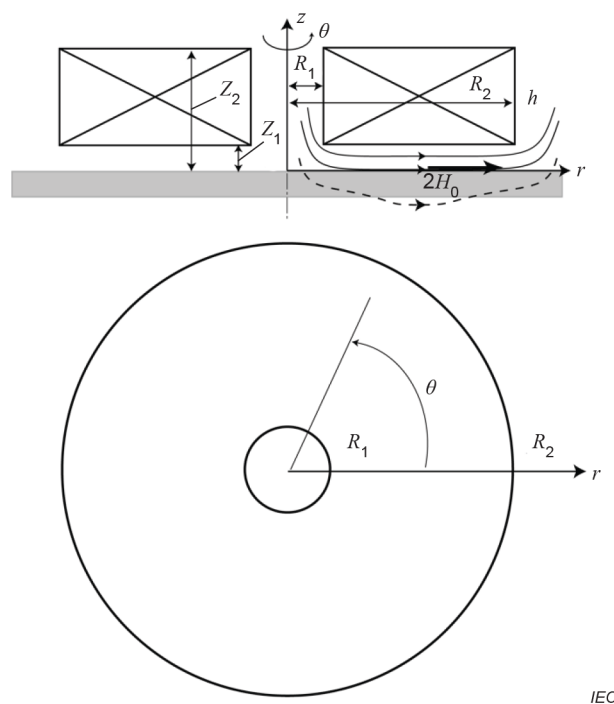
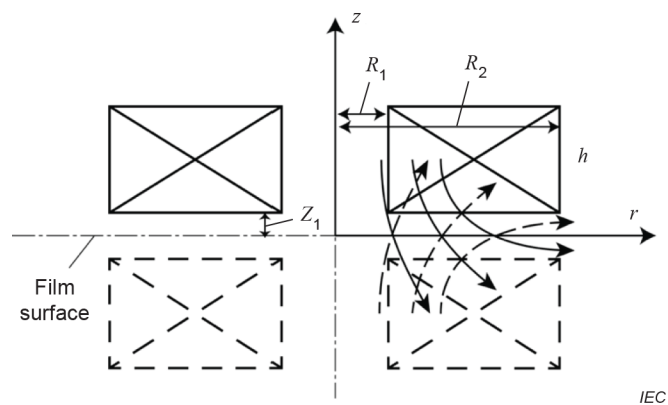


Figure 4 – Illustration of the sample coil and the magnetic field during measurement



NOTE The image coil and its magnetic field generation are shown by the broken line.

Figure 5 – Illustration of the sample coil and its magnetic field generation

Table 1 – Specifications and theoretical coil coefficients k of sample coils

	D_1	D_2	h	Turns	k	r at $F(r) = F_m$
	mm	mm	mm		1/mm	mm
A1	0,8	2,2	1,0	200	62,9	0,74
A2	0,9	2,9	1,0	300	92,2	0,95
A3	1,0	3,6	1,0	400	117,4	1,15
A4	1,0	4,3	1,0	500	135,2	1,35
A5	1,0	4,9	1,0	600	151,5	1,52
A6	1,0	3,6	1,5	600	136,0	1,17
B1	1,0	4,3	1,0	150	34,4	1,35
B2	1,0	5,4	1,0	200	41,9	1,67
B3	1,0	6,5	1,0	250	47,9	1,98
B4	1,0	7,6	1,0	300	52,6	2,31
B5	1,5	5,4	1,5	300	51,5	1,68
Coils A1 to A6 are made of 50- μ m-diameter copper wires (coil-to-film distance $Z_1 = 0,2$ mm), and coils B1 to B5 are made of 100- μ m-diameter copper wires (coil-to-film distance $Z_1 = 0,33$ mm).						

6.2.2 Transport measurements of bridges in the calibration wafer

- a) Measure the E - J characteristics of the transport bridges of the calibration wafer by a four-probe method, and obtain the power-law E - J characteristics,

$$E_t = A_{0t} \times J^n. \quad (3)$$

- b) Repeat the measurement for at least three different bridges. Three sets of data ($n = 20,5$ to 23,8) measured for three bridges are shown in the upper (high- E) part of Figure 6.

6.2.3 U_3 measurements of the calibration wafer

- a) Measure U_3 in the inductive measurement area of the calibration wafer as a function of the coil current with three or four frequencies, and obtain the experimental I_{th} using a constant-inductance criterion; namely, $U_3/fI_0 = 2\pi L_c$ $U_3/fI_{th} = 2\pi L_c$. The criterion L_c should be as small as possible within the range with sufficiently large signal-to-noise (S/N) ratios, in order to use the simple Equation (4) for the electric-field calculation (7.1 c) and Clause C.2). ~~An example of the measurement is shown in Figure 6 with $2\pi L_c = 2 \mu\Omega \cdot \text{sec}$.~~
- b) Repeat the measurement for at least three different points of the film.

6.2.4 Calculation of the E - J characteristics from frequency-dependent I_{th} data

- a) Calculate J_{c0} ($= kI_{th}/d$) and the average E induced in the superconducting film at the full penetration threshold (when $J_c = J_{c0}$) by

$$E_{\text{avg-}U_3} \approx 2,04\mu_0 f d^2 J_c = 2,04\mu_0 k f d I_{th} \quad E_{\text{avg}} \approx 2,04\mu_0 f d^2 J_c = 2,04\mu_0 k f d I_{th}, \quad (4)$$

from the obtained I_{th} at each frequency using the theoretical coefficient k calculated in 6.2.1. The derivation of Equation (4) is described in Clause A.4.

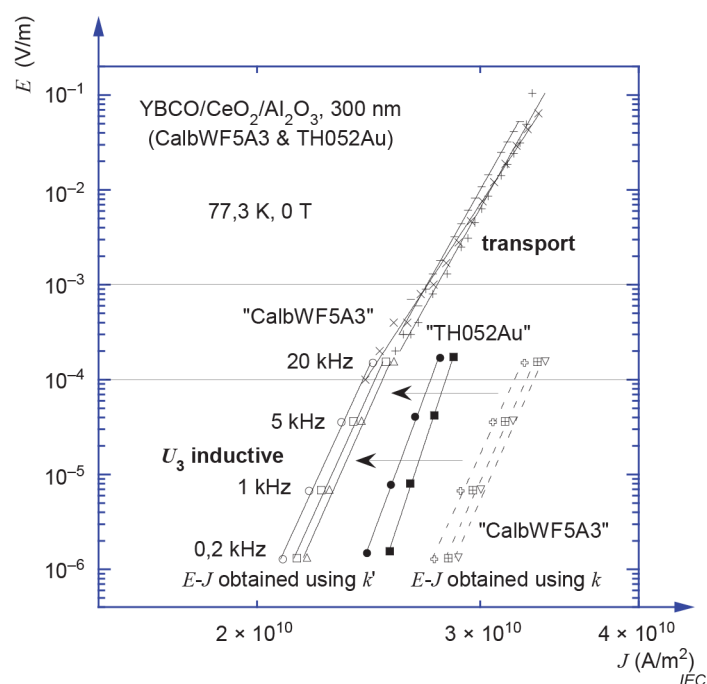
- b) Obtain the E - J characteristics, and the electric fields E_i induced in the superconducting film can be approximated as

$$E_i = A_{0i} \times J^n, \quad (5)$$

from the relation between E_{avg} and J_{c0} , and plot them in the same figure where the transport E - J characteristics data were plotted. Broken lines in Figure 6 show three sets of data measured at different points of the film. Transport data and U_3 inductive data do not yet match at this stage.

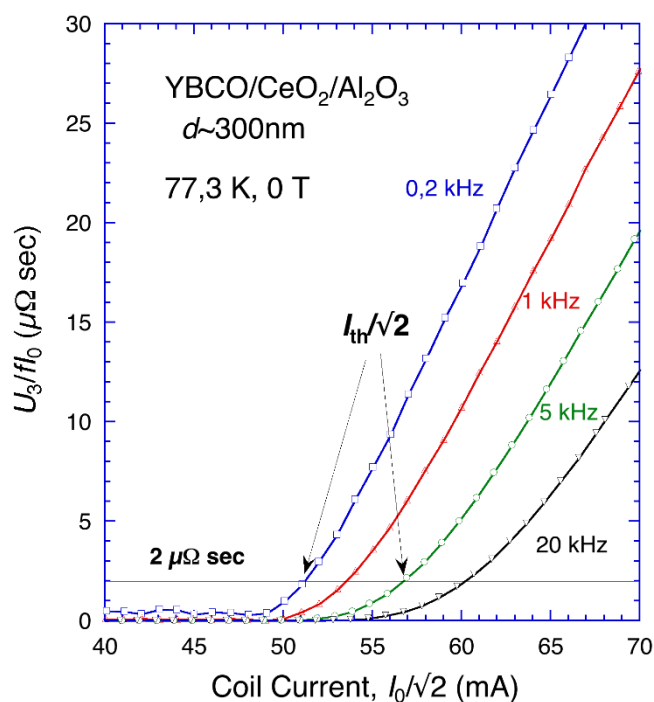
6.2.5 Determination of the k' from J_{ct} and J_{c0} values for an appropriate E

- Choose an appropriate electric field that is within (or near to) both the transport E - J curves and the inductive E - J curves, such as 200 $\mu\text{V/m}$ in Figure 6.
- At this electric field, calculate both the transport critical current densities J_{ct} and the inductive J_{c0} values from Equation (3) and Equation (5), respectively.
- Determine the experimental coil coefficient k' by $k' = (J_{ct}/J_{c0})k$, where J_{ct} and J_{c0} indicate the average values of obtained J_{ct} and J_{c0} values, respectively. If the $J_c (= k'I_{th}/d)$ values are plotted against $E_{\text{avg}} = 2,04 \mu_0 k' d I_{th}$, the E - J characteristics from the U_3 measurement match the transport data well (Figure 6).



NOTE Broken lines show three sets of data measured at different points of the film.

Figure 6 – E - J characteristics measured by a transport method and the U_3 inductive method



IEC 018/13

Figure 6—Example of the normalized third-harmonic voltages (U_3/fI_0) measured with various frequencies

6.3 Measurement of J_c in sample films

- Measure U_3 with two, three or four frequencies in sample films, and obtain I_{th} with the same criterion L_c as used in 6.2.3.
- Use the obtained experimental coil coefficient k' to calculate J_c ($= k'I_{th}/d$) at each frequency, and obtain the relation between J_c and E_{av} E_{avg-U_3} ($= 2,04\mu_0 k f d I_{th}$) using k because of the underestimation as mentioned in 7.1 c). An example of the E - J characteristics is also shown in Figure 6, measured for a sample film (TH052Au, solid symbols) with n -values (36,0 and 40,4) exceeding those of the calibration wafer ($n = 28,0$ to $28,6$).
- From the obtained E - J characteristics, calculate the J_c value with an appropriate electric-field criterion, such as $E_c = 100 \mu V/m$.
- Measurement with three or four frequencies is beneficial to check the validity of the measurement and sample by checking the power-law E - J characteristics. Measurement with two frequencies can be used for routine samples in the interests of time.

6.4 Measurement of J_c with only one frequency

As mentioned in Clause 1 and Clause 3, ~~J_c is a function of electric field, and~~ it is recommended to determine J_c with a constant electric-field criterion using a multi-frequency approach through procedures described in 6.2 and 6.3, ~~because a supercurrent flowing in a superconductor is a function of electric field.~~ However, one frequency measurement is sometimes desired for simplicity and inexpensiveness. In this case, the J_c values are determined with variable electric-field criteria as specified in the following procedures.

- Calculate the theoretical coil coefficient k as described in 6.2.1.
- Obtain the E - J characteristics of the transport bridges of the calibration wafer (Equation (3)) through the procedures of 6.2.2.

- c) Measure U_3 in the inductive measurement area of the calibration wafer as a function of the coil current with one frequency, and obtain the experimental I_{th} using a constant-inductance criterion; namely, $U_3/fI_0 = 2\pi L_c$. The criterion L_c should be as small as possible within the range with sufficiently large S/N ratios, in order to use the simple Equation (4) in 6.2.4 for the electric-field calculation. Calculate J_{c0} ($= kI_{th}/d$) and the average E induced in the superconducting film at the full penetration threshold by Equation (4). Repeat the measurement for at least three different points of the film, and obtain average J_{c0} and E_{avg-U_3} .
- d) Using the transport E - J characteristics of Equation (3), calculate J_{ct} for the average E_{avg-U_3} obtained in c).
- e) Determine the experimental coil coefficient k' by $k' = (J_{ct}/J_{c0})k$.
- f) Measure U_3 with the same frequency in sample films, and obtain I_{th} with the same criterion L_c as used in c). Calculate J_c ($= k'I_{th}/d$) using the obtained experimental coil coefficient k' . Calculate also E_{avg-U_3} with Equation (4), and this value should be accompanied by each J_c value.

6.5 Examples of the theoretical and experimental coil coefficients

Some examples of the theoretical and experimental coil coefficients (k and k') for typical sample coils are shown in Table 2 with the specifications and recommended criteria for the I_{th} determination, $2\pi L_c = U_3/fI_0$. Note that the k' depends on the criterion L_c . ~~Coil 1 is wound with a 50 μ m diameter, self-bonding polyurethane enameled round copper winding wire, and coils 2 and 3 are wound with a 50 μ m diameter, polyurethane enameled round copper winding wire.~~ Coils 1 and 2 are wound with a 50 μ m diameter, polyurethane enamelled round copper winding wire, and coil 3 is wound with a 50 μ m diameter, self-bonding polyurethane enamelled round copper winding wire. Measured resistances at 77,3 K and calculated self-inductances when a superconducting film is placed below the coil are also shown. The coil-to-film distance Z_1 is fixed at 0,2 mm. The images of coils 1 and 3 are shown in Figure 7, and the coil-factor functions $F(r)$ for the three coils show that the peak magnetic field occurs near the mean coil radius (Figure 8).

Table 2 – Specifications and coil coefficients of typical sample coils

	D_1 mm	D_2 mm	h mm	Turns	k 1/mm	k' 1/mm	U_3/fI_0 $\mu\Omega\cdot\text{sec}$	R Ω	L mH
1	0,9	4,2	1,0	400	106,6	82,2	2	4,1	0,165
2	1,0	3,6	1,0	400	117,4	89,1	2	3,4	0,163
3	0,8	2,2	1,0	200	63,2	47,0	0,6	1,6	0,028

	D_1 mm	D_2 mm	h mm	Turns	k 1/mm	k' 1/mm	U_3/fI_0 $\mu\Omega\cdot\text{s}$	R Ω	L mH
1	0,8	2,2	1,0	200	62,9	47,0	0,6	1,6	0,028
2	1,0	3,6	1,0	400	117,4	89,1	2	3,4	0,163
3	0,9	4,2	1,0	400	106,6	82,2	2	4,1	0,165

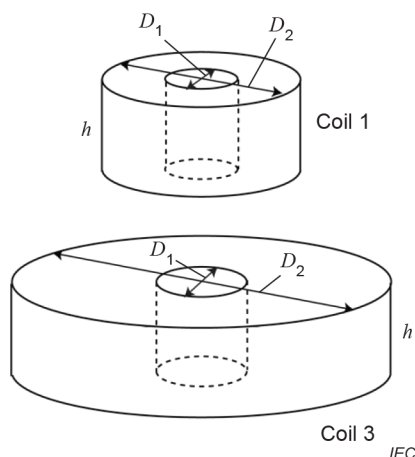


Figure 7 – Illustration of coils 1 and 3 in Table 2

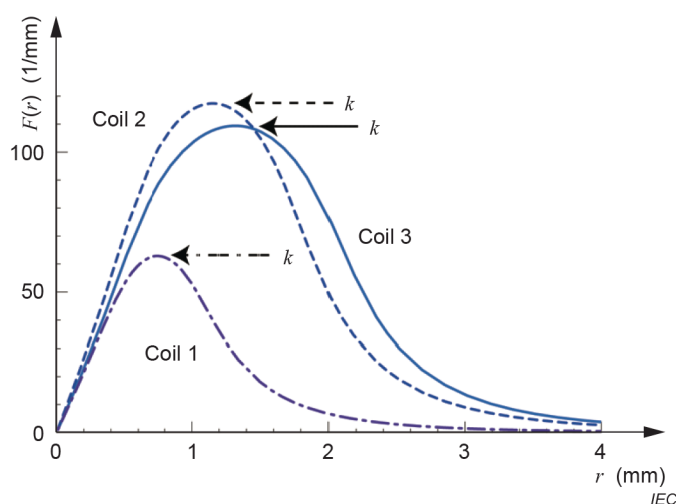


Figure 8 – The coil-factor function $F(r) = 2H_0/I_0$ calculated for the three coils

7 Uncertainty in the test method

7.1 Major sources of systematic effects that affect the U_3 measurement

The most significant systematic effect on the U_3 measurement is due to the deviation of the coil-to-film distance Z_1 from the prescribed value. Because the measured value $J_c d$ in this technique is directly proportional to the magnetic field at the upper surface of the superconducting film, the deviation of the spacing Z_1 directly affects the measurement. The key origins of the uncertainty are listed in a) to c) below. ~~Note that the general concept of the "uncertainty" is summarized in Annex C.~~

a) Inadequate pressing of the coil to the film

As the measurement is performed in liquid nitrogen, the polyimide film placed above the HTS thin film becomes brittle and liquid nitrogen may enter the space between the polyimide and HTS films. Thus, sufficient pressure is ~~necessary~~ needed to keep the polyimide film flat and avoid the deviation of Z_1 . An experiment has shown that the required pressure is about 0,2 MPa [18]. Here it is to be noted that thermal contraction of polyimide films at the liquid nitrogen temperature is less than ~~$0,002 \times (300 - 77) \approx 0,45 \%$~~ $0,008 \times (300 - 77) \approx 1,78 \%$,

which leads to negligible values of ~~0,2 μ m to 0,6 μ m~~ 0,9 μ m to 4,5 μ m compared with the total coil-to-film distance (about 200 μ m) [26].

b) Ice layer formed between the coil and polyimide film

The liquid nitrogen inevitably contains powder-like ice. If the sample coil is moved to scan the large-area HTS film area for an extended period, an ice layer is often formed between the polyimide film and the sample coil, which increases the coil-to-film distance Z_1 from the prescribed value. As shown later in 7.2, this effect reduces coil coefficients (k and k'), and the use of uncorrected k' results in an overestimate in J_c . Special care should be taken to keep the measurement environment as dry as possible. If the measurement system is set in an open (ambient) environment, the J_c values measured after an extended period of time become sometimes greater than those measured before, and the overestimation was as large as 6 % when measured after one hour. If the measurement system is set in almost closed environment and the ambient humidity is kept less than about 5 %, such effect of ice layers can be avoided. We can check this effect by confirming reproducibility. If the same J_c values are obtained after an extended period, it proves that there is negligible effect of ice layers. These two systematic effects (a) and b)) are not considered in the estimate of the uncertainty in the experimental coil coefficient k' in 7.3 and Clause C.1, because they can be eliminated by careful measurements.

c) Underestimation of the induced electric field E by a simple Bean model

The calculation of average induced electric fields ~~E_{avg}~~ E_{avg-U_3} in the superconducting film via Equation (4) is sufficiently accurate provided the magnetic-field penetration below the bottom of the film can be neglected. However, considerable magnetic fields penetrate below the film when the experimental threshold current I_{th} is determined and detectable U_3 has emerged. It was pointed out that the rapid magnetic-field penetration below the film at $I_0 = I_{th}$ may cause a considerable increase of the induced electric field and that the E calculated by Equation (4) might be significantly underestimated [27]. However, several experimental results have shown that the relative standard uncertainty from this effect is usually less than 5 %. The detail is described in Clause C.2.

7.2 Effect of deviation from the prescribed value in the coil-to-film distance

Because the magnetic field arising from the coil depends on the coil-to-film distance Z_1 , the coil coefficient also depends on Z_1 . Figure 9 shows the Z_1 dependence of the theoretical coil coefficient k calculated from Equations (1) and (2). The theoretical coil coefficient k normalized by k_0 is plotted as the function of Z_1 , where k_0 is the theoretical coil coefficient for $Z_1 = 0,2$ mm. Dimensions of coils 1, 2, and 3 are listed in Table 2. The relative effect of deviation on k of coil ~~1~~ 3 is about 2,6 %, when $Z_1 = 0,2$ mm $\pm$ 0,02 mm. Provided the deviation of Z_1 is small (e.g. ≤ 20 %), the deviated experimental coil coefficient k' is proportional to k . Some experimental results that support this are described in Clause C.3. Therefore, use Figure 9 to estimate the systematic effect on k' , if the deviated distance can be reasonably estimated.

The effect of the coil inclination to the superconducting film was theoretically investigated [28]. It was found that the theoretical coil coefficient k decreases approximately 7 % when the sample coil ($D_1 = 1,0$ mm, $D_2 = 3,6$ mm, $h = 1$ mm) is inclined 4 % and the distance between the coil and the superconducting thin film is increased.

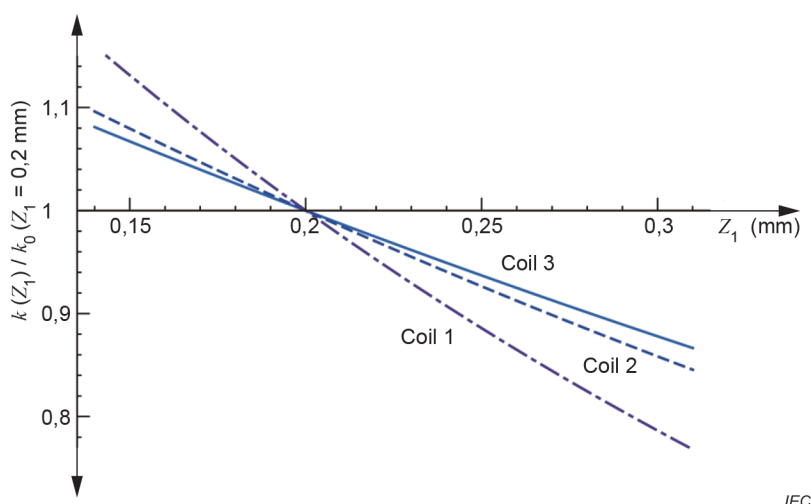


Figure 9 – The coil-to-film distance Z_1 dependence of the theoretical coil coefficient k

7.3 Uncertainty in the experimental coil coefficient and the obtained J_c

Since the proposed method uses a standard sample (the calibration wafer) to determine the experimental coil coefficient k' that directly affects the measured J_c values, the uncertainty in k' is one of the key factors affecting the uncertainty in the measurement, and the homogeneity of the large-area thin film used in the calibration wafer is an important source of such uncertainty. The experimental coil coefficient k' is calculated by $k' = (J_{ct}/J_{c0})k$ at an appropriate electric field, where J_{ct} is the critical current density measured by the transport method and $J_{c0} = kI_{th}/d$ measured by the inductive method (6.2.5). ~~An example of the evaluation of the uncertainty of k' for the coil 1 (Table 1) was shown in D.1. The result is $k' = (J_{ct}/J_{c0})k = (2,5878/3,4437) \times 109,4 = 82,2 \text{ mm}^{-1}$ with the combined standard uncertainty of $u_c(k') = 2,4 \text{ mm}^{-1}$ (2,93 %).~~ An example of the evaluation of the uncertainty in k' for the coil 3 (Table 2) is shown in Clause C.1. The result is $k' = (J_{ct}/J_{c0})k = \{(2,5878 \times 10^{10} \text{ A/m}^2)/(3,3556 \times 10^{10} \text{ A/m}^2)\} \times 106,6 \text{ mm}^{-1} = 82,2 \text{ mm}^{-1}$ with the combined standard uncertainty in $u_c(k') = 2,4 \text{ mm}^{-1}$ (2,93 %). It has been demonstrated that the uncertainty in the transport J_{ct} dominates the combined standard uncertainty in k' .

The uncertainty originating from the underestimation of E_{avg} E_{avg-U_3} by a simple Bean model (Equation (4)) is evaluated in Clause C.2. The relative standard uncertainty (Type B) is evaluated to be $u_B = 6,6/\sqrt{3} \% = 3,8 \%$ for a typical specimen with $n = 25$. In contrast to these Type-B uncertainties, Type-A uncertainty in J_c , originating from the experimental uncertainty in the electric U_3 measurement, is much smaller, typically about 0,3 %, as shown in Clause C.4. The uncertainty in k' and that from the underestimation of E_{avg} E_{avg-U_3} dominate the combined standard uncertainty in the absolute value of J_c , and the relative combined standard uncertainty was 4,7 % for a typical $\text{DyBa}_2\text{Cu}_3\text{O}_7$ (DyBCO) sample film (Clause C.5). This is well below the target value of 10 %. Note that for the purpose of evaluating the homogeneity of J_c distribution in large-area superconducting thin films, the uncertainty in k' does not contribute to the uncertainty in J_c distribution, provided the same sample coil is used. Therefore, the relative standard uncertainty should be less than the target uncertainty of 5 %.

7.4 Effects of the film edge

Figure 8 shows that substantial magnetic fields exist, even outside the coil area, which induce shielding currents in the superconducting film. Therefore, the coil ~~must~~ needs to be apart from the film edge for the precise measurement. The original paper by Claassen *et al.* recommended that the outer diameter of the coil should be less than half of the film width to neglect the edge effect [10]. However, recent numerical calculation with the finite element method indicated that correct measurements can be made when the film width is as small as 6 mm for a coil with an outer diameter of 5 mm and for $Z_1 = 0,2$ mm [29]. The experimental results described in Clause C.6 have shown that precise measurements can be made for either of coils 2 or 3 (Table 2) when the outside of the coil is more than 0,3 mm apart from the film edge. With the uncertainty of 0,1 mm to 0,2 mm in the coil setting in mind, the outside of the coil should be more than 0,5 mm apart from the film edge when coils with an outer diameter of 2 mm to 5 mm are used.

7.5 Specimen protection

Moisture and water sometimes react with the Ba atoms in the YBCO film and cause the superconducting properties to deteriorate. If YBCO films are still used for some purpose after the measurement, they should be warmed up in a moisture-free environment, e.g. a vacuum or He gas to avoid degradation. Some protection measure can also be provided for the specimens. A thin organic coating, with thickness less than several micrometres, does not affect the measurements and can subsequently be removed; thus it can be used for protection.

8 Test report

8.1 Identification of test specimen

The test specimen shall be identified, if possible, by the following:

- a) name of the manufacturer of the specimen;
- b) classification;
- c) lot number;
- d) chemical composition of the thin film and substrate;
- e) thickness and roughness of the thin film;
- f) manufacturing process technique.

8.2 Report of J_c values

The J_c values shall be reported with the electric-field criterion, E_c . If possible, the n -values, the indices of the power-law E - J characteristics, shall be reported together. It is known that the measurement of n -values facilitates the detection of degraded segments within a large-area HTS film [15].

8.3 Report of test conditions

The following test conditions shall be reported:

- a) temperature (atmospheric pressure, or the pressure of liquid nitrogen);
- b) DC magnetic fields (if applied);
- c) test frequencies;
- d) possible effects of the ice layer;
- e) specifications of the sample coil;
- f) thickness of the spacer film.

Annex A (informative)

Additional information relating to Clauses 1 to 8

A.1 Comments on other methods for measuring the local J_c of large-area HTS films

There are several AC inductive methods for the nondestructive measurement of local J_c of large-area superconducting thin films [10] [11] [12] [13] [17], in which some detect third-harmonic voltages $U_3 \cos(3\omega t + \theta)$ [10] [11] [17] and others use only the fundamental voltage [12] [13]. In these inductive methods, AC magnetic fields are generated with AC currents $I_0 \cos \omega t$ in a small coil mounted just above the film, and J_c is calculated from the threshold coil current I_{th} , at which full penetration of the AC magnetic field to the film is achieved [14]. When $I_0 < I_{th}$, the magnetic field below the film is completely shielded, and the superconducting film is regarded as a mirror image coil reflected through the upper surface of the film, carrying the same current but in the opposite direction. The response of the superconducting film to $I_0 \cos \omega t$ is linear and no third-harmonic voltage is induced in the coil.

For the case of the U_3 inductive method, U_3 starts to emerge at $I_0 = I_{th}$, when the superconducting shielding current reaches the critical current and its response becomes nonlinear [17]. In the other methods that use only the fundamental voltage, to detect the breakdown of complete shielding when the critical current is reached, penetrated AC magnetic fields are detected by a pickup coil mounted just below the film [12] or a change of mutual inductance of two adjacent coils is measured [13]. In all these inductive J_c measurements, the scheme is common in that the AC magnetic field $2H_0 \cos \omega t$ at the upper surface of the film is measured at the full penetration threshold. We obtain J_c because the amplitude of the full penetration field $2H_0$ equals $J_c d$ [17]. The electric field E induced in the superconductor can be calculated with the same Equation (4) [14], and a similar procedure to that described in Clause 6 can be used for the precise measurement.

Another inductive magnetic method using Hall probe arrays has been commercialized to measure local J_c of long coated conductors [30] [31]. In this method magnetic field profiles are measured in applied DC magnetic field, and the corresponding current distribution is calculated. This method can also be applied to rectangular large-area HTS films having widths less than several centimetres, and has better spatial resolution over AC inductive methods using small coils.

A.2 Requirements

As the third-harmonic voltages are proportional to the measuring frequency, higher frequencies are desirable to obtain a better S/N ratio. However, there is a limitation due to the frequency range of the measuring equipment (lock-in amplifier and/or filter) and to excessive signal voltages induced in the sample coil when a large $J_c d$ film is measured. It is recommended to use a frequency from 1 kHz to 20 kHz for a film with small $J_c d$ (≤ 1 kA/m), and a frequency from 0,2 kHz to 8 kHz for a film with large $J_c d$ (≥ 20 kA/m). Measurements over a wide frequency range are desirable to obtain the current-voltage characteristics in a wide electric-field range. For the general purpose of the J_c measurement, however, one order of frequency range is sufficient to obtain the n -value and measure J_c precisely.

In this document, the measurement temperature is limited to liquid nitrogen temperatures, namely 77,35 K at 1 013 hPa and 65,80 K at 200 hPa, because a refrigerant is needed to cool the sample coil that generates Joule heat. When measuring at variable temperatures in a gas atmosphere, further investigations are necessary.

The U_3 inductive method is applicable not only to large-area HTS films deposited on insulating substrates (sapphire, MgO, etc.), but also to coated conductors with metallic substrates. However, if the coated conductors have thick metallic protective layers (Ag or Cu) and their thickness exceeds about 10 μm , certain measures are needed to avoid the skin effect. One technique involves limiting measuring frequencies to a sufficiently low extent (e.g. about 8 kHz).

A.3 Theory of the third-harmonic voltage generation

Here we present the response of a superconducting film to a current-carrying coil mounted above the film (Figure A.1) [17]. A superconducting film of thickness d , infinitely extended in the xy plane, is situated at $-d < z < 0$, where the upper surface is at $z = 0$ in the xy plane and the lower surface is at $z = -d$. A drive coil is axially symmetric with respect to the z -axis, and the coil occupies the area of $R_1 < r < R_2$ and $Z_1 < z < Z_2$ in the cylindrical coordinate (r, θ, z) . The coil consists of a wire of ~~winding number N~~ N turns, which carries a sinusoidal drive current $I_0(t) = I_0 \cos \omega t$ along the θ direction. Responding to the magnetic field produced by the coil, the shielding current flows in the superconducting film. The sheet current K_θ (i.e. the current density integrated over the thickness, $-d < z < 0$) in the superconducting film plays crucial roles in the response of the film, and $|K_\theta|$ cannot exceed its critical value, $J_c d$.

The response of the superconducting film is detected by measuring the voltage $U(t)$ induced in the coil, and $U(t)$ is generally expressed as the Fourier series,

$$U(t) = \sum_{n=1}^{\infty} U_n \cos(n\omega t + \theta_n). \quad (\text{A.1})$$

The fundamental voltage U_1 is primarily determined by the coil impedance. The even harmonics, U_n for even n , is generally much smaller than the odd harmonics, U_n for odd n . The third-harmonic voltage, U_3 , is the key, because U_3 directly reflects the nonlinear response (i.e. information on $J_c d$) of the superconducting film.

The coil produces an axially symmetric magnetic field, and its radial component H_r at the upper surface of the superconducting film ($z = 0$) is obtained by

$$H_r(r, t) = -H_0 \cos \omega t = -(I_0 / 2) F(r) \cos \omega t. \quad (\text{A.2})$$

The coil-factor function $F(r)$ is determined by the configuration of the coil as

$$F(r) = \frac{N}{2\pi S} \int_{R_1}^{R_2} dr' \int_0^{2\pi} d\theta \int_{Z_1}^{Z_2} dz \frac{r' z \cos \theta}{(z^2 + r^2 + r'^2 - 2rr' \cos \theta)^{3/2}}, \quad (\text{A.3})$$

where $S = (R_2 - R_1)(Z_2 - Z_1)$ is the cross-sectional area of the coil. The $F(r)$ generally has a maximum $F_m > 0$ at $r = r_m$ [where r_m is roughly close to $(R_1 + R_2)/2$], and $F(0) = F(\infty) = 0$.

When $0 < I_0 < I_{th}$, the magnetic field arising from the coil does not penetrate below the film ($z < -d$). In such cases, the magnetic field distribution above the film ($z > 0$) is simply obtained by the mirror-image technique. The magnetic field arising from the image coil (i.e. from the shielding current flowing in the superconducting film) cancels out the perpendicular component H_z , and the parallel component H_r doubles. The sheet current K_θ in the superconducting film is therefore obtained by $K_\theta(r, t) = 2H_r(r, t) = -I_0 F(r) \cos \omega t$. Because of the linear response of the superconducting film for $0 < I_0 < I_{th}$, the voltage induced in the coil contains no harmonics.

Note that the amplitude of the sheet current density, $|K_\theta| = 2|H_r| = I_0 F(r) \leq I_0 F_m$, cannot exceed the critical value, $J_c d$, when $0 < I_0 \leq I_{th}$. The threshold current I_{th} is determined such that $|K_\theta| \leq I_0 F_m$ reaches $J_c d$ when $I_0 = I_{th}$, and is obtained by

$$I_{th} = J_c d / F_m = J_c d / k, \quad (\text{A.4})$$

where the (theoretical) coil coefficient is obtained by $k = F_m$.

When $I_0 > I_{th}$, the magnetic field penetrates below the superconducting film, and the nonlinear response of K_θ yields the generation of the harmonic voltages in the coil.

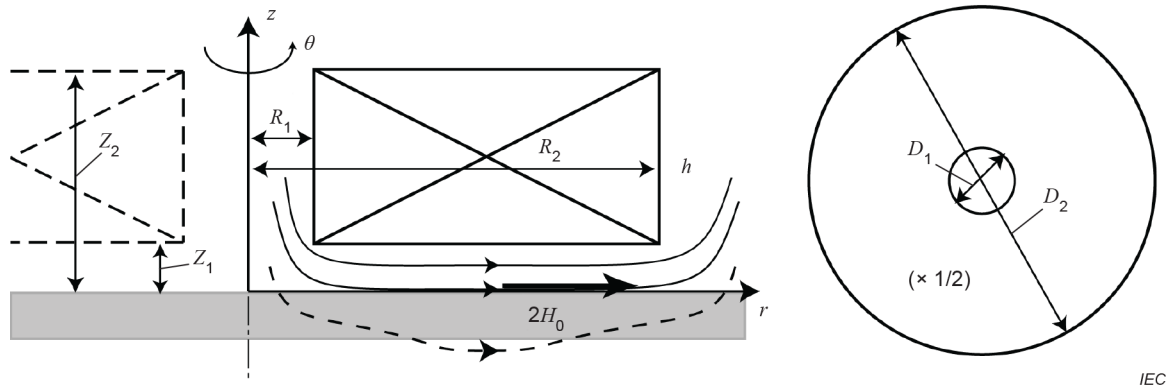


Figure A.1 – Illustration of the sample coil and the magnetic field during measurement

A.4 Calculation of the induced electric fields

Here, we approximate the average E induced in the superconducting film at the full penetration threshold, $I_0 = I_{th}$, using the Bean model [14]. This approximation assumes a semi-infinite superconductor below the xy -plane ($z \leq 0$), and the film is regarded as part of this superconductor ($-d \leq z \leq 0$). When a sinusoidal magnetic field $H_{x0} = 2H_0 \cos \omega t$ ($2H_0 = J_c d$) is applied parallel to the x -direction at the surface of the superconductor, the induced E has only the y -component $E_y(z)$, and $E_y(z \leq -d)$ is zero because the magnetic fluxes just reach the lower surface of the film ($z = -d$). The $E_y(z)$ is calculated by integrating $-\mu_0(dH_x/dt)$ from $z = -d$ to z , yielding $E_y(z) = -\mu_0 \omega d H_0 \sin \omega t (1 - \cos \omega t + 2z/d)$. The time-dependent surface electric field, $|E_y(z=0)|$, peaks at $\omega t = 2\pi/3$, and then, $\max|E_y(0)| = (3\sqrt{3}/4) \mu_0 \omega d H_0$. Because $\max|E_y(z)|$ peaks at $z = 0$ (the upper surface of the film) and is zero at $z = -d$ (the lower surface of the film), the volume average of $\max|E_y(z)|$ is estimated to be half of $\max|E_y(0)|$,

$$E_{avg} \approx (3\sqrt{3}/4) \mu_0 \omega d H_0 \approx 2.04 \mu_0 \omega d^2 J_c = 2.04 \mu_0 k f d I_{th}$$

$$E_{\text{avg-}U_3} \approx (3\sqrt{3}\pi/4)\mu_0 f d H_0 \approx 2,04\mu_0 f d^2 J_c = 2,04\mu_0 k f d I_{\text{th}} \quad (\text{A.5})$$

For typical parameters of the measurement, $f = 1$ kHz, $d = 250$ nm, and $J_c = 10^{10}$ A/m², the calculated E is about 2 $\mu\text{V}/\text{m}$.

A.5 Theoretical coil coefficient k and experimental coil coefficient k'

Here, the basic concept concerning the theoretical coil coefficient $k = J_c d / I_{\text{th}}$ and the experimental coefficient k' for the case of the U_3 inductive method is explained. When the coil current I_0 equals the threshold current I_{th} , the highest magnetic field below the coil $2H_{0,\text{max}} = J_c d$, and the magnetic field just fully penetrates the film. Since $2H_{0,\text{max}}$ can be theoretically calculated, we can calculate the theoretical coil coefficient $k = J_c d / I_{\text{th}}$. However, the above "true I_{th} " corresponds to the coil current at which infinitesimal U_3 is generated in the coil. Because it is impossible to detect $U_3 \approx 0$ to obtain a "true I_{th} ", we need an alternative approach to obtain an "experimental I_{th} " and corresponding experimental coil coefficient k' .

A.6 Scaling of the U_3 – I_0 curves and the constant-inductance criterion to determine I_{th}

For convenience, the (experimental) threshold current I_{th} has been often determined by a constant-voltage criterion, e.g. $U_3/\sqrt{2} = 50$ μV . However, the use of a constant-voltage criterion is problematic. Theoretical analyses on the relationship between I_0 and U_3 showed that there is clear scaling behaviour $U_3/I_{\text{th}} = \omega G(I_0/I_{\text{th}})$, where G is a scaling function that is determined only by the specifications of the sample coil [11] [17]. This equation implies that the U_3 vs. I_0 curves with various I_{th} values ~~should~~ must collapse to one curve if they are normalized with I_{th} . The inset of Figure A.2 a) clearly shows this scaling behaviour. As the third-harmonic resistance $U_3/I_0 = \omega G(I_0/I_{\text{th}})/(I_0/I_{\text{th}})$, the U_3/I_0 itself is already normalized (Figure A.2 b)), and it scales with the scaled current I_0/I_{th} (inset of Figure A.2 b)). Because the third-harmonic voltage U_3 is proportional to I_{th} , the determination of I_{th} by a constant-voltage criterion inherently causes a systematic error; namely, the J_c of a sample with $J_c d$ larger (smaller) than the standard sample is underestimated (overestimated) [16]. From the scaling behaviour observed in the third-harmonic resistance U_3/I_0 (Figure A.2 b)), it is demonstrated that the I_{th} should be determined by a constant-resistance criterion, such as $U_3/I_0 = 2$ m Ω . Furthermore, as the U_3 values are proportional to the measuring frequency, a constant-inductance criterion, such as $U_3/fI_0 = 2$ $\mu\Omega\cdot\text{s}$, should be used if the U_3 measurements are performed with multiple frequencies [16] [32]. It is also to be noted that such scaling behaviour forms the basis of the $J_c d$ measurement, the procedure for which is described in 6.2 to 6.4 using a standard sample (calibration wafer).

Because a supercurrent flowing in a superconductor is a function of electric field, it is recommended to determine J_c with a constant electric-field criterion using a multi-frequency approach through procedures described in 6.2 and 6.3. An example of the multi-frequency measurement is shown in Figure A.3 with $U_3/fI_0 = 2\pi L_c = 2$ $\mu\Omega\cdot\text{s}$.