

INTERNATIONAL STANDARD



**Superconductivity –
Part 7: Electronic characteristic measurements – Surface resistance of
high-temperature superconductors at microwave frequencies**



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IEC Central Office
3, rue de Varembe
CH-1211 Geneva 20
Switzerland

Tel.: +41 22 919 02 11
info@iec.ch
www.iec.ch

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INTERNATIONAL STANDARD



**Superconductivity –
Part 7: Electronic characteristic measurements – Surface resistance of
high-temperature superconductors at microwave frequencies**

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INTERNATIONAL ELECTROTECHNICAL COMMISSION

SUPERCONDUCTIVITY –

**Part 7: Electronic characteristic measurements –
Surface resistance of high-temperature
superconductors at microwave frequencies**

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International Standard IEC 61788-7 has been prepared by IEC technical committee 90: Superconductivity.

This third edition cancels and replaces the second edition, published in 2006. This edition constitutes a technical revision.

This edition includes the following significant technical changes with respect to the previous edition:

- a) informative Annex B, relative combined standard uncertainty for surface resistance measurement has been added;
- b) precision and accuracy statements have been converted to uncertainty;
- c) reproducibility in surface resistant measurement has been added.

The text of this International Standard is based on the following documents:

FDIS	Report on voting
90/447/FDIS	90/452/RVD

Full information on the voting for the approval of this International Standard can be found in the report on voting indicated in the above table.

This document has been drafted in accordance with the ISO/IEC Directives, Part 2.

A list of all parts of the IEC 61788 series, published under the general title *Superconductivity*, can be found on the IEC website.

The committee has decided that the contents of this document will remain unchanged until the maintenance result date indicated on the IEC web site under "<http://webstore.iec.ch>" in the data related to the specific publication. At this date, the document will be

- reconfirmed;
- withdrawn;
- replaced by a revised edition, or
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INTRODUCTION

Since the discovery of some Perovskite-type Cu-containing oxides, extensive research and development (R & D) work on high-temperature ~~oxide~~ superconductors (HTS) has been, and is being, done worldwide, and its application to high-field magnet machines, low-loss power transmission, electronics and many other technologies is in progress.

In various fields of electronics, especially in telecommunication fields, microwave passive devices such as filters using ~~oxide superconductors~~ HTS are being developed and are undergoing on-site testing [1][2]¹.

Superconductor materials for microwave resonators [3], filters [4], antennas [5] and delay lines [6] have the advantage of very low loss characteristics. The parameters of superconductor materials needed for the design of microwave low loss components are the surface resistance, (R_s) and the temperature dependence of the R_s . Knowledge of this parameter is of primary importance for the development of new materials on the supplier side and for the design of superconductor microwave components on the customer side.

~~Recent advances in high T_c superconductor (HTS) thin films with R_s~~ R_s of high quality HTS films is generally several orders of magnitude lower than that of normal metals [7] [8] [9] [10], ~~have~~ which has increased the need for a reliable characterization technique to measure this property [3,4]. Traditionally, the R_s of niobium or any other low-temperature superconducting material was measured by first fabricating an entire three-dimensional resonant cavity and then measuring its Q -value [11]. The R_s could be calculated by solving the electro-magnetic field (EM) distribution inside the cavity. Another technique involves placing a small sample inside a larger cavity. This technique has many forms but usually involves the uncertainty introduced by extracting the loss contribution due to the HTS films from the experimentally measured total loss of the cavity.

The best HTS samples are epitaxial films grown on flat crystalline substrates and no high-quality films have been grown on any curved surface so far. What is needed is a technique that: can use these small flat samples; requires no sample preparation; does not damage or change the film; is highly repeatable; has great sensitivity (down to 1/1 000 the R_s of copper); has great dynamic range (up to the R_s of copper); can reach high internal powers with only modest input powers; and has broad temperature coverage (4,2 K to 150 K).

The dielectric resonator method is selected among several methods ~~[5,6,7]~~ to determine the surface resistance at microwave frequencies because it is considered to be the most popular and practical at present. Especially, the sapphire resonator is an excellent tool for measuring the R_s of HTS materials ~~[8,9]~~ [12] [13] [14].

The test method given in this document can also be applied to other superconductor bulk plates including low T_c materials.

This document is intended to provide an appropriate and agreeable technical base for the time being to engineers working in the fields of electronics and superconductivity technology.

The test method covered in this document is based on the VAMAS (Versailles Project on Advanced Materials and Standards) pre-standardization work on the thin film properties of superconductors.

¹ Numbers in square brackets refer to the bibliography.

SUPERCONDUCTIVITY –

Part 7: Electronic characteristic measurements – Surface resistance of **high-temperature** superconductors at microwave frequencies

1 Scope

This part of IEC 61788 describes measurement of the surface resistance (R_s) of superconductors at microwave frequencies by the standard two-resonator method. The object of measurement is the temperature dependence of R_s at the resonant frequency.

The applicable measurement range of R_s for this method is as follows:

- Frequency: 8 GHz < f < 30 GHz
- Measurement resolution: 0,01 mΩ at 10 GHz

The R_s data at the measured frequency, and that scaled to 10 GHz, assuming the f^2 rule for comparison, is reported.

2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

IEC 60050-815, *International Electrotechnical Vocabulary (IEV) – Part 815: Superconductivity*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in IEC 60050-815 apply.

ISO and IEC maintain terminological databases for use in standardization at the following addresses:

- IEC Electropedia: available at <http://www.electropedia.org/>
- ISO Online browsing platform: available at <http://www.iso.org/obp>

~~In general, surface impedance Z_s for conductors, including superconductors, is defined as the ratio of the electric field E_t to the magnetic field H_t , tangential to a conductor surface:~~

$$\del{Z_s = E_t / H_t = R_s + jX_s}$$

~~where R_s is the surface resistance and X_s is the surface reactance.~~

4 Requirements

The R_s of a superconductor film shall be measured by applying a microwave signal to a dielectric resonator with the superconductor film specimen and then measuring the attenuation of the resonator at each frequency. The frequency shall be swept around the resonant frequency as the centre, and the attenuation–frequency characteristics shall be recorded to obtain the Q -value, which corresponds to the loss.

The target ~~precision~~ relative combined standard uncertainty of this method ~~is a coefficient of variation (standard deviation divided by the average of the surface resistance determinations) that~~ is less than 20 % for the measurement temperature range from ~~30~~ 20 K to 80 K.

It is the responsibility of the user of this document to ~~consult and~~ establish appropriate safety and health practices and to determine the applicability of regulatory limitations prior to use.

Hazards exist in this type of measurement. The use of a cryogenic system is essential to cool the superconductors to allow transition into the superconducting state. Direct contact of skin with cold apparatus components can cause immediate freezing, as can direct contact with a spilled cryogen. The use of an RF generator is also essential to measure high-frequency properties of materials. If its power is too high, direct contact to human bodies can cause an immediate burn.

5 Apparatus

5.1 Measurement system

Figure 1 shows a schematic diagram of the system required for the microwave measurement. The system consists of a network analyzer system for transmission measurement, a measurement apparatus, and a thermometer for monitoring the measuring temperature.

An incident power generated from a suitable microwave source such as a synthesized sweeper is applied to the dielectric resonator fixed in the measurement apparatus. The transmission characteristics are shown on the display of the network analyzer. The measurement apparatus is fixed in a temperature-controlled cryocooler.

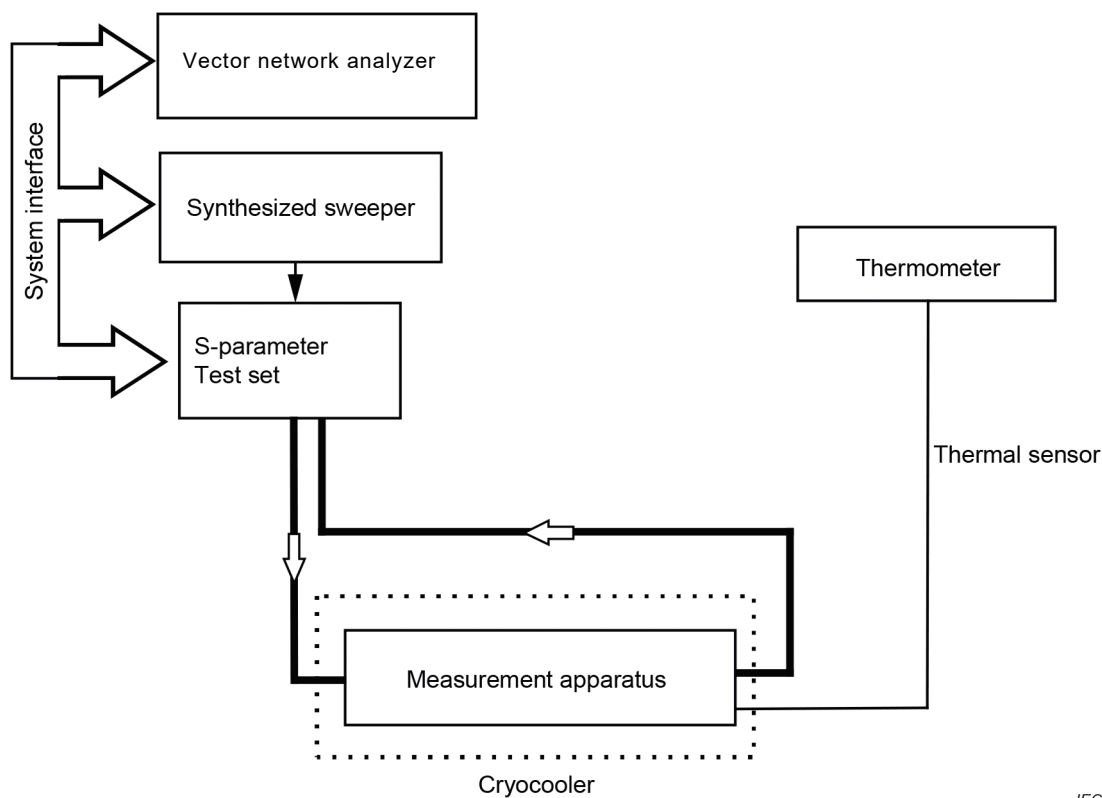


Figure 1 – Schematic diagram of measurement system for temperature dependence of R_s using a cryocooler

~~The measurement apparatus is fixed in a temperature-controlled cryocooler.~~

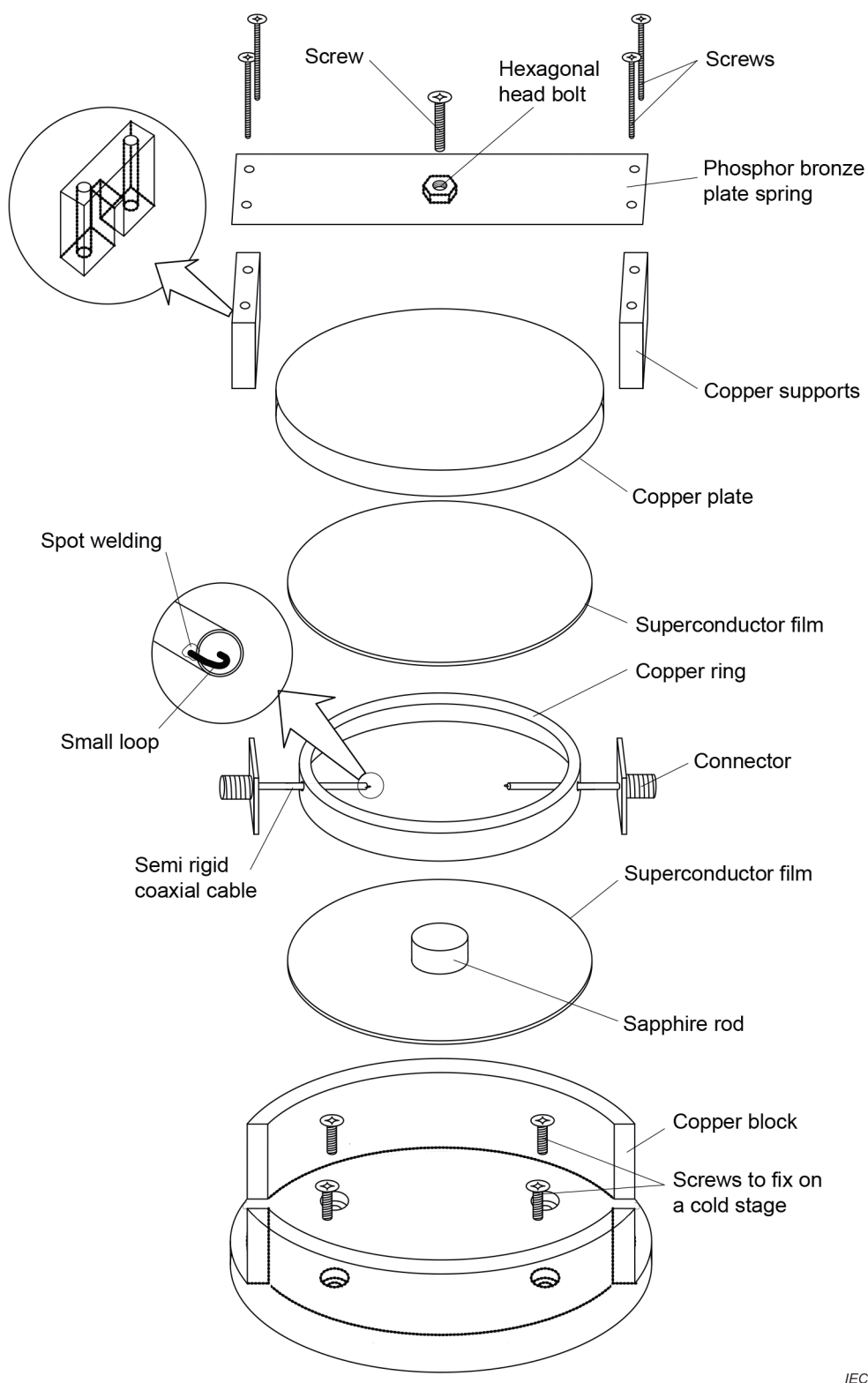
For the measurement of R_s for superconductor films, a vector network analyzer is recommended. A vector network analyzer has better measurement accuracy than a scalar network analyser due to its wide dynamic range. The performance requirements of the vector network analyzer are specified in 7.1.

5.2 Measurement apparatus for R_s

Figure 2 shows a schematic of a typical measurement apparatus (closed type resonator) for the R_s of ~~superconductor~~ HTS films deposited on a substrate with a flat surface. The upper ~~superconductor~~ HTS film is pressed down by a spring, which is made of phosphor bronze. The plate type spring ~~is recommended to~~ should be used for the improvement of measurement ~~accuracy uncertainty~~. This type of spring reduces the friction between the spring and the other part of the apparatus, and allows the smooth movement of superconductor films due to the thermal expansion of the dielectric rod. In order to minimize the measurement ~~error uncertainty~~, the sapphire rod and the copper ring shall be ~~set in coaxial~~ arranged coaxially.

Two semi-rigid cables for measuring transmission characteristics of the resonator shall be attached on both sides of the resonator in an axial symmetrical position ($\phi = 0$ and π , where ϕ is the rotational angle around the central axis of the sapphire rod). Each of the two semi-rigid cables shall have a small loop at the ends. The plane of the loop shall be set parallel to that of the superconductor films in order to suppress the unwanted Transverse Magnetic Wave Modes (TM_{mn0} modes). The coupling loops shall be carefully checked for cracks in the spot weld joint that may have developed upon repeated thermal cycling. These cables can move right and left to adjust the insertion attenuation (IA). In this adjustment, coupling of unwanted cavity modes to the interested dielectric resonance mode shall be suppressed. Unwanted, parasitic coupling to the other modes reduces the high Q -value of the Transverse Electro-Magnetic Mode (TE mode) resonator. For suppressing the parasitic coupling, special attention shall be paid to

designing high- Q resonators. Two other types of resonators along with the closed type shown in Figure 2 can be used. They are explained in A.4.



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Figure 2 – Typical measurement apparatus for R_s

A reference line made of a semi-rigid cable shall be used to measure the full transmission power level, i.e. the reference level. This cable has a length equal to the sum of the two cables of the measurement apparatus. Semi-rigid cable with an outer diameter of 1,20 mm is recommended.

In order to minimize the measurement ~~error~~ uncertainty, two superconductor films shall be set to be parallel to each other. To ensure that the two superconductor films remain in tight contact with the ends of the sapphire rod, without any air gap, both of the surfaces of the films and the ends of the rod shall be cleaned carefully.

5.3 Dielectric rods

Two dielectric rods with the same relative permittivity, ϵ' , and loss factor, $\tan \delta$, preferably cut from one cylindrical dielectric rod, are required. These two rods, standard dielectric rods, shall have the same diameter but different heights: one ~~has a~~ shall have height three times longer than the other.

It is preferable to use ~~standard~~ dielectric rods with low $\tan \delta$ to achieve the requisite measurement ~~accuracy~~ uncertainty on R_s . Recommended dielectric rods are single-crystal sapphire rods with $\tan \delta$ less than ~~10⁻⁶~~ 2×10^{-7} at 77 K. Specifications on the sapphire rods are described in 7.1. In order to minimize the measurement error in R_s of the superconductor films, both ends of the sapphire rods shall be polished parallel to each other and perpendicular to the axis. Specifications for the sapphire rods are given in Clause 7.

The diameter and the heights of the standard sapphire rods shall be carefully designed so that the TE_{011} and TE_{013} modes do not couple to other TM, HE and EH modes, since the coupling between TE mode and other modes causes the degradation of unloaded Q . A design guideline for the standard sapphire rods is described in Clause A.5. Table 1 shows typical examples of dimensions of the standard sapphire rods for 12 GHz, 18 GHz, and 22 GHz resonance. ~~At higher frequencies the unloaded Q value will be lower, which makes the measurement easier, and the error will be lower.~~ In the R_s measurement at 22 GHz, the required film diameter can be set to 20 mm, and the measured Q_L is small, therefore the effect of the dielectric loss of sapphire rod can be reduced.

Table 1 – Typical dimensions of pairs of ~~standard~~ single-crystal sapphire rods for 12 GHz, 18 GHz and 22 GHz

Frequency GHz		Diameter	Height
		d mm	h mm
12	Short rod (TE_{011} resonator)	11,4	5,7
	Long rod (TE_{013} resonator)	11,4	17,1
18	Short rod (TE_{011} resonator)	7,6	3,8
	Long rod (TE_{013} resonator)	7,6	11,4
22	Short rod (TE_{011} resonator)	6,2	3,1
	Long rod (TE_{013} resonator)	6,2	9,3

Frequency GHz		Diameter, d mm	Height, h mm
12	Short rod (TE_{011} resonator)	11,40 ± 0,05	5,70 ± 0,05
	Long rod (TE_{013} resonator)	11,40 ± 0,05	17,10 ± 0,05
18	Short rod (TE_{011} resonator)	7,60 ± 0,05	3,80 ± 0,05
	Long rod (TE_{013} resonator)	7,60 ± 0,05	11,40 ± 0,05
22	Short rod (TE_{011} resonator)	6,20 ± 0,05	3,10 ± 0,05
	Long rod (TE_{013} resonator)	6,20 ± 0,05	9,30 ± 0,05

6 Measurement procedure

6.1 Specimen preparation

From ~~error~~ uncertainty estimation, the film diameter shall be about three times larger than that of the sapphire rods. In this configuration, the ~~reduction~~ increase in ~~precision~~ uncertainty of R_s due to the different radiation losses between TE_{011} and TE_{013} mode can be considered negligible, given the target ~~precision~~ relative combined standard uncertainty of 20 %. The film thickness shall be about ~~three~~ two times larger than the ~~London~~ magnetic-field penetration depth value at ~~each~~ the maximum temperature in the measurement temperature range. If the film thickness is much less than ~~three~~ two times the ~~London~~ magnetic-field penetration depth, the measured R_s should mean the effective surface resistance.

Table 2 shows dimensions of the superconductor films recommended for the standard sapphire rods of 12 GHz, 18 GHz, and 22 GHz.

Table 2 – Dimensions of superconductor film for 12 GHz, 18 GHz, and 22 GHz

Standard dielectric rod		Superconductor film	
Frequency	Diameter	Diameter	Thickness
GHz	d mm	d' mm	μm
12	11,4	>35	$\approx 0,5$
18	7,6	>25	$\approx 0,5$
22	6,2	>20	$\approx 0,5$

c-cut single-crystal sapphire rod		Superconductor film	
Frequency	Diameter, d	Diameter, d'	Thickness
GHz	mm	mm	μm
12	$11,40 \pm 0,05$	> 35	$\approx 0,5$
18	$7,60 \pm 0,05$	> 25	$\approx 0,5$
22	$6,20 \pm 0,05$	> 20	$\approx 0,5$

In case of using closed type resonators, the dimensions of the superconductor films shall also be designed taking into account the dimension of the copper cylinder between the superconductor films. A design guideline for the dimension of the copper cylinder of the closed type resonator is described in Clause A.6.

6.2 Set-up

Set up the measurement equipment as shown in Figure 1. All of the measurement apparatus, standard sapphire rods, and superconductor films shall be kept in a clean and dry state as high humidity may degrade the unloaded Q -value. The specimen and the measurement apparatus shall be fixed in a temperature-controlled cryocooler. The specimen chamber shall be ~~generally~~ totally evacuated. The temperatures of the superconductor films and standard sapphire rods shall be measured by a diode thermometer, or a thermocouple. The temperatures of the upper and lower superconductor films and standard sapphire rods ~~must be kept as close as possible~~ shall not differ by more than 0,5 K. This can be achieved by covering the measurement apparatus with aluminium foil or filling the specimen chamber with helium gas.

6.3 Measurement of reference level

The level of full transmission power (reference level) shall be measured first. Fix the output power of the synthesized sweeper below 10 mW because the measurement ~~accuracy~~ **uncertainty** depends on the measuring signal level. Connect the reference line of semi-rigid cable between the input and output connectors. Then, measure the transmission power level over the entire measurement frequency and temperature range. The reference level can change several decibels when temperature of the apparatus is changed from room temperature to the lowest measurement temperature. Therefore, the temperature dependence of the reference level must be taken into account.

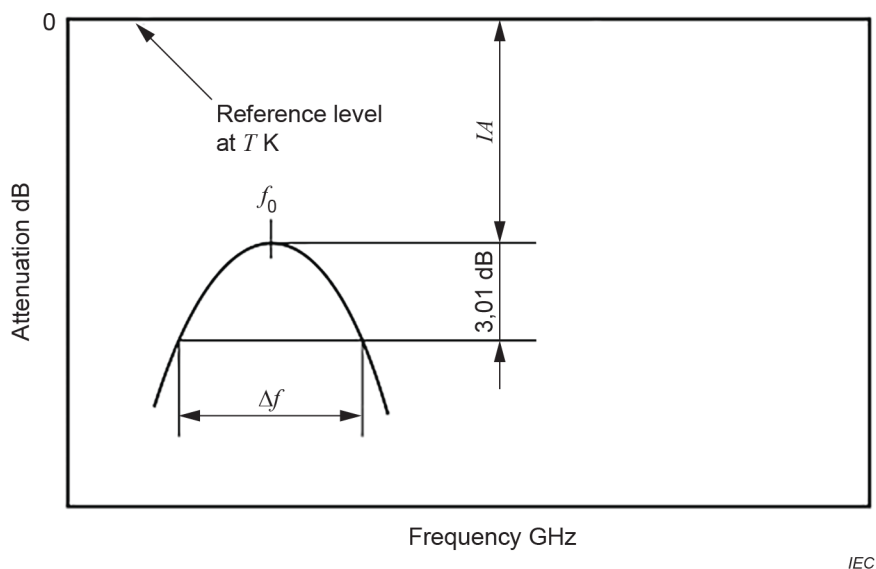


Figure 3 – Insertion attenuation, IA , resonant frequency, f_0 , and half power bandwidth, Δf , measured at T kelvin

6.4 Measurement of the frequency response of resonators

The temperature dependence of the R_s can be obtained through the measurements of resonant frequency, f_0 , and unloaded quality factor, Q_u , for TE_{011} and TE_{013} resonators, which shall be measured as follows.

- Connect the measurement apparatus between the input and output connectors (Figure 1). Insert the standard short sapphire rod near the centre of the lower superconductor film and fix the distance between the rod and each of the loops of the semi-rigid cables to be equal to each other, so that this transmission-type resonator can be under-coupled equally to both loops. Put down the upper superconductor film gently to touch the top face of the rod. Be careful not to damage the surface of the superconductor films by excessive pressure. Evacuate and cool down the specimen chamber below the critical temperature.
- Find the TE_{011} mode resonance peak of this resonator at a frequency nearly equal to the designed value of f_0 .
- Narrow the frequency span on the display so that only the resonance peak of TE_{011} mode can be shown (Figure 3). Confirm that the insertion attenuation, IA , of this mode is larger than 20 dB from the reference level, which depends strongly on the temperature.
- Measure the temperature dependence of f_0 and the half power band width Δf . The loaded **Q-value**, Q_L , of the TE_{011} resonator is given by

$$Q_L = \frac{f_0}{\Delta f}$$

- e) The unloaded Q -value, Q_u , shall be extracted from the Q_L by at least one of the two following techniques.

- 1) One technique for extracting the unloaded Q -value involves measuring the insertion attenuation IA . Q_u is given by

$$Q_u = \frac{Q_L}{1 - A_t}, A_t = 10^{-IA[\text{dB}]/20} \quad (2)$$

The IA of each temperature is determined as follows. In the R_s measurement system shown in Figure 2, a semi-rigid cable is connected instead of the measurement apparatus, and the measurement system is short-circuited to determine the reference level. Measure the reference level at each temperature and use it to determine IA at each temperature.

This technique of using insertion attenuation assumes that the coupling on both sides of the resonator is identical. The coupling loops are difficult to fabricate, the orientation of the loop is difficult to control, and any movement of the sapphire rod during measurement is not known. These assembly-dependent effects are also temperature dependent. This potential asymmetry in coupling can result in large errors in calculating the coupling factor if the coupling is strong ($IA < 10$ dB). If the coupling is weak enough ($IA > 20$ dB), asymmetry in the coupling becomes less important.

- 2) Another technique for extracting the unloaded Q -value involves measuring the reflection scattering parameters at the resonant frequency of both sides of the resonator.

$$Q_u = Q_L (1 + \beta_1 + \beta_2) \quad (3)$$

$$\beta_1 = \frac{1 - |S_{11}|}{|S_{11}| + |S_{22}|} \quad (4)$$

$$\beta_2 = \frac{1 - |S_{22}|}{|S_{11}| + |S_{22}|} \quad (5)$$

~~In the above equations~~ Equations (4) and (5), S_{11} and S_{22} are the reflection scattering parameters as shown in Figure 4, and are measured in linear units of power, not relative dB. β_1 and β_2 are the coupling coefficients.

This technique of using the reflection scattering parameters has two advantages. It does not require the additional step of calibration of the reference level and it gives a measurement of the coupling values for both sides of the resonator. This also has two disadvantages. It only works for a narrow band resonance (which is fortunately the case) and is limited by the dynamic range of the network analyzer in measuring the reflection coefficients [15] [16].

A combination of the two techniques is ~~an excellent "double" check and is therefore recommended~~ useful for a "double" check.

- f) The f_0 and Q_u measured for this short rod are denoted as f_{01} and Q_{u1} for TE_{011} mode resonance. By slowly changing the temperature of the cryocooler the temperature dependence of f_{01} and Q_{u1} shall be measured.
- g) After the temperature dependence measurement of f_{01} and Q_{u1} is finished, the measurement apparatus shall be heated up to room temperature.
- h) Then, replace the TE_{011} resonator in the apparatus with the TE_{013} resonator at room temperature, cool down the apparatus to a temperature lower than the critical temperature, and measure the temperature dependence of f_0 and Q_u of its TE_{013} resonance mode, denoted as f_{03} and Q_{u3} , in a similar way to the TE_{011} resonator case. When the length of

the sapphire rod of the TE₀₁₃ resonator is precisely three times longer than that of the TE₀₁₁ resonator, the f_{03} of the TE₀₁₃ resonator must coincide with f_{01} of the TE₀₁₁ resonator. If carefully designed, the difference between f_{01} and f_{03} is usually very small ($\sim 0,25\%$). We can ~~treat as $f_0 = f_{01} = f_{03}$ in the calculations of 6.5~~ consider that the f_0 in equation (9) can be replaced by f_{01} or f_{03} .

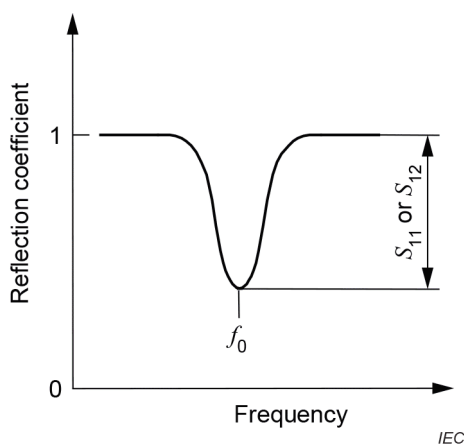


Figure 4 – Reflection scattering parameters (S_{11} and S_{22})

6.5 Determination of surface resistance of the superconductor and ε' and $\tan \delta$ of the standard sapphire rods

~~Calculate the temperature dependence of the surface resistance R_s of the superconductor films, and ε' and $\tan \delta$ of the standard sapphire rods using the temperature dependence of f_{01} , Q_{u1} , f_{03} , and Q_{u3} from Equations (6), (7) and (8).~~

Calculate the temperature dependence of the R_s of the superconductor films, and ε' and $\tan \delta$ of the standard sapphire rods using the temperature dependence of Q_{u1} and Q_{u3} . Q_{u1} and Q_{u3} can be calculated using equations (1) and (2). If the measured temperatures for Q_{u3} appear to be slightly different from those for Q_{u1} , interpolation can be used for preparing adjusted temperature values for the former at the same temperatures for the later. Equations (6) to (12) are established in the open type cavity, but when the superconductor film size is three times larger than sapphire rod diameter, these equations can be used for closed type cavity.

$$R_s = \frac{30\pi^2 \times 3}{(3-1)} \left(\frac{2h_0}{\lambda_0} \right)^3 \frac{\varepsilon' + W}{1+W} \left(\frac{1}{Q_{u1}} - \frac{1}{Q_{u3}} \right) \quad (6)$$

$$\varepsilon' = \left(\frac{\lambda_0}{\pi d} \right)^2 (u^2 + v^2) + 1 \quad (7)$$

$$\tan \delta = \frac{1 + \frac{W}{\varepsilon'}}{(3-1)} \left(\frac{3}{Q_{u3}} - \frac{1}{Q_{u1}} \right) \quad (8)$$

where

$$\lambda_0 = \frac{c}{f_0} \quad (9)$$

$$W = \frac{J_1^2(u) K_0(v) K_2(v) - K_1^2(v)}{K_1^2(v) J_1^2(u) - J_0(u) J_2(u)} \quad (10)$$

$$v^2 = \left(\frac{\pi d}{\lambda_0} \right)^2 \left[\left(\frac{\lambda_0}{2h_0} \right)^2 - 1 \right] \quad (11)$$

$$v^2 = \left(\frac{\pi d}{\lambda_0} \right)^2 \left[\left(\frac{3\lambda_0}{2h_0} \right)^2 - 1 \right] \quad (11)$$

$$u \frac{J_0(u)}{J_1(u)} = -v \frac{K_0(v)}{K_1(v)} \quad (12)$$

In the equations, λ_0 is the free space resonant wavelength, c is the ~~velocity~~ speed of light in vacuum ($c = 2,997\,9 \times 10^8$ m/s), h_0 is the height of the short standard dielectric rod. The value u^2 is given by the transcendental Equation (12) using the value of v^2 , where $J_n(u)$ is the Bessel function of the first kind, and $K_n(v)$ is the modified Bessel function of the second kind. The derivations of the equations are described in Clause A.3.

Generally the thermal expansion coefficient of the rods must be known to determine the temperature dependence of their sizes. However, the thermal expansion effect of the sapphire rods can be neglected for the target ~~precision of the~~ relative standard uncertainty of R_s (20 %).

It is noted that the measured R_s means the effective surface resistance if the film thickness is ~~not much larger~~ less than two times the temperature-dependent penetration depth.

7 ~~Precision and accuracy~~ Uncertainty of the test method

7.1 Surface resistance

The surface resistance (R_s) shall be determined from the Q -value measured with a dielectric resonator technique.

A vector network analyser as specified in Table 3 shall be used to record the frequency dependence of attenuation. The resulting record shall allow the determination of Q to a relative uncertainty of 10^{-2} .

Table 3 – Specifications for vector network analyzer

Dynamic range of S_{21}	above 60 dB
Frequency resolution	below 1 Hz
Attenuation uncertainty	below 0,1 dB
Input power limitation	below 10 dBm

The dielectric resonators shall be provided with two dielectric rods with low $\tan \delta$ of less than 10^{-6} 2×10^{-7} at 77 K and a radius less than 1/3 of the superconducting specimen's radius. The best candidate for the rods is sapphire as specified in Table 4. Term definitions in Table 4 are shown in Figure 5.

Table 4 – Specifications for sapphire rods

Diameter	$\pm 0,05$ mm
Height	$\pm 0,05$ mm
Flatness	below 0,005 mm
Surface roughness	top and bottom surface: below 10 nm RMS cylindrical surface: below 0,001 mm RMS
Perpendicularity	within 0,1 degree
Axis	parallel to c-axis within 0,3°

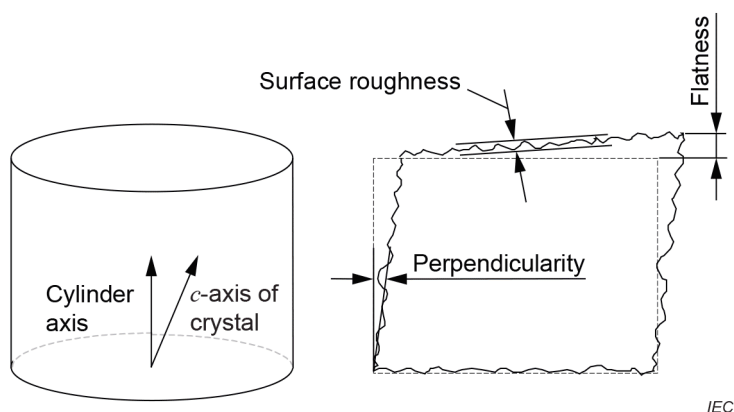


Figure 5 – Term definitions in Table 4

The technique as described assumes that single- and triple-height sapphire rods can be fabricated with the same $\tan \delta$. However, the variation of the $\tan \delta$ between nominally identical rods, cut from the same boule and polished by the same technique, may be as large as two orders of magnitude. To date, the smallest variation in $\tan \delta$ between nominally identical sapphire rods has been a factor of four [9]. Therefore, the uncertainty in the measured $\tan \delta$ is large. The variation of $\tan \delta$ of the present sapphire rod causes an additional uncertainty up to at least 10 % in the surface resistance measurement. This limits the target ~~precision~~ uncertainty of the present technique at ~~20~~ 5 %. If reproducibility of sapphire rods is improved, or a selection method for standard sapphire rods is established, a target ~~precision~~ uncertainty can be improved.

7.2 Temperature

The measurement apparatus is cooled down to the specified temperature by any means during testing. An easy choice would be to immerse the apparatus into a liquid cryogen. This technique is quick and simple and yields a known and stable temperature. Unfortunately, most HTS materials are damaged by the condensation of moisture that occurs when removing the sample from the cryogen. In addition, uncertainties generated by the presence of a gas/liquid mixture within the cavity, and the inability to measure R_s as a function of temperature support the use of other cooling methods. These limitations can be circumvented by the immersion of a vacuum can into a liquid cryogen. If the vacuum can is backfilled with gas, then rapid cooling and uniform temperatures occur. If heaters are attached to the apparatus, then the temperature dependence of the HTS material can be measured. A third and equally good choice is the use of a cryocooler. In this case, the resonator is under vacuum and cooled by conduction through the metallic

package. ~~Care must be taken to avoid~~ Temperature gradients with the apparatus shall be avoided.

A cryostat shall be provided with the necessary environment for measuring R_s and the specimen shall be measured while in a stable and isothermal state. The specimen temperature is assumed to be the same as the sample holder temperature. The holder temperature shall be reported to an accuracy of $\pm 0,5$ K, measured by means of an appropriate temperature sensor.

The difference between the specimen temperature and the holder temperature shall be minimized by using shields with good thermal conductivity.

7.3 Specimen and holder support structure

The support structure shall provide adequate support for the specimen. ~~It is imperative that~~ The two films shall be parallel and mechanically stable throughout the measurement, especially in a cryocooler and over a wide temperature range. The parallelism of the two films can be achieved by using a sapphire rod satisfying the specifications described in Table 4.

7.4 Specimen protection

Condensation of moisture and scratching of the film deteriorate superconducting properties. Some protection measures should be provided for the specimens. Polytetrafluoroethylene (PTFE) or polymethylmethacrylate (PMMA) coating does not affect the measurements, thus they can be used for protection. A coating material thickness of less than several micrometers is recommended.

7.5 Uncertainty of surface resistance measured by standard two-resonator method

For the dielectric resonance method, $\tan \delta$ of the TE_{011} mode and TE_{013} mode sapphire rods are assumed to be absolutely equal. If $\tan \delta$ of two sapphire rods is different, it is not possible to calculate the surface resistance using Equation (6). The effect of $\tan \delta$ deviation on surface resistance was checked (see Annex B). Uncertainty in the microwave surface resistance of superconductors by dielectric resonator method depends on size, dielectric constant, $\tan \delta$ of sapphire rods and uncertainty in measured Q of the resonator. The relative combined standard uncertainty is less than 20 % (see Annex B)

8 Test report

8.1 Identification of test specimen

The test specimen shall be identified, if possible, by the following:

- name of the manufacturer of the specimen;
- classification and/or symbol;
- lot number;
- chemical composition of the thin film and the substrate;
- thickness and roughness of the thin film;
- manufacturing process technique.

8.2 Report of R_s values

The R_s values at the measured frequency, and that scaled to 10 GHz, together with their corresponding f_{01} , f_{03} , Q_{u1} , Q_{u3} , IA and/or (β_1, β_2) , ϵ' and $\tan \delta$ values, and their temperature dependence shall be reported.

8.3 Report of test conditions

The following test conditions shall be reported:

- a) test frequency and resolution of frequency;
- b) test maximum RF power;
- c) test temperature, accuracy of temperature and temperature difference in two plates;
- d) sample history with temperature variation.

Annex A (informative)

Additional information relating to Clauses 1 to 8

A.1 Scope

A.1.1 General

The establishment of the standard measurement method is needed to evaluate film quality of ~~high-Tc superconductor (HTS)~~ films having low surface resistance, R_s , values such as 0,1 mΩ at 10 GHz. Several resonance methods as shown in Figure A.1 have been proposed so far to measure them in microwave and millimetre wave range. These resonator structures are grouped into six types (see A.1.2 to A.1.7).

A.1.2 Cylindrical cavity method ~~[4]²⁾~~ [10] [17]

Figure A.1 a) shows a cavity structure using the TE_{011} mode, which is constructed from a copper cylinder and two HTS films. In the microwave range below 30 GHz, the R_s measurement precision of this method is rather low, because the R_s value of the copper cylinder is 100 times higher than that of HTS films. This method is suitable in the millimetre wave region, because the R_s values increase with f^2 of HTS films and $f^{1/2}$ for copper.

A.1.3 Parallel-plates resonator method ~~[2]~~ [18] [19]

Figure A.1 b) shows a structure of a parallel-plates resonator using the TM_{nm0} mode, which is constructed by inserting a thin low loss dielectric spacer between two rectangular HTS films. This method offers the possibility of measuring extremely small R_s values, but from the viewpoint of the absolute estimation of R_s there are some problems, such as low measurement precision of thickness and $\tan \delta$ of the dielectric spacer, uncertain estimations of radiation loss and an air-gap effect between the plates and the spacer, and critical excitation technique of the resonance modes.

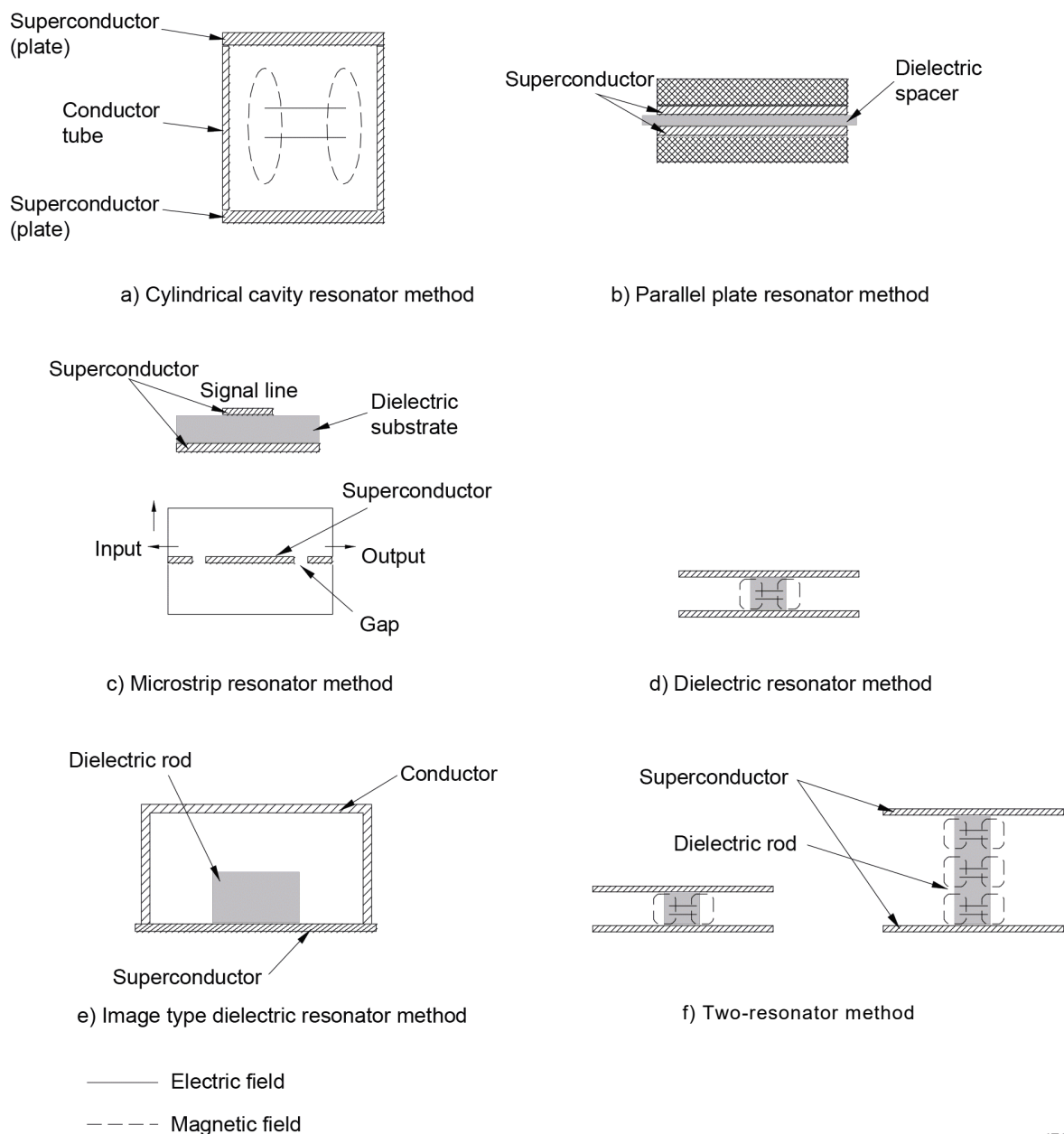
A.1.4 Microstrip-line resonance method ~~[4], [3]~~ [20] [21]

Figure A.1 ~~(c)~~ shows a structure of a microstrip-line resonator using the ~~TEM_n~~ TE_{0mp} mode, which is fabricated by a pattern fabrication process and is close to the real microwave passive devices. However, this method is not suitable for the HTS film estimation because it includes an uncertain influence of the pattern fabrication process.

A.1.5 Dielectric resonator method ~~[4], [5] [7]~~ [22] [23] [24] [25]

Figure A.1 d) shows a structure of a dielectric resonator using a TE_{011} mode, where a low loss sapphire rod is placed between two parallel HTS films. For the TE_{0mp} mode, we can eliminate the air-gap effects, because the normal component of electric field does not exist on the HTS films. In this method the R_s values of the HTS films have been measured by ignoring the effect of $\tan \delta$ on the assumption of $\tan \delta < 1 \times 10^{-8}$ for single-crystal sapphire ~~[4]~~ [24]. However, the $\tan \delta$ values measured for sapphire at 10 GHz and near 50 K, as is well known, take values between 10^{-6} to 10^{-8} due to quantity of lattice defect. Therefore, preparation of very-low-loss sapphire rods with $\tan \delta < 1 \times 10^{-8} < 2 \times 10^{-7}$ (at 80 K and at 10 GHz) is essential for this method.

²⁾—Numbers in brackets in this annex refer to the reference documents in Clause A.9



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Figure A.1 – Schematic configuration of several measurement methods for the surface resistance

A.1.6 Image-type dielectric resonator method ~~[8]~~ [26] [27]

Figure A.1 e) shows a structure of an image-type dielectric resonator using the $TE_{01\delta}$ mode. This structure is available to measure only one HTS film. However, preparation of very-low-loss sapphire rods is essential also for this method, because the R_s is measured by ignoring the effect of $\tan \delta$.

~~In addition, complicated and tedious numerical calculations are needed to determine the R_s values from the measured resonant frequency and unloaded Q .~~

In an improved image-type sapphire rod resonator method, R_s can be obtained with simple calibration process. In this method, the exact R_s of a single HTS film can be obtained by one measurement under the 20 % limitation of variation coefficient.

A.1.7 Two-resonator method ~~[9],[10]~~ [28] [29]

In this method, two sapphire rod resonators with the same $\tan \delta$ values are used; one is a TE_{011} mode resonator and the other is a TE_{013} mode resonator, as shown in Figure A.1 f). Similarly to the dielectric resonator method, the air-gap effects can be eliminated in this method. The $\tan \delta$ value of the sapphire rods and the R_s value of the HTS films can be determined separately from the resonant frequencies and unloaded ~~Q 's~~ Q -values measured for these resonators by using simple formulas derived from the rigorous analysis based on the mode matching method. Thus this method can eliminate the ambiguity of the $\tan \delta$ affected to the R_s measurements. Actually, the results of R_s measured for six YBCO films with $R_s = 0,1 \text{ m}\Omega$ at 12 GHz have verified that the measurement ~~precision~~ uncertainty of ~~10~~ 20 % is realized by this method ~~[10]~~ [22]. However, if there is a difference between two $\tan \delta$ values ($\Delta \tan \delta$), the ~~errors of~~ uncertainty in R_s due to $\Delta \tan \delta$ must be taken into account. This effect ~~will~~ can be estimated from results of the inter-laboratory round robin test of the same HTS films by this method. Recently, the R_s of YBCO film was measured three times and its deviation was less than 20 % (see Figure A.11).

As a result of comparisons among the six methods described above, the two-resonator method is recommended as the standard measurement method of R_s of HTS films, due to the following two advantages:

- absolute measurement of R_s is possible;
- the numerical treatment is simple and easy.

A.2 Requirements

Higher measurement frequencies are desirable for determination of lower R_s because R_s of superconductor films increases with the frequency according to the f^2 rule. The size of the resonators also can be made smaller by using higher measurement frequencies. However, it becomes difficult to set up a microwave measurement system as the measurement frequency becomes higher.

For the measurement temperatures of below ~~30~~ 10 K or above 80 K, this test method is sufficiently applicable, where some new cooling techniques may be required.

A.3 Theory and calculation equations

Figure A.2 shows the configuration of the ~~TE_{0mp}~~ TE_{0mn} mode resonator, which is used to eliminate the air-gap effects. A cylindrical dielectric rod with diameter, d , and height, h , is short-circuited at both ends by surfaces of two parallel superconductor films deposited on dielectric substrates with diameter, d' , thus constituting a resonator. These superconductor films are required to have the same value of R_s . The value of R_s is calculated from the measured resonant frequency f_0 and unloaded quality factor Q_u for the ~~TE_{0mp}~~ TE_{0mn} resonance mode. When the two superconductor films have different values of R_s , the measured R_s value corresponds to the average value of these two films.

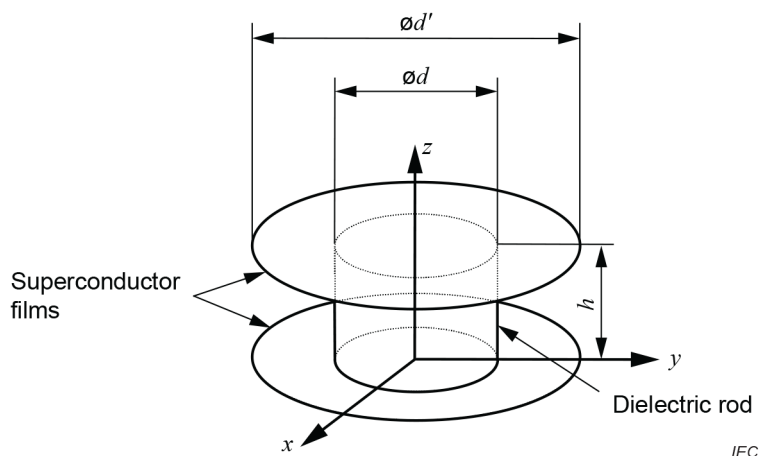


Figure A.2 – Configuration of a cylindrical dielectric rod resonator short-circuited at both ends by two parallel superconductor films deposited on dielectric substrates

The value of R_s is given by [25]

$$R_s = \frac{1}{B} \left(\frac{A}{Q_u} - \tan \delta \right) \quad (\text{A.1})$$

where

$$A = 1 + \frac{W}{\epsilon'} \quad (\text{A.2})$$

$$B = p^2 \left(\frac{\lambda_0}{2h} \right)^3 \frac{1+W}{30\pi^2 \epsilon'}, p = 1, 2, \dots, \quad (\text{A.3})$$

$$\lambda_0 = \frac{c}{f_0} \quad (\text{A.4})$$

$$W = \frac{J_1^2(u) K_0(v) K_2(v) - K_1^2(v)}{K_1^2(v) J_1^2(u) - J_0(u) J_2(u)} \quad (\text{A.5})$$

$$v^2 = \left(\frac{\pi d}{\lambda_0} \right)^2 \left[\left(\frac{p \lambda_0}{2h} \right)^2 - 1 \right] \quad (\text{A.6})$$

$$v^2 = \left(\frac{\pi d}{\lambda_0} \right)^2 \left[\left(\frac{3 \lambda_0}{2h} \right)^2 - 1 \right] \quad (\text{A.6})$$

$$u \frac{J_0(u)}{J_1(u)} = -v \frac{K_0(v)}{K_1(v)} \quad (\text{A.7})$$

In Equations (A.1) and (A.2), ε' and $\tan \delta$ are the relative permittivity and the loss factor of the dielectric rod, respectively. In Equations (A.3) and (A.4), λ_0 is the free space resonant wavelength, and c is the ~~velocity~~ speed of light in vacuum ($c = 2,9979 \times 10^8$ m/s). The function W/ε' equals the ratio of electric-field energy stored outside to that stored inside the dielectric rod. If all of the electric field is concentrated inside the dielectric rod, the value W equals zero.

The value u^2 is given by the transcendental Equation (A.7) using the value of v^2 , where $J_n(u)$ is the Bessel function of the first kind and $K_n(v)$ is the modified Bessel function of the second kind, ~~respectively~~. For any value of v , the m -th solution u exists between u_{0m} and u_{1m} , where $J_0(u_{0m}) = 0$ and $J_1(u_{1m}) = 0$. The first solution ($m = 1$), which is used for easy mode identification, is shown in Figure A.3 by curve (A). The computed result of the W - v relation for $m = 1$ of TE_{0mp} resonance mode is shown in Figure A.3 by curve (B).

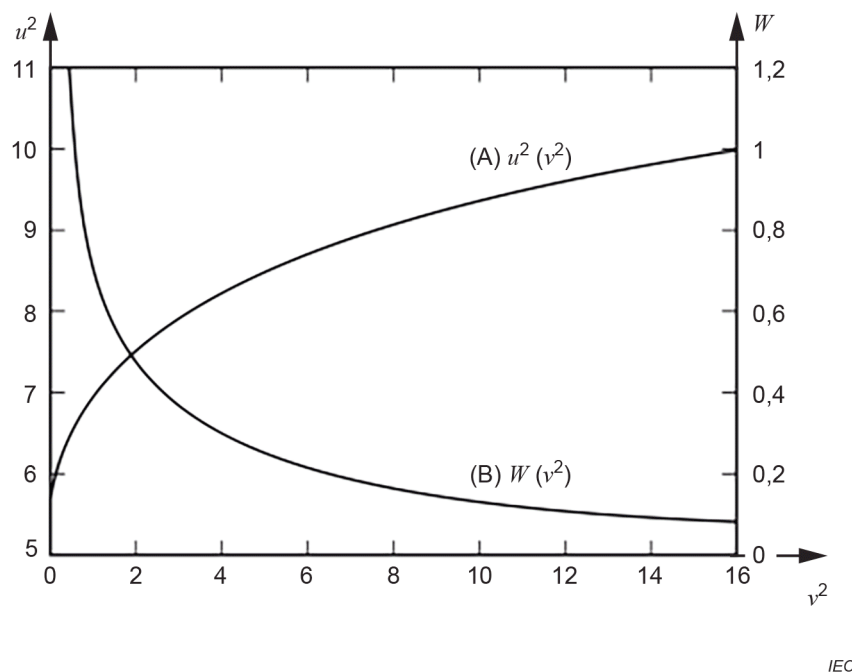


Figure A.3 – Computed results of the u - v and W - v relations for TE_{01p} mode

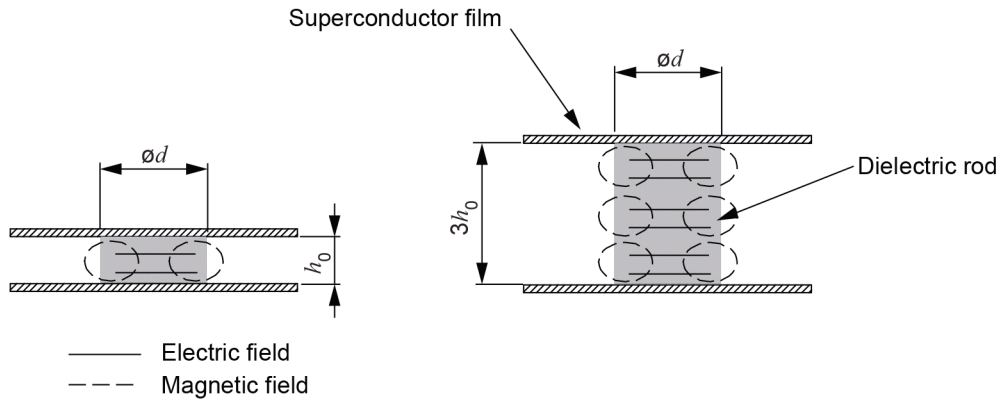
The value of ε' is given by

$$\varepsilon' = \left(\frac{\lambda_0}{\pi d} \right)^2 (u^2 + v^2) + 1 \quad (\text{A.8})$$

using the value of v^2 and u^2 .

In the two-resonator method, a pair of dielectric rods, which are called "standard dielectric rods", are used. These two rods have the same diameter but have different heights. The rod heights are such that one rod is p times the height of the other; p is commonly set equal to three. They are required to have the same values of ε' and $\tan \delta$.

Figure A.4 shows the configuration of the standard dielectric rods in the case of $p = 3$. To avoid confusion, the height of the short standard dielectric rod is denoted by h_0 . Each resonator is called "TE₀₁₁ resonator" and "TE₀₁₃ resonator", respectively. The same superconductor films are used in these resonators. The values of f_0 and Q_u for the TE₀₁₁ mode are measured using the TE₀₁₁ resonator, and those for the TE_{01p} mode are measured using the TE_{01p} resonator. We denote the f_0 and Q_u for each resonator by using the subscripts 1 and p , respectively: f_{01} and Q_{u1} for TE₀₁₁ resonator, and f_{0p} and Q_{up} for TE_{01p} resonator.



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Figure A.4 – Configuration of standard dielectric rods for measurement of R_s and $\tan \delta$

The value of $\tan \delta$ is given from the measured values of Q_u . When the TE_{01p} resonator is precisely p times longer than the TE₀₁₁ resonator, f_{0p} coincides with f_{01} . However, Q_{up} is higher than Q_{u1} according to the different magnitude of the electric field energy stored in the two resonators. Because both dielectric rods are short-circuited at both ends by the same superconductor films, Equation (A.1) yields

$$\tan \delta = \frac{A}{(p-1)} \left(\frac{p}{Q_{up}} - \frac{1}{Q_{u1}} \right) \quad (\text{A.9})$$

$$\tan \delta = \frac{A}{(3-1)} \left(\frac{3}{Q_{up}} - \frac{1}{Q_{u1}} \right) \quad (\text{A.9})$$

As an alternative method, the value of R_s of superconductor films can be directly measured by

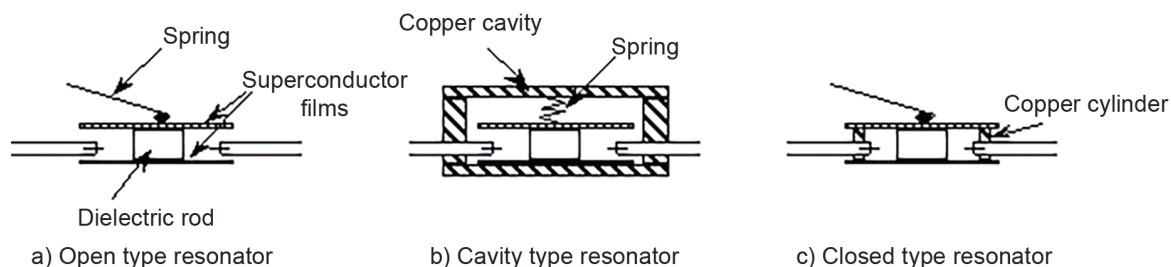
$$R_s = \frac{30\pi^2 p}{(p-1)} \left(\frac{2h_0}{\lambda_0} \right)^3 \frac{\varepsilon' + W}{1+W} \left(\frac{1}{Q_{u1}} - \frac{1}{Q_{up}} \right) \quad (\text{A.10})$$

$$R_s = \frac{30\pi^2 \times 3}{(3-1)} \left(\frac{2h_0}{\lambda_0} \right)^3 \frac{\varepsilon' + W}{1+W} \left(\frac{1}{Q_{u1}} - \frac{1}{Q_{up}} \right) \quad (\text{A.10})$$

where Equation (A.10) is derived by substituting Equation (A.9) into Equation (A.1).

A.4 Apparatus

Unwanted parasitic coupling to the other mode reduces the high Q -value of the TE mode resonator. For suppressing the parasitic coupling, special attention is paid to design high Q -value resonators. Three types of resonators are proposed and shown in Figure A.5.



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Figure A.5 – Three types of dielectric resonators

- Open type resonator: a low loss dielectric rod is placed between two parallel superconductor films. Two semi-rigid cables for the RF input and output magnetic dipole coupling are attached on both sides of the resonator. In this configuration, the vertical position of the coupling cables should be carefully designed so as to prevent the radiation loss propagating along the coupling cables, which degrades the high Q -value of the TE_{0mp} mode and causes the increase in error for the R_s measurements. The coupling cables should be set as close as possible to the lower superconductor film.
- Cavity type resonator: the open type resonator shown in a) is placed in a conductor (copper) cavity.
- Closed type resonator: conductor (copper) cylinder is put between the superconductor films. In this configuration, the radiation loss along the coupling cable is strongly blocked by the copper cylinder. The vertical position of the coupling cables (z) at the highest z -axis magnetic field component (H_z), i.e. $z = h/2$, is recommended.

The measuring apparatus on the cryocooler is protected from mechanical and thermal disturbances, and installed in an X-Y and/or Z-axial manipulator for adjusting sample positions in the range of approximately ± 1 mm.

A loop length of the antenna is designed on the basis of the quarter-wavelength rule for achieving the maximum measuring sensitivity.

A.5 Dimensions of the standard sapphire rods

Figure A.6 and Figure A.7 show the mode charts for designing the TE_{011} and TE_{013} mode resonators, respectively, short-circuited at both ends by parallel superconductor films, in which a uniaxial-anisotropic characteristic of the relative permittivity of the sapphire rod is taken into consideration [14] [28]. ϵ_z is the relative permittivity in the direction of the c -axis, ϵ_r is that in the plane perpendicular to the c -axis, d is the diameter of the sapphire rod, h is the height of the sapphire rod, and λ_0 is the free space resonant wavelength. As shown in Figure A.6 and Figure A.7, TE_{013} resonance mode is apt to be affected by TM or HE mode in comparison with TE_{011} resonance mode. Since the coupling between TE mode and other modes causes the degradation of unloaded Q , the (d/h) of the sapphire rods shall be selected so as to avoid the unwanted couplings.

As shown in Figure A.7, the value $(d/h)^2$ for the TE_{013} resonator is selected from 0,24 to 0,46 in order that the TE_{013} resonance mode is not affected by the other modes. This value

$(d/h)^2 = 0,24$ to $0,46$ leads to a value of $(d/h)^2 = 2,2$ to $4,1$ for the TE_{011} resonator. The TE_{011} resonance mode does not also couple with other modes when the value $(d/h)^2$ is in the region of $2,2$ to $4,1$ as shown in Figure A.6.

As the resonant frequency of TE mode is a function of relative permittivity and dimensions of the sapphire rod, its diameter and height are selected so that the desired f_0 is obtained.

From the curve of TE_{011} mode in Figure A.6, the value of $\varepsilon_r (d/\lambda_0)^2$ can be determined for each $(d/h)^2$ value. When the value $(d/h)^2$ equals $4,0$, for example, the value of $\varepsilon_r (d/\lambda_0)^2$ equals $1,92$. Thus, the resonant frequency of TE_{011} mode for the sapphire rod with dimension of $(d/h)^2 = 4,0$ is calculated from Equation (A.11) by specifying d and ε_r of the sapphire rod:

$$\varepsilon_r (d / \lambda_0)^2 = \varepsilon_r (d \times f_0 / c)^2 = 1,92 \quad (A.11)$$

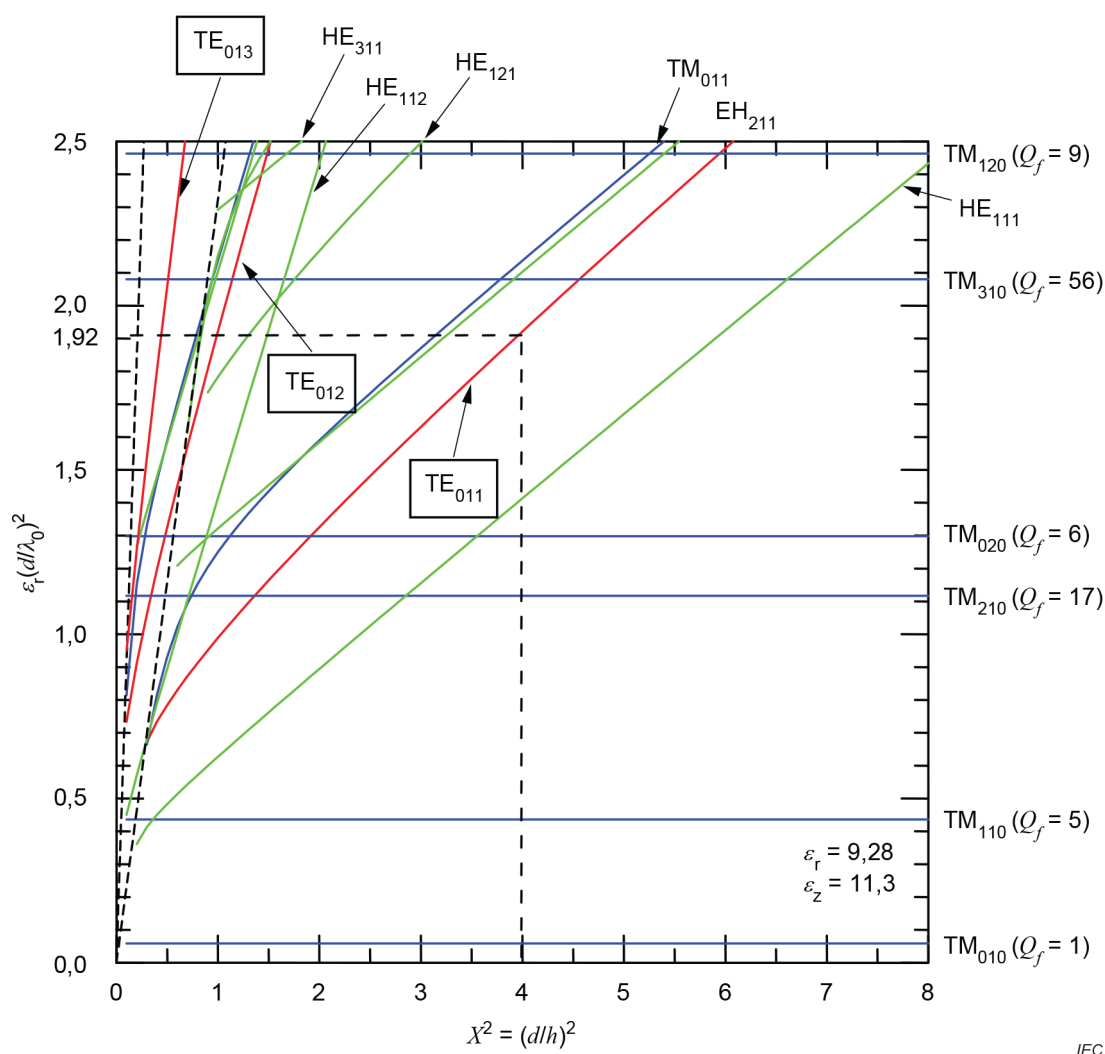


Figure A.6 – Mode chart to design TE_{011} resonator short-circuited at both ends by parallel superconductor films [14] [28]

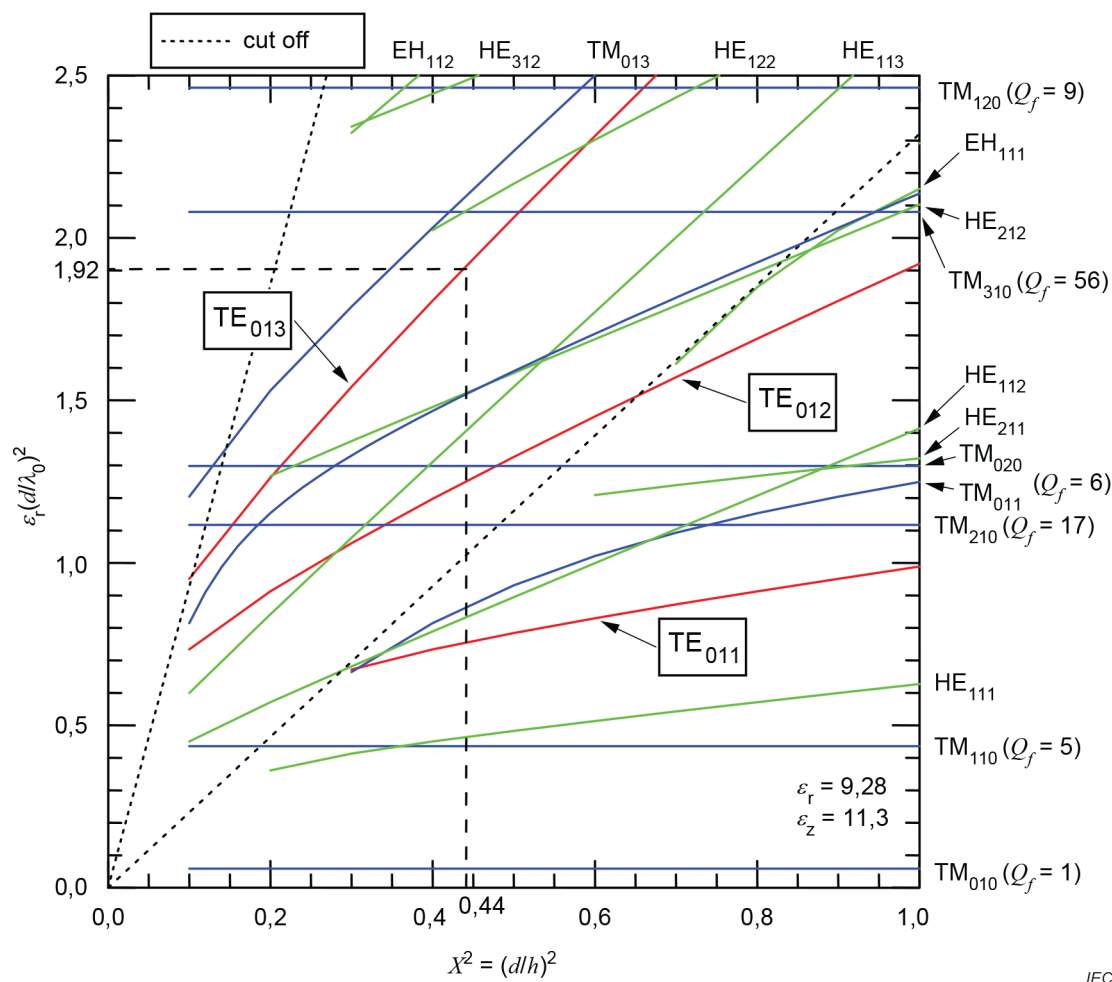


Figure A.7 – Mode chart to design TE₀₁₃ resonator short-circuited at both ends by parallel superconductor films [14] [28]

A.6 Dimension of the closed type resonator

In the closed type resonator, a copper cylinder is put between the superconductor films and a sapphire rod is placed in the center of the copper cylinder (Figure A.5 c)). The resonant frequency of each mode changes with the inner diameter of the copper cylinder, D . Therefore, D shall be selected so as to avoid the unwanted coupling with the other modes. In Figure A.8 and Figure A.9, mode charts of the closed type resonators are shown for the short sapphire rod ($(d/h)^2 = 4$) and the long sapphire rod ($(d/h)^2 = 0,44$), respectively, as a function of $S = D/d$ [14] [28]. The ranges of S to separate the TE₀₁₁ and the TE₀₁₃ modes simultaneously from the others are $S = 1,8$ to $2,8$, $3,8$ to $4,1$ and $4,8$ to $5,2$. The value of $S = D/d = 4$ is recommended because of the ease with which it can be treated.

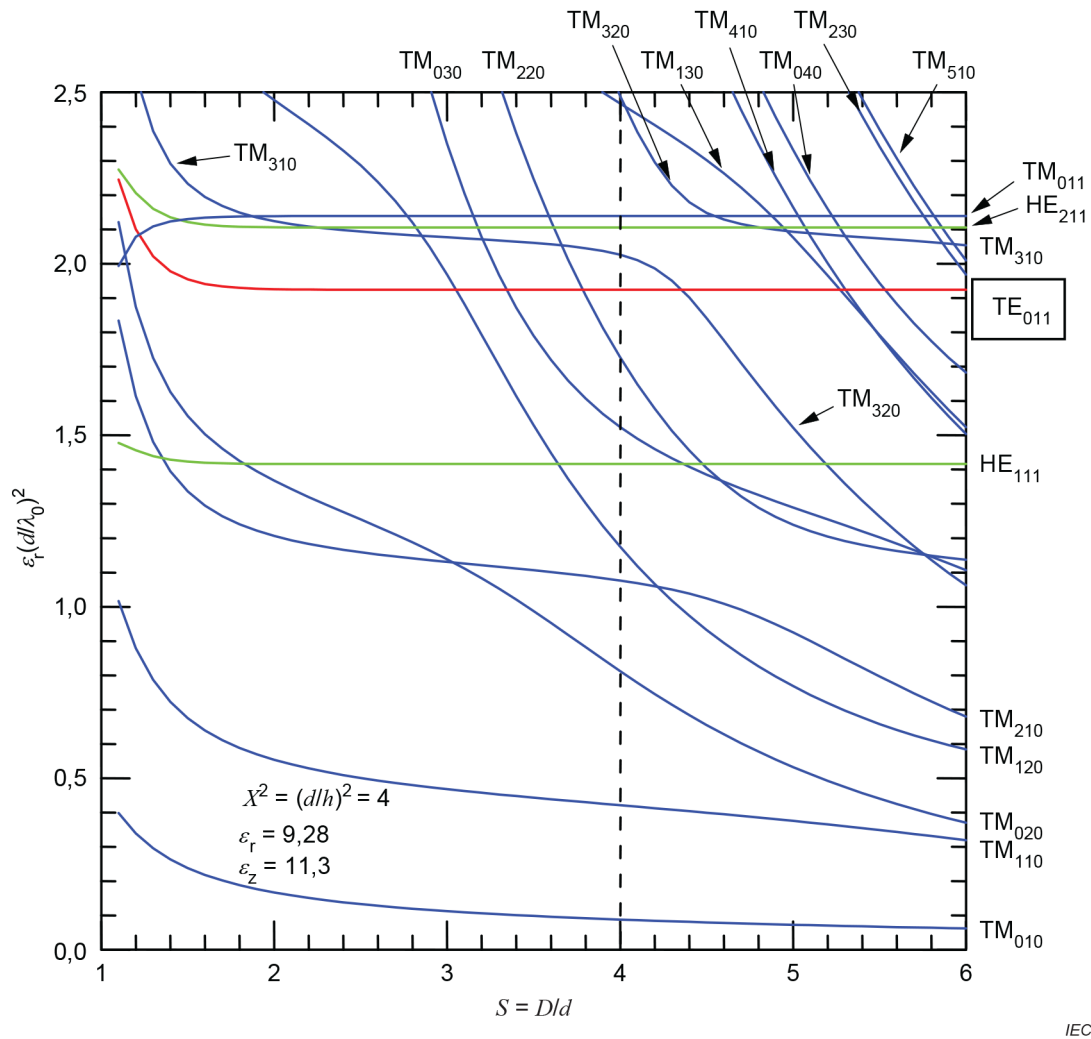


Figure A.8 – Mode chart for TE₀₁₁ closed-type resonator [28]

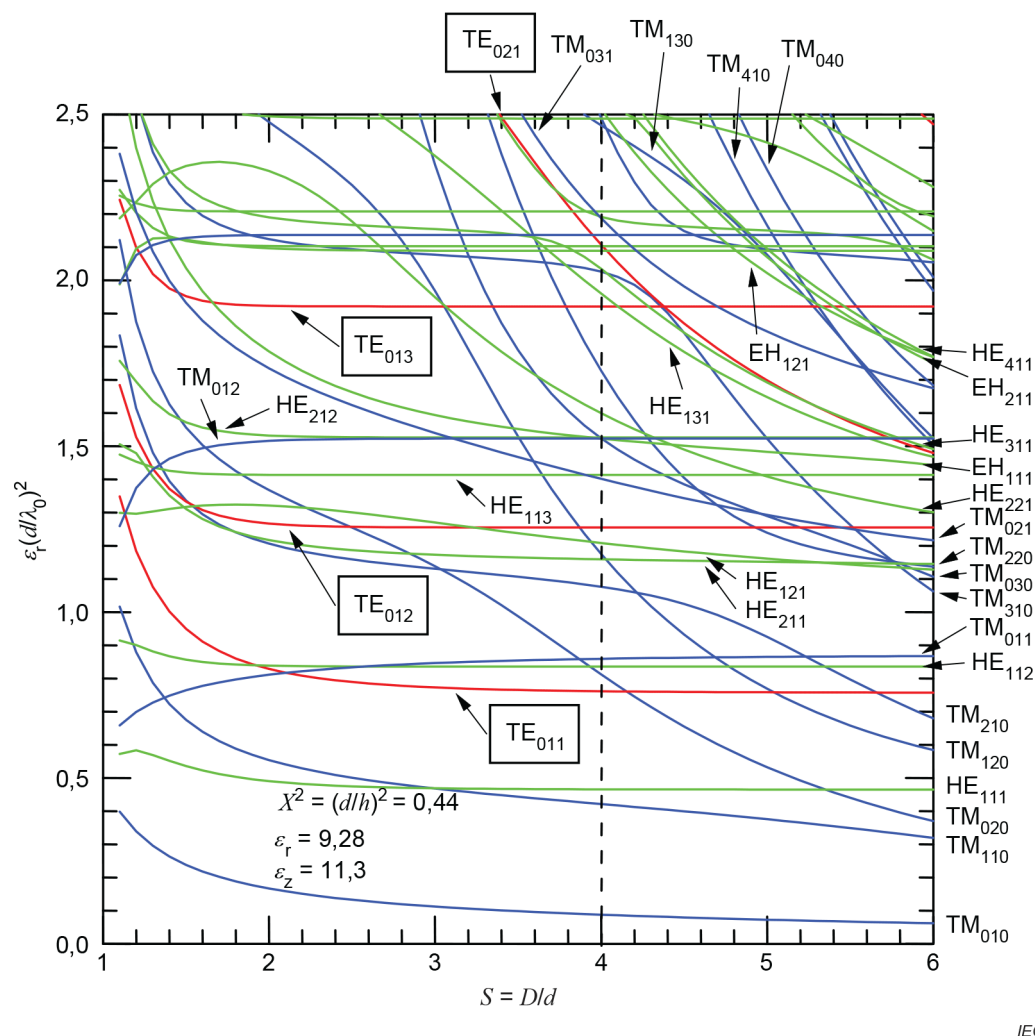


Figure A.9 – Mode chart for TE₀₁₃ closed-type resonator [28]

A.7 Precision and accuracy of the test method

~~Errors estimation [4][10], sensitivity, accuracy and repeatability for these methods can be obtained by the error analysis concerning R_s as well as by the round robin test.~~

A.7 Sapphire rod reproducibility

Knowing the value of $\tan \delta$ of the sapphire rod is a difficult problem. One approach based on the single- and triple-height rods involves the selection of a standard rod for each type. The first step is to compare a large number of sapphire rods by measuring them in a single HTS/sapphire resonator. The sapphire rod with the highest unloaded Q -value is the temporary label of standard. This procedure is repeated for a set of triple-height rods. The “standard” single- and triple-height rods are then used as described above to obtain $\tan \delta$. The electrical properties, ϵ' and $\tan \delta$, of other rods can then be calibrated by a direct comparison with these standard rods. Using a calibrated single-height rod, it is then possible to extract the R_s values of superconductor films under test.

A.8 Test results

Figure A.10 shows the temperature-dependent R_s of a YBCO film with a thickness of 500 nm and size of 25 mm square. The YBCO film was formed on MgO substrate by a co-evaporation method. The measurement was carried out at 21,8 GHz, which was converted to 10 GHz by the f^2 rule.

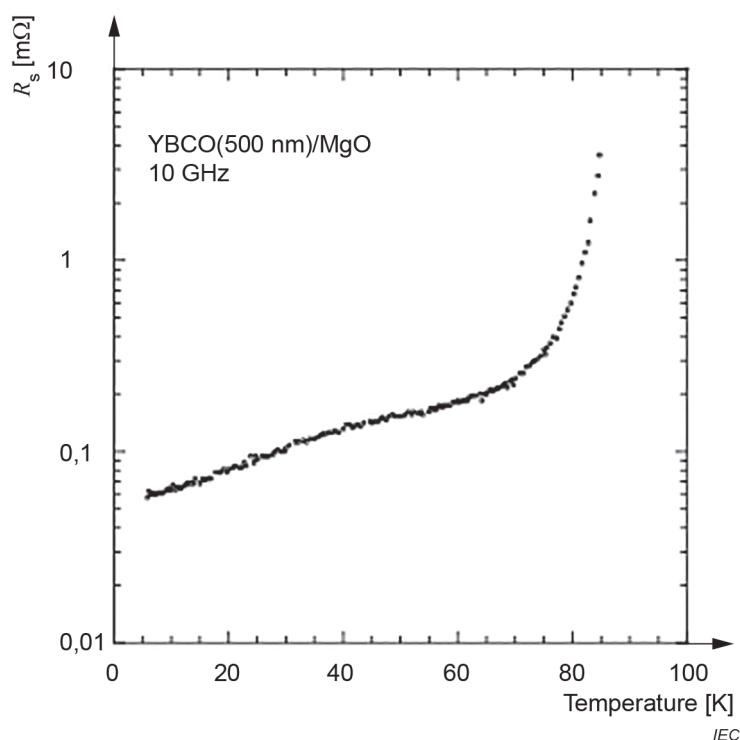


Figure A.10 – Temperature-dependent R_s of YBCO film with a thickness of 500 nm and size of 25 mm square

A.9 Reproducibility of measurement method

In order to evaluate the reproducibility of the surface resistance measurement, R_s of the same sample was measured three times and the variation of R_s was examined. The sample was reset to the measurement cavity each time. Figure A.11 shows the results. Measurement was performed at a frequency of 21,8 GHz and converted to 10 GHz according to the f^2 rule [30]. The variation in R_s was within 10 % in the temperature range of 10 K to 80 K.

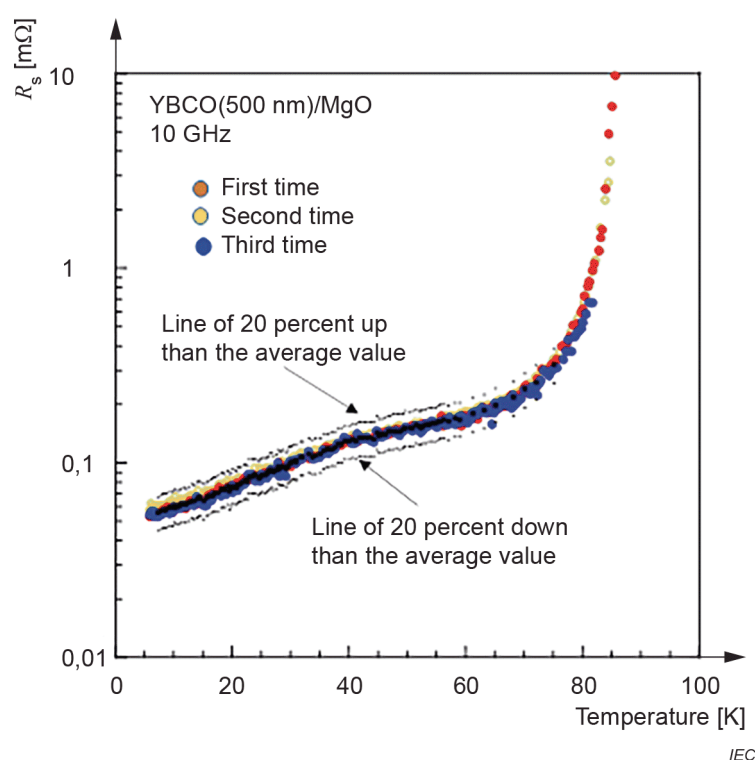


Figure A.11 – Temperature dependent R_s of YBCO film when R_s was measured three times

From the results, the standard deviation of each temperature can be calculated and its values are shown in Table A.1.

Table A.1 – Standard deviation of the surface resistance calculated from the results of Figure A.11

Temperature (K)	20	40	60	80
Standard deviation (%)	0,32	0,23	0,90	6,9

A.10 $\tan \delta$ deviation effect of sapphire rods on surface resistance

For the two-resonator method, $\tan \delta$ of the TE_{011} mode and TE_{013} sapphire rods are assumed to be absolutely equal. If $\tan \delta$ of the two sapphire rods is different, it is not possible to calculate the surface resistance using Equation (6). Therefore, it is necessary to check the effect of $\tan \delta$ deviation on surface resistance. $\tan \delta$ of the high-purity sapphire rod used for the standard surface resistance measurement is extremely small, less than 2×10^{-7} [22]. Using such a sapphire rod, uncertainty in the surface resistance is quite small; however, if sapphire rods with a large $\tan \delta$ are used, uncertainty in the surface resistance increases. $\tan \delta_1$ of the sapphire rod for TE_{011} mode is assumed to be 2×10^{-7} and $\tan \delta_3$ of the sapphire rod for TE_{013} mode changes according to that value. The difference in $\tan \delta$ of the two sapphire rods is defined as in Equation (A.12) and the change in surface resistance as in Equation (A.13).

$$x = \frac{\tan \delta_1 - \tan \delta_3}{\tan \delta_1} \times 100 \quad [\%] \quad (\text{A.12})$$

$$y = \frac{R_s(x) - R_s(0)}{R_s(0)} \quad (\text{A.13})$$

R_s can be calculated using Equation (A.14) when $\tan \delta_1$ and $\tan \delta_3$ are different. The relationship between x and y is shown in Table A.2. When the difference in $\tan \delta$ of the two sapphire rods exceeds ± 50 %, the change in surface resistance exceeds ± 18 %.

$$R_s = \frac{1}{B_1} \left(\frac{A_1}{Q_{u1}} - \tan \delta_1 \right) = \frac{1}{B_3} \left(\frac{A_3}{Q_{u3}} - \tan \delta_3 \right) \quad (\text{A.14})$$

$$A_p = 1 + \frac{W}{\varepsilon_p'} \quad (\text{A.15})$$

$$B_p = p^2 \left(\frac{\lambda_0}{2h_p} \right)^3 \frac{1+W}{30\pi^2 \varepsilon_p'}, (p=1,3) \quad (\text{A.16})$$

$$W = \frac{J_1^2(u) K_0(v) K_2(v) - K_1^2(v)}{K_1^2(v) J_1^2(u) - J_0(u) J_2(u)} \quad (\text{A.17})$$

$$v_0^2 = \left(\frac{\pi d_0}{\lambda_0} \right)^2 \left[\left(\frac{\lambda_0}{2h_0} \right)^2 - 1 \right] \quad (\text{A.18})$$

$$u_0 \frac{J_0(u_0)}{J_1(u_0)} = -v_0 \frac{K_0(v_0)}{K_1(v_0)} \quad (\text{A.19})$$

Table A.2 – Relationship between x , defined by Equation (A.12), and y , defined by Equation (A.13)

x (%)	–50	–40	–30	–20	–10	0	+10	+20	+30	+40	+50
y (%)	18,0	14,4	10,8	7,2	3,6	0	–3,6	–7,2	–10,8	–14,4	–18,0

A.9 — Reference documents

- [1] ~~WILKER, C., SHEN, Z.-Y., PANG, P., FACE, DW., HOLSTEIN, WL., MATTHEWS, AL. and LAUBACHER, D.B. 5 GHz high temperature superconductor resonators with high Q and low power dependence up to 90 K. *IEEE Trans. Microwave Theory Tech.*, 1991, 39, p.1462.~~
- [2] ~~TABER, RG. A parallel plate resonator technique for microwave loss measurements on superconductors. *Rev. Sci. Instrum.*, 1990, No. 61, p.2200-2206.~~
- [3] ~~YOSHITAKE, T. and TSUGE, H. Effects of microstructures on microwave properties in Y-Ba-Cu-O microstrip resonators. *IEEE Trans. Appl. Supercond.*, 1995, 5, p.2571.~~
- [4] ~~MAZIERSKA, J. Dielectric resonator as a possible standard for characterization of high temperature superconducting films for microwave applications. *J. Supercond.*, 1997, 10, p.73.~~
- [5] ~~SHEN, Z.-Y., WILKER, C., PANG, P., HOLSTEIN, WL., FACE, DW. and KOUNTZ, DJ. High Tc superconductor sapphire microwave resonator with extremely high Q-values up to 90 K. *IEEE Trans. Microwave Theory Tech.*, 1992, 40, p.2424.~~
- [6] ~~KRUPKA, J., KLINGER, M., KUHN, M., BARANYAK, A., STILLER, M., HINKEN, J. and MODELSKI, J. Surface resistance measurements of HTS films by means of sapphire dielectric resonators. *IEEE Trans. Appl. Supercond.*, 1993, 3, p.3043.~~

- [7] ~~TELLMANN, N., KLEIN, N., DAHNE, U., SCHOLEN, A., SCHULZ, H. and CHALOUPKA, H. High-Q LaAlO_3 dielectric resonators shielded by YBCO films. *IEEE Trans. Appl. Supercond.*, 1994, 4, p.143.~~
- [8] ~~KOBAYASHI, Y. and KAYANO, H. An improved dielectric resonator method for surface impedance measurement of high-Tc superconductors, *IEEE MTT-S Digest*, 1992, No. IF2 T-3, p.1031.~~
- [9] ~~KOBAYASHI, Y., IMAI, T. and KAYANO, H. Microwave measurement of temperature and current dependences of surface impedance for high-Tc superconductors. *IEEE Trans. Microwave Theory Tech.*, 1991, 39, p.1530.~~
- [10] ~~YOSHIKAWA, H., OKAJIMA, S. and KOBAYASHI, Y. Comparison between BMT ceramic one-resonator method and sapphire two-resonator method to measure surface resistance of YBCO films. *Proc. Asia-Pacific Microwave Conf.*, 1998, 2, p.1083.~~
- [11] ~~HASHIMOTO, T. and KOBAYASHI, Y. Two-Sapphire-Rod-Resonator method to measure the surface resistance of High-Tc superconductor films. *IEICE Trans. Electron.*, 2004, E87-C, No.5, p. 681.~~

Annex B (informative)

Evaluation of relative combined standard uncertainty for surface resistance measurement

B.1 Practical surface resistance measurement

A superconductor shows a significantly increased surface resistance when irradiated by a high frequency current in the microwave range. There are several methods for measuring surface resistance, but the dielectric resonator method can measure surface resistance the most accurately. The method is described in Annex A. Figure B.1 shows these two resonators used to measure the frequency characteristics of TE_{011} and TE_{013} modes.

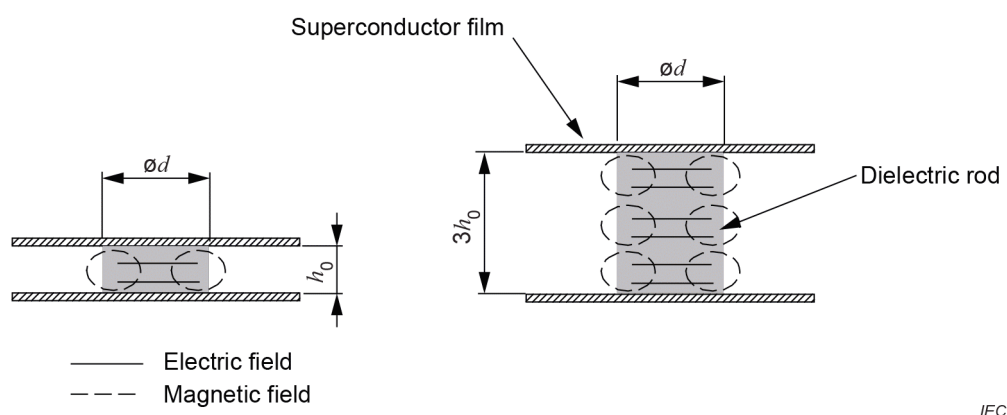
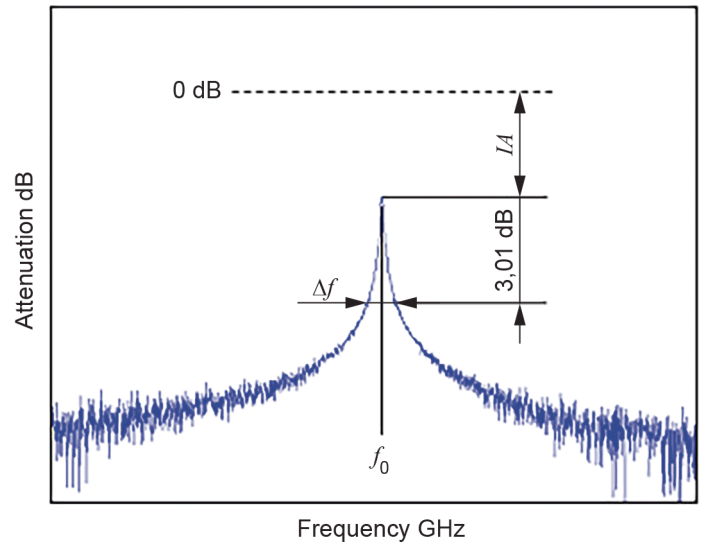


Figure B.1 – Schematic diagram of TE_{011} and TE_{013} mode resonance

Figure B.2 shows the typical frequency characteristics of TE_{011} mode resonance. Q_{L1} is determined using the results of this measurement and Q_{u1} is calculated using Equation (B.2).

$$Q_{L1} = \frac{f_0}{\Delta f} \quad (B.1)$$

$$Q_{u1} = \frac{Q_{L1}}{1 - A_t}, A_t = 10^{-IA[\text{dB}]/20} \quad (B.2)$$



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Figure B.2 – Typical frequency characteristics of TE₀₁₁ mode resonance

B.2 Determination of surface resistance of the superconductor

The microwave surface resistance of the superconductor is obtained from Equations (B.3) to (B.8).

$$R_s = \frac{30\pi^2 \times 3}{3-1} \left[\frac{2h_0}{\lambda_0} \right]^3 \frac{\varepsilon' + W}{1+W} \left[\frac{1}{Q_{u1}} - \frac{1}{Q_{u3}} \right] \quad (\text{B.3})$$

$$\varepsilon' = \left(\frac{\lambda_0}{\pi d} \right)^2 (u^2 + v^2) + 1 \quad (\text{B.4})$$

$$W = \frac{J_1^2(u) K_0(v) K_2(v) - K_1^2(v)}{K_1^2(v) J_1^2(u) - J_0(u) J_2(u)} \quad (\text{B.5})$$

$$v^2 = \left(\frac{\pi d}{\lambda_0} \right)^2 \left[\left(\frac{3\lambda_0}{2h_0} \right)^2 - 1 \right] \quad (\text{B.6})$$

$$u \frac{J_0(u)}{J_1(u)} = -v \frac{K_0(v)}{K_1(v)} \quad (\text{B.7})$$

$$\lambda_0 = \frac{c}{f_0} \quad (\text{B.8})$$

Here, d and h_0 are the diameter and height of the sapphire rod, respectively, ε' is the effective relative permittivity of the sapphire rod, c is the speed of light in vacuum, and f_0 is the resonant frequency of the resonator.

B.3 Combined standard uncertainty

B.3.1 General

By partially differentiating the equation (B.3) with h_0 , ε' , W , Q_{u1} , and Q_{u3} , the equation of uncertainty shown in Equation (B.9) is obtained.

$$\begin{aligned}
 u_c^2 &= \frac{\delta R_s^2}{R_s^2} = \left(\frac{\partial R_s}{\partial h} \right)^2 \left(\frac{\delta h}{R_s} \right)^2 + \left(\frac{\partial R_s}{\partial \varepsilon'} \right)^2 \left(\frac{\delta \varepsilon'}{R_s} \right)^2 + \left(\frac{\partial R_s}{\partial W} \right)^2 \left(\frac{\delta W}{R_s} \right)^2 \\
 &\quad + \left(\frac{\partial R_s}{\partial Q_{u1}} \right)^2 \left(\frac{\delta Q_{u1}}{R_s} \right)^2 + \left(\frac{\partial R_s}{\partial Q_{u3}} \right)^2 \left(\frac{\delta Q_{u3}}{R_s} \right)^2 + c_{\text{unknown}}^2 u_{\text{unknown}}^2 \\
 &\quad (\delta h = hu_1, \delta \varepsilon' = \varepsilon' u_2, \delta W = Wu_3, \delta Q_{u1} = Q_{u1} u_4, \delta Q_{u3} = Q_{u3} u_5) \\
 &= \left(\frac{\partial R_s}{\partial h} \right)^2 \left(\frac{h}{R_s} \right)^2 u_1^2 + \left(\frac{\partial R_s}{\partial \varepsilon'} \right)^2 \left(\frac{\varepsilon'}{R_s} \right)^2 u_2^2 + \left(\frac{\partial R_s}{\partial W} \right)^2 \left(\frac{W}{R_s} \right)^2 u_3^2 \\
 &\quad + \left(\frac{\partial R_s}{\partial Q_{u1}} \right)^2 \left(\frac{Q_{u1}}{R_s} \right)^2 u_4^2 + \left(\frac{\partial R_s}{\partial Q_{u3}} \right)^2 \left(\frac{Q_{u3}}{R_s} \right)^2 u_5^2 + c_{\text{unknown}}^2 u_{\text{unknown}}^2 \\
 &= c_1^2 u_1^2 + c_2^2 u_2^2 + c_3^2 u_3^2 + c_4^2 u_4^2 + c_5^2 u_5^2 + c_{\text{unknown}}^2 u_{\text{unknown}}^2
 \end{aligned} \tag{B.9}$$

The variables c_{unknown} and u_{unknown} in Equation (B.9) represent in the uncertainty of unknown factors of measurement system, for example, noise factor. The variables in Equation (B.9) are expressed by Equations (B.10) to (B.14).

$$c_1 = \frac{\partial R_s}{\partial h} \frac{h}{R_s} = \frac{3R_s}{h_0} \frac{h_0}{R_s} = 3 \tag{B.10}$$

$$c_2 = \frac{\partial R_s}{\partial \varepsilon'} \frac{\varepsilon'}{R_s} = \frac{\varepsilon'}{\varepsilon' + W} \tag{B.11}$$

$$c_3 = \frac{\partial R_s}{\partial W} \frac{W}{R_s} = \frac{(1 - \varepsilon')W}{(1 + W)(\varepsilon' + W)} \tag{B.12}$$

$$c_4 = \frac{\partial R_s}{\partial Q_{u1}} \frac{Q_{u1}}{R_s} = \frac{-Q_{u3}}{Q_{u3} - Q_{u1}} \tag{B.13}$$

$$c_5 = \frac{\partial R_s}{\partial Q_{u3}} \frac{Q_{u3}}{R_s} = \frac{Q_{u1}}{Q_{u3} - Q_{u1}} \tag{B.14}$$

B.3.2 Calculation of c_2 to c_5 (12 GHz resonance at 20 K)

When calculating uncertainty using the Equation (B.9), it is necessary to specify the resonant frequency. Here, we will discuss a typical 12 GHz resonator.

12 GHz resonator ($f_0 = 12$ GHz, $d = 11,4$ mm, $h_0 = 5,7$ mm)

Using the above values, we can calculate v^2 and u^2 by Equations (B.6) and (B.7).

$$v^2 = 7,815, u^2 = 9,09 \quad (\text{B.15})$$

ε' and W are determined by Equations (B.4) and (B.5).

$$\varepsilon' = 9,22, W = 0,16 \quad (\text{B.16})$$

Then c_2 and c_3 can be determined using Equations (B.11) and (B.12).

$$c_2 = 0,983, c_3 = -0,122 \quad (\text{B.17})$$

In order to determine c_4 and c_5 , we have to know the values of unloaded Q_{u1} and Q_{u3} . Y. Kobayashi reported Q_{u1} and Q_{u3} at 20 K, 12 GHz [28]. Their reported values were $Q_{u1} = 2\,000\,000$ ($Q_{L1} = 1\,800\,000$) and $Q_{u3} = 7\,500\,000$ ($Q_{L3} = 6\,750\,000$). Therefore, c_4 and c_{5L} can be calculated.

$$c_4 = 1,364, c_5 = 0,3636 \quad (\text{B.18})$$

B.3.3 Determination of u_1 to u_5

u_1 to u_5 , which are derived from the uncertainty of the diameter, height and dielectric constant of sapphire rod and Q_{u1} and Q_{u3} of the cavity, are calculated by the following procedures.

1) u_1

u_1 is determined by the uncertainty of the height of the sapphire rod. The height of the sapphire rod of the 12 GHz resonator is 5,7 mm, and its uncertainty is 50 μm . Therefore, u_1 is as follows.

$$u_1 = \frac{1}{\sqrt{3}} \frac{0,05}{5,7} = 0,005\,07 \quad (\text{B.19})$$

2) u_2

u_2 is determined by the uncertainty in the effective dielectric constant ε'_δ of the sapphire rod and can be calculated using Equation (B.20).

$$u_2 = \frac{1}{\sqrt{3}} \frac{\varepsilon' - \varepsilon'_\delta}{\varepsilon'} \quad (\text{B.20})$$

where ε'_δ is the relative permittivity induced from uncertainty in the shape of the sapphire rod.

$$\varepsilon' = \left(\frac{\lambda_0}{\pi d_0} \right)^2 (u_0^2 + v_0^2) + 1 \quad (d_0 = 11,4 \text{ mm}, h_0 = 5,7 \text{ mm}) \quad (\text{B.21})$$

$$\varepsilon'_\delta = \left(\frac{\lambda_0}{\pi d} \right)^2 (u_{0\delta}^2 + v_{0\delta}^2) + 1 \quad (d_{0\delta} = 11,45 \text{ mm}, h_{0\delta} = 5,65 \text{ mm}) \quad (\text{B.22})$$

and u_0 , v_0 , $u_{0\delta}$ and $v_{0\delta}$ are calculated using Equations (B.23) and (B.24).

$$v_0^2 = \left(\frac{\pi d_0}{\lambda_0} \right)^2 \left[\left(\frac{3\lambda_0}{2h_0} \right)^2 - 1 \right], v_{0\delta}^2 = \left(\frac{\pi d_{0\delta}}{\lambda_0} \right)^2 \left[\left(\frac{3\lambda_0}{2h_{0\delta}} \right)^2 - 1 \right] \quad (\text{B.23})$$

$$u_0 \frac{J_0(u_0)}{J_1(u_0)} = -v_0 \frac{K_0(v_0)}{K_1(v_0)}, u_{0\delta} \frac{J_0(u_{0\delta})}{J_1(u_{0\delta})} = -v_{0\delta} \frac{K_0(v_{0\delta})}{K_1(v_{0\delta})} \quad (\text{B.24})$$

As a result, we find that $\varepsilon' = 9,22$ and $\varepsilon'_\delta = 9,29$.

$$u_2 = 0,004\ 4 \quad (\text{B.25})$$

3) u_3

u_3 can be determined using Equation (B.26).

$$u_3 = \frac{1}{\sqrt{3}} \frac{W - W_\delta}{W} \quad (\text{B.26})$$

W is determined using Equation (B.5) and W_δ comes from uncertainty in the shape of the sapphire rod and can be determined by using Equation (B.27).

$$W_\delta = \frac{J_1^2(u_\delta) K_0(v_\delta) K_2(v_\delta) - K_1^2(v_\delta)}{K_1^2(v_\delta) J_1^2(u_\delta) - J_0(u_\delta) J_2(u_\delta)} \quad (\text{B.27})$$

By calculating W and W_δ using u_0 , v_0 , $u_{0\delta}$, $v_{0\delta}$ in Equations (B.23) and (B.24), u_3 turns out to have the following value.

$$u_3 = 0,014\ 3 \quad (\text{B.28})$$

4) u_4 and u_5

u_4 and u_5 are determined by measuring the uncertainty in Q_{u1} and Q_{u3} . The measurement of the uncertainty in Q_{u1} and Q_{u3} has the most influence on the dynamic range uncertainty of the vector network analyzer and other factors are negligible. Since the dynamic range uncertainty in the recommended vector network analyzer is $\pm 0,1$ dB, u_4 and u_5 are calculated using this value. Regarding the frequency characteristics of the resonator in Figure B.2, the frequency characteristic near the resonant frequency is assumed to be a Lorentz distribution [30].

$$y = \left| \frac{S_{12}(f)}{S_{12}(f_0)} \right| = \frac{1}{\left(1 + Q^2 \Delta^2(f) \right)^{1/2}} = \frac{1}{\left(1 + \left(\frac{f - f_0}{f_1} \right)^2 \right)^{1/2}},$$

$$\Delta(f) = 1 - \frac{f_0^2}{f^2} = \frac{2(f - f_0)}{f_0} \quad (\text{B.29})$$

The frequency characteristics are shown in Figure B.3. The curve can be approximated by Equation (B.29), where f_0 is the resonant frequency and $2f_1$ is the bandwidth at 3,01 dB. The frequency bandwidth which is 3,01 dB below the resonance peak is defined as Δf and it corresponds to the bandwidth across 0,707 in Figure B.3. The $\pm 0,1$ dB of the dynamic range uncertainty corresponds to $\pm 0,008$ in Figure B.3. Therefore, Q_{L1} , $Q_{L1\delta}$, Q_{L2} and $Q_{L2\delta}$