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Fibre optic communication system design guides –

Part 9: Guidance on polarization mode dispersion measurements and theory



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Fibre optic communication system design guides –

Part 9: Guidance on polarization mode dispersion measurements and theory

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INTERNATIONAL ELECTROTECHNICAL COMMISSION

FIBRE OPTIC COMMUNICATION SYSTEM DESIGN GUIDES –

Part 9: Guidance on polarization mode dispersion
measurements and theory

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IEC 61282-9, which is a technical report, has been prepared by subcommittee 86C: Fibre optic systems and active devices, of IEC technical committee 86: Fibre optics.

The text of this technical report is based on the following documents:

Enquiry draft	Report on voting
86C/696/DTR	86C/703/RVC

Full information on the voting for the approval of this technical report can be found in the report on voting indicated in the above table.

This publication has been drafted in accordance with the ISO/IEC Directives, Part 2.

A list of all parts of IEC 61282 series, published under the general title *Fibre optic communication system design guides*, can be found on the IEC website.

The committee has decided that the contents of this publication will remain unchanged until the maintenance result date indicated on the IEC web site under "<http://webstore.iec.ch>" in the data related to the specific publication. At this date, the publication will be

- reconfirmed,
- withdrawn,
- replaced by a revised edition, or
- amended.

A bilingual version of this publication may be issued at a later date.

Withdrawn

FIBRE OPTIC COMMUNICATION SYSTEM DESIGN GUIDES –

Part 9: Guidance on polarization mode dispersion measurements and theory

1 Scope

This technical report applies to all commercially available fibre optic products sensitive to polarization mode dispersion (PMD).

This report presents general information about PMD, the mathematical formulation related to the application of the generally accepted methods to test PMD, and some considerations related to the sampling theory regarding the use of different light sources and detection systems.

This report is complementary to the International Standards describing the PMD procedures (IEC 60793-1-48, IEC 61280-4-4, IEC 61290-11-1, IEC 61290-11-2 and IEC 61300-3-32) and other design guides on PMD (IEC 61282-3 and IEC 61292-5).

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

IEC 60793-1-48: *Optical fibres – Part 1-48: Measurement methods and test procedures – Polarization mode dispersion*

IEC 61280-4-4: *Fibre optic communication subsystem test procedures – Part 4-4: Cable plants and links – Polarization mode dispersion measurement for installed links*

IEC 61290-11-1: *Optical fibre amplifier test methods – Part 11-1: Polarization mode dispersion – Jones matrix eigenanalysis method (JME)*

IEC 61290-11-2: *Optical amplifiers – Test methods – Part 11-2: Polarization mode dispersion parameter – Poincaré sphere analysis method*

IEC 61300-3-2: *Fibre optic interconnecting devices and passive components – Basic test and measurement procedures – Part 3-2: Examinations and measurements – Polarization dependence of attenuation in a single-mode fibre optic device*

IEC 61300-3-32: *Fibre optic interconnecting devices and passive components – Basic test and measurement procedures – Part 3-32: Examinations and measurements – Polarization mode dispersion for passive optical components*¹

IEC/TR 61282-3: *Fibre optic communication system design guides – Part 3: Calculation of polarization mode dispersion*

IEC/TR 61292-5: *Optical amplifiers – Part 5: Polarization mode dispersion parameter – General information*

¹ To be published

3 Acronyms and abbreviations

ASE	amplified spontaneous emission
AWG	array waveguide grating
BBS	broadband source
BER	bit error rate or bit error ratio
Clh	circular left handed
Crh	circular right handed
CSO	composite second-order beat noise
DAS	differential attenuation slope
DGD	differential group delay
DOP	degree of polarization
DUT	device under test
DVV	polarization dispersion vector velocity
DWDM	dense wavelength division multiplexing
EC	extrema counting
EDFA	erbium doped fibre amplifier
FA	fixed analyser
FAFT	fixed analyser Fourier transform
FAEC	fixed analyser extrema counting
FCFT	fast cosine Fourier transform
ffs	for further study
FT	Fourier transform
FWHM	full width at half the maximum
GINTY	general analysis for the interferometric method
HMD	harmonic distortion
IMD	intermodulation distortion
INTY	interferometry
I/O	input/output
ISI	inter-symbol interference
JME	Jones matrix eigenanalysis
L	linear
LH	linear horizontal
LV	linear vertical
MMA	Mueller matrix analysis
MPS	modulation phase shift
OA	optical amplifier
OFA	optical fibre amplifier
OSA	optical spectrum analyser
OTDR	optical time domain reflectometer
P-	parallel polarization (to the plane of incidence)
PDCD	polarization dependent chromatic dispersion
PDD	polarization dependent delay
PDG	polarization dependent gain

PDL	polarization dependent loss
PDV	polarization dispersion vector
PM	polarization mode
PMD	polarization mode dispersion
PMD_1	first-order polarization mode dispersion
PMD_2	second-order polarization mode dispersion
PMF	polarization maintaining fibre
POTDR	polarization optical time domain reflectometer
POWA	planar optical waveguide amplifier
PPS	polarization phase shift
PS	Poincaré sphere
PSA	Poincaré sphere analysis
PSP	principal states of polarization
RBW	resolution bandwidth
RMS	root mean square
RTM	reference test method
S	source
SMSR	side mode suppression ratio
SOP	state of polarization
SPE	Stokes parameter evaluation
SRM	standard reference material
SSE	source spontaneous emission
TINTY	traditional analysis for the interferometric method
TLS	tuneable laser source
TSSE	total source spontaneous emission
WDM	wavelength division multiplexing

4 General information

The following text provides general information concerning the PMD theory and phenomenon. In that context, the word “device” or “device under test” (DUT) is used throughout the text in the sense of an optical path with an input and an output interface, such as an optical fibre, an optical fibre cable, an optical component, an optical amplifier, etc. The device may be connectorised.

4.1 Polarization modes

The solution of the wave equation has degenerated eigenvalues. This means that even the fundamental solution is degenerated. A single-mode fibre can therefore support several modes, the polarization modes (PM), and by analogy can be considered as a multi-(polarization) mode fibre. In particular, the lowest order mode, namely the fundamental HE_{11} (LP_{01}) mode, can be chosen to have its transverse electric field predominately along the x -direction; the orthogonal polarization is an independent mode, as shown in Figure 1.

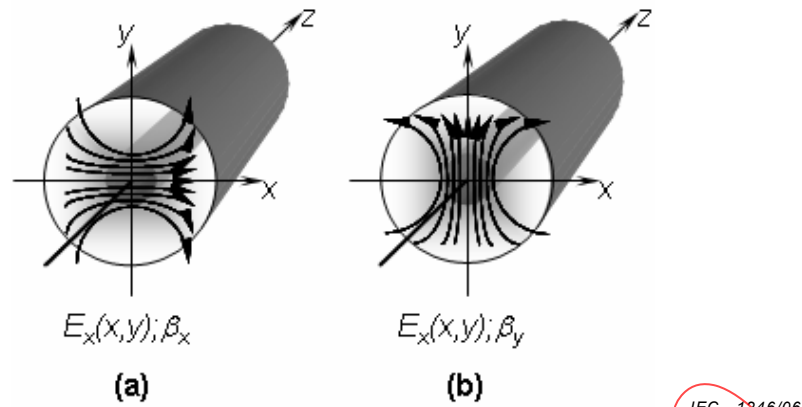


Figure 1 – Two electric field vector polarizations of the HE₁₁ mode in an optical fibre along the a) x-direction and b) y-direction

In a lossless optical fibre, the electric field vector of a monochromatic electromagnetic wave propagating along the z -direction can be described by a linear superposition of these two PM in the x - y transverse plane as shown in Equation (1) and in Figure 1 [1,2]².

$$E = \left[A_x(z) \cdot E_x(x,y) \right] + \left[A_y(z) \cdot E_y(x,y) \right] \cdot e^{-i\omega t} \quad (1)$$

where

$A_x(z) = E_x e^{i\beta_x z}$, is the complex coefficient describing the amplitude E_x and the phase β_x of the PM along the x -direction propagating in the z -direction;

$A_y(z) = E_y e^{i\beta_y z}$, is the complex coefficient describing the amplitude E_y and the phase β_y of the PM along the y -direction propagating in the z -direction;

$E_x(x,y)$ is the spatial variation (in the x - y transverse plane) of the electric field vector of the PM along the x -direction (see Figure 1(a);

$E_y(x,y)$ is the spatial variation (in the x - y transverse plane) of the electric field vector of the PM along the y -direction (see Figure 1(b);

$\beta_x = kn_x$, is the propagation constant (also called effective index or wavenumber) of the PM along the x -direction with the index of refraction n_x ; β_x , through n_x has a dependence on optical angular frequency ω , or optical frequency ν , or wavelength λ ;

$\beta_y = kn_y$, is the propagation constant (also called effective index or wavenumber) of the PM along the y -direction with the index of refraction n_y ; β_y , through n_y has a dependence on angular optical frequency ω , or optical frequency ν , or wavelength λ ;

$k = 2\pi\nu = 2\pi/\lambda = \omega/c$, is the propagation constant with the wavelength λ in vacuum;

ν is the optical frequency in s^{-1} or Hz;

ω is the angular optical frequency in rad/s;

c is the speed of light in vacuum;

z is the distance in the device along the optical axis; $z = L$ at the output of the device with length L .

The complex ratio $A_x(z)/A_y(z)$ describes the state of polarization (SOP) defined in the x - y plane of the wave propagating along the z -direction.

² Figures in square brackets refer to the Bibliography.

In an ideal optical fibre with perfect circular symmetry,

- $\beta_y = \beta_x$;
- the two PM are degenerate;
- consequently, any wave with a defined input SOP will propagate unchanged along the z -direction throughout the output of the fibre.

However, in a practical optical fibre, imperfections produced by the fabrication process, cabling, field installation/use or the environment break the circular symmetry,

- $\beta_y \neq \beta_x$, implying a phase difference, and index of refraction difference Δn and a phase velocity difference Δv between the two PM;
- the degeneracy of the two PM is lifted;
- consequently, the SOP of an input wave will change along the z -direction throughout the output of the fibre.

The difference between β_y and β_x , namely $\Delta\beta$, is called the phase birefringence or simply the birefringence and has units of inverse length. Birefringence may also be referred to as the index difference, Δn . Index differences typically vary between 10^{-7} and 10^{-5} in commonly available single-mode fibres [2].

4.2 Birefringence

Birefringence is produced by an anisotropic distribution of the index of refraction in the propagating region of an optical device medium. As such, any device is susceptible to birefringence. For instance, birefringence is produced by asymmetry in the optical fibre core, meaning when the circular symmetry of the fibre core is broken [1,3].

It is well known that the asymmetry provides an index of refraction that is smaller along an axis, and as such that smaller index provides a faster phase velocity along that axis compared to the other one. That axis is consequently called the fast axis as opposed to the other one, which is called the slow axis, corresponding to a larger index and a slower phase velocity. The slower wave is also said to be retarded compared to the other one.

The asymmetry can result from geometrical deformation of the medium or material anisotropy through various elasto-optic, magneto-optic or electro-optic changes of the index of refraction. Geometrical deformation and asymmetric lateral stress via elasto-optic index changes may be produced during fabrication, for instance, and will typically produce linear birefringence. Bending, kinks and electro-optic Kerr effect will also typically produce linear birefringence. Twist and magneto-optic effect via Faraday effect will produce circular birefringence.

Several of these mechanisms may coexist and may be present in various numbers, strengths and distributions; and may vary with time and over environmental conditions. This makes the SOP along the propagating medium and at the output unpredictable and unstable and, consequently, PMD compensation difficult to achieve.

Birefringence can, however, be imposed, such as is the case for polarization maintaining fibre (PMF). This type of fibre has a strong birefringence maintained along the fibre length which can be created, for example, by introducing stress in one plane. This will keep the two PM non-degenerate. Any SOP launched aligned with the axis of one of these two PM will be maintained throughout the fibre. As opposed to PMF, commonly available single-mode fibres used for fibre optic transmissions in the field are weakly birefringent fibres.

4.3 Beat length

Birefringence makes the two PM slip in phase relative to one another as they propagate at different phase velocities. When the phase difference between the two PM is equal to an integer number of 2π , the two PM beat with one another, and at that periodic point along the z -direction, the input SOP is reproduced. The length corresponding to that periodicity is called the beat length L_b .

$$L_b = \frac{2\pi}{\Delta\beta} \text{ in units of length} \quad (2)$$

Beat lengths typically vary between 2 m and 15 m in commonly available single-mode fibres [2].

Beat length should not be confused with coupling length, which will be explained later.

4.4 Polarization transfer function

Polarized light and its related SOP can be represented using Jones calculus [4] as a complex vector illustrating the x - and y - components of the electric field of the SOP at a point in space (see Figure 2).

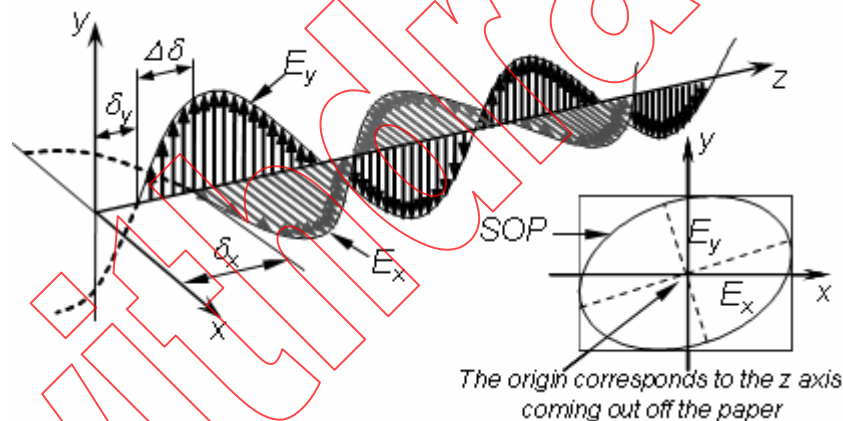


Figure 2 – Cartesian and elliptical representation of a state of polarization

NOTE In the presence of anisotropy, the y axis is the slow axis in Figure 2, while the x axis is the fast axis, and the wave propagating in the y - z plane is retarded. The resulting wave from the vector combination of the y - z plane wave and the x - z plane wave is shown in the x - y plane in the lower right corner of Figure 2

The Jones vector has the form (not considering any unit vector):

$$E = \begin{pmatrix} A_x \\ A_y \end{pmatrix} = \begin{pmatrix} E_x e^{i\delta_x} \\ E_y e^{i\delta_y} \end{pmatrix} \quad (3)$$

Equation (3) can easily be related to Equation (1) in case of anisotropy when δ becomes β . As polarized light traverses any DUT such as an optical fibre, its SOP undergoes a transformation. This transformation is described in Jones calculus by the complex 2x2 Jones matrix, T , so that the output Jones vector E_{out} is related to the input Jones vector E_{in} through $E_{out} = T E_{in}$. The Jones matrix is determined by measuring the output Jones vectors in response to any three unique input vectors. The calculation is simplest when the stimuli are linear horizontal (LH) 0° ($E_y = 0$; $\delta_x = 0$), linear vertical (LV) 90° ($E_x = 0$, $\delta_y = 0$) and linear (L) $+45^\circ$ ($E_y = E_x$; $\delta_y = \delta_x = 0$) SOPs. If the responses to the three stimuli are:

$$\begin{pmatrix} X_1 \\ Y_1 \end{pmatrix}, \begin{pmatrix} X_2 \\ Y_2 \end{pmatrix}, \begin{pmatrix} X_3 \\ Y_3 \end{pmatrix} \quad (4)$$

the Jones matrix is

$$T = C^* \begin{bmatrix} K_1 K_4 & K_2 \\ K_4 & 1 \end{bmatrix} \quad (5)$$

$$K_1 = \frac{X_1}{Y_1}; \quad K_2 = \frac{X_2}{Y_2}; \quad K_3 = \frac{X_3}{Y_3}; \quad K_4 = \frac{K_3 - K_2}{K_1 - K_3} \quad (6)$$

where C is a complex constant.

The magnitude of C can be calculated from intensities measured with the DUT removed from the optical path. Most measurements do not require the calculation of C .

4.5 Stokes parameters and the Poincaré sphere

While Jones calculus is very useful in explaining SOPs, it is problematic in practice to measure the electric field of a light wave. An alternative expression for the SOPs is the Stokes vector, as shown in Equation (7):

$$S = \begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix} \quad (7)$$

where

$S_0 = E_x^2 + E_y^2$ is the total power including the polarized and unpolarized parts;

NOTE The degree of polarization (DOP) is equal to the ratio of the polarized part of the power to S_0 .

$S_1 = E_x^2 - E_y^2$ is the power difference between the LH and LV SOPs;

$S_2 = 2\text{Re}(E_x E_y^*)$ is the power difference between the L +45 ° and the L –45 ° SOPs;

$S_3 = 2\text{Im}(E_x E_y^*)$ is the power difference between the circular right-handed (Crh) and circular left-handed (Clh) SOPs.

The magnitude of the Stokes vector is the square root of the sum of the squares of S_1 , S_2 and S_3 .

The Poincaré sphere uses the normalised S parameters s ($s_1 = S_1/S_0$, $s_2 = S_2/S_0$, $s_3 = S_3/S_0$) as a three-real component unit vector co-ordinate system to represent any possible SOP [5,6]. The LH 0 ° SOP is represented as (1,0,0), LV 90 ° as (–1,0,0), L +45 ° as (0,1,0), L –45 ° as (0,–1,0), Crh as (0,0,1), and Clh as (0,0,–1).

$$E = \begin{pmatrix} E_x * e^{i\delta_x} \\ E_y * e^{i\delta_y} \end{pmatrix} \Rightarrow \begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix} = \begin{bmatrix} E_x^2 + E_y^2 \\ E_x^2 - E_y^2 \\ 2\text{Re}(E_x E_y^*) \\ 2\text{Im}(E_x E_y^*) \end{bmatrix} = \begin{bmatrix} E_x^2 + E_y^2 \\ E_x^2 - E_y^2 \\ 2E_x E_y \cos \Delta\delta \\ 2E_x E_y \sin \Delta\delta \end{bmatrix} \quad (8)$$

$$\begin{bmatrix} s_1 \\ s_2 \\ s_3 \end{bmatrix} = \begin{bmatrix} S_1/S_0 \\ S_2/S_0 \\ S_3/S_0 \end{bmatrix} = \begin{bmatrix} (E_x^2 - E_y^2)/(E_x^2 + E_y^2) \\ 2\text{Re}(E_x E_y^*)/(E_x^2 + E_y^2) \\ 2\text{Im}(E_x E_y^*)/(E_x^2 + E_y^2) \end{bmatrix} = \begin{bmatrix} (E_x^2 - E_y^2)/(E_x^2 + E_y^2) \\ 2E_x E_y \cos \Delta\delta / (E_x^2 + E_y^2) \\ 2E_x E_y \sin \Delta\delta / (E_x^2 + E_y^2) \end{bmatrix} \quad (9)$$

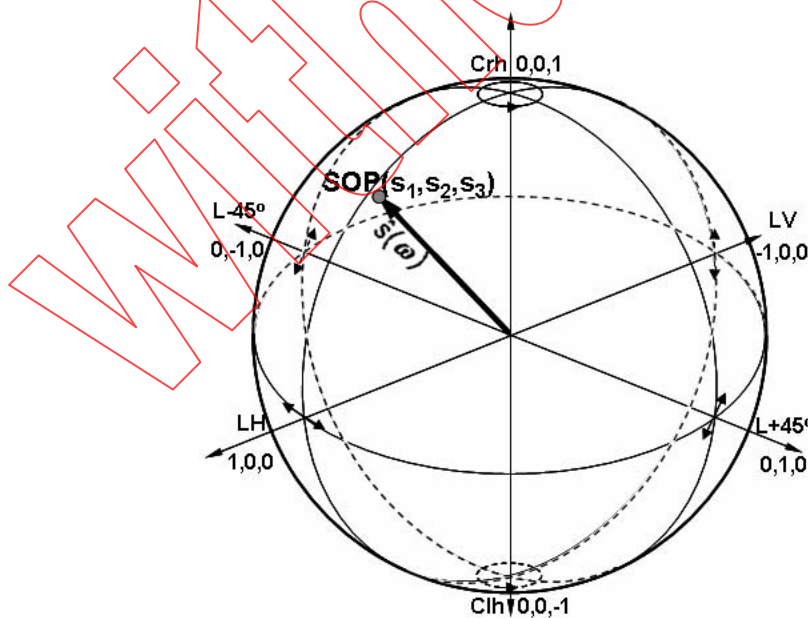
In polar coordinates,

$$E = \begin{bmatrix} (\cos \theta) e^{-i\mu/2} \\ (\sin \theta) e^{i\mu/2} \end{bmatrix} e^{i\phi} \quad (10)$$

$$\begin{bmatrix} s_1 \\ s_2 \\ s_3 \end{bmatrix} = \begin{bmatrix} \cos 2\theta \\ (\sin 2\theta) \cos \mu \\ (\sin 2\theta) \sin \mu \end{bmatrix} \quad (11)$$

where $\Delta\delta = \delta_x - \delta_y$, is the difference in phase between the x (LH) and y (LV) SOPs.

The use of the Stokes vector parameters is best illustrated via the use of the Poincaré sphere (see Figure 3). Stokes vectors can be converted to Jones vectors using the respective E -field formulae.



IEC 1248/06

Figure 3 – Poincaré sphere representation of states of polarization

NOTE The LH axis usually faces the observer/receiver

For a fully polarized field (DOP = 100 %), $\sqrt{s_1^2 + s_2^2 + s_3^2}$ is unitary, and a SOP is represented by a dot on the surface of the sphere (the tip of the SOP vector). The observer looks at the LH SOP. Any SOP laying on the equator of the sphere is linear. The north pole of the sphere is Crh SOP, and the south pole is Clh SOP. Any other SOP anywhere else on the sphere is elliptical.

4.6 Principal states of polarization

In a device that exhibits PMD, there are two SOPs called principal states of polarization (PSPs) [7,8] having the property that the SOP measured at the output of the device is invariant with optical frequency (or wavelength) to first order (over a small optical frequency or wavelength increment) for any output SOP aligned with these PSPs. Of the two PSPs, the one called the fast PSP gives the fastest group velocity and consequently the shortest propagation delay; the other, called the slow PSP, gives the slowest group velocity and consequently the longest propagation delay. These two PSPs will be orthogonal in the absence of polarization dependent loss (PDL) or polarization dependent gain (PDG) and non-linear effects. In a device with polarization-mode coupling such as a single-mode optical fibre used to transport fibre optic telecommunications, the PSPs are, in general, different from the eigenstates of the device; in a non-mode-coupled device such as a PMF, the PSPs are identical to the eigenstates of the device. The eigenstates are the eigenvectors of the Jones matrix, i.e. the two SOPs which propagate through the device without modification.

NOTE 1 The two PSPs are an intrinsic function of the material birefringence and induced external and internal stresses acting on the device.

NOTE 2 An output signal whose SOP is aligned with one of the two PSPs will have been undistorted by PMD, at least to first order.

4.7 Differential group delay

The difference between the two PSP arrival times is the differential group delay (DGD). The resultant pulse broadening or splitting is the source of signal distortion.

The definition provided above is in the context of 1st order PMD without PDL. In the case of PDL, there is an open issue regarding the definition (see also E.2.1).

NOTE Depending on the device, DGD varies with wavelength, and for a randomly mode coupled device, the pattern of variation changes randomly with device deployment and small temperature changes (~0,1 °C). The average DGD across a wavelength range remains moderately stable, depending on the device construction and the wavelength range that is sampled.

4.8 Polarization mode dispersion

PMD is the temporal distortion of an optical signal induced by the interaction of the source polarization and spectral characteristics with the transport of the signal through a device that includes two orthogonal SOPs (i.e. PSPs) with differing group velocities. The term "PMD" is used both in this general sense of a "phenomenon" of pulse width broadening, as well as specific "numeric values" of broadening (in time) that characterise a device.

4.8.1 Phenomenon

When an optical signal pulse passes through a device, the pulse may change. PMD refers to the change in shape, particularly in the root-mean-square (RMS) width of the pulse, due to the average propagation delay difference between the two PSPs, i.e. DGD, and/or waveform distortion for each PSP (see Figure 4). PMD together with PDL and PDG, when applicable, may introduce waveform distortion leading to an unacceptable increase in bit error ratio (BER).

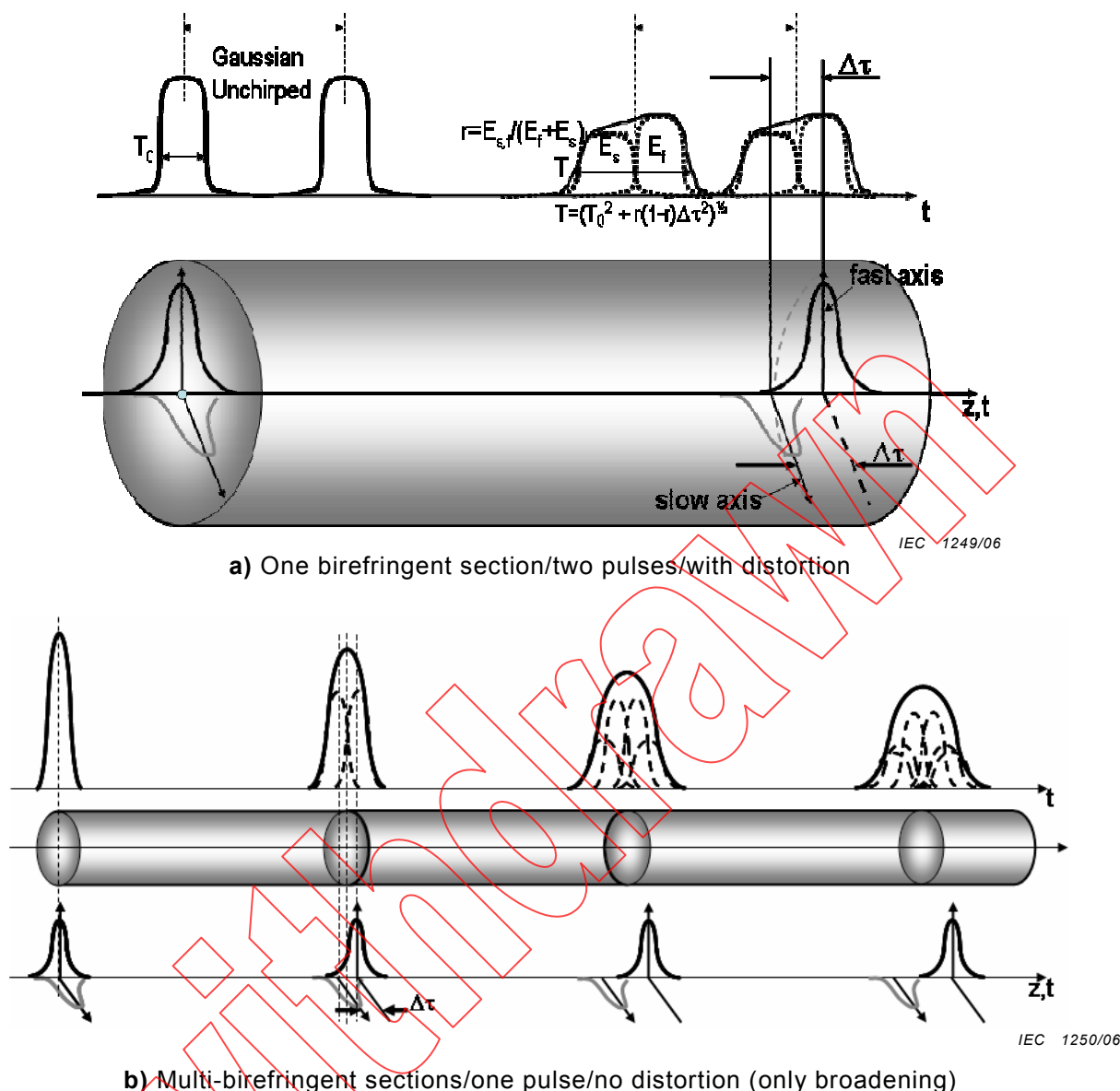


Figure 4 – Effect of polarization mode dispersion on transmission of an information-bit pulse in a device

4.8.2 Numerical value

The PMD value (in units of time) is specified and measured depending on the type of device and its contribution to the overall signal link distortion. In some cases, either the average or RMS DGD over an optical frequency (or wavelength) range is used to define the PMD value. In other cases, the maximum DGD occurring in the optical frequency or wavelength range is used.

4.9 Polarization dispersion and birefringence vectors

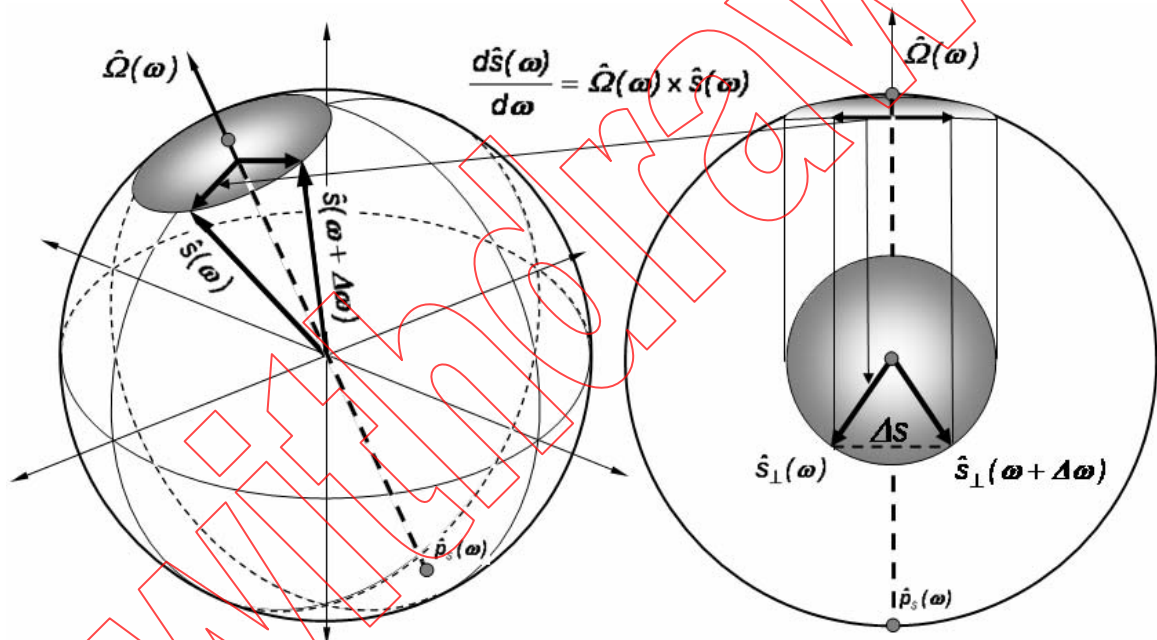
From Poole [6,9,10], the polarization dispersion vector (PDV), $\hat{\Omega}$, and the birefringence vector, $\hat{\Delta}\beta$ are a compact way of representing PMD. The variation of the SOP, $\hat{s}$, as a function of the optical angular frequency, ω and z , inside the device obeys the equations of motion described in (12) and (13) respectively (see Figure 5).

$$\frac{d\hat{s}(\omega)}{d\omega} = \hat{\Omega}(\omega) \times \hat{s}(\omega) \quad (12)$$

$$\frac{d\hat{s}(\omega)}{dz} = \Delta\hat{\beta}(\omega) \times \hat{s}(\omega) \quad (13)$$

where

- $\hat{s}$ is the SOP vector with three components, s_1 , s_2 and s_3 , defined in Equation (9), unitless and depending on ω and z ;
- $\hat{\Omega}$ is the PDV with three components, Ω_1 , Ω_2 and Ω_3 , depending on ω and z , and having units of time; $\hat{\Omega}$ depends on the DUT properties along the entire optical path;
- $\Delta\hat{\beta}$ is the birefringence vector with three components, $\Delta\beta_1$, $\Delta\beta_2$ and $\Delta\beta_3$, depending on ω and z , and having units of inverse length; $\Delta\hat{\beta}$ depends on the DUT properties at position z ;



IEC 1251/06

Figure 5 – Polarization dispersion vector and principal states of polarization

A relationship exists between the PDV and the birefringence vector, and this relationship has important consequences [6,10]:

$$\frac{d\hat{\Omega}}{dz} = \frac{d\Delta\hat{\beta}}{d\omega} + (\Delta\hat{\beta} \times \hat{\Omega}) \quad (14)$$

where

- $d\Delta\hat{\beta}/d\omega$ is the modal birefringence (vector), B , at position z along the DUT and has units of delay time per unit length;
- $\Delta\hat{\beta} \times \hat{\Omega}$ is the term indirectly affecting the growth of the polarization dispersion by causing the PDV to change direction with $d\Delta\hat{\beta}/d\omega$, causing dephasing between the two vectors.

$$B = \frac{1}{v} \left(\frac{\Delta v}{v} \right) \quad (15)$$

where

v is the average group velocity;

Δv is the differential group velocity between the fast and slow PM (PSPs), so that $v + \frac{1}{2}\Delta v$ is the fast group velocity and $v - \frac{1}{2}\Delta v$ is the slow group velocity [11].

Modal birefringence is responsible for the growth of the polarization dispersion along the DUT length.

When looking into the ω domain, the direction of $\hat{\Omega}$ is collinear with the PSP (in the direction of the fast PSP, $\hat{s}_p$, as shown in Figure 5). The magnitude of $\hat{\Omega}$ (its modulus) is equal to the DGD, i.e. (see Figure 5)

$$\hat{\Omega}(\omega) = \Delta\tau(\omega) \cdot \hat{s}_p(\omega) \quad (16)$$

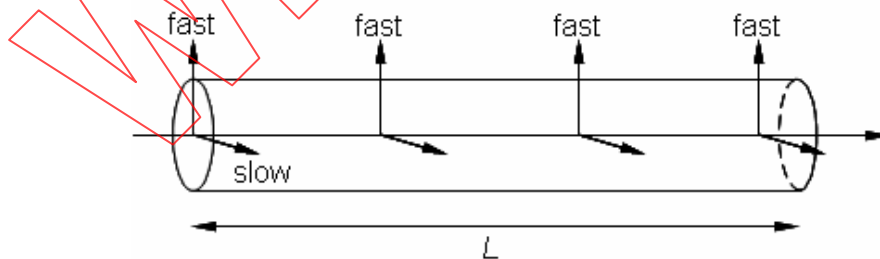
where [7]

$$\hat{s}_p^{slow}(\omega) = -\hat{s}_p^{fast}(\omega) \quad (17)$$

$$|\hat{\Omega}(\omega)| = \Delta\tau(\omega) = DGD \quad (18)$$

4.10 Polarization mode coupling

The two PM and output PSPs of a simple birefringent element result from two input PSPs whose fast and slow axes are maintained throughout the element (see Figure 6). A short optical fibre is often modelled as a simple birefringent element for which the two output PSPs do not change appreciably with optical frequency and throughout the fibre.



IEC 1252/06

Figure 6 – No or negligible mode coupling

In that case, the modal birefringence, B , is modelled as a constant vector $(B,0,0)$. This is the case for PMF in which birefringence is strong.

If two such elements are concatenated, i.e. they are jointed with random axis orientation, the output PSPs of the first element may not align with the input PSPs of the second element, as shown in Figure 7. The distance between such concatenations is associated with the coupling length, h (also called the h -parameter).

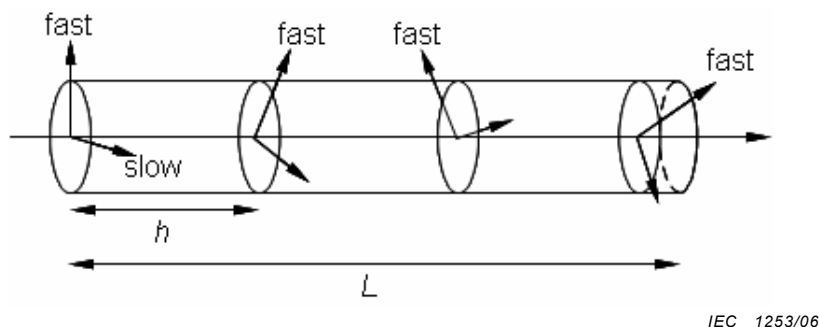


Figure 7 – Random mode coupling

Under this condition, mode coupling occurs: part of the energy of one local PM is transferred to the other one [11]. The mode coupling resulting from the repeated concatenation of many elements produces a continuous variation of the fast and slow axes along the DUT and a random variation, with optical frequency, of the output PSPs and DGD of the concatenation.

In a long single-mode optical fibre, the birefringence is generally too weak to maintain the polarization as compared to a PMF and is typically modelled as a weak but constant background birefringence $(\Delta\beta, 0, 0)$ together with a small random stepsize fluctuating perturbation $(\delta\Delta\beta_1, \delta\Delta\beta_2, \delta\Delta\beta_3)$ [6,10]. The distributions of the $\delta\Delta\beta_i$ are Gaussian and uncorrelated.

The local fast-axis PSP PM and the slow-axis PSP PM can easily couple together under the influence of external factors like fibre twist, or bending. That DUT is often modelled as the concatenation of several randomly oriented simple birefringent elements (short fibres) of randomly varying lengths. The h parameter is a metric used to quantify the average length of these elementary sections as shown in Figure 7.

In presence of random coupling, many PM exist across the frequency spectrum and a natural definition for PMD delay would be the mean square deviation of the time of flight of the various PMs. The many PMs exist because many frequencies exist. If only one frequency is present, only one pair of PMs would be present as well. The output time intensity leads to the telegrapher equation, describing the probability density function (PDF), ρ , of time of flight, τ , as a function of distance, z , in a DUT such as an optical fibre [11-13]:

$$\frac{\partial^2 \rho}{\partial z^2} = \left(B^2 \frac{\partial^2 \rho}{\partial \tau^2} \right) - \left(\frac{2}{h} \frac{\partial \rho}{\partial z} \right) \quad (19)$$

The PMD delay is expressed by the RMS width of the PM time of flight distribution [6,14,15] and is expressed in units of ps:

$$PMD = \frac{Bh}{\sqrt{2}} \left(\frac{2L}{h} - 1 + e^{-2L/h} \right)^{1/2} \quad (20)$$

where h is the PM mean coupling length [11].

Equation (20) has two limit conditions [11]:

a) when $L \ll h$, the PMD distribution function (PDF of times of flight, τ) is flat:

$$PMD = BL \quad (21)$$

and PMD increases linearly with L .

- b) when $L \gg h$, the PMD distribution function (PDF of times of flight, τ) becomes essentially Gaussian:

$$PDF(\tau) = \left(\frac{1}{\sqrt{2\pi\langle\tau^2\rangle}} \right) e^{-\tau^2/2\langle\tau^2\rangle} \quad (22)$$

where $\langle\tau^2\rangle^{1/2}$ is the PMD delay.

In that case,

$$PMD = B\sqrt{hL} \quad (23)$$

and PMD increases with the square root of L .

4.10.1 Random mode coupling

When the DGD statistically varies as a function of wavelength (or optical frequency), the PMD phenomenon is in general considered stochastic. The PMD phenomenon is defined as a stochastic process because of the random mode coupling between the two PSPs.

NOTE The expression *strong mode coupling* is also used in the literature and in some International Standards for *random mode coupling*. As a preference, the expression *random mode coupling* will be used in this document.

At any one optical frequency or wavelength, there are net PSPs for the entire DUT length (such as a long fibre) that depend on the totality of local disturbances, with DGD between fast and slow PSPs. Averaging the PSPs over a wavelength range, thermal range or time then yields the concept of sampling the population of disturbances and their spatial distribution (in relation to the local birefringence). The PMD is then essentially a statistical parameter characterised by the statistics of the PSPs and the statistics of DGD. In the former case, the PDV components ($\Omega_1, \Omega_2, \Omega_3$) are known to obey a PDF following a Gaussian distribution similar to Equation (22), while in the latter case, the DGD (the square root of the sum of the squared Ω_i) is known to obey a PDF following a Maxwell distribution:

$$PDF(\Delta\tau) = \left(\frac{2}{\sqrt{\pi}} \right) \frac{\Delta\tau^2}{\sigma^3} e^{-\Delta\tau^2/2\sigma^2} \quad (24)$$

If the quadratic mean value $\langle\Delta\tau^2\rangle$ is considered, then

$$\sigma = \frac{\langle\Delta\tau^2\rangle^{1/2}}{\sqrt{3}} \quad (25)$$

and [5]

$$PDF(\Delta\tau) = 3 \left(\frac{6}{\pi} \right) \frac{\Delta\tau^2}{\langle\Delta\tau^2\rangle^3} e^{-3\Delta\tau^2/2\langle\Delta\tau^2\rangle} \quad (26)$$

and PMD will be equal to the RMS DGD:

$$PMD = \langle\Delta\tau^2\rangle^{1/2} \quad (27)$$

If the mean value $\langle \Delta \tau \rangle$ is considered, then

$$\sigma = \left(\sqrt{\frac{\pi}{8}} \right) \langle \Delta \tau \rangle \quad (28)$$

and

$$PDF(\Delta \tau) = \frac{32}{\pi^2} \frac{\Delta \tau^2}{\langle \Delta \tau \rangle^3} e^{-4\Delta \tau^2 / \pi \langle \Delta \tau \rangle^2} \quad (29)$$

and the PMD is equal to the mean value of the DGD (see Figure 8).

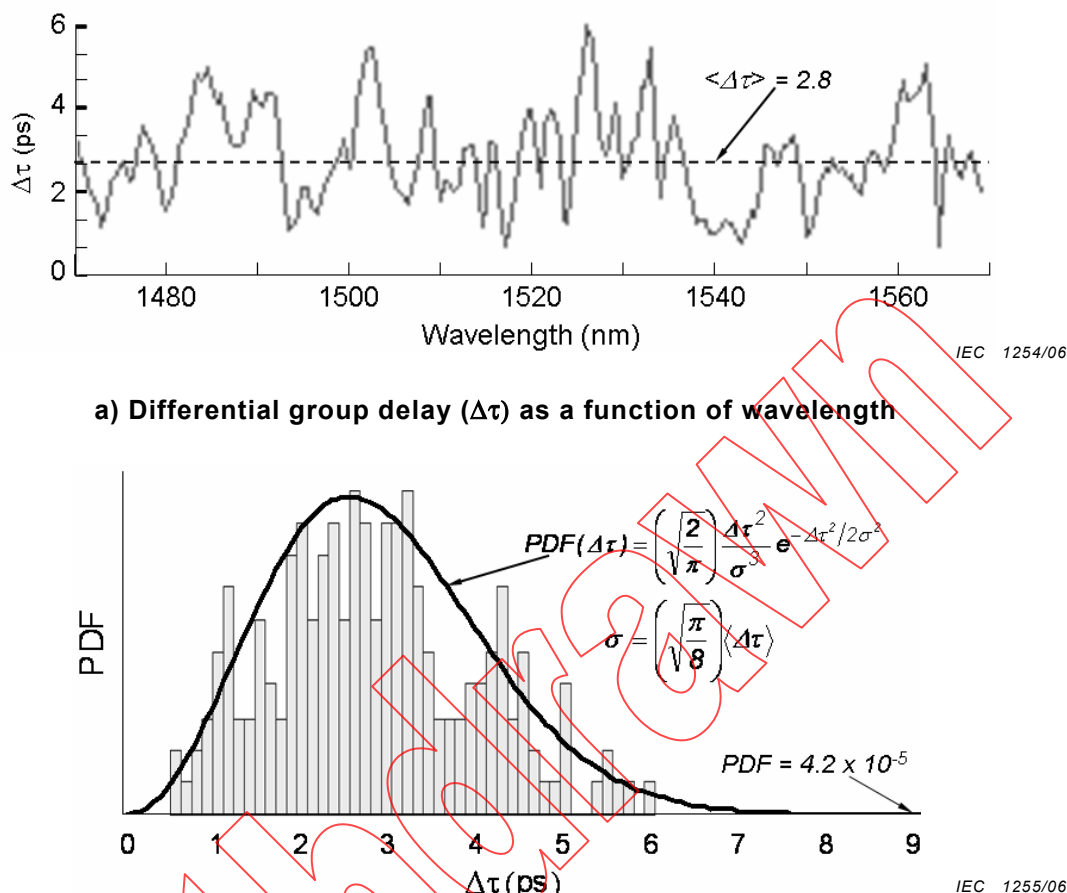
$$PMD = \langle \Delta \tau \rangle \quad (30)$$

The RMS DGD, $\langle \Delta \tau^2 \rangle^{1/2}$, and the mean DGD $\langle \Delta \tau \rangle$ are both related as follows [9, 16]:

$$\langle \Delta \tau^2 \rangle = \frac{3\pi}{8} \langle \Delta \tau \rangle^2 \quad (31)$$

The statistics of mean or RMS value of the DGD variation is analysed over a wavelength (or optical frequency³) range (as wide as possible) (see Figure 8).

³ The RMS value of the spectral variation of the DGD (or PDV length or modulus) $PMD_{RMS} = \sqrt{\langle |\hat{\Omega}(\omega)|^2 \rangle}$ is generally more representative of the mathematics of the PMD.



b) Probability density function (PDF) (histogram) as a function of $\Delta\tau$ from (a).
Maxwell PDF is fit to the experimental results

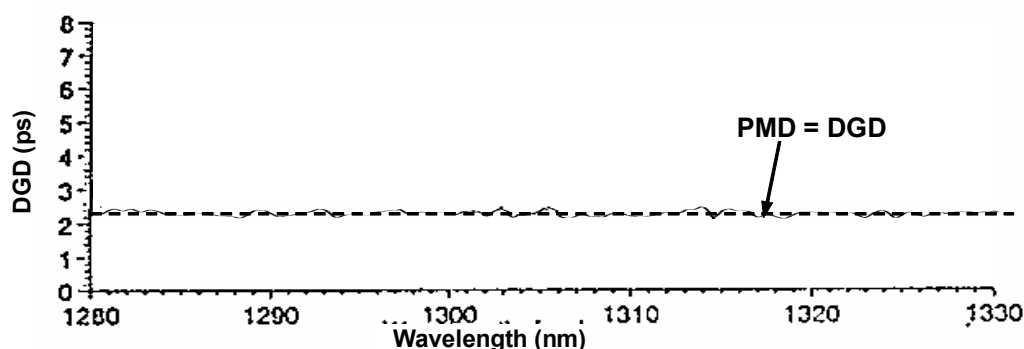
Figure 8 – Statistics of differential group delay and related Maxwell distribution [15]

4.10.2 Negligible mode coupling

In the case where the birefringence inside the DUT remains relatively fixed (fixed PSP orientation along the DUT as shown in Figure 6), the two PSP PMs are not coupled or negligibly coupled together.

NOTE The expression *weak mode coupling* is also used in the literature and in some International Standards for negligible mode coupling. As a preference, the expression *negligible mode coupling* will be used in this document.

As a consequence, DGD does not vary as a function of wavelength (or optical frequency), as shown in Figure 9.



IEC 1256/06

Figure 9 – Polarization mode dispersion and differential group delay in negligible mode coupling

As most optical components have extremely small physical lengths as compared to fibre lengths, PMD of such components may then be reported in units of ps only. In complex components and optical amplifiers, the DGD may vary as a function of wavelength (or frequency) even if this variation is smooth, small or sometimes predictable. In that case, the concept of PMD expressed as the RMS value of the variation of the DGD as a function of wavelength (or optical frequency) and the concept of maximum value of that DGD variation can be used.

There can also be intermediate cases where the device has a few birefringent sections, and the DGD can vary less strongly as a function of wavelength, such as a monotonous or sine wave variation. Some components, such as some array waveguide gratings (AWG), may exhibit such behaviour. In that case, the PMD is still the mean or RMS value of the DGD distribution, and the phenomenon is still somehow deterministic, but the mode coupling is neither negligible nor random.

There are also DUT classes where the issue of random mode coupling or not is irrelevant. Such a device is one with multiple optical paths for different frequencies, such as an optical multiplexer, where the DGD for each frequency is largely independent of the others. Some Bragg grating DUT could also fit into this category. Clause C.4 describes the treatment of such a device.

4.11 Second-order polarization mode dispersion

If the DGD and PSPs were wavelength independent, their knowledge would be sufficient to characterise propagation through a DUT, whatever the spectral width of the input signal. However, the DGD can change within a frequency increment. Moreover, as PMD increases in random mode coupling, the DGD varies more rapidly in its magnitude and the PSPs more rapidly in their orientation. Considering that first-order PMD (PMD_1) is found from Equations (16) and (18), second-order PMD (PMD_2) is this variation of the DGD and PSPs with optical frequency (or wavelength), as shown in Equation (32), when deriving Equation (16).

$$\frac{d\hat{\Omega}(\omega)}{d\omega} = \left(\hat{s}_p \frac{d\Delta\tau(\omega)}{d\omega} \right) + \left(\Delta\tau \frac{d\hat{s}_p(\omega)}{d\omega} \right) \quad (32)$$

PMD₂ term

PMD_2 was identified and described in some detail several years ago [17], but more recent work has provided a deeper appreciation of its potential importance for realistic communications links [5,18].

There has been a common misconception that PMD_2 was simply derived from the slope of a DGD-versus- ω plot. The fact that the derivative is of a vector (see Equation (32)) rather than a scalar introduces an additional term proportional to the frequency dependence of the PSP orientation. For the very practical case of a long telecommunication fibre, this contribution can be shown to account for about 8/9 of the total PMD_2 effect [18].

On the right-hand side of Equation (32), there are two terms present in the definition of PMD_2 . The first term is in the direction of the fast PSP and is often referred to as the parallel term, while the second term is in the direction perpendicular to the PSP and is often referred to as the perpendicular term.

As an example, a single waveplate or PMF does not (ideally) exhibit any PMD_2 , since its DGD does not change significantly over a wavelength range (see Figure 9) (apart from the small variation due to chromatic dispersion), and since the orientation of its fast and slow axes do not change as the input wavelength is varied (see Figure 6). This will be the case for a simple component where the DGD barely varies as a function of wavelength. If the waveplate orientation was changed in response to changes in wavelength, this would introduce PMD_2 , which could be the case for more complex devices.

On a more rigorous mathematical basis, the combined effect of PMD_1 and PMD_2 on the output-SOP can be easily pictured on the Poincaré sphere (see Figure 10).

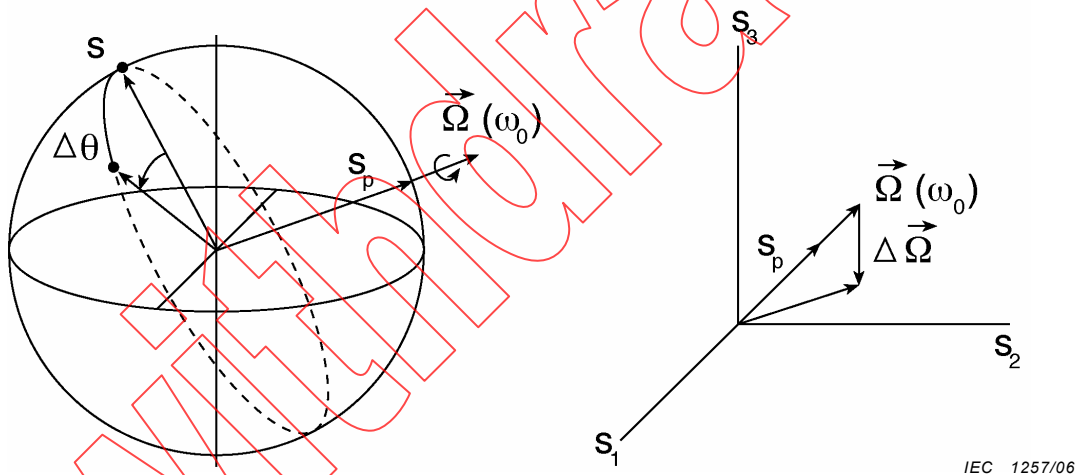


Figure 10 – Effects of first-order polarization mode dispersion (PMD_1) and second-order polarization mode dispersion (PMD_2) on the output state of polarization on the Poincaré sphere

At a given angular optical frequency, ω_0 , PMD_1 is characterised by the PDV as shown in Equations (16) and (18).

The vector, $\hat{\Omega}$, lies along the direction of the output fast PSP, $\hat{s}_p$, and its modulus, $|\hat{\Omega}|$, is equal to the DGD as shown in Equation (18). Specifically, PMD_1 is defined in terms of the RMS value of the modulus of $\hat{\Omega}$, viz

$$PMD_1 = \sqrt{\langle |\hat{\Omega}|^2 \rangle} = \sqrt{\langle \Delta\tau^2 \rangle} \quad (33)$$

On the sphere, $\hat{s}$ "precesses" around $\hat{\Omega}$ at a rate $d\theta/d\omega = \Delta\tau$ (like a spin around a static magnetic field). On the other hand, PMD_2 is characterised by the ω -derivative of $\hat{\Omega}$, its velocity vector, and its modulus is the PDV velocity (DVV). From Equation (32), we note again that both the variations of DGD and PSPs are included. Following [18], PMD_2 is defined in analogy to PMD_1 as proportional to the RMS value of the ω -derivative of $\hat{\Omega}$ Equation (32) or the RMS value of the DVV.

$$PMD_2 = \sqrt{\left\langle \left| \frac{d\hat{\Omega}}{d\omega} \right|^2 \right\rangle} \quad (34)$$

The fundamental units of PMD_2 are s^2 , as for chromatic dispersion. More convenient units are ps^2 or ps/nm with a slightly different formula (see Equation (35)).

$$PMD_2 = \frac{2\pi c}{\lambda^2} \sqrt{\left\langle \left| \frac{d\hat{\Omega}}{d\omega} \right|^2 \right\rangle} \quad (35)$$

As with chromatic dispersion, if the derivative of the PDV with respect to frequency is measured, a minimum and maximum value will be found. The polarization dependent chromatic dispersion (PDCD) is related to the DVV (the modulus of the ω -derivative of $\hat{\Omega}$, the PDV) as follows:

$$PDCD = CD_{\max} - CD_{\min} = DVV = \left| \frac{d\hat{\Omega}}{d\omega} \right| \quad (36)$$

Equation (36) holds in case of negligible PDL/PDG.

From the above discussion, it is clear that it is the behaviour of the $\hat{\Omega}$ vector, both in its magnitude and in its optical-frequency dependence, which underlies both PMD_1 and PMD_2 . Thus, it is not surprising that the measurable effects of both phenomena can be related to each other in terms of this vector. In fact, it can be shown [5,18] that in the (very realistic) limit of fibres with length much longer than the coupling length (random mode coupling), the following relation holds:

$$\left\langle \left| \frac{d\hat{\Omega}}{d\omega} \right|^2 \right\rangle = \frac{1}{3} \langle |\hat{\Omega}|^2 \rangle^2 \quad (37)$$

Combining Equations (34), (35) and (37), the following relation is obtained between PMD_1 and PMD_2 :

$$PMD_2 = \frac{1}{\sqrt{3}} (PMD_1)^2 \quad (38)$$

5 Mathematical formulations of the polarization mode dispersion test methods

This clause is organized to present first the Stokes parameter evaluation (polarimetric), then the methods most similar (in polarimetry), then those in the frequency domain, and finally the time domain method.

There are two generic categories of PMD test methods: one that determines DGD and PMD, and another that only determines PMD.

DGD and PMD determination is covered by the following methods:

- Stokes parameter evaluation (SPE):
 - o Jones matrix eigenanalysis (JME),
 - o Poincaré sphere analysis (PSA);
- Modulation phase shift (MPS);
- Polarization phase shift (PPS);
- Poincaré sphere arc method (PS);
- Poole formula.

PMD-only determination is covered by the following methods:

- Fixed analyser (FA):
 - o Extrema counting (EC),
 - o Fourier transform (FT);
- Interferometric method (INTY).

5.1 Stokes parameter evaluation

The SPE method determines PMD by measuring a polarimetric response of a DUT to a change of narrowband light across a wavelength range. For each wavelength, three distinct and known SOPs, the easiest being the linear (orthogonal on the Poincaré sphere) SOPs at nominally 0°, 45°, and 90°, are launched into the DUT, and the output Stokes vectors transmitted through the DUT are obtained. The change of these Stokes vectors with angular optical frequency, ω , (or wavelength) and, with change in input SOPs, yields the DGD as a function of wavelength through relationships that are based on Equations (12) and (18).

In the JME approach, the output Stokes vectors are transformed into Jones matrices [4], with an appropriate combination of the matrices at adjacent wavelengths. The resulting eigenvalues are calculated followed by the application of an argument function to obtain the DGD at the base frequency.

In the PSA approach, matrix algebra is performed on the normalised output Stokes vectors to infer the output Stokes vector associated with circular birefringence at two adjacent wavelengths, followed by the application of an arcsine formula to obtain the DGD at the base frequency.

For both JME and PSA, the average of the DGD values for a given wavelength range yields the PMD value over that range. The JME and PSA approaches are mathematically and formalistically equivalent.

5.1.1 Jones matrix eigenanalysis

Any Jones vector can be completely specified by a magnitude, an absolute phase as shown in Equation (3), and the Jones matrix of a DUT can be determined following 4.4. Typically, a linear SOP parallel to the x (0°) axis, a linear SOP parallel to the y (90°) axis, and parallel to the bisector of the angle between the positive x and y axes (45°) are used as stimuli to the DUT.

To obtain the output Jones vectors, the output Stokes vectors are measured using the stimuli, then normalised to unit length. These are then converted to Jones vectors using Equations (8) and (9), assuming, for example, that $0 \leq \theta \leq \pi$. Following Equation (6), four complex ratios, not affected by the assumption on θ , and independent of the intensities of the three SOPs, can then be formed

The transmission Jones matrix, T , is then given by Equation (5).

By definition of the PSP, a general device has associated with it a pair of input principal SOPs. $\hat{x}(\omega)$, which, as the input SOP is held constant while the angular optical frequency, ω , is changed by a small increment, results in a pair of output PSPs whose unit vectors are invariant to first order over ω . For a general transmission Jones matrix, $T(\omega)$, an output PSP can be expressed as a magnitude, $\sigma(\omega)$, and absolute phase, $\phi(\omega)$, times a unit vector, $\hat{y}(\omega)$, which specifies the output PSP:

$$y(\omega) = T(\omega)\hat{x}(\omega) = \sigma(\omega)e^{i\phi(\omega)}\hat{y}(\omega) \quad (39)$$

$\sigma(\omega)$ and $\phi(\omega)$ may vary with ω , but $\hat{y}(\omega)$ is frequency-invariant to first order by definition of the output PSP. Using primes to denote differentiation with respect to ω , differentiation of Equation (39) results in:

$$y'(\omega) = T'(\omega)\hat{x}(\omega) = \left(\frac{\sigma'}{\sigma} + i\phi' \right) y + \sigma(\omega)e^{i\phi(\omega)}\hat{y}'(\omega) \quad (40)$$

The first derivative of the absolute phase ϕ' is the group delay, τ_g , through the device. If the device is not perfectly polarizing, its transmission matrix, T , is non-singular, and the input can be expressed in terms of the output as $\hat{x} = T^{-1}y$. Explicitly setting $\hat{y}'$ to zero, the following eigenvalue relation is obtained:

$$\sigma(\omega)e^{i\phi(\omega)}\hat{y}'(\omega) = \left[T'T^{-1} - \left(\frac{\sigma'}{\sigma} + i\tau_g \right) I \right] y = 0 \quad (41)$$

where I is the identity matrix, a square matrix with values of one on the diagonal and zero elsewhere.

The imaginary parts of the eigenvalues of the matrix product are the group delays associated with the PSPs, and the DGD $\Delta\tau$ which leads to PMD is given by the difference of the imaginary parts of the two eigenvalues. The output PSP themselves are the eigenvectors of $T'T^{-1}$, which may be non-unitary for devices with PDL, in which case the output PSPs are not necessarily orthogonal.

Measurement of T' and T , including measurement of the absolute phase, would allow direct calculation of the two group delays and $\Delta\tau$, but in practice, two restrictions are imposed by the Jones matrix measurement technique previously described. Instead of measuring T' directly, T' must be approximated as $T' \approx [T(\omega+\Delta\omega) - T(\omega)]/\Delta\omega$ for a finite $\Delta\omega$. If the angular optical frequency increment $\Delta\omega$ is small enough so that each output PSP suffers nearly the same loss at ω and $\omega + \Delta\omega$, then $\sigma' \Delta\omega / \sigma \approx 0$ and Equation (42) can be rewritten as

$$[T(\omega + \Delta\omega)T^{-1}(\omega) - (1 + i\tau_g\Delta\omega)]y = 0 \quad (42)$$

The second restriction arises from the fact that $T(\omega+\Delta\omega)T^{-1}(\omega)$, and therefore its associated eigenvalues ρ_1 and ρ_2 can be determined only to within a complex constant, preventing determination of the two group delays individually. When the loss through the device is independent of polarization, the eigenvalues of $T(\omega+\Delta\omega)T^{-1}(\omega)$ are determined to be $\beta\rho_1$ and $\beta\rho_2$, where β is a complex constant and $\rho_k = \exp(i\tau_{g,k}\Delta\omega)$. The DGD $\Delta\tau$ can therefore be expressed as

$$\Delta\tau = |\tau_{g,1} - \tau_{g,2}| = \left| \frac{\text{Arg}\left(\frac{\rho_1}{\rho_2}\right)}{\Delta\omega} \right| \quad (43)$$

where ρ_1 and ρ_2 are the eigenvalues of $T(\omega+\Delta\omega)T^{-1}(\omega)$ and Arg denotes the argument function, that is, $\text{Arg}(\alpha e^{i\theta}) = \theta$. In the presence of PDL, the eigenvalues can still be approximated as

$$\rho_k = 1 + i\tau_{g,k}\Delta\omega \approx e^{i\tau_{g,k}\Delta\omega} \quad (44)$$

In practice, the impact of PDL on the measurement of $\Delta\tau$ can be reduced by using smaller $\Delta\omega$ increments. In all cases, the condition $\Delta\tau \bullet \Delta\omega < \pi$ must be satisfied in order to avoid the ambiguities of the multiple-valued argument function.

NOTE The JME method does not correspond to the fundamental Jones mathematical treatment, such as

$$\frac{d|E\rangle}{d\omega} = A|E\rangle \quad (45)$$

where

A is the matrix of equivalent device defined as

$$A = \left(\frac{dT}{d\omega} \right) \bullet T^{-1} \quad (46)$$

where

T is the Jones transfer matrix

By definition,

$$\text{DGD} = |\text{Im}(\text{eigenvalue difference of } A)| \quad (47)$$

eigenvalue difference of $A = a_1 - a_2$

where

a_1 is one eigenvalue of A ;

a_2 is the other eigenvalue of A .

$$\text{PSPs} = 2 \text{ eigenvectors of } A = \pm \frac{\hat{\Omega}}{|\hat{\Omega}|} \quad (48)$$

The JME method uses the following approximation:

$$\left[\frac{T(\omega + \Delta\omega) - T(\omega)}{\Delta\omega} \right] \bullet T(\omega)^{-1} = \frac{\{T(\omega + \Delta\omega) \bullet T(\omega)^{-1}\} - 1}{\Delta\omega} = A_{\text{measured}} \approx A \quad (49)$$

JME is measuring $T(\omega + \Delta\omega)T(\omega)^{-1}$ and by definition,

$$DGD_{\text{measured}} = |\text{Im}(\text{eigenvalue difference of } A_{\text{measured}})| \quad (50)$$

$$\text{eigenvalue difference of } A_{\text{measured}} = (a_1 - a_2)_{\text{measured}} = \rho_1 - \rho_2$$

The JME method is not based on finite differences.

JME measures the polarizations, not the intensities, nor the phases and does not obtain the true Jones matrix, but with a factor β (complex number, not constant with λ):

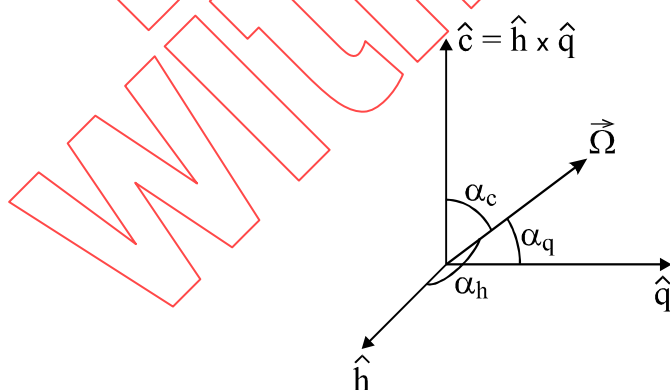
$$\begin{aligned} a_1 (\text{or } \rho_1) &= \beta_1 e^{i\left(\frac{DGD\Delta\omega}{2}\right)} \\ a_2 (\text{or } \rho_2) &= \beta_2 e^{i\left(\frac{DGD\Delta\omega}{2}\right)} \\ \frac{a_1}{a_2} &= \left(\frac{\beta_1}{\beta_2}\right) e^{iDGD\Delta\omega} = \left|\frac{\beta_1}{\beta_2}\right| e^{i\left[\arg\left(\frac{a_1}{a_2}\right)\right]} \end{aligned} \quad (51)$$

then

$$DGD = |\text{Im}(a_1 - a_2)| = \frac{\left|\arg\left(\frac{a_1}{a_2}\right)\right|}{\Delta\omega} \quad (52)$$

5.1.2 Poincaré sphere analysis

Let $\hat{h}$ and $\hat{q}$ be the output Stokes vectors measured for two linear input SOPs oriented at 45° from each other (90° on the Poincaré sphere), and $\hat{c} = \hat{h} \times \hat{q}$ the cross product of the two measured response vectors, as shown in Figure 11 [19].



IEC 1258/06

Figure 11 – Rectangular system of co-ordinates defined by the response Stokes vectors, and direction angles of the polarization dispersion vector

Assuming negligible PDL, the transformation of Stokes vectors from input to output is a rotation (angles between vectors are preserved), and the equation of motion of any one of the response Stokes vector (including $\hat{c}$) is given by Equation (12) with

$$\hat{s} = \hat{h}, \hat{q} \text{ or } \hat{c} \quad (53)$$

The square modulus of the derivative in Equation (12) can be written as

$$\left(\frac{d\hat{s}}{d\omega} \right)^2 = |\hat{\Omega}|^2 \sin^2(\alpha) \quad (54)$$

where α is the angle between $\hat{s}$ and $\hat{\Omega}$.

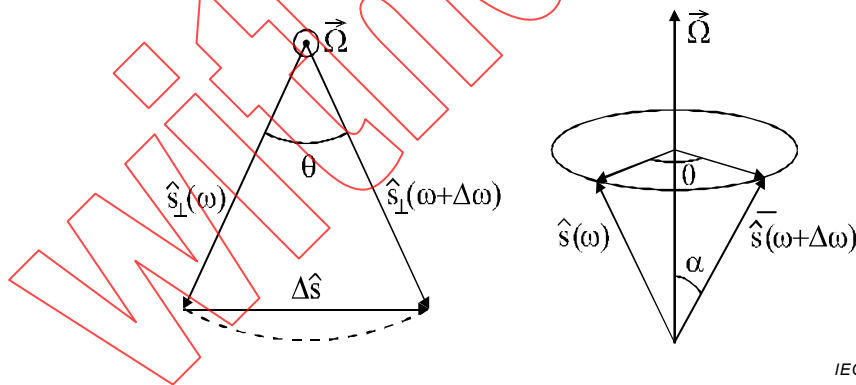
But $\hat{h}$ and $\hat{q}$ are mutually orthogonal, since angles are preserved from input to output. Thus $\hat{h}$, $\hat{q}$ and $\hat{c}$ define a rectangular system of co-ordinates (see Figure 11) in which $\cos(\alpha_h)$, $\cos(\alpha_q)$ and $\cos(\alpha_c)$ are the direction cosines of $\hat{\Omega}$. Using the sum rule,

$$\cos^2(\alpha_h) + \cos^2(\alpha_q) + \cos^2(\alpha_c) = 1 \quad (55)$$

In Equation (56), the following expression of DGD is obtained:

$$|\hat{\Omega}| = \sqrt{\frac{1}{2} \left[\left(\frac{d\hat{h}}{d\omega} \right)^2 + \left(\frac{d\hat{q}}{d\omega} \right)^2 + \left(\frac{d\hat{c}}{d\omega} \right)^2 \right]} \quad (56)$$

If only finite differences $\Delta\hat{s}/\Delta\omega$ are known instead of derivatives ($\Delta\omega \rightarrow 0$), an approximation of DGD must be obtained that is more accurate than Equation (56). Assuming that the PDV is independent of ω over the increment $[\omega, \omega + \Delta\omega]$, the output SOP follows an arc of a circle as shown in Figure 12.



IEC 1259/06

Figure 12 – Arc of a circle described by the output state of polarization in the increment $[\omega, \omega + \Delta\omega]$

In Figure 12, $\hat{s}_\perp$ is the component of $\hat{s}$ perpendicular to $\hat{\Omega}$ and θ the angle swept by $\hat{s}_\perp$. Also, as a consequence of the equation of motion (12), we have

$$|\hat{s}_\perp| = |\sin(\alpha)| \quad (57)$$

$$\theta = |\hat{\Omega}| \Delta\omega \quad (58)$$

Through simple geometry (see Figure 12),

$$|\Delta \hat{s}| = 2|\hat{s}_\perp| \left| \sin\left(\frac{\theta}{2}\right) \right| \quad (59)$$

is obtained, so that

$$|\Delta \hat{s}| = 2 \left| \sin(\alpha) \sin\left(\frac{|\hat{\Omega}| \Delta \omega}{2}\right) \right| \quad (60)$$

The sum of the square modulus of $\Delta \hat{h}$, $\Delta \hat{q}$ and $\Delta \hat{c}$ is then written as

$$\Delta \hat{h}^2 + \Delta \hat{q}^2 + \Delta \hat{c}^2 = 4 \sin^2\left(\frac{|\hat{\Omega}| \Delta \omega}{2}\right) \cdot [\sin^2(\alpha_h) + \sin^2(\alpha_q) + \sin^2(\alpha_c)] \quad (61)$$

Finally, we apply the sum rule of Equations (56) to (61) to obtain the following expression of DGD:

$$|\hat{\Omega}| = \frac{2}{\Delta \omega} \arcsin \left[\frac{1}{2} \sqrt{\frac{1}{2} (\Delta \hat{h}^2 + \Delta \hat{q}^2 + \Delta \hat{c}^2)} \right] \quad (62)$$

$$PSP = \frac{\pm \hat{u}}{|\hat{u}|} \text{ with } \hat{u} = (\bar{c} \cdot \Delta \hat{q}) \bar{h} + (\bar{h} \cdot \Delta \hat{c}) \bar{q} + (\bar{q} \cdot \Delta \hat{h}) \bar{c} \quad (63)$$

where $\bar{c}$, $\bar{h}$ and $\bar{q}$ are the mean of the Stokes vectors measured at ω and $(\omega + \Delta \omega)$, i.e.

$$\bar{s} = \frac{\hat{s}(\omega) + \hat{s}(\omega + \Delta \omega)}{2}$$

5.1.3 Calculation of differential group delay

In the following example, the DGD of a DUT is found from the Jones matrix or directly from the Stokes vectors obtained at a pair of input optical frequencies or wavelengths. The hypothetical DUT is described in terms of Mueller matrices at these optical frequencies or wavelengths.

The DUT chosen for this example has linear PSPs at $+45^\circ$ and -45° in the wavelength range of interest. The 4x4 Mueller matrix, M , for such DUT is [20]

$$M(\omega) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \delta & 0 & -\sin \delta \\ 0 & 0 & 1 & 0 \\ 0 & \sin \delta & 0 & \cos \delta \end{bmatrix} \quad (64)$$

where δ is the retardance at the angular optical frequency ω in rad/s.

Assuming the DUT is non-dispersive for the two PSPs, the retardance is related to the DGD $\Delta\tau$ by

$$\delta = \omega\Delta\tau \quad (65)$$

Assuming that the Jones matrix or the Stokes vectors of the DUT are obtained at frequencies $\omega_1 = 1,21 \times 10^{15}$ rad/s (1 556,734 nm) and $\omega_2 = \omega_1 + \Delta\omega = 1,22 \times 10^{15}$ rad/s (1 543,974 nm), and that the DGD of the DUT is $\Delta\tau(\omega_1) = 100$ fs, the Mueller matrices at these optical frequencies are

$$M_1 = M(\omega_1) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 121 & 0 & -\sin 121 \\ 0 & 0 & 1 & 0 \\ 1 & \sin 121 & 0 & \cos 121 \end{bmatrix} \quad (66)$$

$$M_2 = M(\omega_2) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 122 & 0 & -\sin 122 \\ 0 & 0 & 1 & 0 \\ 0 & \sin 122 & 0 & \cos 122 \end{bmatrix}$$

Of course, these matrices are unknown beforehand.

5.1.3.1 Jones matrix eigenanalysis

In the JME approach, PMD determination requires a Jones matrix at each of two wavelengths. The Jones matrix at a particular wavelength is found from the output SOPs corresponding to each of three input SOPs. The three chosen input SOPs are as follows:

$$0^\circ \Rightarrow x_a = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \quad 45^\circ \Rightarrow x_b = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \quad 90^\circ \Rightarrow x_c = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix} \quad (67)$$

The three output SOPs, related to the input ones by $y = M_1 x$, are obtained at ω_1 :

$$y_{a1} = \begin{bmatrix} 1 \\ \cos 121 \\ 0 \\ \sin 121 \end{bmatrix} \quad y_{b1} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \quad y_{c1} = \begin{bmatrix} 1 \\ -\cos 121 \\ 0 \\ -\sin 121 \end{bmatrix} \quad (68)$$

The three other output SOPs, related to the input ones by $y = M_2 x$, are obtained at ω_2 :

$$y_{a2} = \begin{bmatrix} 1 \\ \cos 122 \\ 0 \\ \sin 122 \end{bmatrix} \quad y_{b2} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \quad y_{c2} = \begin{bmatrix} 1 \\ -\cos 122 \\ 0 \\ -\sin 122 \end{bmatrix} \quad (69)$$

The Jones matrix T_1 is calculated at ω_1 , starting by converting the three output SOPs at ω_1 to the following Jones vectors:

$$v_{a1} = \begin{bmatrix} 0,689687 \\ 0,724108i \end{bmatrix} \quad v_{b1} = \begin{bmatrix} 0,707107 \\ 0,707107 \end{bmatrix} \quad v_{c1} = \begin{bmatrix} 0,724108 \\ 0,689687i \end{bmatrix} \quad (70)$$

From the Jones vectors, the following complex ratios are formed in order to populate the Jones matrix:

$$k_1 = \frac{0,689687}{0,724108i} = -0,952465i \quad k_2 = \frac{0,707107}{0,707107} = 1 \quad (71)$$

$$k_3 = \frac{0,724108}{-0,689687i} = 1,049910i \quad k_4 = \frac{k_3 - k_2}{k_1 - k_3} = -0,524332 - 0,499408i$$

The following Jones matrix T_1 is obtained, where β_1 is an unknown complex constant:

$$T_1 = \beta_1 \begin{bmatrix} k_1 k_4 & k_2 \\ k_4 & 1 \end{bmatrix} = \begin{bmatrix} (-0,475668 + 0,499408i) & 1 \\ (-0,524332 - 0,499408i) & 1 \end{bmatrix} \quad (72)$$

Similarly, the Jones matrix T_2 is obtained by converting the Stokes output vectors at ω_2 to Jones vectors:

$$v_{a2} = \begin{bmatrix} 0,258102 \\ 0,966118i \end{bmatrix} \quad v_{b2} = \begin{bmatrix} 0,707107 \\ 0,707107 \end{bmatrix} \quad v_{c2} = \begin{bmatrix} 0,966118 \\ 0,258102i \end{bmatrix} \quad (73)$$

Again, from the Jones vectors, the following complex ratios are formed in order to populate the found Jones matrix T_2 where β_2 is an unknown complex constant:

$$k_1 = \frac{0,258102}{0,966118i} = -0,267153i \quad k_2 = \frac{0,707107}{0,707107} = 1 \quad (74)$$

$$k_3 = \frac{0,966118}{-0,258102i} = 3,74317i \quad k_4 = \frac{k_3 - k_2}{k_1 - k_3} = -0,933384 - 0,249357i$$

$$T_2 = \beta_2 \begin{bmatrix} k_1 k_4 & k_2 \\ k_4 & 1 \end{bmatrix} = \begin{bmatrix} (-0,0666165 + 0,249357i) & 1 \\ (-0,933384 - 0,249357i) & 1 \end{bmatrix} \quad (75)$$

Since β_1 and β_2 do not affect the eigenvalue ratio or the eigenvectors, they are dropped.

Next, the product $T_2 T_1^{-1}$ is calculated:

$$A = T_2 T_1^{-1} = \begin{bmatrix} (0,770151 - 0,420735i) & (0,229849 + 0,420735i) \\ (0,229849 + 0,420735i) & (0,770151 - 0,420735i) \end{bmatrix} \quad (76)$$

The eigenvalues of A are $\rho_1 = 1$ and $\rho_2 = 0,540302 - 0,841471 i$.

The corresponding eigenvectors are

$$z_1 = \begin{bmatrix} 0,707107 \\ 0,707107 \end{bmatrix} \quad z_2 = \begin{bmatrix} 0,707107 \\ -0,707107 \end{bmatrix} \quad (77)$$

This can be seen by confirming that $\rho_1 z_1 = A z_1$ and $\rho_2 z_2 = A z_2$ (the set of solutions is unique).

Eigenvectors z_1 and z_2 are Jones vector representations of $+45^\circ$ and -45° linear SOPs, so they correctly identify the output PSPs.

Now the DGD $\Delta\tau$ is found from

$$\Delta\tau(\omega_1) = \left| \frac{\text{Arg}\left(\frac{\rho_1}{\rho_2}\right)}{\omega_2 - \omega_1} \right| = \left| \frac{\text{Arg}\left(\frac{1}{0,540302 - 0,841471i}\right)}{0,01 \times 10^{15} \text{ rad/s}} \right| \quad (78)$$

The expression $1/(0,540302 - 0,841471i)$ has magnitude 1 and polar angle 1 rad.

$$\Delta\tau(\omega_1) = \left| \frac{1 \text{ rad}}{0,01 \times 10^{15} \text{ rad/s}} \right| = 100 \text{ fs} \quad (79)$$

The DGD has correctly been found.

5.1.3.2 Poincaré sphere analysis

From the Stokes vectors $\hat{h}$, $\hat{v}$ and $\hat{q}$, the vector products $\hat{c} = \hat{h} \times \hat{q}$ and $\hat{c}' = \hat{q} \times \hat{v}$ are formed at each wavelength. For each wavelength increment, the finite differences are computed:

$$\begin{aligned} \Delta\hat{h} &= \hat{h}(\omega + \Delta\omega) - \hat{h}(\omega) & \Delta\hat{q} &= \hat{q}(\omega + \Delta\omega) - \hat{q}(\omega) & \Delta\hat{v} &= \hat{v}(\omega + \Delta\omega) - \hat{v}(\omega) \\ \Delta\hat{c} &= \hat{c}(\omega + \Delta\omega) - \hat{c}(\omega) & \Delta\hat{c}' &= \hat{c}'(\omega + \Delta\omega) - \hat{c}'(\omega) \end{aligned} \quad (80)$$

The DGD $\Delta\tau$ is found for a particular wavelength increment from the following expression:

$$\Delta\tau = \frac{1}{\Delta\omega} \cdot \left\{ \arcsin \left[\frac{1}{2} \sqrt{\frac{1}{2} (\Delta\hat{h}^2 + \Delta\hat{q}^2 + \Delta\hat{c}^2)} \right] + \arcsin \left[\frac{1}{2} \sqrt{\frac{1}{2} (\Delta\hat{q}^2 + \Delta\hat{v}^2 + \Delta\hat{c}'^2)} \right] \right\} \quad (81)$$

Each DGD value is taken to represent the DGD at the midpoint of the corresponding wavelength increment.

Now the mean Stokes vectors are formed as follows:

$$\begin{aligned} \hat{h} &= \frac{\hat{h}(\omega) + \hat{h}(\omega + \Delta\omega)}{2} & \hat{q} &= \frac{\hat{q}(\omega) + \hat{q}(\omega + \Delta\omega)}{2} & \hat{v} &= \frac{\hat{v}(\omega) + \hat{v}(\omega + \Delta\omega)}{2} \\ \hat{c} &= \frac{\hat{c}(\omega) + \hat{c}(\omega + \Delta\omega)}{2} & \hat{c}' &= \frac{\hat{c}'(\omega) + \hat{c}'(\omega + \Delta\omega)}{2} \end{aligned} \quad (82)$$

and the scalar products are computed as follows:

$$u_h = \hat{c} \cdot \Delta\hat{q} \quad u_q = \hat{h} \cdot \Delta\hat{c} \quad u_c = \hat{q} \cdot \Delta\hat{h} \quad (83)$$

The PSPs are found from the following expression:

$$PSP = \frac{\pm \hat{u}}{|\hat{u}|} \quad \hat{u} = u_h \hat{h} + u_q \hat{q} + u_c \hat{c} \quad (84)$$

Now in the case of the hypothetical DUT defined previously, the six output Stokes vectors are

$$\begin{aligned}\hat{h}(\omega) &= \begin{bmatrix} \cos(121) \\ 0 \\ \sin(121) \end{bmatrix} & \hat{v}(\omega) &= \begin{bmatrix} -\cos(121) \\ 0 \\ -\sin(121) \end{bmatrix} & \hat{q}(\omega) &= \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \\ \hat{h}(\omega + \Delta\omega) &= \begin{bmatrix} \cos(122) \\ 0 \\ \sin(122) \end{bmatrix} & \hat{v}(\omega + \Delta\omega) &= \begin{bmatrix} -\cos(122) \\ 0 \\ -\sin(122) \end{bmatrix} & \hat{q}(\omega + \Delta\omega) &= \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}\end{aligned}\quad (85)$$

From Equation (85), the vector products $\hat{c} = \hat{h} \times \hat{q}$ and $\hat{c}' = \hat{q} \times \hat{v}$ are computed at ω and $(\omega + \Delta\omega)$:

$$\begin{aligned}\hat{c}(\omega) &= \begin{bmatrix} -\sin(121) \\ 0 \\ \cos(121) \end{bmatrix} & \hat{c}(\omega + \Delta\omega) &= \begin{bmatrix} -\sin(122) \\ 0 \\ \cos(122) \end{bmatrix} \\ \hat{c}'(\omega) &= \begin{bmatrix} -\sin(121) \\ 0 \\ \cos(121) \end{bmatrix} & \hat{c}'(\omega + \Delta\omega) &= \begin{bmatrix} -\sin(122) \\ 0 \\ \cos(122) \end{bmatrix}\end{aligned}\quad (86)$$

and the square modulus of the finite differences is found as follows:

$$\begin{aligned}\Delta\hat{v}^2 &= |\hat{v}(\omega + \Delta\omega) - \hat{v}(\omega)|^2 = [\cos(122) - \cos(121)]^2 + [\sin(122) - \sin(121)]^2 = 4 \sin^2(1/2) \\ \Delta\hat{q}^2 &= |\hat{q}(\omega + \Delta\omega) - \hat{q}(\omega)|^2 = 0 \\ \Delta\hat{c}^2 &= |\hat{c}(\omega + \Delta\omega) - \hat{c}(\omega)|^2 = [\sin(122) - \sin(121)]^2 + [\cos(122) - \cos(121)]^2 = 4 \sin^2(1/2) \\ \Delta\hat{c}'^2 &= |\hat{c}'(\omega + \Delta\omega) - \hat{c}'(\omega)|^2 = [\sin(122) - \sin(121)]^2 + [\cos(122) - \cos(121)]^2 = 4 \sin^2(1/2) \\ \Delta\hat{h}^2 &= |\hat{h}(\omega + \Delta\omega) - \hat{h}(\omega)|^2 = [\cos(122) - \cos(121)]^2 + [\sin(122) - \sin(121)]^2 = 4 \sin^2(1/2)\end{aligned}\quad (87)$$

and the DGD is found from

$$\begin{aligned}\Delta\tau &= \frac{1}{\Delta\omega} \cdot \left(\arcsin\left\{ \frac{1}{2} \sqrt{\frac{1}{2} \left[4\sin^2\left(\frac{1}{2}\right) + 0 + 4\sin^2\left(\frac{1}{2}\right) \right]} \right\} + \arcsin\left\{ \frac{1}{2} \sqrt{\frac{1}{2} \left[0 + 4\sin^2\left(\frac{1}{2}\right) + 4\sin^2\left(\frac{1}{2}\right) \right]} \right\} \right) \\ \Delta\tau &= \frac{1}{\Delta\omega} \cdot 2 \cdot \arcsin\left[\sin\left(\frac{1}{2}\right) \right] = \frac{1}{\Delta\omega} = 100 \text{ fs}\end{aligned}\quad (88)$$

This calculation yields the correct value of $\Delta\tau = 100$ fs.

The PSPs can also be computed from the following mean Stokes vectors:

$$\hat{h} = \frac{\hat{h}(\omega) + \hat{h}(\omega + \Delta\omega)}{2} = \begin{pmatrix} -0,458 \\ 0 \\ 0,749 \end{pmatrix} \quad \hat{q} = \frac{\hat{q}(\omega) + \hat{q}(\omega + \Delta\omega)}{2} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}\quad (89)$$

$$\hat{c} = \frac{\hat{c}(\omega) + \hat{c}(\omega + \Delta\omega)}{2} = \begin{pmatrix} -0,749 \\ 0 \\ -0,458 \end{pmatrix}$$

and the scalar products:

$$u_h = (\hat{c} \cdot \Delta\hat{q}) = 0 \quad u_q = (\hat{h} \cdot \Delta\hat{c}) = -0,841 \quad u_c = (\hat{q} \cdot \Delta\hat{h}) = 0 \quad (90)$$

The PSPs are found:

$$PSP = \frac{\pm \hat{u}}{|\hat{u}|} = \begin{pmatrix} 0 \\ \pm 1 \\ 0 \end{pmatrix} \quad (91)$$

The linear SOPs, PSPs have correctly been found at +45 ° and –45 °.

5.1.4 Formalistic equivalence of Poincaré sphere analysis and Jones matrix eigenanalysis

The test procedures for PSA and JME are the same. The two methods determine the DGD and the PSPs from the same raw data, i.e. the normalised Stokes vectors at the output of the device under test as a function of the angular optical frequency ω for a number of input SOPs. The PSA is performed directly in the Stokes parameter representation of the SOP, while the JME is performed in the SOP vector (or normalised Jones vector) representation, after transformation of the Stokes vectors into SOP vectors (see Figure 13).

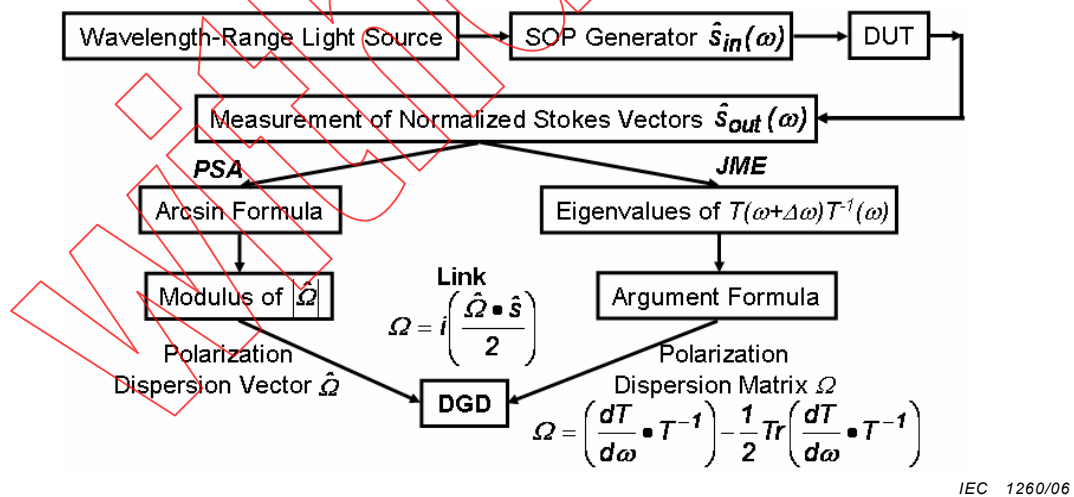


Figure 13 – Functional diagram of Stokes parameter evaluation

In the following sections, PSA and JME are demonstrated to be formally equivalent [21]. The *polarization dispersion matrix* is identified in the Jones vector representation that is equivalent to the *complex or generalised PDV* in the Stokes vector representation. A simple expression of the polarization dispersion matrix as a function of the PDV establishes the conceptual link between the two analyses. A formula is also provided for a simple calculation of DGD in low PDL conditions, which is the equivalent of the "exponential approximation" used in JME.

5.1.4.1 Fundamental link between Poincaré sphere analysis and Jones matrix eigenanalysis

The normalised Jones vector can be viewed as the quantum mechanical state-vector representing the SOP of each individual photon of a fully polarized wave. On the other hand, the normalised Stokes vector, $\hat{s}$, can be defined as the mean value of the photon angular momentum (in units of $\hbar$). Hence, $\hat{s} = \langle e | \hat{\sigma} | e \rangle$ where $|e\rangle$ is the normalised Jones vector, and $\hat{\sigma}$ is the Pauli operator.

JME and PSA are based on the equations of motion of the SOP at the output of the device under test, as a function of angular optical frequency, ω . The derivative of $\hat{s}$, $d\hat{s}/d\omega$ can be found from $d|e\rangle/d\omega$, starting from $\hat{s} = \langle e | \hat{\sigma} | e \rangle$. In order to obtain formally identical equations, it is convenient to use the polarization density matrix $\rho = |e\rangle\langle e|$ instead of the SOP-vector. This eliminates the cumbersome phase factor that appears in $|e\rangle$. The equation of motions can be written as

$$\frac{d\hat{s}}{d\omega} = \frac{1}{2} [\hat{\Omega} - i(\hat{\Omega} \times \hat{s})] \times \hat{s} + c.c. \quad \frac{d\rho}{d\omega} = [\Omega - \text{Tr}(\Omega\rho)]\rho + h.c. \quad (92)$$

where

c.c. is a complex conjugate expression;

h.c. is the hermitian conjugate.

The complex or generalised polarization dispersion vector $\hat{\Omega}$ and the polarization dispersion matrix Ω are related to the Jones matrix (T) through the following relations:

$$W = \frac{dT}{d\omega} T^{-1} \quad \Omega = W - \frac{1}{2} \text{Tr}(W) I \quad (93)$$

$$\Omega = \frac{i}{2} (\hat{\Omega} \cdot \hat{\sigma}) \quad (94)$$

where

Tr is the trace of the matrix;

I is the identity matrix.

In the basic theory of JME, the PSPs and DGD are defined as the eigenvectors and difference-of-eigenvalues of W (imaginary part). From Equation (93), the eigenvectors and difference-of-eigenvalues of Ω can be seen to be the same as those of W . The link establishing the conceptual equivalence of PSA and JME is the expression of Ω as a function of $\hat{\Omega}$ Equation (94). Simply, knowing $\hat{\Omega}$, the vector obtained via PSA, one can deduce Ω , the matrix that defines the DGD and PSPs in JME.

5.1.4.2 Comparison of results obtained through Poincaré sphere analysis and Jones matrix eigenanalysis

To illustrate the identity of the two analyses, the DGD was calculated from both PSA and JME, as a function of the optical frequency. The simulation was performed with a concatenation of 20 waveplates with random delays and orientations of the birefringence axes. In PSA, the DGD was deduced from the finite differences $\Delta\hat{s}_a$ corresponding to input SOP's $a = i, j, k$ through the following formula:

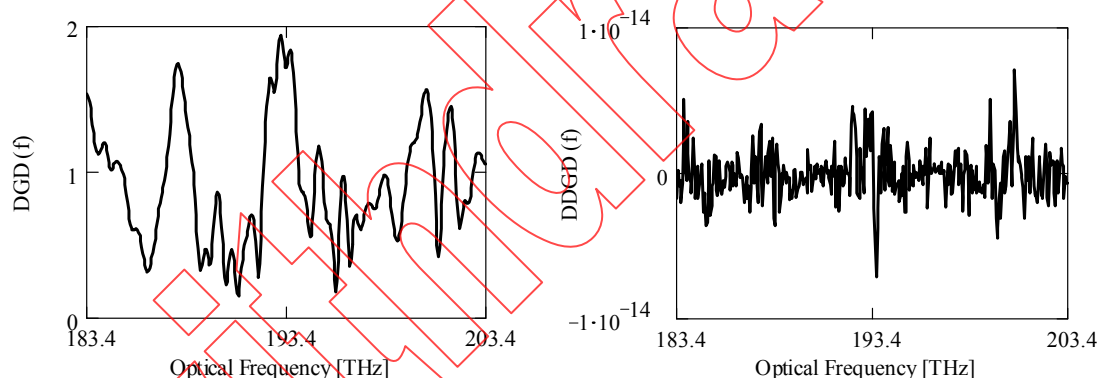
$$DGD = \frac{2}{\Delta\omega} \arcsin \left[\frac{1}{2} \sqrt{\frac{1}{2} \sum_a \Delta \hat{s}_a^2} \right] \quad (95)$$

where $\Delta\omega$ is the angular optical frequency increment.

The relative frequency increment was $\Delta\omega \cdot \Delta\tau_{\max} = 0,2\pi$. The following section provides the derivation of Equation (95). Equation (95) is obtained using the equivalent of the JME exponential approximation, which consists of assuming that the polarization dispersion matrix (vector) is constant over the frequency increment, so that the Jones matrix can be written as an exponential, i.e.

$$T(\omega + \Delta\omega) = T(\omega) \cdot e^{W\Delta\omega} \quad (96)$$

The equivalent in PSA is to take into account that the path followed by the SOP between two points is not along $\Delta\hat{s}$ but along an arc of a circle subtended by $\Delta\hat{s}$. Angles between Stokes vectors being preserved in absence of PDL (rotation), the third result (k) corresponding to circular input SOP can be deduced from the first two measurements (i, j) with linear input SOP's oriented at 0° and 45° from a given reference axis. The DGD curves obtained from the two analyses are shown in Figure 14(a).



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NOTE This simulation uses 20 concatenated waveplates with random delays and orientations of the birefringence axes. The PMD (RMS DGD) is found to be 1,01 ps when averaged over a range of 20 THz centred at 193,4 THz (1 550 nm). The relative frequency increment is $\delta f \cdot DGD_{\max} = 0,1$.

Figure 14 – a) Differential group delay (DGD) as a function of the optical frequency (f) obtained through Poincaré sphere analysis (PSA) and Jones matrix analysis (JME), and b) Difference of DGDs.

The two curves are superimposed on the graph. The difference of PMD values is $\Delta PMD = 2,2 \times 10^{-16}$. The difference between DGD's (ΔDGD) is shown in Figure 14(b) as a function of the optical frequency. Its RMS value is $\Delta DGD_{RMS} = 1,6 \times 10^{-15}$. These residual differences are accounted for by rounding errors.

The PSPs obtained from both analyses are shown in Figure 15 (trajectories).