

An American National Standard

IEEE Guide for the Statistical Analysis of Thermal Life Test Data

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of the
IEEE Dielectrics and Electrical Insulation Society

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Foreword

(This Foreword is not a part of ANSI/IEEE Std 101-1987, IEEE Guide for the Statistical Analysis of Thermal Life Test Data.)

ANSI/IEEE Std 101-1987 is substantially revised from IEEE Std 101-1972. Extensive revision was deemed necessary to reflect the widespread access of users of this document to advanced calculators and computers. Thus the tedious workbook approach used in previous versions was no longer necessary. However, Annex 3 of this document does contain a detailed worked example to provide guidance to those individuals without advanced calculators or computers.

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IEEE Guide for the Statistical Analysis of Thermal Life Test Data

1. Introduction

This revision of IEEE Std 101-1972 describes statistical analyses for data from thermally accelerated aging tests. It explains the basis and use of statistical calculations for an engineer or scientist. Statistical methods discussed in [2]¹ through [6] provide more information. Life test data analysis is dealt with more specifically in [7] through [13]. The subject of this guide is treated extensively in [11] and [12].

1.1 General Background

ANSI/IEEE Std 1-1986 (see Annex 1) discusses the principles for temperature rating of electrical insulation. These principles are carried forward in ANSI/IEEE Std 98-1972 and ANSI/IEEE Std 99-1980 (see Annex 1) which respectively outline test procedures for the experimental estimation of the life of insulating materials and systems. Life test procedures for specific insulating materials and systems are outlined in other IEEE publications (see Annex 1). Insulation life test procedures are also described in ASTM (American Society for Testing and Materials), NEMA (National Electrical Manufacturers Association), and IEC (International Electrotechnical Commission) standards (see Annex 1). Also, proposed life test procedures continue to appear in the literature. All such publications assume that the life of insulation with organic materials is a decreasing function of temperature.

Accelerated test procedures usually call for a number of specimens to be aged at each of several temperatures appreciably above normal operating temperatures. High temperatures are chosen to produce specimen failures (according to specified failure criteria) in typically one week to one year. The test objective is to determine the dependence of median life on temperature from the data and to estimate, by extrapolation, the median life to be expected at service temperature. This guide presents methods for analyzing such data and for comparing test data on different materials.

1.2 The Arrhenius Model

The Arrhenius equation gives the rate of a chemical reaction as a function of temperature. It has been adapted [1] to approximate the relationship between insulation life and temperature as follows.

¹The numbers in brackets refer to those of the bibliographic entries listed in Section 5..

The Arrhenius equation for a chemical reaction rate is

$$k = D \exp(-E/RT) \quad (1)$$

where

- k = specific reaction rate
- E = activation energy of the reaction (assumed to be constant), in calories/mol, or J/mol or electron volts
- R = Boltzmann gas constant = 1.987 calories/mol/K or 8.314 J/mol/K, or electron volts/K
- T = absolute temperature in degrees Kelvin (273 + temperature in °C)
- D = frequency factor, a quantity that is assumed to be constant; it depends on the number of collisions of the molecules reacting to produce chemical deterioration of the insulation.

The median life L of insulation specimens is assumed to be inversely proportional to the chemical reaction rate k . This yields

$$\log(L) = \text{constant} + ((E/RT)/2.303) \quad (2)$$

where $\log$ is the base 10 logarithm throughout.

This “Arrhenius” equation has the algebraic form

$$M(X) = A + BX \quad (3)$$

in which

- $M(X)$ = $\log(L)$ = the mean log life
- X = $1/T$
- A = a constant characteristic of the insulation population, test specimen, test method, and failure mode
- B = $E/(2.303R)$, another constant characteristic of the insulation population, test specimen, test method, and failure mode

The coefficients A and B are estimated by fitting the above equation to experimental data. This fitting can be done graphically or, more precisely, by the method of least squares. These methods are presented in sections 2 and 3. Section 3 gives confidence limits that indicate the uncertainty in estimates from data. Throughout, population values are denoted by capital letters (A , B , $M(X)$, etc), and their sample estimates based on experimental data by lower-case letters (a , b , $m(X)$, etc) to distinguish them.

Theoretically, Eq 3 is valid only if a single chemical reaction and failure mode control the insulation failure mechanism. Other reactions may occur, but if they are not dominant, application of the Arrhenius equation may still be valid. The Arrhenius equation application is often valid in practice. Sometimes one reaction and failure mode dominates over a temperature range, but another reaction with a different temperature coefficient and/or failure mode dominates at lower or higher temperatures. Deviations from the simple Arrhenius relation may be caused by different failure modes dominant at different temperatures or by variations in mechanical stress, with temperature, that affect the life. Therefore, while the Arrhenius relation will often fit insulation life-temperature data, it will not always apply. Presented in [8] are valid analyses of such data with two or more failure modes identified by examination of failed specimens.

With these reservations, this guide assumes the Arrhenius equation applies and outlines statistical calculations to fit this equation to test data.

1.3 Test Data Recommendations

Statistical analysis will not compensate for invalid test data. The following recommendations on test procedures ensure correct test data. Some test conditions needed for validity of the Arrhenius relation were mentioned in section 1.2.

Periodic “proof tests” are often used to measure an insulation property that deteriorates. Failure to withstand the specified proof test level is then only detected after a “cycle” of aging. It is more rigorous to treat the failure as if it occurred at the midpoint of the cycle. If there are different modes of failure, the data require special data analysis methods [8].

In planning and carrying out life tests, one should aim toward valid failure time data. One obtains more accurate life estimates at design temperatures from a test with a larger number of specimens at the low end of the test temperature range and fewer at the high end and the middle of this range [11]. It is also best to select the lowest accelerated test temperature as close to the anticipated service use temperature as is possible—consistent with a reasonable estimated test time and enough failures in the allotted test time. Further comments on planning the aging test are given in ANSI/IEEE Std 1-1986 (see Annex 1).

2. Data Analysis

2.1 Graphical Analysis of Data for Individual Temperatures

At any particular test temperature, the log times to failure will be distributed about a mean. A quick, easy, and informative analysis of data at each test temperature is to plot the life times on log-normal probability paper, since the distribution is assumed to be log-normal [11].

First, arrange the n failure times at a temperature in order from lowest to highest (see Table 1). Then calculate the corresponding cumulative probability (cumulative fraction failed), P_j , for the j th failure time:

$$P_j = 100j/(n+1)$$

where j varies from 1 to n . The P_j appear in Table 1.

Then, on log-normal probability paper, plot each failure time versus its P_j , as in the example of Fig 1. Then a line is drawn by eye to best fit the points.

The same procedure is followed for the data at each test temperature. Use log-normal probability paper with enough decades so that the data for all test temperatures fit on the same graph, as shown in Fig 1.

Table 1—Test Data

Temperature (°C)	Specimen No	Life (hours)	Cumulative Probability, P_j
150	1	900	0.09
150	2	900	0.18
150	3	972	0.27
150	4	1260	0.36
150	5	1260	0.45
150	6	1476	0.54
150	7	1476	0.64
150	8	1476	0.73
150	9	1980	0.88
150	10	2196	0.91
175	1	264	0.14
175	2	312	0.29
175	3	312	0.43
175	4	408	0.57
175	5	504	0.71
175	6	552	0.86
200	1	84	0.09
200	2	84	0.18
200	3	108	0.27
200	4	132	0.36
200	5	156	0.45
200	6	156	0.54
200	7	156	0.64
200	8	156	0.73
200	9	204	0.88
200	10	228	0.91

The plotted points for each temperature should be approximately linear. The plots for the different temperatures should be nearly parallel, indicating equal standard deviations of log life. The analysis of such data according to the Arrhenius model is based on the assumptions that the distribution lines for the true populations are straight and parallel. Some departure from strict linearity or parallelism may be acceptable, due to expected randomness in small samples. Gross departure from linearity or parallelism, however, may indicate significant uncontrolled differences in test conditions, specimens, or failure modes at the different temperatures. This would lead to incorrect estimates of life at lower temperatures and incorrect confidence limits.

If the above described plotting is fitted well by straight parallel lines, as illustrated in Fig 1, then the 50% points on the lines give an estimate of the median life at each test temperature. An estimate s of the standard deviation σ at temperature i is the difference between the log of median life and the log of the 16% life on the line for temperature i .

2.2 Calculation of the Mean and Standard Deviation of Data at a Temperature

It is sometimes desirable to analyze the data at individual temperatures to compare two sets of data on different insulations or duplicate tests on the same insulation.

The sample mean and standard deviation of the log life times are calculated for each temperature. Such estimates will generally be close to the graphical estimates. Graphs may, however, reveal important information that the calculations do not. These calculations should be made after converting the life times L_{ij} , of specimen j at temperature i , to their base 10 logs:

$$Y_{ij} = \log(L_{ij})$$

The log average, $\bar{Y}_i$, of the Y_{ij} values at a temperature i is

$$\bar{Y}_i = \frac{\sum_{j=1}^n Y_{ij}}{n_i} \quad (4A)$$

where n_i is the number of specimens at that temperature i , and the sum runs over all n values at the temperature. The subscript j denotes a specimen value within one temperature, i .

It is important to note that if the mean and standard deviation are calculated with the log values (assuming a normal distribution for the log values) that the antilogs of the upper and lower standard deviation interval from the mean will not be symmetric about the mean. The asymmetry will not be great, however, unless the distribution is quite wide.

The sample standard deviation of the log failure times at one temperature is

$$s_i = \sum_{j=1}^n (Y_{ij} - \bar{Y}_i)^2 / (n_i - 1)^{0.5} \quad (4B)$$

$$= \left[n_i \sum_{j=1}^n Y_{ij}^2 - \left(\sum_{j=1}^n Y_{ij} \right)^2 \right] / [n_i(n_i - 1)]^{0.5} \quad (4C)$$

Equation 4B gives explicitly the sum of the squares of the individual deviation Y_{ij} from the mean $\bar{Y}_i$, but Eq 4C is easier to use in calculators and computers, where sums are easily accumulated as data are entered. Some calculators have programs for doing this. Equation 4C is prone to roundoff errors.

The calculated s should be close to the graphical estimate. The calculated values for s from the data of Table 1 are plotted with the failure times in Fig 1. The antilogs of the log mean are plotted at the 50% line, the antilogs of the mean plus the standard deviation at the 84% line, and the antilogs of the mean minus the standard deviation at the 16% line.

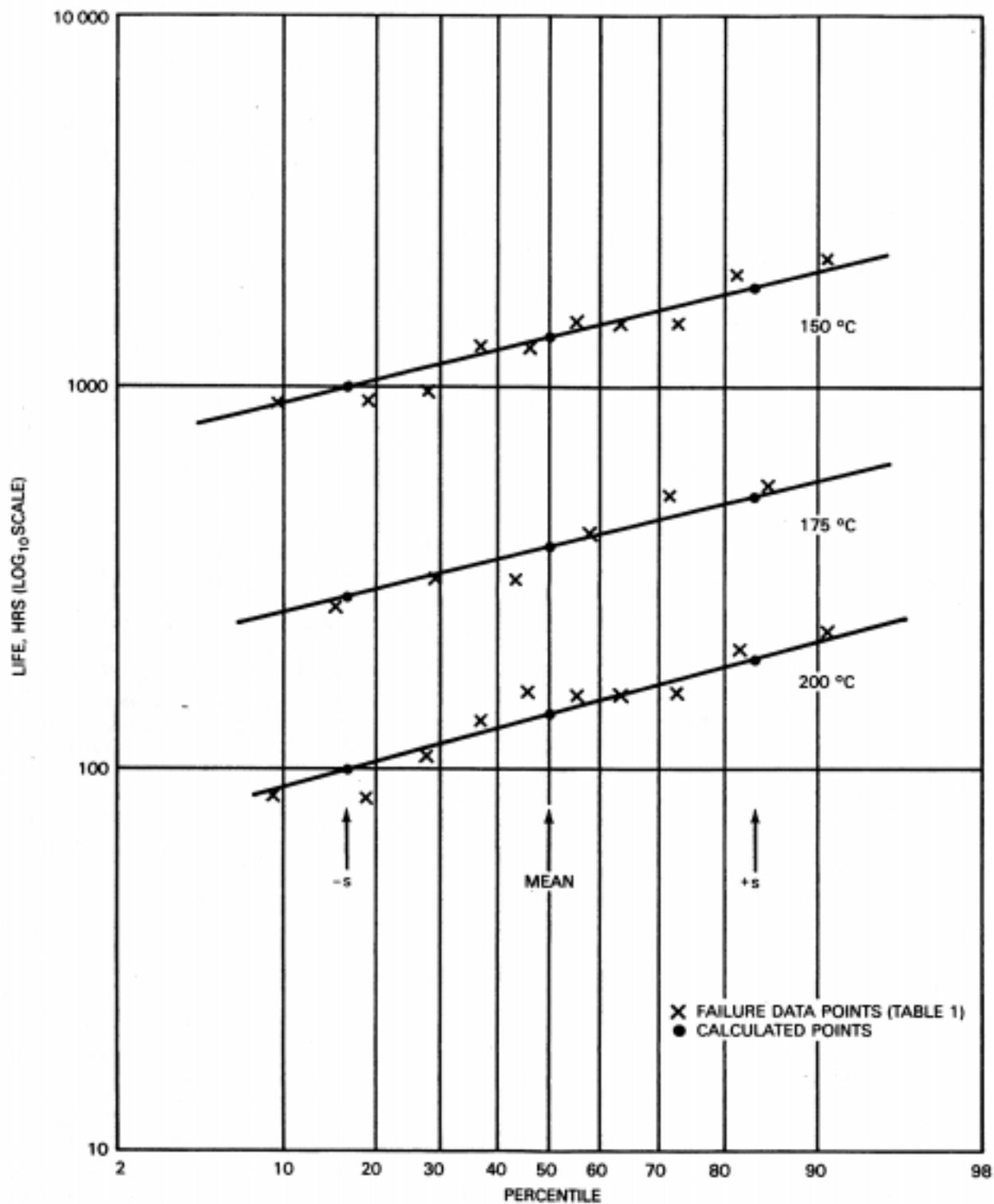


Figure 1—Log Life Versus Normal Probability Graph

Calculation of the standard deviation at individual temperatures permits a statistical comparison of those standard deviations. The statistical significance of their differences can be assessed with Bartlett's test, which is outlined in [11]².

2.3 Analysis of Incomplete Test Data

It is often desirable to analyze data from a partially completed life test at one or more temperatures [7]. For example, only half of a set of test specimens at one temperature may have failed. Thus, failure times are established for part of the set and a running time for the remainder. Such data with unfailed specimens are called "censored." By analyzing data before all specimens fail, one can save considerable time and cost and still get reliable estimates. Many applications require early analyses. Censored life data at a temperature are called "singly censored" if the unfailed units have a common running time and all the failure times are below that. Data for a temperature are called "multiply censored" if failure and running times differ and are intermixed. The latter type of data result when specimens are put on test at different times.

It is very useful to graph incomplete (censored) test data on log normal probability paper, as in Fig 2. The cumulative probability, P , is calculated for each failure time, as in section 2.1, using the total number of failed and unfailed specimens for n . This yields a plot of the failure times in the lower tail of the sample. A line is drawn through these points parallel to the line(s) for other temperature(s); this line estimates the distribution.

The example in Fig 2 uses the complete data at 200 °C (from Fig 1), the lowest 4 of 6 points at 175 °C, and the lowest 4 of 10 points at 150 °C. The lines were drawn parallel by eye through the data points. The 50% points on these lines give estimates of medians that are close to those obtained in Fig 1 using the complete data.

More extensive graphical and numerical analyses of such incomplete test data are discussed in [7], [9], [10], which in turn give further references.

3. Estimation of the Relationship Between Life and Temperature

3.1 Estimation by Plotting

As the first step, plot the lives (and the antilogs of the averages of log lives, calculated in section 2.2) against temperature on Arrhenius paper, as shown in Fig 3. Arrhenius paper³ has a log scale for time and a reciprocal Kelvin scale for temperature. Figure 3 was plotted using the data of Table 1.

The plotted points should scatter around a straight line whose algebraic form was given in Eq 3. It is better to fit the line by eye to the median life times obtained from Fig 1 or to the antilogs of the log averages calculated from Eq 4A. This line is used to obtain an estimate of the median life at a temperature, particularly, a design temperature. If a good graphical fit to a straight line is obtained, only slightly greater accuracy in fitting the line is gained by the more objective least squares regression, described in the next section. However, the least squares fit yields much more information, particularly confidence limits.

3.2 Estimation by Numerical Methods

The graphical method does not provide confidence limits or objective statistical comparisons, such as comparing data from two insulation systems.

²See Part III, pp 108–111.

³This and other special graph papers are available from TEAM, Box 25, Tamworth, NH 03886.