

IEEE Std 1057-1994 (R2001)
(Revision of Trial-Use IEEE Std 1057-1989)

IEEE Standard for Digitizing Waveform Recorders

Sponsor
**Waveform Measurements and Analysis Committee
of the
IEEE Instrumentation and Measurement Society**

Approved June 14, 1994
Reaffirmed September 25, 2001
IEEE Standards Board

Abstract: Terminology and test methods for describing the performance of waveform recorders are provided.

Keywords: digitizers, digitizing oscilloscopes, error sources, performance, terminology, test methods, waveform recorders

The Institute of Electrical and Electronics Engineers, Inc.
345 East 47th Street, New York, NY 10017-2394, USA
Copyright © 1994 by the Institute of Electrical and Electronics Engineers, Inc.
All rights reserved. Published 1994. Printed in the United States of America.
ISBN 1-55937-488-8

No part of this publication may be reproduced in any form, in an electronic retrieval system or otherwise, without the prior written permission of the publisher.

IEEE Standards documents are developed within the Technical Committees of the IEEE Societies and the Standards Coordinating Committees of the IEEE Standards Board. Members of the committees serve voluntarily and without compensation. They are not necessarily members of the Institute. The standards developed within IEEE represent a consensus of the broad expertise on the subject within the Institute as well as those activities outside of IEEE that have expressed an interest in participating in the development of the standard.

Use of an IEEE Standard is wholly voluntary. The existence of an IEEE Standard does not imply that there are no other ways to produce, test, measure, purchase, market, or provide other goods and services related to the scope of the IEEE Standard. Furthermore, the viewpoint expressed at the time a standard is approved and issued is subject to change brought about through developments in the state of the art and comments received from users of the standard. Every IEEE Standard is subjected to review at least every five years for revision or reaffirmation. When a document is more than five years old and has not been reaffirmed, it is reasonable to conclude that its contents, although still of some value, do not wholly reflect the present state of the art. Users are cautioned to check to determine that they have the latest edition of any IEEE Standard.

Comments for revision of IEEE Standards are welcome from any interested party, regardless of membership affiliation with IEEE. Suggestions for changes in documents should be in the form of a proposed change of text, together with appropriate supporting comments.

Interpretations: Occasionally questions may arise regarding the meaning of portions of standards as they relate to specific applications. When the need for interpretations is brought to the attention of IEEE, the Institute will initiate action to prepare appropriate responses. Since IEEE Standards represent a consensus of all concerned interests, it is important to ensure that any interpretation has also received the concurrence of a balance of interests. For this reason IEEE and the members of its technical committees are not able to provide an instant response to interpretation requests except in those cases where the matter has previously received formal consideration.

Comments on standards and requests for interpretations should be addressed to:

Secretary, IEEE Standards Board
445 Hoes Lane
P.O. Box 1331
Piscataway, NJ 08855-1331
USA

IEEE standards documents may involve the use of patented technology. Their approval by the Institute of Electrical and Electronics Engineers does not mean that using such technology for the purpose of conforming to such standards is authorized by the patent owner. It is the obligation of the user of such technology to obtain all necessary permissions.

Introduction

(This introduction is not a part of IEEE Std 1057-1994, IEEE Standard for Digitizing Waveform Recorders.)

Work on this standard unofficially began with the “Seminar on Waveform Recorder Measurement Needs and Techniques for Evaluation/Calibration,” hosted by the National Bureau of Standards (now National Institute of Standards and Technology [NIST]) in 1981. Dissatisfied with the state of measurement standards and test techniques for waveform recorders, a group of concerned users, manufacturers, and researchers who attended the seminar formed a committee to crystallize waveform recorder specifications and achieve consensus on them. This effort gathered additional members and became affiliated with the IEEE Instrumentation and Measurement Society in 1983. The initial work was completed in 1988, and a Trial-Use Standard was issued in July of 1989. Since that time, the standard has been substantially revised, resulting in the current standard, which now has full-use status.

The purpose of this standard is to provide common terminology and test methods for describing the performance of waveform recorders. Since these devices give digital answers that are intended to be sent to a computer for analysis, this committee found it necessary to adopt an approach that is perhaps more highly analytical than approaches used by other instrumentation standards committees. A report on the user’s reaction to this approach and any additional feedback will be welcomed.

At the time of the approval of this standard, the working group had the following membership:

Michael Souders, *Chair*

Jerome Blair
William Boyer
Philip Crosby
Philip Green

James Hyatt
Dan Knierim
Robert Lawton*
Thomas Linnenbrink
Bruce Peetz

William Sapankevych
Otis Solomon
Stuart Streiff
Steven Tilden

* Chair 1982–1986

The following persons were on the balloting committee:

Jerome Blair
William Boyer
Steve Broadstone
Willard Bucklen
John P. Deyst
Laszlo Dobos
Steven Ems
William L. Gans
Philip Green
James Hyatt
Y. C. Jenq

Dan Knierim
Istvan Kollar
Richard Kromer
John Kuffel
Robert Lawton
Tom Linnenbrink
T. R. McComb
Norris Nahman
Phil Painchaud
Sangil Park

Bruce Peetz
Edward Platner
Sedki M. Riad
Kenneth Rush
W. Sapankevych
Andrew Sexton
Otis M. Solomon
Michael Souders
Stuart Streiff
Robert Talambiras
Steve Tilden

When the IEEE Standards Board approved this standard on June 14, 1994, it had the following membership:

Wallace S. Read, *Chair*

Donald C. Loughry, *Vice Chair*

Andrew G. Salem, *Secretary*

Gilles A. Baril
Bruce B. Barrow
José A. Berrios de la Paz
Clyde R. Camp
James Costantino
Stephen L. Diamond
Donald C. Fleckenstein
Jay Forster*
Ramiro Garcia

Donald N. Heirman
Richard J. Holleman
Jim Isaak
Ben C. Johnson
Sonny Kasturi
Lorraine C. Kevra
E. G. "Al" Kiener
Ivor N. Knight

Joseph L. Koepfinger*
D. N. "Jim" Logothetis
L. Bruce McClung
Marco W. Migliaro
Mary Lou Padgett
Arthur K. Reilly
Ronald H. Reimer
Gary S. Robinson
Leonard L. Tripp

*Member Emeritus

Also included are the following nonvoting IEEE Standards Board liaisons:

Satish K. Aggarwal
James Beall
Richard B. Engelman
David E. Soffrin

Rochelle Stern
IEEE Standards Project Editor

Contents

CLAUSE	PAGE
1. Overview.....	1
1.1 Scope.....	1
1.2 Purpose.....	1
1.3 Guidance to the user.....	1
1.4 Manufacturer supplied information	3
2. References.....	4
3. Definitions and symbols	4
3.1 Definitions.....	4
3.2 Symbols	8
4. Test methods	12
4.1 General methods	12
4.2 Input impedance.....	31
4.3 Gain and offset.....	32
4.4 Linearity, harmonic distortion, and spurious response	33
4.5 Noise	38
4.6 Analog bandwidth.....	41
4.7 Frequency response.....	42
4.8 Step response parameters.....	43
4.9 Time base errors.....	46
4.10 Triggering	49
4.11 Crosstalk	52
4.12 Monotonicity.....	52
4.13 Hysteresis.....	53
4.14 Overvoltage recovery.....	53
4.15 Word error rate.....	54
4.16 Cycle time	55
4.17 Differential input specifications.....	55

ANNEXES

Annex A Derivation of the three parameter (known frequency) sine wave curvefit algorithm (informative)	58
Annex B Derivation of the four parameter (general case) sine wave curvefit algorithm (informative)	62
Annex C Comments on errors associated with word-error-rate measurement (informative).....	69
Annex D Measurement of random noise below the quantization level (informative).....	71
Annex E Bibliography (informative).....	73

IEEE Standard for Digitizing Waveform Recorders

1. Overview

1.1 Scope

Instruments covered by this standard are electronic digitizing waveform recorders, waveform analyzers, and digitizing oscilloscopes with digital outputs. This standard is directed toward, but not restricted to, general-purpose waveform recorders and analyzers. Special applications may require additional manufacturer information and verification tests.

1.2 Purpose

The purpose of this standard is to provide common terminology and test methods for describing the performance of waveform recorders. It is meant for users and manufacturers of these devices. The main body presents many performance features, sources of error, and test methods. The tests are meant to be used by manufacturers to locate and correct sources of error in the hardware.

The information in this standard can also be applied by users to correct some of the errors by software processing after the signal has been recorded. Another planned use for this standard is to aid users in writing specifications for purchasing new waveform recorders.

1.3 Guidance to the user

This standard presents a large number of possible waveform recorder error sources and test methods for quantifying them. Some of these methods are cumbersome in that they require sophisticated test equipment and signal processing, or require a considerable amount of time. It is generally not necessary to perform all of these tests to adequately characterize a particular waveform recorder for a particular application. Also, the existence of some errors can preclude measuring others. For many applications, the two relatively easy tests for determining effective bits and step response may suffice or at least point the user in a direction to pursue other tests. These tests require a high purity sine-wave source and a clean, fast leading-transition step generator. Recommended algorithms for performing the sine fit used in determining effective bits are described in 4.1.3.

The sine-fit test will detect such errors as random noise, nonlinearities, and aperture uncertainty. Some insight into these errors may be gained by examining a plot of the residuals as defined in equations (23) or (53), as appropriate. The direction of further testing may be suggested by whether the residuals appear random or systematic.

The sine-fit test does not measure amplitude flatness or phase linearity. Deviations from the ideal in these parameters may appear as aberrations in the recorder's step response. The step response parameters should be determined over a range of levels to detect amplitude nonlinearity and slew limit effects. When the recorder is not too noisy, the impulse response may be approximated by differentiating the step response. For more information, see [B14], [B17], and [B16].¹

Waveform recorders often have various controllable settings such as gain, offset, filtering, sampling rate, trigger level, and trigger slope. The performance parameters and test methods described in this standard apply to a single set of settings. Changing a setting can affect the recorder's performance. This is particularly true of gain, offset, and sampling rate. Thus, a waveform recorder should be characterized over the range of settings at which the instrument will be used. Also, some waveform recorders have the capability to automatically self-calibrate, and the data taken before the calibration may differ from the data taken after the calibration. If a test procedure is affected by a self-calibration, the user may have to defeat the self-calibration feature before the test procedure has begun.

Drift in performance over time and temperature are not specifically addressed for most parameters in this standard. An exception is long-term sample time drift. The manufacturer should specify the operating temperature range and recalibration intervals.

Many test methods call for test equipment whose performance is better than that expected from the waveform recorder being measured. The required test equipment may not be available. This is especially the case for recorders at the state of the art in sampling speed or accuracy, or both. A paradox may exist in that high performance waveform recorders are required to develop and evaluate new test equipment. When this is the case, clever engineering is required to develop techniques to measure the parameters of interest using available equipment.

Waveform recorders and digital oscilloscopes may employ either real-time or equivalent-time sampling, or both, at different times. With real-time sampling, samples of the input signal are acquired sequentially, at the stated sampling rate. However, with equivalent-time sampling, the samples may be acquired sequentially or otherwise, at a rate usually much lower than the stated equivalent-time sampling rate. Equivalent-time sampling requires that input waveforms be repetitive, since a waveform sampled in equivalent-time is reconstructed from samples taken over many repetitions. This restriction does not apply for recorders employing real-time sampling.

In this standard, the recommended test methods were designed with real-time sampling instruments specifically in mind. Nevertheless, the test methods will generally still be valid when applied to recorders that use equivalent-time sampling. A few additional points should be kept in mind in such cases.

A common feature of equivalent-time sampling instruments is the ability to perform waveform averaging. This is useful in reducing noise components, both originating in the waveform source and also in the recorder. Therefore, tests that in some way measure the recorder's noise processes will be affected by the amount of averaging applied in the recorder. Examples of such tests are effective bits, signal-to-noise ratio, peak error, random noise, and aperture uncertainty. The results of any such tests performed on recorders employing equivalent-time sampling should include the number of samples or records averaged.

As noted in 4.9.2, the use of equivalent-time sampling also affects the time over which the aperture uncertainty is measured. Therefore, the results of this measurement should include the time required to capture a record, or equivalently, the real sampling rate and the record length.

Finally, some tests of equivalent-time sampling recorders will actually be easier to perform, or will have less uncertainty because the sampling is often phase-locked to the trigger signal. As an example, the measurement of trigger delay and jitter may be more accurate (see 4.10.1.3).

¹The numbers in brackets correspond to those bibliographic items listed in annex E.

1.4 Manufacturer supplied information

1.4.1 General information

Manufacturers shall supply the following general information:

- a) Model number
- b) Dimensions and weight
- c) Power requirements
- d) Environmental conditions: Safe operating, non-operating, and specified performance temperature range; altitude limitations; humidity limits, operating and storage; vibration tolerance; and compliance with applicable electromagnetic interference specifications
- e) Any special or peculiar characteristics
- f) Compliance with military and other specifications
- g) Available options and accessories, including their impact on equipment performance
- h) Exceptions to the above parameters where applicable
- i) Calibration interval, if required

1.4.2 Performance specifications

The manufacturer shall provide the minimum specifications (see clause 1.4.2.1). See clause 3 for definitions.

1.4.2.1 Minimum specifications

- a) Number of digitized bits
- b) Sample rates
- c) Memory length
- d) Input impedance
- e) Analog bandwidth
- f) Input signal ranges

1.4.2.2 Additional specifications

- a) Gain
- b) Offset
- c) Differential nonlinearity
- d) Harmonic distortion and spurious response
- e) Integral nonlinearity
- f) Maximum static error
- g) Signal-to-noise ratio
- h) Effective bits
- i) Peak error
- j) Random noise
- k) Frequency response
- l) Settling time
- m) Transition duration of step response (rise time)
- n) Slew limit
- o) Overshoot and precursors
- p) Aperture uncertainty (short-term time- base stability)
- q) Long-term time-base stability
- r) Fixed error in sample time
- s) Trigger delay and jitter
- t) Trigger sensitivity
- u) Trigger hysteresis band
- v) Trigger minimum rate of change
- w) Trigger coupling to signal
- x) Crosstalk
- y) Monotonicity
- z) Hysteresis
- aa) Overvoltage recovery

- ab) Word error rate
- ac) Cycle time
- ad) Common mode rejection ratio
- ae) Maximum common mode signal level
- af) Differential input impedance
- ag) Maximum operating common mode signal level

2. References

IEEE Std 181-1977, IEEE Standard on Pulse Measurement and Analysis by Objective Techniques.²

3. Definitions and symbols

3.1 Definitions

3.1.1 amplitude flatness: The variation in output amplitude as a function of frequency in response to a constant amplitude sine wave input.

3.1.2 aperture uncertainty: The standard deviation of the sample instant in time. *Syn:* short-term timing instability; timing jitter; timing phase noise.

3.1.3 CMRR: See 3.1.11, common-mode rejection ratio.

3.1.4 code bin k: A digital output that corresponds to a particular set of input values.

Code Transition Level	Code Bin	Code Bin Width
$T[2^N - 1]$	2^{N-1}	$W[2^N - 2]$
$T[2^N - 2]$	2^{N-2}	
$\vdots$	$\vdots$	
$\vdots$	$\vdots$	
$T[k + 2]$	$k + 1$	$W[k + 1]$
$T[k + 1]$	k	
$T[k]$	$k - 1$	
$T[k - 1]$	$k - 1$	
$\vdots$	$\vdots$	$W[1]$
$\vdots$	$\vdots$	
$\vdots$	$\vdots$	
$\vdots$	$\vdots$	
$T[2]$	1	
$T[1]$	0	

Figure 1—Definitions pertaining to input quantization

²IEEE Std 181-1977 has been withdrawn; however, copies can be obtained from the IEEE Standards Department, 445 Hoes Lane, P.O. Box 1331, Piscataway, NJ 08855-1331, USA.

3.1.5 code bin width $W[k]$: The difference of the code transition levels that delimit the bin.

$$W[k] = T[k + 1] - T[k] \quad (1)$$

3.1.6 code transition level: The boundary between two adjacent code bins.

3.1.7 code transition level $T[k]$: The value of the recorder input parameter at the transition point between two given, adjacent code bins. The transition point is defined as the input value that causes 50% of the output codes to be greater than or equal to the upper code of the transition, and 50% to be less than the upper code of the transition. The transition level $T[k]$ lies between code bin $k-1$ and code bin k . *See:* 3.1.61, transition point.

3.1.8 coherent sampling: Sampling of a periodic waveform in which there is an integer number of cycles in the data record. In other words, coherent sampling occurs when the following relationship exists:

$$f_s \cdot M_c = f_0 \cdot M \quad (2)$$

where

f_s is the sampling frequency
 M_c is the integer number of cycles in the data record
 f_0 is the frequency of the input
 M is the number of samples in the record

3.1.9 common-mode overvoltage: A signal level whose magnitude is less than the specified maximum safe common-mode signal but greater than the maximum operating common-mode signal.

3.1.10 common-mode overvoltage recovery time: The time required for the recorder to return to its specified characteristics after the end of a common-mode overvoltage pulse.

3.1.11 common-mode rejection ratio (CMRR): The ratio of the input common-mode signal to the effect produced at the output of the recorder in units of the input.

3.1.12 common mode signal: The average value of the signals at the positive and negative inputs of a differential input waveform recorder. If the signal at the positive input is designated V_+ , and the signal at the negative input is designated V_- , then the common mode signal (V_{cm}) is

$$V_{cm} = \frac{V_+ + V_-}{2} \quad (3)$$

3.1.13 crosstalk (multichannel): The ratio of the signal induced in one channel to a common signal applied to all other channels.

3.1.14 cycle time: The real time elapsed (with a recorder continually taking records of data) between the beginning of two records taken in succession.

3.1.15 data window: A set of coefficients by which corresponding samples in the data record are multiplied to more accurately estimate certain properties of the signal, particularly frequency domain properties. Generally, the coefficient values increase smoothly toward the center of the record.

3.1.16 differential input impedance to ground: The impedance between either the positive input and ground or the negative input and ground.

3.1.17 differential nonlinearity: The difference between a specified code bin width and the average code bin width, divided by the average code bin width.

3.1.18 fixed error in sample time: A nonrandom error in the instant of sampling. A fixed error in sample time may be fixed with respect to the data samples acquired or correlated with an event that is detected by the sampling process. Unless otherwise specified, usually taken to mean the maximum fixed error that may be observed.

3.1.19 full-scale range: The difference between the maximum and the minimum recordable input values as specified by the manufacturer.

3.1.20 full-scale signal: A signal that spans the entire manufacturer's specified amplitude range of the instrument.

3.1.21 gain and offset: (A) (independently based). Gain and offset are the values by which the input values are multiplied and then to which the input values are added, respectively, to minimize the mean squared deviation from the output values. (B) (terminal-based). Gain and offset are the values by which the input values are multiplied and then to which the input values are added, respectively, to cause the deviations from the output values to be zero at the terminal points, that is, at the first and last codes.

3.1.22 harmonic distortion: For a pure sine wave input, output components at frequencies that are an integer multiple of the applied sine wave frequency.

3.1.23 hysteresis: The maximum difference in values for a digitizer code transition level when the transition level is approached from either side of the transition.

3.1.24 ideal code bin width Q : The full-scale range divided by the total number of code states.

3.1.25 incoherent sampling: Sampling of a waveform such that the relationship between the input frequency, sampling frequency, number of cycles in the data record, and the number of samples in the data record does not meet the definition of coherent sampling.

3.1.26 input impedance: The impedance between the signal input of the waveform recorder and ground.

3.1.27 integral nonlinearity: The maximum difference between the ideal and actual code transition levels after correcting for gain and offset.

3.1.28 k th code transition level $T[k]$: The input value corresponding to the transition between codes $k-1$ and k .

3.1.29 large signal: A signal whose peak-to-peak amplitude is as large as practical but is recorded by the instrument within, but not including, the maximum and minimum amplitude data codes. As a minimum, the signal must span at least 90% of the full-scale range of the waveform recorder.

3.1.30 long-term settling error: The absolute difference between the final value specified for short-term settling time, and the value 1 s after the beginning of the step, expressed as a percentage of the step amplitude.

3.1.31 long-term timebase stability: The change in time base frequency (usually given in parts per million) over a specified period of time at a specified sampling rate.

3.1.32 maximum common-mode signal level: The maximum level of the common-mode signal at which the common mode rejection ratio is still valid.

3.1.33 maximum operating common-mode signal: The largest common-mode signal for which the waveform recorder will meet its effective bits specifications in recording a simultaneously-applied, normal-mode signal.

3.1.34 maximum static error (MSE): The maximum difference between any code transition level and its ideal value.

3.1.35 monotonic recorder: A recorder that has output codes that do not decrease (increase) for a uniformly increasing (decreasing) input signal, disregarding random noise.

3.1.36 noise: Any deviation between the output signal (converted to input units) and the input signal, except deviations caused by linear time invariant system response (gain and phase shift), a dc level shift, or an error in the sample rate. For example, noise includes the effects of random errors, fixed pattern errors, nonlinearities and time base errors (fixed error in sample time and aperture uncertainty).

3.1.37 normal mode signal (or differential signal): The difference between the signal at the positive input and the negative input of a differential input waveform recorder. If the signal at the positive input is designated V_+ , and the signal at the negative input is designated V_- , then the normal mode (or differential) signal (V_{dm}) is

$$V_{dm} = V_+ - V_- \quad (4)$$

3.1.38 normalized peak error: Peak error divided by three times the standard deviation of the differences discussed in 4.5.1.1. See 3.1.42, peak error.

3.1.39 offset: See 3.1.21, gain and offset.

3.1.40 overshoot: The maximum amount by which the step response exceeds the topline, specified as a percentage of (recorded) pulse amplitude.

3.1.41 overvoltage: Any voltage whose magnitude is less than the maximum safe input voltage of the recorder but greater than the full-scale value for the selected range.

3.1.42 peak error: The residual with the largest absolute value.

3.1.43 precursor: A deviation from the baseline prior to the pulse transition.

3.1.44 random noise: A nondeterministic fluctuation in the output of a waveform recorder, described by its frequency spectrum and its amplitude statistical properties.

3.1.45 record of data: A sequential collection of samples acquired by the waveform recorder.

3.1.46 relatively prime: Describes integers whose greatest common divisor is 1.

3.1.47 residuals: The differences between the recorded data and the fitted sine wave for sine-wave curve fitting.

3.1.48 reverse coupling: The ratio of the spurious signal generated by a signal at some other input to the recorder and the signal recorded at the specified input of the recorder.

3.1.49 settling time: Measured from the mesial point (50%) of the output, the time at which the step response enters and subsequently remains within a specified error band around the final value. The final value is defined to occur 1 s after the beginning of the step.

3.1.50 short-term settling time: Measured from the mesial point (50%) of the output, the time at which the step response (see 4.1.4) enters and subsequently remains within a specified error band around the final value. The final value is defined to occur at a specified time less than one second after the beginning of the step.

3.1.51 short-term timing instability: See 3.1.2, aperture uncertainty.

3.1.52 signal-to-noise ratio: The ratio of a signal to the noise.

3.1.53 single-ended recorder: A non-differential waveform recorder, i.e., one that does not subtract the signals at two input terminals.

3.1.54 slew limit: The value of output transition rate of change for which an increased amplitude input step causes no change.

3.1.55 spurious components: Persistent sine waves at frequencies other than the harmonic frequencies. See 3.1.22, harmonic distortion.

3.1.56 step response: The recorded output response for an ideal input step with designated baseline and topline.

3.1.57 timing jitter: See 3.1.2, aperture uncertainty.

3.1.58 timing phase noise: See 3.1.2, aperture uncertainty.

3.1.59 total harmonic distortion: The root sum square of all harmonic distortion components including their aliases.

3.1.60 transition duration of a step response: The duration between the proximal point (10%) and the distal point (90%) on the recorded output response transition, for an ideal input step with designated baseline and topline.

3.1.61 transition point: The input value that causes 50% of the output codes to be greater than or equal to the upper code of the transition, and 50% to be less than the upper code of the transition.

3.1.62 trigger delay: The elapsed time from the occurrence of a trigger pulse at the trigger input connector to the time at which the first or a specified data sample is recorded.

3.1.63 trigger jitter: The standard deviation in the trigger delay time over multiple records.

3.1.64 trigger minimum rate of change: The slowest rate of change of the leading edge of a pulse of a specified level that will trigger the recorder.

3.1.65 trigger signal coupling: The ratio of the spurious signal level (that is recorded by an input to the recorder) to the trigger signal level.

3.2 Symbols

$$\alpha_n = \cos(\omega t_n + \theta)$$

$\bar{\alpha}$ = average of α_n over M samples

$$\beta_n = \sin(\omega t_n + \theta)$$

$\bar{\beta}$ = average of β_n over M samples

ε = error, used for total error and error band

ε_{rms} = root-mean-square value of ε

$\varepsilon[f_s]$ = error in sampling rate

$\varepsilon[f_r]$ = measured error in input reference frequency

$\varepsilon[k]$ = difference between $T[k]$ and ideal value of $T[k]$ computed from G and V_{os}

θ = phase, expressed as radians

π = constant, ratio of the circumference of a circle to the diameter

ρ = reflection coefficient

σ = standard deviation; sometimes used as noise amplitude, which is the standard deviation of the random component of a signal

σ_{σ} = standard deviation of the standard deviation (for example, standard deviation of the noise amplitude)

σ_t = aperture uncertainty

τ = sampling period, the inverse of f_s

$\Phi[X]$ = cumulative density function for a Gaussian distribution (the fraction of the total distribution covered from negative infinity to X)

ϕ = phase estimate, expressed in radians

ψ = frequency estimate, expressed in radians per second

ω = frequency, expressed in radians per second

A = sinusoidal amplitude

A_1 = amplitude estimate of a sinusoid

A_D = amplitude denominator, when A (or A_1) is expressed as a fraction

A_N = amplitude numerator, when A (or A_1) is expressed as a fraction

$A[i]$ = average of a record

$a_{11}, a_{12}, a_{21}, a_{22}$ = linear equation matrix elements for four parameter fit iteration

B = test tolerance in fractions of an ideal code-bin width (Q). Also used as an amplitude

B_1 = amplitude estimate of a cosine wave

B_D = amplitude denominator, when B (or B_1) is expressed as a fraction

B_N = amplitude numerator, when B (or B_1) is expressed as a fraction

C = offset

DNL = maximum differential nonlinearity over all k

$DNL[k]$ = differential nonlinearity of code k

E = number of effective bits

$ENBW$ = equivalent noise bandwidth

$e_m[f]$ = aliasing and first differencing magnitude errors

$e_p[f]$ = aliasing and first differencing phase errors

f = frequency, Hz

f_0 = frequency of the input signal

f_{co} = upper frequency for which the amplitude response is -3 dB

f_{eq} = equivalent sampling rate

f^* = Approximate desired input frequency

$f[k]$ = correction function converting sine wave probability distribution function to uniform distribution function

f_{opt} = optimal input frequencies for the sine wave curve fit

f_r = input signal reference frequency or input signal repetition rate

f_s = sampling frequency

G = gain of waveform recorder, ideally = 1

$hn(i)$ = Hann window time coefficients

INL = integral nonlinearity

K = number of data records

k = code bin

L = equivalent time sampling parameter

M = number of sequential samples

M_c = number of input sine wave cycles

MSE = maximum static error

m = root-mean-square of rising and falling slopes of selected portions of a waveform

mse = mean square error

N = number of digitized bits

n = sample index within a record

P_+ , P_- = maximum and minimum peak regions of a sine wave

p = probability

Q = ideal code-bin width, expressed in input units

Q' = average code bin width

R = error parameter, or number of records

R_+ , R_- = rising and falling regions of a sine wave

S = set of samples collected over more than one record, also used as an error parameter

snr = signal-to-noise ratio

Δt = interval between equivalent time samples

$T[k]$ = code transition level between code k and code $k-1$

T_k = ideal value of $T[k]$ or $[Q \cdot (k - 1) + T_1]$

T_1 = ideal value of $T[1]$

T_{MIN} = minimum trigger delay required for a large pulse

T_{OD} = trigger delay for a pulse value V_{OD}

t_n = discrete sample times

u = confidence expressed as a fraction

V = full-scale input range of waveform recorder

V_{HYST} = the trigger hysteresis level

$V(t)$ = the settled value after a specified time

V_O = overdrive voltage

V_{OD} = a small trigger pulse level

V_{TH} = minimum level that will trigger the recorder

V_{os} = input offset of waveform recorder, ideally = 0

$W[k]$ = code bin width of code bin k

w = estimated word error rate

w' = worst-case word error rate

$w(i)$ = window function (for a Discrete Fourier Transform [DFT])

X = number of standard deviations of a Gaussian distribution

x = number of errors detected

$\bar{y}$ = average of y_n over M samples

y_n = data samples within a record

y'_n = best fit points to a data record

Z_0 = transmission line impedance

Z_{τ} = waveform recorder input impedance

$Z_{u/2}$ = number of standard deviations that encompass 100 (1- u)% of a Gaussian distribution about the center

4. Test methods

4.1 General methods

4.1.1 Taking a record of data

A record of data is a sequential collection of samples acquired by the waveform recorder. The action of taking a record of data is defined as creating the record of data in the recorder by making a measurement, and then transferring the data record to a computer for analysis.

4.1.2 Locating code transition levels

Since a waveform recorder digitizes the signal for storage, many of the specifications describe the fidelity of the digitization process. The fidelity of digitization depends on how well the code bins (into which the samples fall) match the ideal. The only way of directly measuring the code bins is to identify the end points of the bins. The boundary between two adjacent code bins is called the code transition level. Measurement of the code transition levels is used to determine conformance to several specifications.

The integer k is the recorder's output code, which ranges from 0 to 2^N-1 , where N is the number of digitized bits. The k th code transition level $T[k]$ is defined to be the input value corresponding to the transition between codes $k-1$ and k . Therefore $T[k]$ is the input value for which 50% of the output codes will be less than k , and 50% will be greater than or equal to k .

4.1.2.1 Static test method

A programmable source (e.g., a digital to analog converter) is required whose range and output parameter (voltage or current) is compatible with the waveform recorder, and whose resolution is at least four times that of the waveform recorder. The static transfer characteristic of the source should be known. The code transition levels are then determined as follows:

The output of the source is connected to the input of the waveform recorder. The following steps should be performed:

- a) Begin with $k = 1$.
- b) Apply an input level slightly lower than the expected code transition level. For $k = 1$, begin with a value slightly lower (for example 2%) than the minimum level recordable by the waveform recorder.
- c) Take a record of data. Evaluate the record to determine the percentage of codes in the record that are less than k . Temporarily store the percentage along with the corresponding input value.
- d) If the percentage from step c) is greater than 50%, then raise the input value by $1/4 Q$ (of the waveform recorder) or less, and repeat c), updating the stored input value and percentage.
- e) The first time the percentage drops to 50% or less, the transition has been crossed. The code transition level is computed by linear interpolation based on the recorded percentages at this level and the previous level. Record the interpolated value as the code transition level $T[k]$.
- f) After finding $T[k]$, repeat b) to find $T[k + 1]$ (the final level of step e) for the k th code transition level will generally be a satisfactory starting point for code transition level $k + 1$, step b).
- g) Repeat step f) until $T[k]$ has been found for all k .

Care should be taken to ensure the impedance of the source and the input impedance of the recorder do not affect this measurement.

4.1.2.1.1 Comment on significance of record length and the presence of noise

The location of code transition levels is a probabilistic process because of the inevitable presence of noise. As a consequence, the percentages estimated by the measurements of 4.1.2.1 have an associated standard deviation. Corresponding uncertainties in the estimates of the code transition levels result. The uncertainties can be reduced by choosing larger record lengths.

Assuming the noise to have zero mean and a normal distribution, the size of the record required for a given precision in the estimates of code transition levels can be computed. Table 1 gives the precision with a 99.87% (3σ) confidence level, expressed as a percentage of the root-mean-square (rms) noise value, computed for several record lengths.

If the precision needs to be known absolutely (that is, in terms of input units) rather than as a percentage of the noise, then the noise level must be determined. Subclause 4.5.5 provides procedures for measuring the noise level if the dominant source of noise is the waveform recorder rather than the programmable source. Alternatively, the noise can be estimated as the code transitions are located by examining the probability distribution vs. input level on either side of a code transition (see [B4]). Note that there is limited value in determining the code transition levels with a precision very much smaller than the noise level, unless the results of an intended application can be used in conjunction with noise-reducing signal processing.

Table 1—Precision of estimates of code transition level

Record length (samples)	Precision of estimates of code transition level (% of rms noise)
64	45%
256	23%
1024	12%
4096	6%

4.1.2.2 Dynamic test method

The following method is often easier to implement, especially if one is only interested in determining non-linearities. A sine wave of amplitude sufficient to slightly overdrive the recorder is recorded many times, and a histogram is constructed. If the input range of the recorder is not symmetrical around 0, then a constant must be added to the sine wave so that the peaks of the combined signal are equidistant from the center of the range. The triggering of the recorder should be asynchronous with the sine wave, and the frequency of the sine wave must be specified. The amount of overdrive required depends on the noise level of the recorder and on the accuracy required; this will be covered in 4.1.2.2.2. If the amplitude and offset of the sine wave are precisely known, this method gives the transition levels to the same precision. If the amplitude of the sine wave and the offset are unknown, this method gives the transition levels to within a gain and offset error. That is, the calculated transition levels, $T'[k]$, will be related to the true transition levels, $T[k]$, by the relation

$$T'[k] = aT[k] + b \quad (5)$$

where a and b are constants.

The sine wave frequency must be chosen as described in 4.1.2.2.1. Take many records of data (the required amount will be covered in 4.1.2.2.2) and keep track of the total number of samples received in each code bin. The transition levels are then given by

$$T[k] = C - A \cos \left[\frac{\pi \cdot H_c[k-1]}{S} \right] \quad (6)$$

where

- A is the amplitude of the sine wave
- C is the offset (dc level) of the applied signal
- $H_c[j]$ is $\sum_{i=0}^j H[i]$
- $H[i]$ is the total number of samples received in code bin i
- S is the total number of samples

If A and C are unknown, they can be determined from the data and an independent estimate of any two of the transition levels. Errors in the values of A and/or C do not induce any errors in the determination of differential or integral nonlinearity from the calculated transition levels because they only induce gain and offset errors in the transition levels as shown in equation (5). These results are derived in [B21].

4.1.2.2.1 Comment on the selection of the frequency and record length

The frequency of the sine wave and the record length of the data collected should be carefully selected in order for the error estimates of the following clause to apply. There should be an exact integer number of cycles in a record, and the number of cycles in a record should be relatively prime to the number of samples in the record. This guarantees that the samples in each record are uniformly distributed in phase from 0 to 2π . If the frequency is low enough that dynamic errors do not arise, this method will give the same results as the static test method. If the frequency is chosen large enough that the dynamic errors are significant, the user should be warned that some dynamic errors will appear in the results, while others will be averaged out by the histogram calculations.

A frequency that meets the above requirements can be selected as follows. Choose the number of cycles per record, M_c , and a record length, M , such that M_c and M have no common factors. Choose the frequency by the following formula:

$$f = \frac{M_c}{M} f_s \quad (7)$$

where

- f is the signal frequency
- f_s is the sampling frequency

In order for the test tolerances in the derivations in 4.1.2.2.2 to be valid, the accuracy of the signal frequency is given by

$$\frac{\Delta f}{f} \leq \frac{1}{4(M-1)} \quad \text{if } M_c = 1$$

$$\frac{\Delta f}{f} \leq \frac{1}{4(M_c - 1)M} \quad \text{if } M_c > 1 \quad (8)$$

where

Δf is the allowable error in the signal frequency

With larger values of M , fewer total samples (the number of records times the number of samples per record) will be required to obtain any given accuracy, but greater accuracy will be required of the signal frequency. The best approach is to use the largest value of M compatible with the frequency accuracy obtainable. The frequency accuracy specified by equation (8) guarantees that the phase separation between samples is within $\pm 25\%$ of the ideal. This tolerance was assumed in the derivation of equation (11).

4.1.2.2.2 Comment on the amount of overdrive and the number of records required

The minimum amount of overdrive required depends on the combined noise level of the signal source and the recorder. In the absence of noise, the overdrive need only be sufficient to receive at least one count in each of the first and last code bins. If noise is present, it will modify the probabilities of samples falling in various code bins, and the effect will be largest near the peaks where the curvature of the probability density is greatest. This effect can be made as small as desired by making the overdrive large enough. The amount of overdrive required to obtain a specified accuracy also depends on whether the specified accuracy is for the code bin widths (i.e., differential nonlinearity) or for the transition levels (i.e., integral nonlinearity).

The overdrive required to obtain a specified tolerance in code bin widths is given by

$$V_O \geq \text{maximum of } (3 \sigma) \text{ or } \sigma \sqrt{\frac{3}{2B}} \quad (9)$$

where

σ is the rms value of the random noise in input units

B is the desired tolerance as a fraction of the code bin width

V_O is the overdrive voltage, the difference between the positive (negative) peaks of the sine wave, and the most positive (negative) transition level of the recorder

This amount of overdrive guarantees that the error caused by the noise is $\leq 1/3$ of the desired tolerance.

The overdrive required to obtain a specified tolerance in transition levels is given by

$$V_O \geq \text{maximum of } (2\sigma) \text{ or } \frac{\sigma^2 2^N}{VB} \quad (10)$$

where

V is the full scale voltage of the instrument in input units

σ is the rms value of the random noise in input units

B is desired tolerance as a fraction of the code bin width

N is the number of bits of the waveform recorder

V_O is the overdrive voltage

The values of overdrive in equations (9) and (10) are adequate to keep the errors due to noise to equal to or less than $B/3$ code bin widths so that these errors are negligible when added to the statistical errors due to taking a finite number of samples.

The number of records required depends on several factors. It depends on the combined noise level of the recorder and the signal source, on the desired test tolerance and confidence level, on whether the tolerance and confidence level is for INL (transition levels) or DNL (code bin widths), and on whether one wants to obtain the desired confidence for a particular width or transition level or for the worst case for all widths or transition levels. The number of records required for a given test tolerance and a given confidence in code bin widths is given by

$$R = J \left[\frac{2^{N-1} K_u}{B} \right]^2 \left[\frac{c\pi}{M} \right] \left\{ 1.1 \left[\frac{\sigma^*}{V} \right] + 0.2 \left[\frac{c\pi}{M} \right] \right\} \quad (11)$$

where

- R is minimum required number of records
- J is 1 for INL and $J = 2$ for DNL
- M is the number of samples per record
- c is $1 + 2(V_O/V)$
- V is the full scale voltage of the instrument in volts
- V_O is overdrive voltage
- K_u is $Z_{u/2}$ for obtaining the specified confidence in an individual transition level or code bin width
- K_u is $Z_{N,u/2}$ for obtaining the specified confidence in the worst case transition level or code bin width
- u is $1-v$, with v the desired confidence level expressed as a fraction
- σ^* is σ , the rms random noise level in volts, for INL
- σ^* is the minimum of σ or $Q/1.1$ for DNL (Q = code bin width)
- N is number of bits of the waveform recorder
- B is desired test tolerance as a fraction of the code bin width

The values for $Z_{u/2}$ and $Z_{N,u/2}$ can be obtained from table 2. For values of N between those in the table, use linear interpolation. Expression $Z_{u/2}$ is defined such that the probability is $1-u$ that the absolute value of a Gaussian distributed random variable with a mean of zero and a standard deviation of one is $\leq Z_{u/2}$. Expression $Z_{N,u/2}$ is defined such that the probability is $1-u$ that the maximum of the absolute values of 2^N Gaussian distributed random variables with a mean of zero and a standard deviation of one will be $\leq Z_{N,u/2}$.

Table 2—Values of $Z_{u/2}$ and $Z_{N, u/2}$

u	$Z_{u/2}$	$Z_{4, u/2}$	$Z_{8, u/2}$	$Z_{12, u/2}$	$Z_{16, u/2}$	$Z_{20, u/2}$	$Z_{24, u/2}$
0.2	1.28	2.46	3.33	4.04	4.64	5.19	5.68
0.1	1.64	2.72	3.53	4.21	4.80	5.33	5.81
0.05	1.96	2.95	3.72	4.37	4.94	5.46	5.93
0.02	2.33	3.22	3.95	4.57	5.12	5.62	6.08
0.01	2.58	3.42	4.11	4.71	5.25	5.74	6.19
0.005	2.81	3.60	4.27	4.85	5.38	5.85	6.30
0.002	3.09	3.84	4.47	5.03	5.54	6.01	6.44
0.001	3.29	4.00	4.62	5.16	5.66	6.12	6.54

Equations (9) through (11) and the values in table 2 are derived in [B2]. For further information, see [B15].

4.1.3 Fitting sine waves to recorded sine wave data

Apply a sine wave with specified parameters to the input of the recorder. Take a record of data. Fit a sine wave function to the record by varying the phase, amplitude, DC value, and (if needed) frequency of the fit function to minimize the sum of the squared difference between the data and the function. Two suggested approaches for performing the least squares fit are given below—one for known frequency solutions, which can be used when the sample clock and input frequencies are both known and stable, and one for unknown frequency solutions for all other cases. Two algorithms are included for each approach—one that requires matrix operations, and another that does not. For the known frequency approach, the two algorithms give identical results. For the approach used when the frequency is unknown, the two algorithms give the same results when the initial conditions are the same, although the convergence properties of the two differ. The matrix algorithm converges faster than the non-matrix algorithm, especially for records that contain five or fewer periods of the sine wave. For this reason, the matrix algorithm is recommended for use when the host computer supports matrix operations.

NOTE—The three parameter algorithms of 4.1.3.1 (for known frequency) are closed form solutions that will always produce an answer, but the answer will be a poorer fit than the four parameter algorithms of 4.1.3.2 if the actual frequency applied to the recorder differs from the frequency used by the algorithm. The algorithms of 4.1.3.2, on the other hand, employ an iterative solution that may diverge for bad initial estimates, or especially corrupted data. The method used should be stated.

4.1.3.1 An algorithm for three parameter (known frequency) least squares fit to sine wave data using matrix operations

Assuming the data record contains the sequence of samples $y_1, y_2, \dots, y_m$, taken at times $t_1, t_2, \dots, t_m$, this algorithm finds the values of A_0 , B_0 , and C_0 that minimize the following sum of squared differences:

$$\sum_{n=1}^M [y_n - A_0 \cos(\omega_0 t_n) - B_0 \sin(\omega_0 t_n) - C_0]^2 \quad (12)$$

where

ω_0 is the frequency applied to the waveform recorder input

To find the values for A_0 , B_0 , and C_0 , first create the following matrices:

$$D_0 = \begin{bmatrix} \cos(\omega_0 t_1) & \sin(\omega_0 t_1) & 1 \\ \cos(\omega_0 t_2) & \sin(\omega_0 t_2) & 1 \\ \vdots & \vdots & \vdots \\ \cos(\omega_0 t_M) & \sin(\omega_0 t_M) & 1 \end{bmatrix} \quad (13)$$

$$y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_M \end{bmatrix} \quad (14)$$

$$x_0 = \begin{bmatrix} A_0 \\ B_0 \\ C_0 \end{bmatrix} \quad (15)$$

In matrix notation, the sum of squared differences in equation (12) is given by

$$(y - D_0 x_0)^T (y - D_0 x_0) \quad (16)$$

where

$(*)^T$ designates the transpose of $(*)$

Compute the least-squares solution, $\hat{x}_0$, that minimizes equation (16) using

$$\hat{x}_0 = (D_0^T D_0)^{-1} (D_0^T y) \quad (17)$$

The fitted function is then given by

$$y_n' = A_0 \cos(\omega_0 t_n) + B_0 \sin(\omega_0 t_n) + C_0 \quad (18)$$

To convert the amplitude and phase to the form

$$y_n' = A \cos(\omega_0 t_n + \theta) + C \quad (19)$$

use

$$A = \sqrt{A_0^2 + B_0^2} \quad (20)$$

$$\theta = \tan^{-1} \left[\frac{-B_0}{A_0} \right] \quad \text{if } A_0 \geq 0 \quad (21)$$

$$\theta = \tan^{-1} \left[\frac{-B_0}{A_0} \right] + \pi \quad \text{if } A_0 < 0 \quad (22)$$

The residuals, r_n , of the fit are given by

$$r_n = y_n - A_0 \cos(\omega_0 t_n) - B_0 \sin(\omega_0 t_n) - C_0 \quad (23)$$

and the rms error is given by

$$\epsilon_{\text{rms}} = \sqrt{\frac{1}{M} \sum_{n=1}^M r_n^2} \quad (24)$$

4.1.3.2 A non-matrix algorithm for three parameter (known frequency) least squared fit to sine wave data

Assuming the data record contains the sequence of samples y_n taken at times t_n , the following nine sums are computed using the angular frequency applied to the waveform recorder input as ω :

Define

$$\alpha_n = \cos(\omega t_n) \quad (25)$$

$$\beta_n = \sin(\omega t_n) \quad (26)$$

Then calculate

$$\begin{array}{lll} \sum_{n=1}^M y_n & \sum_{n=1}^M \alpha_n & \sum_{n=1}^M \beta_n \\ \sum_{n=1}^M \alpha_n \beta_n & \sum_{n=1}^M \alpha_n^2 & \sum_{n=1}^M \beta_n^2 \\ \sum_{n=1}^M y_n \alpha_n & \sum_{n=1}^M y_n \beta_n & \sum_{n=1}^M y_n^2 \end{array}$$

Using these sums, compute

$$A_1 = \frac{A_N}{A_D} \quad (27)$$

where

$$A_N = \frac{\sum_{n=1}^M y_n \alpha_n - \bar{y} \sum_{n=1}^M \alpha_n}{\sum_{n=1}^M \alpha_n \beta_n - \bar{\beta} \sum_{n=1}^M \alpha_n} - \frac{\sum_{n=1}^M y_n \beta_n - \bar{y} \sum_{n=1}^M \beta_n}{\sum_{n=1}^M \beta_n^2 - \bar{\beta} \sum_{n=1}^M \beta_n} \quad (28)$$

$$A_D = \frac{\sum_{n=1}^M \alpha_n^2 - \bar{\alpha} \sum_{n=1}^M \alpha_n}{\sum_{n=1}^M \alpha_n \beta_n - \bar{\beta} \sum_{n=1}^M \alpha_n} - \frac{\sum_{n=1}^M \alpha_n \beta_n - \bar{\alpha} \sum_{n=1}^M \beta_n}{\sum_{n=1}^M \beta_n^2 - \bar{\beta} \sum_{n=1}^M \beta_n} \quad (29)$$

$$B_1 = \frac{B_N}{B_D} \quad (30)$$

where