



IEEE Standard for Digitizing Waveform Recorders

IEEE Instrumentation and Measurement Society

Sponsored by the
Waveform Generation, Measurements and Analysis Committee (TC-10)



IEEE
3 Park Avenue
New York, NY 10016-5997, USA

18 April 2008

IEEE Std 1057™-2007
(Revision of
IEEE 1057-1994)

This is a Redline Document produced by Techstreet, a business of Thomson Reuters.

This document is intended to provide users with an indication of changes from one edition to the next. It includes a full-text version of the new document, plus an indication of changes from the previous version.

Redlines are designed to save time and improve efficiencies by using the latest software technology to find and highlight document changes. More professionals are using valuable new technologies like redlines, to help improve outcomes in a fastpaced global business world.

Because it may not be technically possible to capture all changes accurately, it is recommended that users consult previous editions as appropriate. In all cases, only the current base version of this publication is to be considered the official document.

Redline Processing Notes:

1. ~~Red Text~~ - Red strikethrough text denotes deletions.
2. Blue Text - Blue underlined text denotes modifications and additions.

IEEE Standard for Digitizing Waveform Recorders

Sponsored by

Waveform Generation, Measurements and Analysis Committee (TC-10)
of the
IEEE Instrumentation and Measurement Society

Approved 15 April 2008

American National Standards Institute

Approved 5 December 2007

IEEE-SA Standards Board

Abstract: Terminology and test methods for describing the performance of waveform recorders are presented.

Keywords: effective number of bits, frequency response, noise, sine fitting, step pulse response, total harmonic distortion, transitions levels, waveform recorders

The Institute of Electrical and Electronics Engineers, Inc.
3 Park Avenue, New York, NY 10016-5997, USA

Copyright © 2008 by the Institute of Electrical and Electronics Engineers, Inc.
All rights reserved. Published 18 April 2008. Printed in the United States of America.

IEEE is a registered trademark in the U.S. Patent & Trademark Office, owned by the Institute of Electrical and Electronics Engineers, Incorporated.

PDF: ISBN 978-0-7381-5350-6 STD95748
Print: ISBN 978-0-7381-5351-3 STDPD95748

No part of this publication may be reproduced in any form, in an electronic retrieval system or otherwise, without the prior written permission of the publisher.

IEEE Standards documents are developed within the IEEE Societies and the Standards Coordinating Committees of the IEEE Standards Association (IEEE-SA) Standards Board. The IEEE develops its standards through a consensus development process, approved by the American National Standards Institute, which brings together volunteers representing varied viewpoints and interests to achieve the final product. Volunteers are not necessarily members of the Institute and serve without compensation. While the IEEE administers the process and establishes rules to promote fairness in the consensus development process, the IEEE does not independently evaluate, test, or verify the accuracy of any of the information or the soundness of any judgments contained in its standards.

Use of an IEEE Standard is wholly voluntary. The IEEE disclaims liability for any personal injury, property or other damage, of any nature whatsoever, whether special, indirect, consequential, or compensatory, directly or indirectly resulting from the publication, use of, or reliance upon this, or any other IEEE Standard document.

The IEEE does not warrant or represent the accuracy or content of the material contained herein, and expressly disclaims any express or implied warranty, including any implied warranty of merchantability or fitness for a specific purpose, or that the use of the material contained herein is free from patent infringement. IEEE Standards documents are supplied “**AS IS**.”

The existence of an IEEE Standard does not imply that there are no other ways to produce, test, measure, purchase, market, or provide other goods and services related to the scope of the IEEE Standard. Furthermore, the viewpoint expressed at the time a standard is approved and issued is subject to change brought about through developments in the state of the art and comments received from users of the standard. Every IEEE Standard is subjected to review at least every five years for revision or reaffirmation. When a document is more than five years old and has not been reaffirmed, it is reasonable to conclude that its contents, although still of some value, do not wholly reflect the present state of the art. Users are cautioned to check to determine that they have the latest edition of any IEEE Standard.

In publishing and making this document available, the IEEE is not suggesting or rendering professional or other services for, or on behalf of, any person or entity. Nor is the IEEE undertaking to perform any duty owed by any other person or entity to another. Any person utilizing this, and any other IEEE Standards document, should rely upon his or her independent judgment in the exercise of reasonable care in any given circumstances or, as appropriate, seek the advice of a competent professional in determining the appropriateness of a given IEEE standard.

Interpretations: Occasionally questions may arise regarding the meaning of portions of standards as they relate to specific applications. When the need for interpretations is brought to the attention of IEEE, the Institute will initiate action to prepare appropriate responses. Since IEEE Standards represent a consensus of concerned interests, it is important to ensure that any interpretation has also received the concurrence of a balance of interests. For this reason, IEEE and the members of its societies and Standards Coordinating Committees are not able to provide an instant response to interpretation requests except in those cases where the matter has previously received formal consideration. A statement, written or oral, that is not processed in accordance with the IEEE-SA Standards Board Operations Manual shall not be considered the official position of IEEE or any of its committees and shall not be considered to be, nor be relied upon as, a formal interpretation of the IEEE. At lectures, symposia, seminars, or educational courses, an individual presenting information on IEEE standards shall make it clear that his or her views should be considered the personal views of that individual rather than the formal position, explanation, or interpretation of the IEEE.

Comments for revision of IEEE Standards are welcome from any interested party, regardless of membership affiliation with IEEE. Suggestions for changes in documents should be in the form of a proposed change of text, together with appropriate supporting comments. Comments on standards and requests for interpretations should be submitted to the following address:

Secretary, IEEE-SA Standards Board
445 Hoes Lane
Piscataway, NJ 08854
USA

Authorization to photocopy portions of any individual standard for internal or personal use is granted by The Institute of Electrical and Electronics Engineers, Inc., provided that the appropriate fee is paid to Copyright Clearance Center. To arrange for payment of licensing fee, please contact Copyright Clearance Center, Customer Service, 222 Rosewood Drive, Danvers, MA 01923 USA; +1 978 750 8400. Permission to photocopy portions of any individual standard for educational classroom use can also be obtained through the Copyright Clearance Center.

Introduction

This introduction is not part of IEEE Std 1057-2007, IEEE Standard for Digitizing Waveform Recorders.

This standard is a complete revision and modernization of the previous standard, IEEE Std 1057-1994. It presents recommended methods for the test and evaluations of digitizing waveform recorders approved by balloting committee.

Notice to users

Laws and regulations

Users of these documents should consult all applicable laws and regulations. Compliance with the provisions of this standard does not imply compliance to any applicable regulatory requirements. Implementers of the standard are responsible for observing or referring to the applicable regulatory requirements. IEEE does not, by the publication of its standards, intend to urge action that is not in compliance with applicable laws, and these documents may not be construed as doing so.

Copyrights

This document is copyrighted by the IEEE. It is made available for a wide variety of both public and private uses. These include both use, by reference, in laws and regulations, and use in private self-regulation, standardization, and the promotion of engineering practices and methods. By making this document available for use and adoption by public authorities and private users, the IEEE does not waive any rights in copyright to this document.

Updating of IEEE documents

Users of IEEE standards should be aware that these documents may be superseded at any time by the issuance of new editions or may be amended from time to time through the issuance of amendments, corrigenda, or errata. An official IEEE document at any point in time consists of the current edition of the document together with any amendments, corrigenda, or errata then in effect. In order to determine whether a given document is the current edition and whether it has been amended through the issuance of amendments, corrigenda, or errata, visit the IEEE Standards Association Web site at <http://ieeexplore.ieee.org/xpl/standards.jsp>, or contact the IEEE at the address listed previously.

For more information about the IEEE Standards Association or the IEEE standards development process, visit the IEEE-SA Web site at <http://standards.ieee.org>.

Errata

Errata, if any, for this and all other standards can be accessed at the following URL: <http://standards.ieee.org/reading/ieee/updates/errata/index.html>. Users are encouraged to check this URL for errata periodically.

Interpretations

Current interpretations can be accessed at the following URL: <http://standards.ieee.org/reading/ieee/interp/index.html>.

Patents

Attention is called to the possibility that implementation of this standard may require use of subject matter covered by patent rights. By publication of this standard, no position is taken with respect to the existence or validity of any patent rights in connection therewith. The IEEE is not responsible for identifying Essential Patent Claims for which a license may be required, for conducting inquiries into the legal validity or scope of Patents Claims or determining whether any licensing terms or conditions provided in connection with submission of a Letter of Assurance, if any, or in any licensing agreements are reasonable or nondiscriminatory. Users of this standard are expressly advised that determination of the validity of any patent rights, and the risk of infringement of such rights, is entirely their own responsibility. Further information may be obtained from the IEEE Standards Association.

Participants

At the time this standard was submitted to the IEEE-SA Standards Board for approval, the Waveform Recorder Subcommittee had the following membership:

William Boyer, *Chair*
Thomas Linnenbrink, *TC-10 Chair*
Jerome Blair, *Secretary*
Francisco Alegria, *Symbols Editor*

Eulalia Balestrieri
David Bergman
Pasquale Daponte

Robert Graham
Donald Larson
Solomon Max
Nicholas Paulter

Sergio Rapuano
Steven Tilden
Fang Xu

In addition the following people made significant contributions to the standard:

Istvan Kollar

Dan Knierem

The following individual members of the balloting committee voted on this standard. Balloters may have voted for approval, disapproval, or abstention.

Francisco Alegria
Ali Al Awazi
David Bergman
Jerome Blair
William Boyer
James S. Case
Keith Chow
Tommy P. Cooper
Pasquale Daponte
Gary Engmann
Chris C. Gorringer
Robert Graham
Randall C. Groves
Scott A. Gudgel

Werner Hoelzl
Efthymios G. Karabetsos
J. L. Koepfinger
Donald Larson
Yeou Song Lee
Thomas Linnenbrink
G. Luri
Solomon Max
Michael J. Maytum
Edward M. McCall
Gary L. Michel
Scott Misha
Jerry R. Murphy
Bruce Muschlitz

Michael S. Newman
Donald M. Parker
Ulrich Pohl
Sergio Rapuano
Gordon D. Robinson
Robert A. Robinson
Bartien Sayogo
Joseph J. Stanco
Walter Struppler
Steven Tilden
John A. Vergis
Fang Xu
Oren Yuen
Waldemar Ziomek

The following people were on the working group of the original standard:

Michael Souders, *Chair*

Jerome Blair
William Boyer
Philip Crosby
Philip Green

James Hyatt
Dan Knierim
Robert Lawton*
Thomas Linnenbrink
Bruce Peetz

William Sapankevych
Otis Solomon
Stuart Streiff
Steven Tilden

*Chairman 1982-1986

The working group thanks the corporations, laboratories, and universities that provided significant committee support and resources during the development of this standard.

When the IEEE-SA Standards Board approved this standard on 5 December 2007, it had the following membership:

Steve M. Mills, *Chair*
Robert M. Grow, *Vice Chair*
Don Wright, *Past Chair*
Judith Gorman, *Secretary*

Richard DeBlasio
Alex Gelman
William R. Goldbach
Arnold M. Greenspan
Joanna N. Guenin
Julian Forster*
Kenneth S. Hanus
William B. Hopf

Richard H. Hulett
Hermann Koch
Joseph L. Koepfinger*
John Kulick
David J. Law
Glenn Parsons
Ronald C. Petersen
Tom A. Prevost

Narayanan Ramachandran
Greg Ratta
Robby Robson
Anne-Marie Sahazizia
Virginia C. Sulzberger
Malcolm V. Thaden
Richard L. Townsend
Howard L. Wolfman

*Member Emeritus

Also included are the following nonvoting IEEE-SA Standards Board liaisons:

Satish K. Aggarwal, *NRC Representative*
Alan H. Cookson, *NIST Representative*

Don Messina
IEEE Standards Program Manager, Document Development

Contents

1	Overview	1
1.1	Scope	1
1.2	Purpose	2
2	Normative references.....	2
3	Definitions, symbols, and acronyms.....	2
3.1	Definitions	2
3.2	Symbols and acronyms	8
4	Test parameters and methods.....	11
4.1	Manufacturer supplied information	11
4.2	Test selection	13
4.3	Test setup	14
4.4	Equivalent-time sampling	18
4.5	Discrete Fourier transform (DFT).....	21
4.6	Sinewave testing and fitting.....	30
4.7	Locating code transition levels	36
4.8	Step function response measurements	53
4.9	Tests using a dc input.....	56
5	Input impedance	56
5.1	Test method	56
5.2	Alternate test method using a time domain reflectometer (TDR)	56
5.3	Input impedance for out-of-range signals	57
6	Gain and offset	59
6.1	Independently based gain and offset.....	59
6.2	Terminal-based gain and offset.....	60
7	Linearity	61
7.1	Integral nonlinearity (<i>INL</i>)	61
7.2	Maximum static error (<i>MSE</i>)	61
7.3	Differential nonlinearity (<i>DNL</i>) and missing codes	62
7.4	Example <i>INL</i> and <i>DNL</i> data	63
7.5	Monotonicity.....	64
7.6	Hysteresis.....	64
7.7	Total harmonic distortion (<i>THD</i>)	65
7.8	Intermodulation distortion (<i>IMD</i>)	70
7.9	Noise power ratio (<i>NPR</i>).....	72
8	Noise.....	77
8.1	Comments on noise.....	77
8.2	Ratio of signal to noise and distortion (<i>SINAD</i>).....	77
8.3	Signal to noise ratio (<i>SNR</i>).....	78

8.4	Comments on <i>SINAD</i> and <i>SNR</i>	79
8.5	Effective number of bits (<i>ENOB</i>)	80
8.6	Random noise	82
8.7	Spurious components	84
8.8	Spurious-free dynamic range (<i>SFDR</i>)	85
9	Step response parameters	86
9.1	Settling parameters	86
9.2	Transition duration of the step response	89
9.3	Slew rate limit	91
9.4	Overshoot and precursors	91
9.5	Aperture duration	91
9.6	Limitations on the use of step responses	92
10	Frequency response parameters	94
10.1	Analog bandwidth	94
10.2	Gain error (gain flatness)	95
10.3	Frequency response and gain from step response	95
11	Interchannel parameters	100
11.1	Crosstalk	100
11.2	Multiple input reverse coupling	100
12	Time base parameters	101
12.1	Fixed error in sample time	101
12.2	Aperture uncertainty	101
12.3	Long-term stability	103
13	Out-of-range recovery	104
13.1	Test method for absolute out-of-range voltage recovery	105
13.2	Test method for relative out-of-range voltage recovery	105
13.3	Comments on test method	105
14	Word error rate	105
14.1	Test method	105
14.2	Comment on the number of samples required for word error rate	106
14.3	Comments on test equipment and making measurements	106
15	Differential input specifications	106
15.1	Differential input impedance to ground	107
15.2	Common-mode rejection ratio (<i>CMRR</i>) and maximum common-mode signal level	107
15.3	Maximum operating common-mode signal	107
15.4	Common-mode out-of-range signal recovery time	108
16	Cycle time	108
16.1	Test method	108
16.2	Comment	109

17	Triggering.....	109
17.1	Trigger delay and trigger jitter.....	109
17.2	Trigger sensitivity.....	110
17.3	Trigger minimum rate of change	111
17.4	Trigger coupling to signal.....	111
	Annex A (informative) Sine fitting algorithms.....	112
A.1	Algorithm for three-parameter (known frequency) least squares fit to sinewave data using matrix operations	112
A.2	Algorithm for four-parameter (general use) least squares fit to sinewave data using matrix operations.....	113
	Annex B (informative) Phase noise.....	116
B.1	What is phase noise?.....	116
B.2	Phase noise measurements.....	118
B.3	Phase noise effect on test results.....	118
B.4	Phase noise effects on specific tests.....	121
	Annex C (informative) Comment on errors associated with word-error-rate measurement.....	124
	Annex D (informative) Measurement of random noise below the quantization level	126
D.1	Derivation of equations.....	126
D.2	Alternate approach using a servo loop.....	128
	Annex E (informative) Software considerations	129
E.1	Motivation.....	129
E.2	Test of software to fit waveforms	129
E.3	Test of DFT software.....	129
E.4	Software toolkit	129
	Annex F (informative) Excitation with precision source with ramp vernier: determination of the test parameters	130
F.1	Triangular wave amplitude (A)	130
F.2	DC source output voltages (V_j)	131
F.3	Number of samples (K and M) and triangular wave frequency (f).....	132
	Annex G (informative) Presentation of sinewave data.....	133
G.1	$ENOB$ presentation	133
G.2	Presentation of residuals	134
G.3	Other examples of presentations of sinewave test results	135
	Annex H (informative) Bibliography	140

IEEE Standard for Digitizing Waveform Recorders

IMPORTANT NOTICE: This standard is not intended to assure safety, security, health, or environmental protection in all circumstances. Implementers of the standard are responsible for determining appropriate safety, security, environmental, and health practices or regulatory requirements.

This IEEE document is made available for use subject to important notices and legal disclaimers. These notices and disclaimers appear in all publications containing this document and may be found under the heading “Important Notice” or “Important Notices and Disclaimers Concerning IEEE Documents.” They can also be obtained on request from IEEE or viewed at <http://standards.ieee.org/IPR/disclaimers.html>.

1. Overview

1.1 Scope

This standard defines specifications and describes test methods for measuring the performance of electronic digitizing waveform recorders, waveform analyzers, and digitizing oscilloscopes with digital outputs. The standard is directed toward, but not restricted to, general-purpose waveform recorders and analyzers.

Special applications can require additional manufacturer information and verification tests not covered in this standard.

IEEE Std 1057™ has many similarities to IEEE Std 1241™-2000 [B24],¹ which applies to analog-to-digital converters (ADCs). However, IEEE Std 1057 shall be used for waveform recorders, and IEEE Std 1241-2000 shall be used for ADCs.

1.2 Purpose

The main purpose of this standard is to ensure that manufactures provide common terminology and users of waveform recorders have a well-defined set of specifications and test methods so they can understand, describe, and compare the performance of these recorders using a common language.

~~The main body pre-sents many performance features, sources of error, and test methods. The tests are meant to be used by manufacturers to locate and correct sources of error in the hardware.~~

~~The information in this standard can also be applied by users to correct some of the errors by software processing after the signal has been recorded. Another planned use for this standard is to aid users in writing specifications for purchasing new waveform.~~

1.3 Guidance to the user

This standard presents a large number of possible waveform recorder error sources and test methods for quantifying them. Some of these methods are cumbersome in that they require sophisticated test equipment and signal processing, or require considerable amount of time. It is generally not necessary to perform all of these tests to adequately characterize a particular waveform recorder for a particular application. Also, the existence of some errors can preclude measuring others. For many applications, the two relatively easy tests for determining effective bits and step response may suffice or at least point the user in a direction to pursue other tests. These tests require a high purity sine wave source and a clean, fast leading transition step generator. Recommended algorithms for performing the sine fit used in determining effective bits are described in 4.1.

The sine fit test will detect such errors as random noise, nonlinearities, and aperture uncertainty. Some insight into these errors may be gained by examining a plot of the residuals as defined in equations (23) or (53), as appropriate. The direction of further testing may be suggested by whether the residuals appear random or systematic.

The sine fit test does not measure amplitude flatness or phase linearity. Deviations from the ideal in these parameters may appear as aberrations in the recorder's step response. The step response parameters should be determined over a range of levels to detect amplitude nonlinearity and slew limit effects. When the recorder is not too noisy, the impulse response may be approximated by differentiating the step response. For more information, see [B14], [B17], and [B16].⁴

Waveform recorders often have various controllable settings such as gain, offset, filtering, sampling rate, trigger level, and trigger slope. The performance parameters and test methods described in this standard apply to a single set of settings. Changing a setting can affect the recorder's performance. This is particularly true of gain, offset, and sampling rate. Thus, a waveform recorder should be characterized over the range of settings at which the instrument will be used. Also, some waveform recorders have the capability to automatically self-calibrate, and the data taken before the calibration may differ from the data taken after the calibration. If a test procedure is affected by a self-calibration, the user may have to defeat the self-calibration feature before the test procedure has begun.

Drift in performance over time and temperature are not specifically addressed for most parameters in this standard. An exception is long term sample time drift. The manufacturer should specify the operating temperature range and recalibration intervals.

Many test methods call for test equipment whose performance is better than that expected from the waveform recorder being measured. The required test equipment may not be available. This is especially the case for recorders at the state of the art in sampling speed or accuracy, or both. A paradox may exist in that high performance waveform recorders are required to develop and evaluate new test equipment. When this is the case, clever engineering is required to develop techniques to measure the parameters of interest using available equipment.

Waveform recorders and digital oscilloscopes may employ either real time or equivalent time sampling, or both, at different times. With real time sampling, samples of the input signal are acquired sequentially, at the stated sampling rate. However, with equivalent time sampling, the samples may be acquired sequentially or otherwise, at a rate usually much lower than the stated equivalent time sampling rate. Equivalent time sampling requires that input waveforms be repetitive, since a waveform sampled in equivalent time is reconstructed from samples taken over many repetitions. This restriction does not apply for recorders employing real time sampling.

In this standard, the recommended test methods were designed with real time sampling instruments specifically in mind. Nevertheless, the test methods will generally still be valid when applied to recorders that use equivalent time sampling. A few additional points should be kept in mind in such cases:

A common feature of equivalent time sampling instruments is the ability to perform waveform averaging. This is useful in reducing noise components, both originating in the waveform source and also in the recorder. Therefore, tests that in some way measure the recorder's noise processes will be affected by the amount of averaging applied in the recorder. Examples of such tests are effective bits, signal to noise ratio, peak error, random noise, and aperture uncertainty. The results of any such tests performed on recorders employing equivalent time sampling should include the number of samples or records averaged.

As noted in 4.9.2, the use of equivalent time sampling also affects the time over which the aperture uncertainty is measured. Therefore, the results of this measurement should include the time required to capture a record, or equivalently, the real sampling rate and the record length.

Finally, some tests of equivalent time sampling recorders will actually be easier to perform, or will have less uncertainty because the sampling is often phase locked to the trigger signal. As an example, the measurement of trigger delay and jitter may be more accurate (see 4.10.1.3).

[†]The numbers in brackets correspond to those bibliographic items listed in annex E.

1.4 Manufacturer supplied information

1.4.1 General information

Manufacturers shall supply the following general information:

- a) Model number
- b) Dimensions and weight
- c) Power requirements
- d) Environmental conditions: Safe operating, non operating, and specified performance temperature range; altitude limitations; humidity limits; operating and storage; vibration tolerance; and compliance with applicable electromagnetic interference specifications
- e) Any special or peculiar characteristics
- f) Compliance with military and other specifications
- g) Available options and accessories, including their impact on equipment performance h)
- Exceptions to the above parameters where applicable
- i) Calibration interval, if required

1.4.2 Performance specifications

The manufacturer shall provide the minimum specifications (see clause 1.4.2.1). See clause 3 for definitions.

1.4.2.1 Minimum specifications

- a) Number of digitized bits
- b) Sample rates
- c) Memory length
- d) Input impedance
- e) Analog bandwidth
- f) Input signal ranges

1.4.2.2 Additional specifications

- a) Gain
- b) Offset
- c) Differential nonlinearity
- d) Harmonic distortion and spurious response
- e) Integral nonlinearity
- f) Maximum static error
- g) Signal to noise ratio
- h) Effective bits
- i) Peak error
- j) Random noise
- k) Frequency response
- l) Settling time
- m) Transition duration of step response (rise time)
- n) Slew limit
- o) Overshoot and precursors
- p) Aperture uncertainty (short term time base stability)
- q) Long term time base stability
- r) Fixed error in sample time
- s) Trigger delay and jitter
- t) Trigger sensitivity
- u) Trigger hysteresis band
- v) Trigger minimum rate of change
- w) Trigger coupling to signal
- x) Crosstalk
- y) monotonicity
- z) Hysteresis
- aa) Overvoltage recovery
- ab) Word error rate
- ac) Cycle time
- ad) Common mode rejection ratio
- ae) Maximum common mode signal level
- af) Differential input impedance
- ag) Maximum operating common mode signal level

2. Normative references

The following referenced documents are indispensable for the application of this standard. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments or corrigenda) applies.

IEEE Std 181-~~1977~~ ^{TM-2003}, IEEE Standard on Transitions, ~~Measurement~~ Pulses, and ~~Objective~~ Related Waveforms. ~~Analysis by Techniques.~~²

3. Definitions, symbols , and acronyms

For the purposes of this standard, the following terms and definitions apply. *The Authoritative Dictionary of IEEE Standards Terms* [B23] shall be referenced for terms not defined in this clause.

3.1 Definitions

3.1.1 ac coupled: A recorder whose input circuitry does not pass dc. Such recorders use either a series capacitance or a transformer at the input.

~~**3.1.1 amplitude flatness:** The variation in output amplitude as a function of frequency in response to a constant amplitude sine wave~~

3.1.2 aliasing: The phenomenon in sampled data systems whereby signal components at input frequencies greater than half the sampling frequency are converted to frequencies at less than half the sampling frequency in the output data record.

3.1.3 analog bandwidth (BW): Unless otherwise specified, the difference between the upper and lower frequency at which the amplitude response as seen in the data record is 0.707 (−3 dB) of the response as seen in the data record at the specified reference frequency .

~~**3.1.3 CMRR:** See 3.1.11, common mode rejection ratio~~

3.1.4 aperture duration: The period of time over which the input signal is averaged or weighted for an output value produced by a waveform recorder.

3.1.5 aperture uncertainty: The standard deviation of the sample instant in time. *Syn:* **short-term timing instability; timing jitter; timing phase noise.**

3.1.6 code bin k : A digital output that is designed to correspond to a particular set of input values.

NOTE—See Figure 1.²

3.1.7 code bin width $W[k]$: The difference of the code transition levels that delimit the bin.

$$W[k] = T[k + 1] - T[k] \quad \text{for } 1 \leq k \leq 2^N - 2 \quad (1)$$

3.1.8 code transition level: The boundary between two adjacent code bins.

~~In other words, coherent sampling occurs when the following relationship exists:~~

$$f_s \times M_c = f_0 \times M \quad (2)$$

~~f_s is the sampling frequency~~

~~M_c is the integer number of cycles in the data record~~

~~f_0 is the frequency of the input~~

~~M is the number of samples in the record~~

3.1.9 code transition level $T[k]$: The value of the recorder input parameter at the transition point between two given, adjacent code bins. The transition point is defined as the input value that causes 50% of the output codes to be greater than or equal to the upper code of the transition, and 50% to be less than the upper code of the transition. The transition level $T[k]$ lies between code bin $k-1$ and code bin k . See [also: transition point](#).

² Notes in text, tables, and figures are given for information only, and do not contain requirements needed to implement the standard.

Code Transition Level	Code Bin	Code Bin Width
$T[2^N-1]$	2^N-1	
$T[2^N-2]$	2^N-2	$W[2^N-2]$
	$\vdots$	$\vdots$
$T[k+2]$	$k+1$	$W[k+1]$
$T[k+1]$	k	$W[k]$
$T[k]$	$k-1$	$W[k-1]$
$T[k-1]$	$\vdots$	$\vdots$
$T[2]$	1	$W[1]$
$T[1]$	0	

Figure 1—Definitions pertaining to input quantization

3.1.10 coherent sampling: Sampling of a periodic waveform in which there is an integer number of cycles in the data record.

~~**3.1.9 common-mode overvoltage:** A signal level whose magnitude is less than the specified maximum safe common-mode signal but greater than the maximum operating common-mode signal.~~

~~**3.1.10 common-mode overvoltage recovery time:** The time required for the recorder to return to its specified characteristics after the end of a common-mode overvoltage pulse.~~

~~**3.1.12 common mode signal:** The average value of the signals at the positive and negative inputs of a differential input waveform recorder. If the signal at the positive input is designated V_+ , and the signal at the negative input is designated V_- , then the common mode signal (V_{cm}) is~~

$$V_{cm} = \frac{V_+ + V_-}{2} \quad (3)$$

~~**3.1.14 cycle time:** The real time elapsed (with a recorder continually taking records of data) between the beginning of two records taken in succession.~~

3.1.11 common-mode maximum operating signal: The largest common-mode signal for which a waveform recorder with differential inputs will meet its recording specifications while recording a normal-mode signal.

3.1.12 common-mode maximum signal level: For recorders with differential inputs, the maximum level of the common-mode signal at which the common-mode rejection ratio (CMRR) is still valid.

3.1.13 common-mode out-of-range: A signal level whose magnitude is less than the specified maximum safe common-mode signal but greater than the maximum operating common-mode signal.

3.1.14 common-mode out-of-range recovery time: The time required for the recorder to return to its specified characteristics after the end of a common-mode overvoltage signal.

3.1.15 common-mode rejection ratio (CMRR): The ratio of the input common-mode signal to the effect produced at the output of the recorder in units of the input.

3.1.16 common-mode signal: The average value of the signals at the positive and negative inputs of a differential input waveform recorder.

3.1.17 crosstalk (multichannel): The ratio (usually expressed in decibels) of the signal induced in one channel to a common signal applied to all other channels.

3.1.18 cumulative distribution function (cdf): For a continuous random variable, the function $F_X(x)$, which is the probability that the random variable, X , is less than or equal to x . This function is positive and nondecreasing monotonic.

3.1.19 cycle time: The real time elapsed (with a recorder continually taking records of data) between the beginning of two successive records.

3.1.20 data window: A set of coefficients by which corresponding samples in the data record are multiplied to more accurately estimate certain properties of the signal, particularly frequency domain properties. Generally, the coefficient values increase smoothly and symmetrically towards a maximum at the center of the record.

3.1.21 dc coupled: [A recorder whose input circuitry passes dc.](#)

3.1.22 decibels full scale (dBFS): The root-mean-square (rms) amplitude of any signal in decibels relative to a sinewave that spans the entire input range of the recorder.

~~**3.1.16 differential input impedance to ground:** The impedance between either the positive input and ground or the negative input and ground.~~

3.1.23 differential input impedance: [For a differential-input waveform recorder, the impedance between the positive input and the negative input with grounds and shields terminated as specified.](#)

3.1.24 differential input recorder: [A waveform recorder with differential inputs, which produces output codes that are a function of the difference between two input signals. The two input signals are typically called positive \(or plus\) and negative \(or minus\).](#)

3.1.25 differential nonlinearity (DNL): The difference between a specified code bin width and the average code bin width, divided by the average code bin width.

3.1.26 discrete Fourier transform (DFT): [A mathematical operation that converts sampled data from the time domain to the frequency domain. Contrast: inverse discrete Fourier transform \(IDFT\).](#)

3.1.27 distortion: [Aberrations in the output of the waveform recorder caused by nonlinearities other than quantization. Distortion effects appear as harmonics and intermodulation products of the input signal frequencies in the frequency domain of the output signal.](#)

3.1.28 dynamic range: [See: effective number of bits \(ENOB\), ratio of signal to noise and distortion \(SINAD\), signal-to-noise ratio \(SNR\), and spurious free dynamic range.](#)

NOTE— [The term *dynamic range* has a number of meanings depending on the application. It is not used by itself in this standard.](#)

3.1.29 effective number of bits (ENOB): [For an input sinewave of specified frequency and amplitude, the number of bits of an ideal waveform recorder for which the root-mean-square \(rms\) quantization error is equal to the rms noise and distortion of the waveform recorder under test.](#)

3.1.30 equivalent-time sampling: [A process by which consecutive samples of a repetitive waveform are acquired and assembled from multiple repetitions of the waveform, to produce a record of samples representing a single repetition of the waveform sampled at a higher rate.](#)

3.1.31 fixed error in sample time: A nonrandom error in the instant of sampling. A fixed error in sample time can be fixed with respect to the data samples acquired or correlated with an event that is detected by the sampling process. Unless otherwise specified, the term is usually taken to mean the maximum systematic fixed error that can be observed.

3.1.32 frequency response: [The complex gain \(magnitude and phase\) as a function of input frequency, or the Fourier transform of the impulse response.](#)

3.1.33 full-scale input range (FSR): The difference between the maximum and the minimum recordable input values as specified by the manufacturer exclusive of quantization limitations.

3.1.34 full-scale signal: A signal that spans the entire manufacturer's specified amplitude range of the instrument.

3.1.35 gain and offset: **(A)** (*independently based*). The values by which the transition levels are multiplied and then to which the transition levels are added, respectively, to minimize the mean squared deviation from the ideal transition levels. **(B)** (*terminal-based*). The values by which the transition levels are multiplied and then to which the transition levels are added, respectively, to cause the deviations from the ideal transition levels to be zero at the terminal points, that is, at the first and last code transitions.

3.1.36 gain error: **(A)** (*dynamic*). The difference between the dynamic gain, $G(f)$, of the waveform recorder at a given frequency and its gain at a specified reference frequency divided by its gain at the reference frequency. **(B)** (*static*) The difference between the actual gain and the nominal gain.

3.1.37 gain flatness: Variation of gain over a specified frequency range, usually expressed in decibels.

3.1.38 harmonic distortion: For a pure sinewave input, output components at frequencies that are an integer multiple of the applied sinewave frequency.

3.1.39 hysteresis: The maximum systematic difference in values for a digitizer code transition level when the transition level is approached from either side of the transition with a specified limit on the slew rate of the input signal.

3.1.40 ideal code bin width (Q): The full-scale input range divided by the total number of code ~~states~~ bins.

~~**3.1.25 incoherent sampling:** Sampling of a waveform such that the relationship between the input frequency, sampling frequency, number of cycles in the data record, and the number of samples in the data record does not meet the definition of coherent sampling.~~

3.1.41 input impedance: The impedance between a signal input ~~of the waveform recorder and ground terminal and a specified reference terminal.~~

3.1.42 integral nonlinearity (INL): The maximum systematic difference between the ideal and actual code transition levels after correcting for gain and offset.

3.1.43 intermodulation distortion (IMD): When there is an input signal with multiple frequencies at the input to the recorder, the generation of distortion signals at the sum and difference frequencies of the inputs due to nonlinearities in the waveform recorder response.

3.1.44 inverse discrete Fourier transform (IDFT): A mathematical operation that converts sampled data from the frequency domain to the time domain. Contrast: discrete Fourier transform (DFT).

3.1.45 k^{th} code transition level $T[k]$: The input value corresponding to the transition between codes $k-1$ and k .

NOTE—See Figure 1.

3.1.46 large signal: A signal whose peak-to-peak amplitude (including noise) is as large as can be practically recorded by the instrument within, but not including, the maximum and minimum amplitude data codes. As a minimum, the signal shall span at least -1 dBFS ($\sim 89\%$) of the waveform recorder range.

3.1.47 least significant bit (LSB): One integer count in the digitized output of the waveform recorder. (If the recorder outputs data in a floating point format, e.g., input units, the data can be converted to LSBs using the manufacturer's instructions.) The nominal value associated with an LSB is the ideal code bin width, Q .

NOTE—See Figure 1.

3.1.48 linear time invariant (LTI) system: A system in which the output can be computed from the following convolution integral:

$$y(t) = \int_0^t x(\tau)h(t-\tau)d\tau \quad (2)$$

where

$x(t)$ is the input signal as a function of time
 $y(t)$ is the output signal as a function of time
 $h(t)$ is the impulse response of the system or the inverse Fourier transform of the complex frequency response of the system. The impulse response is not a function of the amplitude of the input $x(t)$ (linearity); nor is it a function of the time at which the input occurs (time invariance).

NOTE—See Oppenheim and Willsky [B35] for more information on LTI systems.

3.1.49 long-term settling error: The absolute difference between the final value specified for short-term transition settling duration, and the value 1 s after the beginning of the step, expressed as a percentage of the step amplitude.

3.1.50 long-term time base stability: The change in time base frequency (usually given in parts in 10^6) over a specified period of time at a specified sampling rate.

~~**3.1.32 maximum common-mode signal level:** The maximum level of the common-mode signal at which the common-mode rejection ratio is still valid.~~

~~**3.1.33 maximum operating common-mode signal:** The largest common-mode signal for which the waveform recorder will meet its effective bits specifications in recording a simultaneously-applied, normal-mode signal.~~

3.1.51 maximum static error: The maximum difference between any code transition level and its ideal value.

3.1.52 mod(x , m): A function that gives the remainder when x is divided by m , e.g., $\text{mod}(8.1, 2.5) = 0.6$. The sign of the result is the same as the sign of m .

3.1.53 monotonic recorder: A recorder that has output codes that do not ~~disregarding random noise, systematically~~ decrease(increase) for a uniformly increasing (decreasing) input signal.

~~3.1.38 normalized peak error: Peak error divided by three times the standard deviation of the differences discussed in 4.5.1.1. See 3.1.42, peak error.~~

3.1.54 noise: Any deviation between the output signal (converted to input units) and the input signal except deviations caused by linear time invariant system response (gain and phase shift), a dc level shift, total harmonic distortion (*THD*), or an error in the sample rate.

~~For example, noise includes the effects of random errors, fixed pattern errors, nonlinearities and time base errors (fixed error in sample time and aperture uncertainty).~~

NOTE—Noise has historically been an ambiguous term. In this standard, noise is defined as indicated above.

3.1.55 noise and distortion (NAD): The sum of the effects of noise and total harmonic distortion (*THD*).

3.1.56 noise floor: Maximum power level over a given frequency band in the frequency domain of components that are not due to the applied signal, harmonics, or spurious signals. The level could be absolute (expressed in decibels with respect to a milliwatt per unit of bandwidth) or relative to the waveform recorder input full scale (expressed in decibels with respect to full scale per unit of bandwidth).

3.1.57 noise power ratio (NPR): The ratio of the average out-of-notch to the average in-notch power spectral density magnitudes for the discrete Fourier transform (DFT) spectrum of the waveform recorder output sample set measured using a notch-filtered broadband white-noise generator as the input to the waveform recorder under test.

3.1.58 noncoherent sampling: Sampling of a periodic waveform where there is no integer number of cycles in the data.

3.1.59 normalized noise power gain (NNPG): Sum of squares of the values of a data window function divided by the number of samples.

3.1.60 normalized signal-to-noise (SNR1Hz): Signal-to-noise ratio (*SNR*) normalized to a reference bandwidth of 1 Hz. *Syn:* noise spectral density.

3.1.61 normal-mode signal (or differential signal): The difference between the signal at the positive input and the negative input of a differential input waveform recorder

~~If the signal at the positive input is designated V_+ , and the signal at the negative input is designated V_- , then the normal mode (or differential) signal (V_{dm}) is~~

$$V_{dm} = V_+ - V_- \quad (4)$$

3.1.62 Nyquist frequency: Frequency equal to one-half the sampling frequency.

3.1.63 offset: *See: gain and offset.*

~~3.1.41 overvoltage: Any voltage whose magnitude is less than the maximum safe input voltage of the recorder but greater than the full scale value for the selected range.~~

3.1.64 out of range (overvoltage): Any voltage whose magnitude is less than the maximum safe input voltage of the recorder but greater than the full-scale value for the selected range.

3.1.65 overshoot: The maximum amount by which the step response is above the high state for a positive-going transition or is below the low state for a negative-going transition, specified as a percentage of (recorded) pulse ~~percentage of (recorded) pulse exceeds the topline, specified as a~~ amplitude.

NOTE—See IEEE Std 181-2003³ for more information on low and high states.

~~**3.1.42 peak error:** The residual with the largest absolute value.~~

3.1.66 Parseval relationship: The relationship where the sum (or integral) of the square of a function is equal to the sum (or integral) of the square of its Fourier transform.

3.1.67 passband: The band of input signal frequencies that the waveform recorder is intended to digitize with nominally constant gain.

3.1.68 phase nonlinearity: The deviation in phase response from a perfectly linear-phase response as a function of frequency.

³ Information on references can be found in Clause 2.

3.1.69 precursor: Any deviations from the low ~~baseline prior to the pulse~~ state for a positive-going transition or from the high state for a negative-going transition prior to the step transition.

NOTE—See IEEE Std 181-2003 for more information on low and high states.

3.1.70 probability density function (pdf): For a continuous random variable, the function $f_X(x)$, which has the interpretation that $f_X(x)dx$ is the probability that the random variable, X , lies in the interval $(x, x + dx)$.

3.1.71 quantization: A process in which the continuous range of values of an input signal is divided into subranges, and to each subrange a discrete value of the output is uniquely assigned. In the ideal case, these subranges are nonoverlapping and equal in size. Whenever the signal value falls within a given subrange, the ideal output has the corresponding discrete value.

3.1.72 quantization error: The error caused by conversion of a variable having a continuous range of values to a quantized form having only discrete values, as in analog-to-digital conversion. The error is the difference between the original (analog) value and its quantized (digital) representation translated into input units.

3.1.73 quantization noise: Equivalent noise produced during the quantization of a continuous range variable. A quantization process of step size Q produces a root-mean-square (rms) quantization noise of $Q/12$ when the continuous variable range is much larger than Q .

3.1.74 random noise: A nondeterministic fluctuation in the output of a waveform recorder, usually described by its frequency spectrum and its amplitude statistical properties.

3.1.75 ratio of signal to noise and distortion (SINAD): For a pure sinewave input of specified amplitude and frequency, the ratio of root-mean-square (rms) signal to rms noise and distortion (NAD).

3.1.76 record of data: A sequential collection of samples acquired by the waveform recorder.

3.1.77 relatively prime: [A descriptor of two](#) integers when their greatest common divisor is 1.

3.1.78 residuals: [For least squares curve fitting of data to a mathematically defined function, the differences between the recorded data and the fitted function. The mathematically defined function is often a sinewave.](#)

3.1.79 reverse coupling: The ratio of the spurious signal generated by a signal at some other input to the recorder and the signal recorded at the specified input of the recorder.

3.1.80 root-mean-square (rms): [For a given set of data, the square root of the arithmetic mean of the squared values of each of the data.](#)

3.1.81 sampling: [The process of extracting values at discrete time instants from a continuous input signal.](#)

~~**3.1.49 settling time:** [Measured from the mesial point \(50%\) of the output, the time at which the step response enters and subsequently remains within a specified error band around the final value. The final value is defined to occur 1 s after the beginning of the step](#)~~

3.1.82 short-term settling error: [The maximum absolute difference between the waveform and the final value at all times between a specified time: measured from the 50% reference level instant of the waveform and the end of the data record expressed as a percentage of step amplitude. The final value is defined to occurs \(unless otherwise specified 1\) after the beginning of the step.](#)

3.1.83 short-term timing instability: *See: aperture uncertainty.*

~~**3.1.52 signal to noise ratio:** [The ratio of a signal to the noise](#)~~

3.1.84 short-term transition settling duration: [Measured from the 50% reference level of the output, the time at which the step response enters and subsequently remains within a specified error band around the final value. The final value is defined to occur at 1 s after the beginning of the step unless otherwise specified.](#)

3.1.87 single-ended recorder : [A recorder that measures the voltage difference between a single input connection and ground. ~~A non-differential waveform recorder, i.e., one that does not subtract the signals at two input terminals.~~](#)

3.1.85 signal-to-noise ratio (SNR): [The ratio of a signal to the noise where noise is any deviation between the output signal \(converted to input units\) and the input signal except deviations caused by linear time invariant system response \(gain and phase shift\), a dc level shift, total harmonic distortion \(THD\), or an error in the sample rate.](#)

3.1.86 sinewave histogram: [A histogram where each bin contains the total number of occurrences of each digitized level of an input sinewave signal.](#)

3.1.88 slew rate limit: The value of [rate of change of](#) output ~~transition rate of change~~ codes (referenced to the input) for which an increased amplitude input step causes no change.

3.1.89 spectral leakage: [Spectral lines adjacent to the input line produced by the discrete Fourier transform \(DFT\) when the input is a pure sinewave whose frequency does not satisfy the coherent sampling condition and thus is not at the center of a DFT frequency bin.](#)

3.1.90 spurious components: Persistent sin waves at [any](#) frequencies other than the [input or](#) harmonic frequencies.

3.1.91 spurious free dynamic range: [The frequency domain difference in decibels between the full-scale signal level and the largest spurious component \(including harmonics\) for a large, pure sinewave signal input.](#)

3.1.92 step response: The recorded output response for an ideal input step with designated ~~baseline and topline~~ low state and high state.

~~3.1.57 timing jitter:~~ See 3.1.2, NOTE—See IEEE Std 181-2003 for more information on low and high states.

3.1.93 timing jitter: See: aperture uncertainty.

3.1.94 timing phase noise: See: aperture uncertainty.

3.1.95 total harmonic distortion (THD): The scaled square root of the sum of squares of a specified set of harmonic distortion components including their aliases for a pure sinewave input of specified amplitude and frequency.

3.1.96 transition duration of the step response: The duration between the instants corresponding to two reference levels. Unless otherwise specified, the two reference levels are the 10% and 90% reference levels.

3.1.97 transition level: The input value that causes 50% of the output codes to be greater than or equal to the upper code of the transition, and 50% to be less than the upper code of the transition.

~~duration of a step response: The duration between the proximal point (10%) and the distal point (90%) on the recorded output response transition, for an ideal input step with designated baseline and topline.~~

3.1.98 transition settling duration: The time at which the step response enters and subsequently remains within a specified error band around the final value, measured from the 50% point of the response.

3.1.99 trigger delay: The elapsed time from the occurrence of a trigger pulse at the trigger input connector, to the time at which the first or a specified data sample is recorded.

3.1.100 trigger jitter: The standard deviation in the trigger delay time over multiple records.

3.1.101 trigger minimum rate of change: The slowest rate of change of the transition duration ~~leading edge~~ of a pulse of a specified level that will consistently trigger the recorder.

3.1.102 trigger signal coupling: The ratio of the spurious signal level (that is recorded by an input to the recorder) to the trigger signal level.

3.1.103 waveform: A manifestation or representation (e.g., graph, plot, oscilloscope presentation, discrete time series, equations, table of coordinates or statistical data) or a visualization of a signal.

3.1.104 waveform epoch: An interval to which consideration of a waveform is restricted for a particular calculation, procedure, or discussion. Except when otherwise specified, the waveform epoch is assumed to be the span over which the waveform is measured or defined.

3.1.105 window: See: data window.

3.2 Symbols and acronyms

Below are the acronyms used in this standard and the symbols that are used in more than one subclause in this standard. Other symbols are declared and defined locally as needed.

ε	<u>general symbol for error, uncertainty, or tolerance</u>
ε_f	<u>error in frequency</u>
ε_{fs}	<u>error in sampling rate</u>
ε_G	<u>gain error</u>
$\varepsilon[k]$	<u>residual error in the least squares fit method of computing gain and offset</u>
ε_m	<u>upper bound on aliasing error magnitude spectrum</u>
ε_Q	<u>quantization error</u>
ε_{ms}	<u>mean square error</u>
ε_w	<u>measured word error rate</u>
ε_{wMax}	<u>upper bound on word error rate</u>
η	<u>root-mean-square (rms) amplitude noise</u>
$\eta(f)$	<u>noise power per hertz relative to the carrier</u>
η_ϕ	<u>phase noise</u>
π	<u>constant, ratio of the circumference of a circle to the diameter</u>
ρ	<u>reflection coefficient</u>
σ	<u>standard deviation</u>
σ^2	<u>variance</u>
σ_t	<u>aperture uncertainty</u>
τ	<u>time constant</u>
$\Phi[s]$	<u>cumulative density function for a Gaussian distribution (the fraction of the total distribution covered from negative infinity to s)</u>
$\Phi[\sigma]$	<u>confidence level as a function of standard deviations for a Gaussian distribution, expressed as a fraction</u>
ϕ	<u>phase, expressed in radians</u>
A_{rms}	<u>root-mean-square (rms) value of a fitted sinewave, rms signal ac alternating current</u>
ADC	<u>analog-to-digital converter</u>
BW	<u>bandwidth</u>
CMRR	<u>common-mode rejection ratio</u>
DAC	<u>digital-to-analog converter dc direct current</u>
DFT	<u>discrete Fourier transform</u>
DNL	<u>differential nonlinearity (When given as one number without a code bin specification, it is the maximum differential nonlinearity of the entire range.)</u>
DNL_{rms}	<u>root-mean-square (rms) value of $DNL[k]$</u>
$DNL[k]$	<u>differential nonlinearity of code k</u>
E_{avm}	<u>residual spectrum after the bins at dc and test frequencies have all been set to zero (deleted from the spectrum)</u>
ENOB	<u>effective number of bits</u>

f	frequency, expressed in hertz
f_{eq}	equivalent sampling rate
f_h	subset of harmonics used in computing total harmonic energy f_i input signal reference frequency or input signal repetition rate f_m discrete Fourier transform (DFT) basis frequency
f_{ref}	reference frequency
f_s	sampling frequency
Δf	separation between two frequencies
Δf_{rms}	root-mean-square (rms) frequency deviation over one record length
$FDHM$	full duration at half of maximum
FM	frequency modulation
FSR	full-scale range of the recorder
G	gain of waveform recorder, ideally = 1
$G(f)$	dynamic gain of waveform recorder as a function of input frequency f
$H(f)$	frequency response of the waveform recorder
$h(t)$	impulse response
IDFT	inverse discrete Fourier transform
IMD	intermodulation distortion
INL	integral nonlinearity
K	number of data records
LSB	least significant bit
LTi	linear time invariant
M	number of sequential samples in a data record
MSE	maximum static error
N	number of bits digitized by the waveform recorder
$N_{CHG}[i]$	cumulative number of samples up to bin i in sinewave histogram code transition level test N_H number of the highest order harmonic used for computing total harmonic distortion (THD) $N_{HG}[i]$ average number of histogram samples received in each of the code bins that share the given transition level
N_S	total number of samples processed over several records
NAD	noise and distortion
NPR	noise power ratio
NPR_{max}	maximum noise power ratio
$NNPG$	normalized noise power gain P probability
pdf	probability density function
PSD	power spectral distribution
Q	ideal code bin width, expressed in input units
Q'	actual average code bin width
$SFDR$	spurious free dynamic range
$SINAD$	ratio of signal to noise and distortion
SNR	signal-to-noise ratio
T	time between digitized data samples or sample period
T_R	record length of the recorder in seconds
T_{eq}	interval between equivalent-time samples

t	<u>time, in seconds unless otherwise specified</u>
t_c	<u>transition duration</u>
$T[k]$	<u>code transition level between code k and code $k-1$</u>
$T[k_{in}]$	<u>measured level used to fine tune location of a code transition level</u>
TDR	<u>time domain reflectometer</u>
THD	<u>total harmonic distortion</u>
THE	<u>total harmonic energy</u>
$u(t)$	<u>unit step function</u>
$u'(t)$	<u>non-ideal step-like function</u>
$U'(f)$	<u>Fourier transform of $u'(t)$</u>
V_{in}	<u>amplitude of the input signal to the waveform recorder</u>
$V_{in\ rms}$	<u>root-mean-square (rms) of the input signal to the waveform recorder</u>
V_{os}	<u>input offset of waveform recorder, ideally = 0</u>
V_{out}	<u>peak-to-peak amplitude of the signal in the digitized data</u>
$VSWR$	<u>voltage standing wave ratio</u>
$W[k]$	<u>code bin width of code bin k</u>
$W[n]$	<u>Fourier transform of the data window function $w[k]$</u>
$w[k]$	<u>window function [for a discrete Fourier transform (DFT)]</u>
$X[k]$	<u>frequency domain samples of the input sequence $x[i]$ computed via the discrete Fourier transform (DFT)</u>
$X_{avg}[m]$	<u>averaged spectral magnitude of the discrete Fourier transforms (DFTs) of multiple data records</u>
X_{avgfh}	<u>averaged discrete Fourier transform (DFT) spectrum, X_{avg}, with the components (bins) at dc, the test frequencies f_i and (f_s-f_i), and the specified harmonic frequencies $f_{h[n]}$, all set to zero (deleted)</u>
$X_{wavg}[m]$	<u>averaged spectral magnitude of the discrete Fourier transforms (DFTs) of multiple windowed data records</u>
$X_w[i]$	<u>Fourier transform of a windowed input sequence $w[i] x[i]$</u>
$X_{w,k}[i]$	<u>windowed Fourier transform of the k^{th} record when multiple records are taken</u>
$x[i]$	<u>sequence of digitized input time domain samples</u>
$x'[i]$	<u>best fit points to a data record</u>
$x(t)$	<u>continuous input signal as a function of time</u>
$\bar{x}$	<u>average value of the input signal over the record</u>
$y[i]$	<u>filtered input sequence of digitized time domain samples</u>
Z_0	<u>transmission line impedance</u>
Z_i	<u>waveform recorder input impedance</u>
$\alpha_n = \cos(\omega t_n + \theta)$	
α	<u>average of α_n over M samples</u>
$\beta_n = \sin(\omega t_n + \theta)$	
β	<u>average of β_n over M samples</u>
ϵ	<u>error, for total error and error band</u>
ϵ_{rms}	<u>root mean square value of ϵ</u>
ϵ/f_s	<u>error sampling rate</u>

$e[f_s]$ = measured error input reference frequency

$e[k]$ = difference between $T[k]$ and ideal value of $T[k]$ computed from G and V_{os}

θ = phase, expressed as radians

π = constant, ratio of the circumference of a circle to the diameter

ρ = reflection coefficient

σ = standard deviation; sometimes used as noise amplitude, which is deviation of the random component of a signal

σ_σ = standard deviation of the standard deviation (for example, standard deviation of the noise amplitude)

σ_x = aperture uncertainty

τ = sampling period, the inverse of f_s

$\Phi[X]$ = cumulative density function for a Gaussian distribution (the fraction of the total distribution covered from negative infinity to X)

ϕ = phase estimate, expressed in radians

ψ = frequency estimate, expressed in radians per second

ω = frequency, expressed in radians per second

A = sinusoidal amplitude

A_+ = amplitude estimate of a sinusoid

A_D = amplitude denominator, when A (or A_+) is expressed as a fraction

A_N = amplitude numerator, when A (or A_+) is expressed as a fraction

$A[i]$ = average of a record

$\alpha_{11}, \alpha_{12}, \alpha_{21}, \alpha_{22}$ = linear equation matrix elements for four parameter fit iteration

B = test tolerance in fractions of an ideal code bin width (Q). Also used as an amplitude

B_+ = amplitude estimate of a cosine wave

B_D = amplitude denominator, when B (or B_+) is expressed as a fraction

B_N = amplitude numerator, when B (or B_+) is expressed as a fraction

C = offset

DNL = maximum differential nonlinearity over all k

$DNL[k]$ = differential nonlinearity of code k

E = number of effective bits

$ENBW$ = equivalent noise bandwidth

e_{mfff} = aliasing and first differencing magnitude errors

e_{pfff} = aliasing and first differencing phase errors

f = frequency, Hz

f_0 = frequency of the input signal

f_{eo} = upper frequency for which the amplitude response is -0.3 dB

f_{eq} = equivalent sampling rate

f^* = Approximate desired input frequency

$ffk/$ = correction function converting sine wave probability distribution function to uniform distribution function

f_{opt} = optimal input frequencies for the sine wave curve fit

f_r = input signal reference frequency or input signal repetition rate

f_s = sampling frequency

G = gain of waveform recorder, ideally = 1

$hn(i)$ = Hann window time coefficients

INL = integral nonlinearity

K = number of data records

k = code bin

L = equivalent time sampling parameter

M = number of sequential samples

M_e = number of input sine wave cycles

MSE = maximum static error

m = root-mean-square of rising and falling slopes of selected portions of a waveform mse N = number of digitized bits

n = sample index within a record

P_+, P_- = maximum and minimum peak regions of a sine wave

p = probability

Q = ideal code bin width, expressed in input units

Q' = average code bin width

R = error parameter, or number of records

$R+$, RD = rising and falling regions of a sine wave

S = set of samples collected over more than one record, also used as an error parameter

snr = signal-to-noise ratio

Δt = interval between equivalent time samples

$T[k]$ = code transition level between code k and code $k+1$

T_k = ideal value of $T[k]$ or $[Q \cdot (k+1) + T_+]$

T_+ = ideal value of $T[1]$

T_{MIN} = minimum trigger delay required for a large pulse

T_{OD} = trigger delay for a pulse value V_{OD}

t_n = discrete sample times

u = confidence expressed as a fraction

V = full-scale input range of waveform recorder

V_{HYST} = the trigger hysteresis level

$V(t)$ = the settled value after a specified time

V_O = overdrive voltage

V_{OD} = a small trigger pulse level

V_{TH} = minimum level that will trigger the recorder

V_{os} = input offset of waveform recorder, ideally = 0

$W[k]$ = code bin width of code bin k

w = estimated

w' = worst case

$w(i)$ = window Discrete Fourier Transform (DFT)

X = number of standard deviations of a

x = number of errors detected

y = average of y_n over M samples

y_n = data samples within a record

y'_n = best fit points to a data record

Z_u = transmission line impedance

Z_{π} = waveform recorder input impedance

$Z_{u/2}$ = number of standard deviations that encompass 100 (1- α)% of a Gaussian distribution about the center

4. Test parameters and methods

4.1 Manufacturer supplied information

4.1.1. General **methods** information

Manufacturers shall supply the following general information:

- Model number
- Dimensions and weight
- Power requirements
- Environmental conditions: Safe operating, nonoperating, and specified performance temperature range; altitude limitations; humidity limits, operating and storage; vibration tolerance; and compliance with applicable electromagnetic interference specifications
- Any special or peculiar characteristics
- Compliance with military and other specifications
- Available options and accessories, including their impact on equipment performance
- Exceptions to the above parameters where applicable
- Calibration interval, if required

4.1.2 Performance specifications

The manufacturer shall provide the minimum specifications defined in 4.1.2.1. Additional potentially useful specifications are given in 4.1.2.2. See Clause 3 for definitions.

4.1.2.1 Minimum specifications

- Number of digitized bits
- Sample rates
- Memory lengths
- Input impedances
- Analog bandwidths
- Input signal ranges
- Power supply voltage ranges
- Power dissipation

4.1.2.2 Additional specifications

- Gain
- Offset
- Differential nonlinearity (*DNL*)
- Integral nonlinearity (*INL*)
- Maximum static error
- Monotonicity
- Hysteresis
- Total harmonic distortion (*THD*) and spurious response
- Intermodulation distortion (*IMD*)
- Noise power ratio, (*NPR*)
- Ratio of signal to noise and distortion (*SINAD*)
- Signal-to-noise ratio (*SNR*)
- Effective number of bits (*ENOB*)

- [Random noise](#)
- [Gain error versus frequency](#)
- [Frequency response](#)
- [Transition settling duration](#)
- [Transition duration of step response \(rise time\)](#)
- [Overshoot and precursor](#)
- [Slew limit](#)
- [Crosstalk](#)
- [Aperture uncertainty \(short-term time base stability\)](#)
- [Long-term time base stability](#)
- [Fixed error in sample time](#)
- [Trigger delay and jitter](#)
- [Trigger sensitivity](#)
- [Trigger hysteresis band](#)
- [Trigger minimum rate of change](#)
- [Trigger coupling to signal](#)
- [Overvoltage recovery](#)
- [Word error rate](#)
- [Cycle time](#)
- [Common-mode rejection ratio \(CMRR\)](#)
- [Maximum common-mode signal level](#)
- [Differential input impedance](#)
- [Maximum operating common-mode signal level](#)

4.1.3 Fitting sine waves to recorded sine wave data

Apply a sine wave with specified parameters to the input of the recorder. Take a record of data. Fit a sine wave function to the record by varying the phase, amplitude, DC value, and (if needed) frequency of the fit function to minimize the sum of the squared difference between the data and the function. Two suggested approaches for performing the least squares fit are given below one for known frequency solutions, which can be used when the sample clock and input frequencies are both known and stable, and one for unknown frequency solutions for all other cases. Two algorithms are included for each approach one that requires matrix operations, and another that does not. For the known frequency approach, the two algorithms give identical results. For the approach used when the frequency is unknown, the two algorithms give the same results when the initial conditions are the same, although the convergence properties of the two differ. The matrix algorithm converges faster than the non-matrix algorithm, especially for records that contain five or fewer periods of the sine wave. For this reason, the matrix algorithm is recommended for use when the host computer supports matrix operations.

NOTE The three parameter algorithms of 4.1.3.1 (for known frequency) are closed form solutions that will always produce an answer, but the answer will be a poorer fit than the four parameter algorithms of 4.1.3.2 if the actual frequency applied to the recorder differs from the frequency used by the algorithm. The algorithms of 4.1.3.2, on the other hand, employ an iterative solution that may diverge for bad initial estimates, or especially corrupted data. The method used should be stated.

4.1.3.1 An algorithm for three parameter (known frequency) least squares fit to sine wave data using matrix operations

Assuming the data record contains the sequence of samples $y_1, y_2, \dots, y_m$, taken at times $t_1, t_2, \dots, t_m$, this algorithm finds the values of A_0 , B_0 , and C_0 that minimize the following sum of squared differences:

$$\sum_{n=1}^M [y_n - D A_0 \cos(\omega_0 t_n) - D B_0 \sin(\omega_0 t_n) - D C_0]^2 \quad (12)$$

Where

ω_0 is the frequency applied to the waveform recorder input

To find the values for A_0 , B_0 , and C_0 , first create the following matrices:

$$D_0 = \begin{bmatrix} \cos(\omega_0 t_1) & \sin(\omega_0 t_1) & 1 \\ \cos(\omega_0 t_2) & \sin(\omega_0 t_2) & 1 \\ \vdots & \vdots & \vdots \\ \cos(\omega_0 t_M) & \sin(\omega_0 t_M) & 1 \end{bmatrix} \quad (13)$$

$$y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_M \end{bmatrix} \quad (14)$$

$$x_0 = \begin{bmatrix} A_0 \\ B_0 \\ C_0 \end{bmatrix} \quad (15)$$

In matrix notation, the sum of squared differences in equation (12) is given by

$$(y - D_0 x_0)^T (y - D_0 x_0) \quad (16)$$

where

$(*)^T$ designates the transpose of $(*)$

Compute the least squares solution, $\hat{x}_0$, that minimizes equation (16) using

$$\hat{x}_0 = (D_0^T D_0)^{-1} D_0^T y \quad (17)$$

The fitted function is then given by

$$y_{\#} = A_0 \cos(\omega_0 t_{\#}) + B_0 \sin(\omega_0 t_{\#}) + C_0 \quad (18)$$

To convert the amplitude and phase to the form

$$y_n' = A \cos(\omega_0 t_n + \theta) + C \quad (19)$$

use

$$A = \sqrt{A_0^2 + B_0^2} \quad (20)$$

$$\theta = \tan^{-1} \left[\frac{-B_0}{A_0} \right] \quad \text{if } A_0 \geq 0 \quad (21)$$

$$\theta = \tan^{-1} \left[\frac{-B_0}{A_0} \right] + \pi \quad \text{if } A_0 < 0 \quad (22)$$

The residuals, r_n , of the fit are given by

$$r_n = y_n - A_0 \cos(\omega_0 t_n) - B_0 \sin(\omega_0 t_n) - C_0 \quad (23)$$

and the rms error is given by

4.1.3.2 A non-matrix algorithm for three parameter (known frequency) least squared fit to sine wave data

Assuming the data record contains the sequence of samples y_n taken at times t_n , the following nine sums are computed using the angular frequency applied to the waveform recorder input as ω :

Define

$$\alpha_n = \cos(\omega t_n) \quad (25)$$

$$\beta_n = \sin(\omega t_n) \quad (26)$$

Then calculate

$$\begin{array}{lll} \sum_{n=1}^M y_n & \sum_{n=1}^M \alpha_n & \sum_{n=1}^M \beta_n \\ \sum_{n=1}^M \alpha_n \beta_n & \sum_{n=1}^M \alpha_n^2 & \sum_{n=1}^M \beta_n^2 \\ \sum_{n=1}^M y_n \alpha_n & \sum_{n=1}^M y_n \beta_n & \sum_{n=1}^M y_n^2 \end{array}$$

Using these sums, compute

$$A_1 = \frac{A_N}{A_D} \quad (27)$$

Where

$$A_N = \frac{\sum_{n=1}^M y_n \alpha_n - \bar{y} \sum_{n=1}^M \alpha_n}{\sum_{n=1}^M \alpha_n \beta_n - \bar{\beta} \sum_{n=1}^M \alpha_n} - \frac{\sum_{n=1}^M y_n \beta_n - \bar{y} \sum_{n=1}^M \beta_n}{\sum_{n=1}^M \beta_n^2 - \bar{\beta} \sum_{n=1}^M \beta_n} \quad (28)$$

$$A_D = \frac{\sum_{n=1}^M \alpha_n^2 - \bar{\alpha} \sum_{n=1}^M \alpha_n}{\sum_{n=1}^M \alpha_n \beta_n - \bar{\beta} \sum_{n=1}^M \alpha_n} - \frac{\sum_{n=1}^M \alpha_n \beta_n - \bar{\alpha} \sum_{n=1}^M \beta_n}{\sum_{n=1}^M \beta_n^2 - \bar{\beta} \sum_{n=1}^M \beta_n} \quad (29)$$

$$B_1 = \frac{B_N}{B_D} \quad (30)$$

Where

$$B_N = \frac{\sum_{n=1}^M y_n \alpha_n - \bar{y} \sum_{n=1}^M \alpha_n}{\sum_{n=1}^M \alpha_n^2 - \bar{\alpha} \sum_{n=1}^M \alpha_n} - \frac{\sum_{n=1}^M y_n \beta_n - \bar{y} \sum_{n=1}^M \beta_n}{\sum_{n=1}^M \alpha_n \beta_n - \bar{\alpha} \sum_{n=1}^M \beta_n} \quad (31)$$

$$B_D = \frac{\sum_{n=1}^M \alpha_n \beta_n - \bar{\beta} \sum_{n=1}^M \alpha_n}{\sum_{n=1}^M \alpha_n^2 - \bar{\alpha} \sum_{n=1}^M \alpha_n} - \frac{\sum_{n=1}^M \beta_n^2 - \bar{\beta} \sum_{n=1}^M \beta_n}{\sum_{n=1}^M \alpha_n \beta_n - \bar{\alpha} \sum_{n=1}^M \beta_n} \quad (32)$$

$$C = \bar{y} - A_1 \bar{\alpha} - B_1 \bar{\beta} \quad (33)$$

Where

$$\bar{y} = \frac{1}{M} \sum_{n=1}^M y_n \quad (34)$$

$$\bar{\alpha} = \frac{1}{M} \sum_{n=1}^M \alpha_n \quad (35)$$

$$\bar{\beta} = \frac{1}{M} \sum_{n=1}^M \beta_n \quad (36)$$

This produces a fitted function of the form

$$y'_n = A_1 \cos(\omega t_n) + B_1 \sin(\omega t_n) + C \quad (37)$$

where

y'_n is fitted points

Now the rms error can be computed as follows:

$$\epsilon_{\text{rms}} = \sqrt{\frac{\epsilon}{M}} \quad (38)$$

where

$$\begin{aligned} \epsilon = & \sum_{n=1}^M y_n^2 + A_1^2 \sum_{n=1}^M \alpha_n^2 + B_1^2 \sum_{n=1}^M \beta_n^2 + MC^2 \\ & - 2A_1 \sum_{n=1}^M \alpha_n y_n - 2B_1 \sum_{n=1}^M \beta_n y_n - 2C \sum_{n=1}^M y_n \\ & + 2A_1 B_1 \sum_{n=1}^M \alpha_n \beta_n + 2A_1 C \sum_{n=1}^M \alpha_n + 2B_1 C \sum_{n=1}^M \beta_n \end{aligned} \quad (39)$$

To convert the amplitude and phase to the form

$$y'_n = A \cos(\omega t_n + \theta) + C \quad (40)$$

use

$$A = \sqrt{A_1^2 + B_1^2} \quad (41)$$

$$\theta = \tan^{-1} \left(-\frac{B_1}{A_1} \right) \quad \text{if } A_1 \geq 0$$

$$\theta = \tan^{-1} \left(-\frac{B_1}{A_1} \right) + \pi \quad \text{if } A_1 < 0 \quad (42)$$

4.1.3.3 An algorithm for four parameter (general use) least squares fit to sine wave data using matrix operations

Assuming the data record contains the sequence of samples $y_1, y_2, \dots, y_M$ taken at times $t_1, t_2, \dots, t_M$, this algorithm uses an iterative process to find the values of A_i, B_i, C_i and ω_i that minimize the following sum of squared differences:

$$\sum_{n=1}^M [y_n - A_i \cos(\omega_i t_n) - B_i \sin(\omega_i t_n) - C_i]^2 \quad (43)$$

where

ω_i is the frequency applied to the waveform recorder input

Perform the following steps:

a) Set index $i = 0$. Make an initial estimate of the angular frequency ω_0 of the recorded data. The frequency may be estimated by using a Discrete Fourier Transform (DFT) (either on the full record or a portion of it), or by counting zero crossings, or simply by using the applied input frequency. Perform a prefit using the three parameter matrix algorithm given in 4.1.3.1 to determine A_0, B_0 , and C_0 .

b) Set $i = i + 1$ for the next iteration.

c) Update the angular frequency estimate using:

$$\omega_i = \omega_{i-1} + \Delta\omega_{i-1} \quad (\Delta\omega_{i-1} = 0 \text{ for } i = 1) \quad (44)$$

d) Create the following matrices:

$$y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_M \end{bmatrix} \quad (45)$$

$$D_i = \begin{bmatrix} \cos(\omega_i t_1) & \sin(\omega_i t_1) & 1 & -A_{i-1} t_1 \sin(\omega_i t_1) + B_{i-1} t_1 \cos(\omega_i t_1) \\ \cos(\omega_i t_2) & \sin(\omega_i t_2) & 1 & -A_{i-1} t_2 \sin(\omega_i t_2) + B_{i-1} t_2 \cos(\omega_i t_2) \\ \vdots & \vdots & \vdots & \vdots \\ \cos(\omega_i t_M) & \sin(\omega_i t_M) & 1 & -A_{i-1} t_M \sin(\omega_i t_M) + B_{i-1} t_M \cos(\omega_i t_M) \end{bmatrix} \quad (46)$$

$$x_i = \begin{bmatrix} A_i \\ B_i \\ C_i \\ \Delta\omega_i \end{bmatrix} \quad (47)$$



e) Compute the least squares solution, $\hat{x}$, that minimizes equation (43) using

$$\hat{x}_i = (D_i^T D_i)^{-1} (D_i^T y) \quad (48)$$

f) Compute the amplitude, A , and phase, θ , for the form

$$y_n' = A \cos(\omega t_n + \theta) + C \quad (49)$$

—using

$$A = \sqrt{A_i^2 + B_i^2} \quad (50)$$

and

$$\theta = \tan^{-1} \left[\frac{B_i}{A_i} \right] \text{ if } A_i \geq 0 \quad (51)$$

$$\theta = \tan^{-1} \left[\frac{B_i}{A_i} \right] + \pi \text{ if } A_i < 0 \quad (52)$$

g) Repeat steps b) through f), recomputing the model based on the new values of A , B , and ω , calculated from the previous iteration. Continue to iterate until the changes in A , ω , θ , and C are suitably small.

The residuals, r_n , of the fit are given by

$$r_n = y_n - A_i \cos(\omega_i t_n) - B_i \sin(\omega_i t_n) - C_i \quad (53)$$

and the rms error is given by

4.1.3.4 A non-matrix algorithm for four parameter (general use) least squared fit to sine wave data

Make estimates of frequency ω and phase θ of the recorded data, where phase is referred to the first point in the record.

The frequency expressed in radians per second, may be estimated by using a DFT (either on the full record or a portion of it), or by counting zero crossings. Alternatively, the frequency can be estimated by using the applied input frequency.

The phase expressed in radians, may be estimated by closed form calculation using the algorithm of 4.1.3.1 given a frequency estimate. Another method is to use a linear least squares fit to determine the phase as the intercept of the relation between the record's zero crossing times and consecutive integer multiples of π . A two-point estimate of phase can be arrived at by evaluating

where

$\text{sign}(y_2 - y_1) = 1$ for $y_2 \geq y_1$, or -1 for $y_2 < y_1$

y_1 is the first sample in record where $t = 0$

y_2 is the sample immediately following y_1

C, A is sine wave offset and amplitude, respectively

To make amplitude estimates for this purpose, half of the algebraic difference between the maximum and minimum values in the data record can be used if the recorder is free from large random errors. Alternatively, a histogram representing the probability density function of the digital codes can be employed. To make offset estimates for this purpose, half the sum of the maximum and minimum value of the record should be taken. Another means for offset estimation is to take the average value of the data points over an integer number of cycles (or over the entire record if it contains many cycles). Note that $\text{sign}(y_2 - y_1)$ can give an erroneous sign if there are too few points per cycle, particularly for values of $\cos^2$ close to 0 or π .

For more information on making initial estimates, see [B10].

Assuming the data record contains the sequence of samples y_n taken at times t_n , and using the estimates for ω and θ , compute the following 16 sums:

$$\begin{array}{cccc} \sum_{n=1}^M \alpha_n & \sum_{n=1}^M \beta_n & \sum_{n=1}^M y_n & \sum_{n=1}^M \alpha_n y_n \\ \sum_{n=1}^M \beta_n y_n & \sum_{n=1}^M \alpha_n \beta_n & \sum_{n=1}^M \beta_n^2 & \sum_{n=1}^M \alpha_n^2 \\ \sum_{n=1}^M y_n^2 & \sum_{n=1}^M \beta_n t_n y_n & \sum_{n=1}^M \alpha_n \beta_n t_n & \sum_{n=1}^M \beta_n^2 t_n \\ \sum_{n=1}^M \beta_n^2 t_n^2 & \sum_{n=1}^M \beta_n t_n & \sum_{n=1}^M \alpha_n t_n & \sum_{n=1}^M \beta_n t_n^2 \end{array}$$

Now using the estimates for ω and θ , compute

$$\psi = \omega + \frac{a_{22}R - a_{12}S}{a_{11}a_{22} - a_{12}a_{21}} \quad (58)$$

$$\phi = \theta + \frac{a_{11}S - a_{21}R}{a_{11}a_{22} - a_{12}a_{21}} \quad (59)$$

where

$$a_{11} = \frac{\sum_{n=1}^M \beta_n t_n (\alpha_n - \bar{\alpha}) \sum_{n=1}^M \alpha_n t_n (\beta_n - \bar{\beta})}{\left[\sum_{n=1}^M \alpha_n (\alpha_n - \bar{\alpha}) \right]^2}$$

$$\frac{\sum_{n=1}^M \alpha_n (\alpha_n - \bar{\alpha}) \sum_{n=1}^M \beta_n t_n^2 (\beta_n - \bar{\beta})}{\left[\sum_{n=1}^M \alpha_n (\alpha_n - \bar{\alpha}) \right]^2}$$

(60)

$$a_{12} = \frac{\sum_{n=1}^M \beta_n t_n (\alpha_n - \bar{\alpha}) \sum_{n=1}^M \alpha_n (\beta_n - \bar{\beta})}{\left[\sum_{n=1}^M \alpha_n (\alpha_n - \bar{\alpha}) \right]^2}$$

$$\frac{\sum_{n=1}^M \alpha_n (\alpha_n - \bar{\alpha}) \sum_{n=1}^M \beta_n t_n (\beta_n - \bar{\beta})}{\left[\sum_{n=1}^M \alpha_n (\alpha_n - \bar{\alpha}) \right]^2} \quad (61)$$

$$a_{21} = \frac{\sum_{n=1}^M \beta_n (\alpha_n - \bar{\alpha}) \sum_{n=1}^M \alpha_n t_n (\beta_n - \bar{\beta})}{\left[\sum_{n=1}^M \alpha_n (\alpha_n - \bar{\alpha}) \right]^2}$$

$$\frac{\sum_{n=1}^M \alpha_n (\alpha_n - \bar{\alpha}) \sum_{n=1}^M \beta_n t_n (\beta_n - \bar{\beta})}{\left[\sum_{n=1}^M \alpha_n (\alpha_n - \bar{\alpha}) \right]^2} \quad (62)$$

$$a_{22} = \frac{\sum_{n=1}^M \beta_n (\alpha_n - \bar{\alpha}) \sum_{n=1}^M \alpha_n (\beta_n - \bar{\beta})}{\left[\sum_{n=1}^M \alpha_n (\alpha_n - \bar{\alpha}) \right]^2}$$

$$\frac{\sum_{n=1}^M \alpha_n (\alpha_n - \bar{\alpha}) \sum_{n=1}^M \beta_n (\beta_n - \bar{\beta})}{\left[\sum_{n=1}^M \alpha_n (\alpha_n - \bar{\alpha}) \right]^2} \quad (63)$$

$$R = \frac{\sum_{n=1}^M (y_n - \bar{y}) t_n \beta_n}{\sum_{n=1}^M (y_n - \bar{y}) \alpha_n} - \frac{\sum_{n=1}^M (\alpha_n - \bar{\alpha}) t_n \beta_n}{\sum_{n=1}^M (\alpha_n - \bar{\alpha}) \alpha_n} \quad (64)$$

$$S = \frac{\sum_{n=1}^M (y_n - \bar{y})\beta_n}{\sum_{n=1}^M (y_n - \bar{y})\alpha_n} - \frac{\sum_{n=1}^M (\alpha_n - \bar{\alpha})\beta_n}{\sum_{n=1}^M (\alpha_n - \bar{\alpha})\alpha_n} \quad (65)$$

$$\bar{y} = \frac{1}{M} \sum_{n=1}^M y_n \quad (66)$$

$$\bar{\alpha} = \frac{1}{M} \sum_{n=1}^M \alpha_n \quad (67)$$

$$\bar{\beta} = \frac{1}{M} \sum_{n=1}^M \beta_n \quad (68)$$

$$\alpha_n = \cos(\omega t_n + \theta) \quad (69)$$

$$\beta_n = \sin(\omega t_n + \theta) \quad (70)$$

Repeat this procedure, using ψ and ϕ as new estimates for ω and θ , until the difference between the two is suitably small. This produces a fitted function of the form

$$y_n' = A \cos(\psi t_n + \phi) + C \quad (71)$$

where

A is the fitted amplitude

C is the fitted offset

and may be computed from

$$A = \frac{\sum_{n=1}^M (y_n - \bar{y})(\alpha_n + \beta_n + \beta_n t_n)}{\sum_{n=1}^M (\alpha_n - \bar{\alpha})(\alpha_n + \beta_n + \beta_n t_n)} \quad (72)$$

$$C = \bar{y} - A\bar{\alpha} \quad (73)$$

The actual rms error is $\left(\frac{\varepsilon}{M}\right)^{\frac{1}{2}}$

$$\frac{\varepsilon}{M} = \frac{1}{M} \sum_{n=1}^M y_n^2 + \frac{A^2}{M} \sum_{n=1}^M \alpha_n^2 - 2 \frac{A}{M} \sum_{n=1}^M \alpha_n y_n + C^2 - 2C\bar{y} + 2AC\bar{\alpha} \quad (74)$$

Since this is an iterative process, it can diverge for initial estimates of ω and θ that are grossly incorrect.

4.1.3.5 Comment on choice of input frequency, significance of record size and number of cycles per record

The precise selection of the frequency used is very important. There are frequencies for which there is a potential for hiding errors, and there are frequencies that provide particularly good error coverage. The best and worst frequency can differ by only a fraction of a percent. Input frequency selection becomes more important as the input frequency becomes higher with respect to the sampling frequency.

An optimum frequency is one for which there are M distinct phases sampled which are uniformly distributed between 0 and 2π radians, where M is the number of samples in a record. It can be shown that the optimum frequencies are given by

where

J is an integer which is relatively prime to M
 f_s is the sampling frequency

Note that the condition of being relatively prime means that M and J have no common factors and that for the optimum frequency there are exactly J cycles in a record. If M is a power of two, then any odd value for J meets this condition.

As an example, a frequency, f , which gives poor error coverage, is

$$f = \frac{f_s}{L} \quad (76)$$

where

L is a small integer

In this case there are only L distinct phases sampled.

Sometimes the optimum frequencies are too far apart to adequately approximate the desired frequency. The following is a simple approach for calculating a near-optimum frequency that is close to any desired frequency:

- a) Find an integer, n , such that the desired frequency is approximately f_d/n
- b) Let $D = \text{int}(M/n) =$ the number of full cycles that can be recorded at this frequency
- c) Let $f^* = Df_d/(nDD1) =$ the near optimum frequency

Using this frequency guarantees $nDD1$ distinct samples with phases uniformly distributed between 0 and 2π radians.

For example, if the sampling frequency is 1G sample/s, the desired input frequency is 10 MHz and the record length is 1024, then $n = 100$, $D = 10$ and $f^* = \lfloor 10/((100)(10) - 1) \rfloor \lfloor 1000 \rfloor = 10.01001$ MHz. This frequency guarantees 999 distinct uniformly distributed phases and differs from the desired frequency by 0.1%. The nearest optimum frequency is $(11/1024)1000 = 10.742$ MHz. This differs from the desired frequency by 7.4% but guarantees 1024 distinct uniformly distributed phases. The desired frequency yields only 100 distinct phases because its period is exactly 100 samples.

For an ideal transfer characteristic in the absence of random noise, the minimum record size that will ensure a representative sample of every code bin is $2\pi 2^{N/2}$ when the input frequency is chosen as above.

The number of cycles of the sine wave contained in the record also affects the estimates obtained in the curve fit. This is especially true for the four parameter sine fit algorithms, in which frequency is not fixed. As the number of cycles in the record increases, the estimated frequency becomes more and more tightly constrained by the data since the number of zero crossings increases. Therefore, for records containing large numbers of cycles, the sine fit will return frequency estimates that are very close to the fundamental frequency of the input signal. However, for records containing only a few cycles, e.g., < 5 , the estimated frequency is not strongly constrained by the fundamental frequency and therefore can also change to accommodate error components due to noise, harmonics, or jitter. The result is a fit that returns incorrect estimates, i.e., that do not exactly correspond to the parameters of the fundamental component of the signal, but nevertheless gives smaller mean squared error, and thus smaller residuals. Estimates of all four Parameters—frequency, phase, amplitude and offset—are affected. The differences in the estimates depend on the amount of noise, harmonics, and jitter present, as well as on the accuracy of the initial frequency estimate; if the signal is essentially free of contamination, the estimates will be quite good even for records containing only one cycle. A good rule of thumb is to use records containing at least five cycles. If the harmonics and noise are each less than $5Q$ (where Q is the ideal code bin width of the recorder), then the estimates will have sufficient accuracy for most applications.

4.1.4 Step response

The recorded output response for an ideal input step with designated baseline and topline. If unspecified, the baseline of the input step is 10% and the topline is 90% of full scale.

4.1.4.1 Test method

Apply a suitable input step and take a record sufficiently long to include all features of interest, e.g., precursors, and electrical and thermal settling. A suitable step is one that has transition duration, overshoot, and settling time no greater than one fourth of those expected from the recorder under test.

4.1.5 Equivalent time sampling

The sampling rate of a real time recorder limits the measurement bandwidth. If the sampling rate is not at least twice the frequency of the highest frequency component of the input signal, then aliasing errors can result. If the input signal is repetitive, these limitations can be reduced by equivalent time sampling. Several methods of equivalent time sampling exist. The method described here consists of extracting equivalent time samples from a single record using the recorder's internal time base, provided that the input signal's repetition rate is selected appropriately.

4.1.5.1 Extraction method

If the input signal is repetitive, sampling rate limitations can be reduced by using the principle of equivalent time sampling to multiply the real time sampling rate of the waveform recorder by an integer, D . By

choosing the repetition rate of the input signal (f_r) appropriately, at least D periods of the input waveform are recorded in a single record; then, upon rearranging the samples with a simple algorithm, a single repetition of the input signal is obtained which is effectively sampled at D times the real time sampling rate. This is illustrated in figure 2 for $D = 4$. To implement this method, choose integer D based on the required equivalent sample rate, f_{eq} , such that $f_{eq} = Df_s$, where f_s is the real time sampling rate of the waveform recorder. Next, L , the number of real time samples taken during each repetition of the input waveform, is given by $L = \text{INT}(M/D)$, i.e., the integer part of M/D , where M is the number of samples in a record. Finally, the input signal's repetition rate, f_r , is set such that

$$f_r = f_s \frac{D}{LD-1}, \text{ with } LD \leq M \quad (77)$$

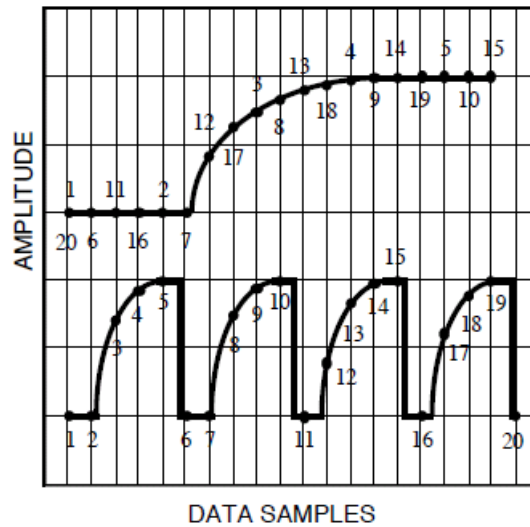


Figure 2—Equivalent time sampling extraction method

For the example of figure 2, $D=4$, $L=5$, M could be 20, 21, 22, 23, or 24, and $f_r=0.2105263f_s$. (Note that data points from LD through M are redundant.)

The following BASIC program implements the algorithm to rearrange the samples in equivalent time:

```

10 F=0
20 FOR I=1 TO L
30   FOR J=1 TO D
40     F=F+1
50     I2=I+(J-1)*L
60     E(F)=R(I2)
70   NEXT J
80 NEXT I

```

where—

D is the sample rate multiplier

M is the record length

L is $\text{INT}(M/D)$ where $\text{INT}(*)$ designates the integer part of $*$

$E(*)$ is the array containing $(L \times D)$ equivalent time samples

F is the equivalent time sampling index

$R(*)$ is the array containing real time

samples $I2$ is the real time sampling index