

IEEE Standard for Digitizing Waveform Recorders

IEEE Instrumentation Society

Sponsored by the
Waveform Generation, Testing, and Measurement Committee

IEEE
3 Park Avenue
New York, NY 10016-5997
USA

IEEE Std 1057™-2017
(Revision of IEEE Std 1057-2007)

IEEE Standard for Digitizing Waveform Recorders

Sponsor

Waveform Generation, Testing, and Measurement Committee
of the
IEEE Instrumentation Society

Approved 28 September 2017

IEEE-SA Standards Board

Abstract: Terminology and test methods for describing the performance of waveform recorders are presented in this standard.

Keywords: effective number of bits, frequency response, noise, IEEE 1057™, sine fitting, step pulse response, total harmonic distortion, transitions levels, waveform recorders

The Institute of Electrical and Electronics Engineers, Inc.
3 Park Avenue, New York, NY 10016-5997, USA

Copyright © 2018 by The Institute of Electrical and Electronics Engineers, Inc.
All rights reserved. Published 26 January 2018. Printed in the United States of America.

IEEE is a registered trademark in the U.S. Patent & Trademark Office, owned by The Institute of Electrical and Electronics Engineers, Incorporated.

PDF: ISBN 978-1-5044-4301-2 STD22763
Print: ISBN 978-1-5044-4302-9 STDPD22763

IEEE prohibits discrimination, harassment, and bullying.

For more information, visit <http://www.ieee.org/web/aboutus/whatis/policies/p9-26.html>.

No part of this publication may be reproduced in any form, in an electronic retrieval system or otherwise, without the prior written permission of the publisher.

Important Notices and Disclaimers Concerning IEEE Standards Documents

IEEE documents are made available for use subject to important notices and legal disclaimers. These notices and disclaimers, or a reference to this page, appear in all standards and may be found under the heading “Important Notices and Disclaimers Concerning IEEE Standards Documents.” They can also be obtained on request from IEEE or viewed at <http://standards.ieee.org/IPR/disclaimers.html>.

Notice and Disclaimer of Liability Concerning the Use of IEEE Standards Documents

IEEE Standards documents (standards, recommended practices, and guides), both full-use and trial-use, are developed within IEEE Societies and the Standards Coordinating Committees of the IEEE Standards Association (“IEEE-SA”) Standards Board. IEEE (“the Institute”) develops its standards through a consensus development process, approved by the American National Standards Institute (“ANSI”), which brings together volunteers representing varied viewpoints and interests to achieve the final product. IEEE Standards are documents developed through scientific, academic, and industry-based technical working groups. Volunteers in IEEE working groups are not necessarily members of the Institute and participate without compensation from IEEE. While IEEE administers the process and establishes rules to promote fairness in the consensus development process, IEEE does not independently evaluate, test, or verify the accuracy of any of the information or the soundness of any judgments contained in its standards.

IEEE Standards do not guarantee or ensure safety, security, health, or environmental protection, or ensure against interference with or from other devices or networks. Implementers and users of IEEE Standards documents are responsible for determining and complying with all appropriate safety, security, environmental, health, and interference protection practices and all applicable laws and regulations.

IEEE does not warrant or represent the accuracy or content of the material contained in its standards, and expressly disclaims all warranties (express, implied and statutory) not included in this or any other document relating to the standard, including, but not limited to, the warranties of: merchantability; fitness for a particular purpose; non-infringement; and quality, accuracy, effectiveness, currency, or completeness of material. In addition, IEEE disclaims any and all conditions relating to: results; and workmanlike effort. IEEE standards documents are supplied “AS IS” and “WITH ALL FAULTS.”

Use of an IEEE standard is wholly voluntary. The existence of an IEEE standard does not imply that there are no other ways to produce, test, measure, purchase, market, or provide other goods and services related to the scope of the IEEE standard. Furthermore, the viewpoint expressed at the time a standard is approved and issued is subject to change brought about through developments in the state of the art and comments received from users of the standard.

In publishing and making its standards available, IEEE is not suggesting or rendering professional or other services for, or on behalf of, any person or entity nor is IEEE undertaking to perform any duty owed by any other person or entity to another. Any person utilizing any IEEE Standards document, should rely upon his or her own independent judgment in the exercise of reasonable care in any given circumstances or, as appropriate, seek the advice of a competent professional in determining the appropriateness of a given IEEE standard.

IN NO EVENT SHALL IEEE BE LIABLE FOR ANY DIRECT, INDIRECT, INCIDENTAL, SPECIAL, EXEMPLARY, OR CONSEQUENTIAL DAMAGES (INCLUDING, BUT NOT LIMITED TO: PROCUREMENT OF SUBSTITUTE GOODS OR SERVICES; LOSS OF USE, DATA, OR PROFITS; OR BUSINESS INTERRUPTION) HOWEVER CAUSED AND ON ANY THEORY OF LIABILITY, WHETHER IN CONTRACT, STRICT LIABILITY, OR TORT (INCLUDING NEGLIGENCE OR OTHERWISE) ARISING IN ANY WAY OUT OF THE PUBLICATION, USE OF, OR RELIANCE UPON ANY STANDARD, EVEN IF ADVISED OF THE POSSIBILITY OF SUCH DAMAGE AND REGARDLESS OF WHETHER SUCH DAMAGE WAS FORESEEABLE.

Translations

The IEEE consensus development process involves the review of documents in English only. In the event that an IEEE standard is translated, only the English version published by IEEE should be considered the approved IEEE standard.

Official statements

A statement, written or oral, that is not processed in accordance with the IEEE-SA Standards Board Operations Manual shall not be considered or inferred to be the official position of IEEE or any of its committees and shall not be considered to be, or be relied upon as, a formal position of IEEE. At lectures, symposia, seminars, or educational courses, an individual presenting information on IEEE standards shall make it clear that his or her views should be considered the personal views of that individual rather than the formal position of IEEE.

Comments on standards

Comments for revision of IEEE Standards documents are welcome from any interested party, regardless of membership affiliation with IEEE. However, IEEE does not provide consulting information or advice pertaining to IEEE Standards documents. Suggestions for changes in documents should be in the form of a proposed change of text, together with appropriate supporting comments. Since IEEE standards represent a consensus of concerned interests, it is important that any responses to comments and questions also receive the concurrence of a balance of interests. For this reason, IEEE and the members of its societies and Standards Coordinating Committees are not able to provide an instant response to comments or questions except in those cases where the matter has previously been addressed. For the same reason, IEEE does not respond to interpretation requests. Any person who would like to participate in revisions to an IEEE standard is welcome to join the relevant IEEE working group.

Comments on standards should be submitted to the following address:

Secretary, IEEE-SA Standards Board
445 Hoes Lane
Piscataway, NJ 08854 USA

Laws and regulations

Users of IEEE Standards documents should consult all applicable laws and regulations. Compliance with the provisions of any IEEE Standards document does not imply compliance to any applicable regulatory requirements. Implementers of the standard are responsible for observing or referring to the applicable regulatory requirements. IEEE does not, by the publication of its standards, intend to urge action that is not in compliance with applicable laws, and these documents may not be construed as doing so.

Copyrights

IEEE draft and approved standards are copyrighted by IEEE under US and international copyright laws. They are made available by IEEE and are adopted for a wide variety of both public and private uses. These include both use, by reference, in laws and regulations, and use in private self-regulation, standardization, and the promotion of engineering practices and methods. By making these documents available for use and adoption by public authorities and private users, IEEE does not waive any rights in copyright to the documents.

Photocopies

Subject to payment of the appropriate fee, IEEE will grant users a limited, non-exclusive license to photocopy portions of any individual standard for company or organizational internal use or individual, non-commercial use only. To arrange for payment of licensing fees, please contact Copyright Clearance Center, Customer Service, 222 Rosewood Drive, Danvers, MA 01923 USA; +1 978 750 8400. Permission to photocopy portions of any individual standard for educational classroom use can also be obtained through the Copyright Clearance Center.

Updating of IEEE Standards documents

Users of IEEE Standards documents should be aware that these documents may be superseded at any time by the issuance of new editions or may be amended from time to time through the issuance of amendments, corrigenda, or errata. An official IEEE document at any point in time consists of the current edition of the document together with any amendments, corrigenda, or errata then in effect.

Every IEEE standard is subjected to review at least every 10 years. When a document is more than 10 years old and has not undergone a revision process, it is reasonable to conclude that its contents, although still of some value, do not wholly reflect the present state of the art. Users are cautioned to check to determine that they have the latest edition of any IEEE standard.

In order to determine whether a given document is the current edition and whether it has been amended through the issuance of amendments, corrigenda, or errata, visit the IEEE Xplore at <http://ieeexplore.ieee.org/> or contact IEEE at the address listed previously. For more information about the IEEE-SA or IEEE's standards development process, visit the IEEE-SA Website at <http://standards.ieee.org>.

Errata

Errata, if any, for all IEEE standards can be accessed on the IEEE-SA Website at the following URL: <http://standards.ieee.org/findstds/errata/index.html>. Users are encouraged to check this URL for errata periodically.

Patents

Attention is called to the possibility that implementation of this standard may require use of subject matter covered by patent rights. By publication of this standard, no position is taken by the IEEE with respect to the existence or validity of any patent rights in connection therewith. If a patent holder or patent applicant has filed a statement of assurance via an Accepted Letter of Assurance, then the statement is listed on the IEEE-SA Website at <http://standards.ieee.org/about/sasb/patcom/patents.html>. Letters of Assurance may indicate whether the Submitter is willing or unwilling to grant licenses under patent rights without compensation or under reasonable rates, with reasonable terms and conditions that are demonstrably free of any unfair discrimination to applicants desiring to obtain such licenses.

Essential Patent Claims may exist for which a Letter of Assurance has not been received. The IEEE is not responsible for identifying Essential Patent Claims for which a license may be required, for conducting inquiries into the legal validity or scope of Patents Claims, or determining whether any licensing terms or conditions provided in connection with submission of a Letter of Assurance, if any, or in any licensing agreements are reasonable or non-discriminatory. Users of this standard are expressly advised that determination of the validity of any patent rights, and the risk of infringement of such rights, is entirely their own responsibility. Further information may be obtained from the IEEE Standards Association.

Participants

At the time this IEEE standard was completed, the Waveform Recorder Working Group had the following membership:

William B. Boyer, *Chair*

Francisco Alegria
Eualia Balestrieri
Niclas Bjorsell
Jerome Blair
Andrea Cataldo
Lorenzo Ciani
Dominique Dallet

Pasquale Daponte
Egidio De Benedetto
Luca De Vito
Izzet Kale
Donald Larson
Dennis Lia

Thomas Linnenbrink
Sol Max
Kruno Miličević
Antonio Moschitta
Sergio Rapuano
Steve Tilden

The following members of the balloting committee voted on this standard. Balloters may have voted for approval, disapproval, or abstention.

Ali AlAwazi
Charles Barest
Niclas Bjorsell
Jerome Blair
William Boyer
Demetrio Bucaneg Jr.
Andrea Cataldo
Luca De Vito
Sourav Dutta

Gearold O. H. Eidhin
Randall Groves
Werner Hoelzl
Ronald Jarrett
Donald Larson
Dennis Lia
Thomas Linnenbrink
Sol Max
Edward McCall

Kruno Miličević
R. Murphy
Sergio Rapuano
Jeremy Smith
Joseph Stanco
Walter Struppler
Steven Tilden
Lisa Ward
Jian Yu

When the IEEE-SA Standards Board approved this standard on 28 September 2017, it had the following membership:

Jean-Philippe Faure, *Chair* **Gary Hoffman, *Vice Chair*** **John D. Kulick, *Past Chair*** **Konstantinos Karachalios, *Secretary***

Chuck Adams
Masayuki Ariyoshi
Ted Burse
Stephen Dukes
Doug Edwards
J. Travis Griffith
Michael Janezic

Thomas Koshy
Joseph L. Koepfinger*
Kevin Lu
Daleep Mohla
Damir Novosel
Ronald C. Petersen
Annette D. Reilly

Robby Robson
Dorothy Stanley
Adrian Stephens
Mehmet Ulema
Phil Wennblom
Howard Wolfman
Yu Yuan

*Member Emeritus

Introduction

This introduction is not part of IEEE Std 1057-2017, IEEE Standard for Digitizing Waveform Recorders.

This standard is a revision and modernization of the previous standard, IEEE Std 1057-2007. It presents methods for the test and evaluations of digitizing waveform recorders approved by the balloting committee.

Contents

1. Overview	15
1.1 Scope	15
1.2 Waveform recorder background	15
2. Normative references	16
3. Definitions	17
3.1 Definitions	17
3.2 Symbols and acronyms	24
4. Test parameters and methods	28
4.1 Manufacturer supplied information	28
4.2 Test selection	31
4.3 Test setup	32
4.4 Equivalent-time sampling	39
4.5 Discrete Fourier transform (DFT)	43
4.6 Sine wave testing and fitting	52
4.7 Locating code transition levels	59
4.8 Step function response measurements	77
4.9 Tests using a dc input	80
5. Input impedance	81
5.1 Test method	81
5.2 Alternate test method using a time domain reflectometer (TDR)	81
5.3 Input impedance for out-of-range signals	81
6. Static gain and offset	83
6.1 Independently based gain and offset	83
6.2 Terminal-based gain and offset	85
7. Linearity	85
7.1 Integral nonlinearity (INL)	85
7.2 Maximum static error (MSE)	86
7.3 Differential nonlinearity (DNL) and missing codes	86
7.4 Example INL and DNL data	87
7.5 Monotonicity	89
7.6 Hysteresis	90
7.7 Total harmonic distortion (THD)	91
7.8 Intermodulation distortion (IMD)	96
7.9 Noise power ratio (NPR)	98
8. Noise ⁷	104
8.1 Comments on noise	105
8.2 Ratio of signal to noise and distortion (SINAD)	105
8.3 Signal to noise ratio (SNR)	106
8.4 Comments on SINAD and SNR	106
8.5 Effective number of bits (ENOB)	107
8.6 Random noise	110
8.7 Spurious components	111
8.8 Spurious-free dynamic range (SFDR)	112

9.2	Transition duration of the step response.....	117
9.3	Slew rate limit.....	119
9.4	Overshoot and precursors	119
9.5	Aperture duration.....	119
9.6	Limitations on the use of step responses	124
10.	Frequency response parameters.....	127
10.1	Analog bandwidth	127
10.2	Gain error (gain flatness)	127
10.3	Frequency response and gain from step response	128
11.	Interchannel parameters	133
11.1	Crosstalk.....	133
11.2	Multiple input reverse coupling.....	133
12.	Time base parameters	134
12.1	Fixed error in sample time	134
12.2	Aperture uncertainty	134
12.3	Long-term stability	136
13.	Out-of-range recovery	137
13.1	Test method for absolute out-of-range voltage recovery.....	138
13.2	Test method for relative out-of-range voltage recovery	138
13.3	Comments on test method	138
14.	Word error rate	138
14.1	Test method for word error rate.....	138
14.2	Comment on the number of samples required for word error rate.....	139
14.3	Comments on test equipment and making measurements.....	139
15.	Differential input specifications.....	139
15.1	Differential input impedance to ground	139
15.2	Common-mode rejection ratio (CMRR) and maximum common-mode signal level	140
15.3	Maximum operating common-mode signal.....	140
15.4	Common-mode out-of-range signal recovery time.....	141
16.	Cycle time	141
16.1	Test method	141
16.2	Comment.....	142
17.	Triggering.....	142
17.1	Trigger delay and trigger jitter	142
17.2	Trigger sensitivity.....	143
17.3	Trigger minimum rate of change.....	144
17.4	Trigger coupling to signal.....	145
	Annex A (informative) Sine fitting algorithms	146
	Annex B (informative) Phase noise.....	150
	Annex C (informative) Comment on errors associated with word-error-rate measurement	159
	Annex D (informative) Measurement of random noise below the quantization level.....	161
	Annex E (informative) Software consideration	164

Annex F (informative) Excitation with precision source with ramp verneir: determination of the test parameters	165
Annex G (informative) Presentation of sine wave data	168
Annex H (informative) Bibliography	177

List of Figures

Figure 1—Block diagram of a common waveform recorder	16
Figure 2—Definitions pertaining to input quantization	18
Figure 3—Block diagram of waveform recorder “ground” nodes.....	35
Figure 4—Generic waveform recorder with features that facilitate testing	37
Figure 5—General sine wave test setup.....	38
Figure 6—General setup for test using an arbitrary signal generator.....	38
Figure 7—General setup for pulse and step signal testing	39
Figure 8—Equivalent-time sampling extraction example	40
Figure 9—Example DFT spectrum using positive and negative frequencies	44
Figure 10—Example DFT showing the negative frequencies mapped up to the frequencies between the Nyquist frequency and twice the Nyquist frequency	44
Figure 11—Plot of DFT detected frequencies versus input frequency for a waveform recorder	45
Figure 12—Plot showing how negative DFT frequencies are mapped up to the frequencies between the Nyquist frequency and twice the Nyquist frequency	46
Figure 13—Fourier transform of the Hann and the rectangular windows.....	47
Figure 14—Fourier transform of the Hann and the Blackman windows	48
Figure 15—Plot of signals and samples that generate the spectrum shown in Figure 9	51
Figure 16—Spectrum analyzer display of signal with DFT results shown in Figure 9	52
Figure 17—Block diagram of feedback technique for determining waveform recorder code thresholds.....	61
Figure 18—Plot of perfect waveform recorder-computed transition level deviations using sine wave histogram with additive harmonic distortion (Effect of 2nd, 3rd, 4th, 5th harmonics on transition levels are shown. The harmonic amplitude is 1 LSB, sine and cosine components.).....	68
Figure 19—Plot of acceptable ramp test input signals.....	69
Figure 20—Plot of transition uncertainty	71
Figure 21—Ramp vernier test signals applied to waveform recorder.....	72
Figure 22—Test procedure for excitation using a high-precision dc source with ramp vernier	74
Figure 23—Representation of the cumulative histograms computed in each step in the case of a 5-bit converter and a test with four steps.....	75
Figure 24—Example data for limited accuracy triangle wave measurement of code transition levels (Data are for an input voltage of 5.2 V, $Q = 1$ V/count, and a noise level of $1 V_{rms}$	76
Figure 25—Setup for a simple step response measurement.....	79

Figure 26—Test setup for measuring input impedance for out-of-range signals by time domain reflectometry	82
Figure 27—Example of calculated INL from a terminal-based calibrated 12-bit device.....	88
Figure 28—DNL from a terminal-based calibrated 12-bit device	89
Figure 29—Block diagram of hysteresis and alternation.....	91
Figure 30—Plot of distribution of calculated values of fundamental and harmonic amplitudes for noncoherent sampling with random phase between the 2nd harmonic and the fundamental	95
Figure 31—THD deviation from true value as a function of number of cycles in the record.....	96
Figure 32—Test setup for measuring NPR	99
Figure 33—Spectrum of NPR test signal.....	100
Figure 34—Time domain representation of NPR test signal	101
Figure 35—Plot of ideal and measured NPR for a Gaussian noise input from Equation (71)	103
Figure 36—Plot of ideal and measured NPR for a uniform noise input from Equation (72)	103
Figure 37—Frequency domain AWG generated NPR test input to the waveform recorder.....	104
Figure 38—Histogram from 16 K sample set.....	104
Figure 39—Illustration of the use of the histogram to remove the sine wave peaks	108
Figure 40—Example of potential errors in measuring the final value of a step pulse response waveform when a final time shorter than 1 s is used (In the plot, ϵ_1 is the short-term settling error, and ϵ_2 is settling error. In this case, the final value at 1 s is 0.0 V.)	115
Figure 41—Impulse response of a sample-and-hold amplifier while in the sample mode.....	120
Figure 42—The step response corresponding to the impulse response of Figure 41	121
Figure 43—View of start of transition duration for $p = 99.9\%$	121
Figure 44—Aperture duration for $p = 50\%$ and $p = 80\%$	122
Figure 45—Evaluation of aperture-duration stop time for $p = 99.9\%$	122
Figure 46—Ringing impulse response	123
Figure 47—Ringing step response	123
Figure 48—Zoomed ringing transition.....	124
Figure 49—Example of noise reduction using an expanding-window moving-average filter on step data for different values of R (The waveform contains two time constants separated by a factor of 10).....	131
Figure 50—Example of a network for simultaneously applying a normal-mode signal and a common-mode signal for measuring maximum operating common-mode signal level.....	141
Figure 51—Trigger sensitivity parameters.....	144
Figure B.1—Phase noise plot	151
Figure B.2—Two examples of phase noise as displayed on a spectrum analyzer.....	151

Figure B.3—Mathematica program to compute phase noise integral.....	153
Figure B.4—Function $D(f)$ with record length $T_R = 1$ s	155
Figure B.5—Mathematica program to compute frequency deviation integral	155
Figure G.1—Example plot of ENOB versus frequency for a two-channel waveform recorder.....	168
Figure G.2—Example plots of sine fit residuals (the left side shows a modulo time plot, and the right side shows a PSD. Both represent the same data)	169
Figure G.3—Same as Figure G.2 , but for channel 1 of the recorder (this figure shows the interleaving error)	171
Figure G.4—Plot of 1024-point record containing 7 cycles of the fundamental from a 10-bit data recorder	172
Figure G.5—Plot of the magnitude of the spectrum of the signal shown in Figure G.4	173
Figure G.6—Plot of the residual signal	174
Figure G.7—Plot of unwound data (original record is reordered)	175
Figure G.8—Plot of unfolded residual	176

List of Tables

Table 1—Critical waveform recorder parameters	30
Table 2—Properties of various data windows	50
Table 3—Uncertainty of estimates of code transition level	62
Table 4—Values of $Z_{u/2}$ and $Z_{N,u/2}$	66
Table 5—Step response parameters.....	78
Table 6—Maximum NPR for Gaussian and uniform noise sources	101
Table 7—Maximum effective bits versus normalized jitter (fraction of sine wave period)	110
Table 8—Worst-case aliasing error in transition duration	127
Table 9—Worst-case aliasing error in frequency response magnitude and phase.....	130
Table B.1—Frequency tolerance and deviation as a function of record length.....	156
Table B.2—Noise level	157
Table B.3—Step response time jitter results for various upper limits of integration.....	157
Table C.1—Examples of the number of standard deviations covered by confidence $\Phi[k]$ in a Gaussian distribution	159

IEEE Standard for Digitizing Waveform Recorders

1. Overview

1.1 Scope

This standard defines specifications and describes test methods for measuring the performance of electronic digitizing waveform recorders, waveform analyzers, and digitizing oscilloscopes with digital outputs. The standard is directed toward, but not restricted to, general-purpose waveform recorders and analyzers.

Special applications can require additional manufacturer information and verification tests not covered in this standard.

IEEE Std 1057™ has many similarities to IEEE Std 1241™-2010 [B24], which applies to analog-to-digital converters (ADCs).¹ However, IEEE Std 1057 shall be used for waveform recorders, and IEEE Std 1241-2010 shall be used for ADCs.

1.2 Waveform recorder background

A waveform recorder is a device for capturing an amplitude versus time portion of a possibly time-varying signal, such as a voltage, by digitizing it and storing the results in an internal memory. The data capture is normally done in real time and in the time domain. Non-real time sampling recorders and frequency domain data recorders do also exist but are not specifically covered in this standard.

A generic waveform recorder is shown in [Figure 1](#).

¹The numbers in brackets correspond to those of the bibliography in [Annex H](#).

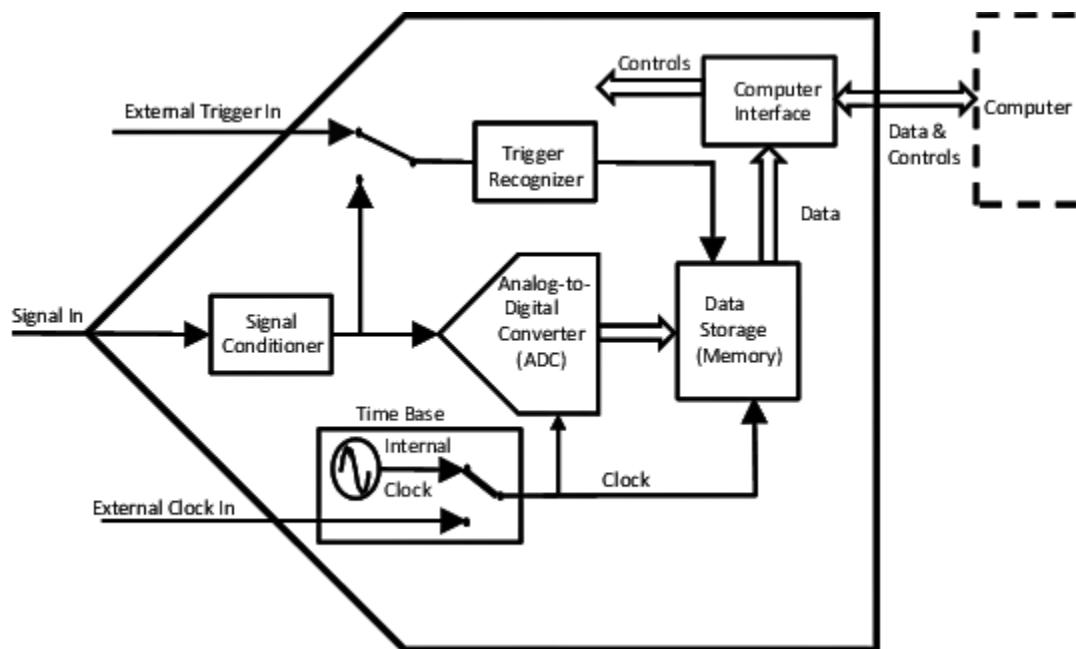


Figure 1—Block diagram of a common waveform recorder

Some key components of common waveform recorders are:

- Signal conditioner – provides impedance matching, gain/attenuation, filtering, etc. for the Signal In.
- Analog-to-digital converter – samples the input and converts each sample to a digital word. The ADC can consist of multiple interleaved ADCs.
- Time base – controls the digitizing sample times.
- Trigger recognizer – determines the start of the digitized epoch.
- Data storage (memory) – stores the digitized waveform.
- Computer interface – provides for computer control of various recorder functions and for the readout of the digitized waveform data.

2. Normative references

The following referenced documents are indispensable for the application of this document (i.e., they must be understood and used, so each referenced document is cited in text and its relationship to this document is explained). For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments or corrigenda) applies.

IEEE Std 181™-2011, IEEE Standard on Transitions, Pulses, and Related Waveforms.^{2,3}

²The IEEE standards or products referred to in [Clause 2](#) are trademarks owned by the Institute of Electrical and Electronics Engineers, Incorporated.

³IEEE publications are available from the Institute of Electrical and Electronics Engineers (<http://standards.ieee.org/>).

3. Definitions

For the purposes of this document, the following terms and definitions apply. The *IEEE Standards Dictionary Online* should be consulted for terms not defined in this clause.⁴

3.1 Definitions

ac coupled recorder: A recorder whose input circuitry does not pass dc. Such recorders use either a series capacitance or a transformer at the input.

aliasing: The phenomenon in sampled data systems whereby signal components at input frequencies greater than half the sampling frequency are converted to frequencies at less than half the sampling frequency in the output data record.

analog bandwidth (BW): Unless otherwise specified, the difference between the upper and lower frequency at which the amplitude response as seen in the data record is 0.707 (–3 dB) of the response as seen in the data record at the specified reference frequency.

aperture: The interval during which the input to the waveform recorder affects the output or the weighting function that determines the sampled output from the input signal.

aperture duration ($p\%$): The $[50 - (p/2)]\%$ to $[50 + (p/2)]\%$ transition duration of the step response of the ADC. If ringing of the step response causes multiple crossings of either of the levels, the $p\%$ aperture duration is the time from the first crossing of the first level to the last crossing of the second level.

NOTE—The significance is that the output of the waveform recorder’s ADC is determined, with an error of $(100 - p)\%$ or less, by the input signal in an interval of this duration. Common values of p are 50, 80, and 99.9. For $p = 80$, this is the 10% to 90% transition duration of the step response.

aperture uncertainty: The standard deviation of the sample instant in time. *Syn:* **short-term timing instability; timing jitter; timing phase noise.**

code bin k : A digital output that is designed to correspond to a particular set of input values.

NOTE—See Equation (1) in **code bin width $W[k]$ definition.**⁵

code bin width $W[k]$: The difference of the code transition levels that delimit the bin.

$$W[k] = T[k+1] - T[k] \quad \text{for } 1 \leq k \leq 2^N - 2 \quad (1)$$

code transition level: The boundary between two adjacent code bins.

code transition level $T[k]$: The value of the recorder input parameter at the transition point between two given, adjacent code bins. The transition point is defined as the input value that causes 50% of the output codes to be greater than or equal to the upper code of the transition, and 50% to be less than the upper code of the transition. The transition level $T[k]$ lies between code bin $k-1$ and code bin k . N is the number of digitized bits in a recorder data word. *See:* **transition point.**

⁴IEEE Standards Dictionary Online is available at: <http://dictionary.ieee.org>.

⁵Notes in text, tables, and figures of a standard are given for information only and do not contain requirements needed to implement this standard.

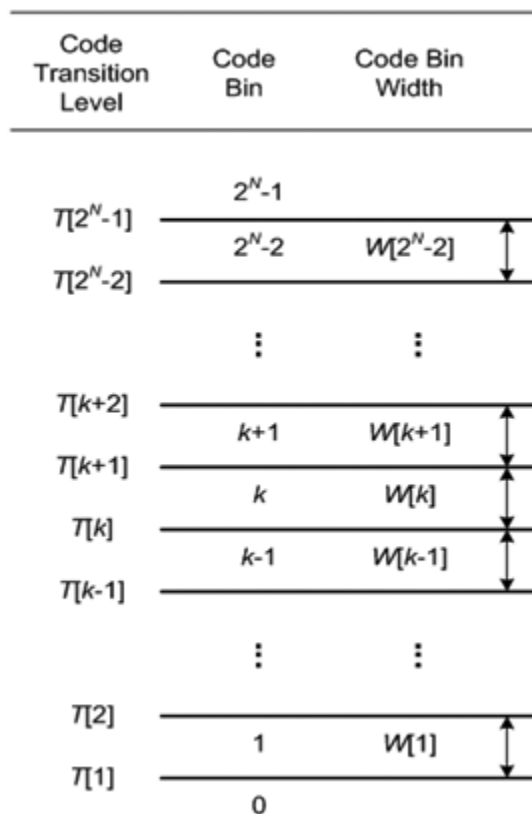


Figure 2—Definitions pertaining to input quantization

coherent sampling: Sampling of a periodic waveform in which there is an integer number of cycles in the data record.

common-mode maximum operating signal: The largest common-mode signal for which a waveform recorder with differential inputs will meet its recording specifications while recording a normal-mode signal.

common-mode maximum signal level: For recorders with differential inputs, the maximum level of the common-mode signal at which the common-mode rejection ratio (CMRR) is still valid.

common-mode out-of-range: A signal level whose magnitude is less than the specified maximum safe common-mode signal but greater than the maximum operating common-mode signal.

common-mode out-of-range recovery time: The time required for the recorder to return to its specified characteristics after the end of a common-mode overvoltage signal.

common-mode rejection ratio (CMRR): The ratio of the input common-mode signal to the effect produced at the output of the recorder in units of the input.

common-mode signal: The average value of the signals at the positive input and the negative input of a differential-input analog-to-digital converter. If the signal at the positive input is designated V_+ , and the signal at the negative input is designated V_- , then the common-mode signal V_{cm} is given in Equation (2):

$$V_{cm} = \frac{V_+ + V_-}{2} \quad (2)$$

crosstalk (multichannel): The ratio (usually expressed in decibels) of the signal induced in one channel to a common signal applied to all other channels.

cumulative distribution function (cdf): For a continuous random variable, the function $F_X(x)$, which is the probability that the random variable, X , is less than or equal to x . This function is positive and nondecreasing.

cycle time: The real time elapsed (with a recorder continually taking records of data) between the beginning of two successive records.

data window: A set of coefficients by which corresponding samples in the data record are multiplied to more accurately estimate certain properties of the signal, particularly frequency domain properties. Generally, the coefficient values increase smoothly and symmetrically toward a maximum at the center of the record.

dc coupled recorder: A recorder whose input circuitry passes dc.

decibels full scale (dBFS): The root-mean-square (rms) amplitude of any signal in decibels relative to a sine wave that spans the entire input range of the recorder.

differential input impedance: For a differential-input waveform recorder, the impedance between the positive input and the negative input with grounds and shields terminated as specified.

differential input recorder: A waveform recorder with differential inputs, which produces output codes that are a function of the difference between two input signals. The two input signals are typically called positive (or plus) and negative (or minus). If the signal at the positive input is designated V_+ , and the signal at the negative input is designated V_- , then the differential signal (V_{dm}) is given in [Equation \(3\)](#):

$$V_{dm} = V_+ - V_- \quad (3)$$

differential nonlinearity (DNL): The difference between a specified code bin width and the average code bin width, divided by the average code bin width.

discrete Fourier transform (DFT): A mathematical operation that converts sampled data from the time domain to the frequency domain. *Contrast:* **inverse discrete Fourier transform (IDFT).**

distortion: Aberrations in the output of the waveform recorder caused by nonlinearities. Distortion effects appear as harmonics and intermodulation products of the input signal frequencies in the frequency domain of the output signal.

dynamic range: *See:* **effective number of bits (ENOB), ratio of signal to noise and distortion (SINAD), signal-to-noise ratio (SNR), spurious free dynamic range (SFDR).**

NOTE—The term *dynamic range* has a number of meanings depending on the application. It is not used by itself in this standard.

effective number of bits (ENOB): For an input sine wave of specified frequency and amplitude, the number of bits of an ideal waveform recorder for which the rms quantization error is equal to the rms noise and distortion of the waveform recorder under test.

equivalent-time sampling: A process by which consecutive samples of a repetitive waveform are acquired and assembled from multiple repetitions of the waveform, to produce a record of samples representing a single repetition of the waveform sampled at a higher rate.

fixed error in sample time: A nonrandom error in the instant of sampling. A fixed error in sample time can be fixed with respect to the data samples acquired or correlated with an event that is detected by the sampling process. Unless otherwise specified, the term is usually taken to mean the maximum systematic fixed error that can be observed.

frequency response: The complex gain (magnitude and phase) as a function of input frequency, or the Fourier transform of the impulse response.

full-scale [input] range (FSR): The difference between the maximum and the minimum recordable input values as specified by the manufacturer exclusive of quantization limitations.

full-scale signal: A signal whose peak-to-peak amplitude spans the entire range of input values recordable by the analog-to-digital converter under test.

gain error: (A) (*dynamic*). The difference between the dynamic gain, $G(f)$, of the waveform recorder at a given frequency and its gain at a specified reference frequency divided by its gain at the reference frequency. (B) (*static*) The difference between the actual gain and the nominal gain.

gain flatness: Variation of gain over a specified frequency range, usually expressed in decibels.

harmonic distortion: For a pure sine wave input, output components at frequencies that are an integer multiple of the applied sine wave frequency.

hysteresis: The maximum systematic difference in values for a digitizer code transition level when the transition level is approached from either side of the transition with a specified limit on the slew rate of the input signal.

ideal code bin width (Q): The full-scale input range divided by the total number of code bins.

input impedance: The impedance between a signal input terminal and a specified reference terminal.

integral nonlinearity (INL): The maximum systematic difference between the ideal and actual code transition levels after correcting for gain and offset.

intermodulation distortion (IMD): When there is an input signal with multiple frequencies at the input to the recorder, the generation of distortion signals at the sum and difference frequencies of the inputs due to nonlinearities in the waveform recorder response.

inverse discrete Fourier transform (IDFT): A mathematical operation that converts sampled data from the frequency domain to the time domain. *Contrast:* **discrete Fourier transform (DFT)**.

k^{th} code transition level $T[k]$: The input value corresponding to the transition between codes $k-1$ and k .

NOTE—See [Figure 2](#).

large signal: A signal whose peak-to-peak amplitude (including noise) is as large as can be practically recorded by the instrument within, but not including, the maximum and minimum amplitude data codes. As a minimum, the signal shall span at least -1 dBFS ($\sim 89\%$) of the waveform recorder range.

least significant bit (LSB): One integer count in the digitized output of the waveform recorder. (If the recorder outputs data in a floating point format, e.g., input units, the data can be converted to LSBs using the manufacturer's instructions.) The nominal value associated with an LSB is the ideal code bin width, Q .

long-term settling error: The absolute difference between the final value specified for short-term transition settling duration, and the value 1 s after the beginning of the step, expressed as a percentage of the step amplitude.

long-term time base stability: The change in time base frequency (usually given in parts in 10^6) over a specified period of time at a specified sampling rate.

maximum static error: The maximum difference between any code transition level and its ideal value.

mod(x, m): A function that gives the remainder when x is divided by m , e.g., $\text{mod}(8.1, 2.5) = 0.6$. The sign of the result is the same as the sign of m .

monotonic recorder: A recorder that has output codes that do not systematically decrease (increase) for a uniformly increasing (decreasing) input signal.

noise: Any deviation between the output signal (converted to input units) and the input signal except deviations caused by linear time invariant system response (gain and phase shift), a dc level shift, total harmonic distortion (THD), or an error in the sample rate.

NOTE—Noise has historically been an ambiguous term. In this standard, noise is defined as indicated above.

noise and distortion (NAD): The sum of the effects of noise and total harmonic distortion (THD).

noise floor: Maximum power level over a given frequency band in the frequency domain of components that are not due to the applied signal, harmonics, or spurious signals. The level could be absolute (expressed in decibels with respect to a milliwatt per unit of bandwidth) or relative to the waveform recorder input full scale (expressed in decibels with respect to full scale per unit of bandwidth).

noise power ratio (NPR): The ratio of the average out-of-notch to the average in-notch power spectral density magnitudes for the discrete Fourier transform (DFT) spectrum of the waveform recorder output sample set measured using a notch-filtered broadband white-noise generator as the input to the waveform recorder under test.

noncoherent sampling: Sampling of a periodic waveform where there is no integer number of cycles in the data.

normalized noise power gain (NNPG): Sum of squares of the values of a data window function divided by the number of samples.

normalized signal-to-noise (SNR1Hz): Signal-to-noise ratio (SNR) normalized to a reference bandwidth of 1 Hz. *Syn:* **noise spectral density.**

normal-mode signal (or differential signal): The difference between the signal at the positive input and the negative input of a differential input waveform recorder.

Nyquist frequency: Frequency equal to one-half the sampling frequency.

offset: *See:* **static gain and offset.**

out of range (overvoltage): Any voltage whose magnitude is less than the maximum safe input voltage of the recorder but greater than the full-scale value for the selected range.

overshoot: The maximum amount by which the step response is above the high state for a positive-going transition or is below the low state for a negative-going transition, specified as a percentage of (recorded) pulse amplitude.

NOTE—See IEEE Std 181-2011 for more information on low and high states.⁶

Parseval relationship: The relationship where the sum (or integral) of the square of a function is equal to the sum (or integral) of the square of its Fourier transform.

passband: The band of input signal frequencies that the waveform recorder is intended to digitize with nominally constant gain.

phase nonlinearity: The deviation in phase response from a perfectly linear-phase response as a function of frequency.

precursor: Any deviations from the low state for a positive-going transition or from the high state for a negative-going transition prior to the step transition.

NOTE—See IEEE Std 181-2011 for more information on low and high states.

probability density function (pdf): For a continuous random variable, the function $f_X(x)$, which has the interpretation that $f_X(x)dx$ is the probability that the random variable, X , lies in the interval $(x, x+dx)$.

quantization: A process in which the continuous range of values of an input signal is divided into subranges, and to each subrange a discrete value of the output is uniquely assigned. In the ideal case, these subranges are nonoverlapping and equal in size. Whenever the signal value falls within a given subrange, the ideal output has the corresponding discrete value.

quantization error: The error caused by conversion of a variable having a continuous range of values to a quantized form having only discrete values, as in analog-to-digital conversion. The error is the difference between the original (analog) value and its quantized (digital) representation translated into input units.

quantization noise: Equivalent noise produced during the quantization of a continuous range variable. A quantization process of step size Q produces a root-mean-square (rms) quantization noise of $Q/\sqrt{12}$ when the continuous variable range is much larger than Q .

random noise: A nondeterministic fluctuation in the output of a waveform recorder, usually described by its frequency spectrum and its amplitude statistical properties.

ratio of signal to noise and distortion (SINAD): For a pure sine wave input of specified amplitude and frequency, the ratio of root-mean-square (rms) signal to rms noise and distortion (NAD).

record of data: A sequential collection of samples acquired by the waveform recorder.

relatively prime: A descriptor of two integers when their greatest common divisor is 1.

residuals: For least squares curve fitting of data to a mathematically defined function, the differences between the recorded data and the fitted function. The mathematically defined function is often a sine wave.

reverse coupling: The ratio of the spurious signal generated by a signal at some other input to the recorder and the signal recorded at the specified input of the recorder.

⁶Information on references can be found in [Clause 2](#).

root-mean-square (rms): For a given set of data, the square root of the arithmetic mean of the squared values of each of the data.

sampling: The process of extracting values at discrete time instants from a continuous input signal.

short-term settling error: The maximum absolute difference between the waveform and the final value at all times between a specified time measured from the 50% reference level instant of the waveform and the end of the data record expressed as a percentage of step amplitude. The final value is defined to occur 1 s (unless otherwise specified) after the beginning of the step.

short-term timing instability: *See:* **aperture uncertainty**.

short-term transition settling duration: Measured from the 50% reference level of the output, the time at which the step response enters and subsequently remains within a specified error band around the final value. The final value is defined to occur at 1 s after the beginning of the step unless otherwise specified.

signal-to-noise ratio (SNR): The ratio of a signal to the noise where noise is any deviation between the output signal (converted to input units) and the input signal except deviations caused by linear time invariant system response (gain and phase shift), a dc level shift, total harmonic distortion (THD), or an error in the sample rate.

sine wave histogram: A histogram where each bin contains the total number of occurrences of each digitized level of an input sine wave signal.

single-ended recorder: A recorder that measures the voltage difference between a single input connection and ground.

slew rate limit: The value of rate of change of output codes (referenced to the input) for which an increased amplitude input step causes no change.

spectral leakage: Spectral lines adjacent to the input line produced by the discrete Fourier transform (DFT) when the input is a pure sine wave whose frequency does not satisfy the coherent sampling condition and thus is not at the center of a DFT frequency bin.

spurious components: Persistent sine waves at any frequencies other than the input or harmonic frequencies.

spurious free dynamic range: The frequency domain difference in decibels between the full-scale signal level and the largest spurious component (including harmonics) for a large, pure sine wave signal input.

static (parameter, error, or test): A static test is a test in which the input signal is essentially constant, meaning the signal is varying slowly enough that the results are the same as that obtained by a constant signal. A static parameter or error is the value obtained when measured with a static test.

static gain and offset: **(A)** (*independently based*). The values by which the transition levels are multiplied and then to which the transition levels are added, respectively, to minimize the mean squared deviation from the ideal transition levels. **(B)** (*terminal-based*). The values by which the transition levels are multiplied and then to which the transition levels are added, respectively, to cause the deviations from the ideal transition levels to be zero at the terminal points, that is, at the first and last code transitions.

step response: The recorded output response for an ideal input step with designated low state and high state.

NOTE—See IEEE Std 181-2011 for more information on low and high states.

timing jitter: *See:* **aperture uncertainty**.

timing phase noise: *See:* **aperture uncertainty.**

total harmonic distortion (THD): The scaled square root of the sum of squares of a specified set of harmonic distortion components including their aliases for a pure sine wave input of specified amplitude and frequency.

transfer curve: The representation of the average digital output code of a waveform recorder as a function of the input signal value.

transition duration of the step response: The duration between the instants corresponding to two reference levels. Unless otherwise specified, the two reference levels are the 10% and 90% reference levels.

transition level: The input value that causes 50% of the output codes to be greater than or equal to the upper code of the transition, and 50% to be less than the upper code of the transition.

transition settling duration: The time at which the step response enters and subsequently remains within a specified error band around the final value, measured from the 50% point of the response.

trigger delay: The elapsed time from the occurrence of a trigger pulse at the trigger input connector, to the time at which the first or a specified data sample is recorded.

trigger jitter: The standard deviation in the trigger delay time over multiple records.

trigger minimum rate of change: The slowest rate of change of the transition duration of a pulse of a specified level that will consistently trigger the recorder.

trigger signal coupling: The ratio of the spurious signal level (that is recorded by an input to the recorder) to the trigger signal level.

waveform: A manifestation or representation (e.g., graph, plot, oscilloscope presentation, discrete time series, equations, table of coordinates, or statistical data) or a visualization of a signal.

waveform epoch: An interval to which consideration of a waveform is restricted for a particular calculation, procedure, or discussion. Except when otherwise specified, the waveform epoch is assumed to be the span over which the waveform is measured or defined. The duration of time corresponding to a data record. For instance, for an M -sample record acquired at the uniform sampling period T_s , the epoch is MT_s .

window: *See:* **data window.**

3.2 Symbols and acronyms

Below are the acronyms used in this standard and the symbols that are used in more than one subclause in this standard. Other symbols are declared and defined locally as needed.

ε	general symbol for error, uncertainty, or tolerance
ε_f	error in frequency
ε_{fs}	error in sampling rate
ε_G	gain error
$\varepsilon[k]$	residual error in the least squares fit method of computing gain and offset
ε_m	upper bound on aliasing error magnitude spectrum
ε_Q	quantization error
ε_{ms}	mean square error

ε_w	measured word error rate
ε_{wMax}	upper bound on word error rate
η	root-mean-square (rms) amplitude noise
$\eta(f)$	noise power per hertz relative to the carrier
η_ϕ	phase noise
π	constant, ratio of the circumference of a circle to the diameter
ρ	reflection coefficient
σ	standard deviation
σ^2	variance
σ_t	aperture uncertainty
τ	time constant
$\Phi[s]$	cumulative density function for a Gaussian distribution (the fraction of the total distribution covered from negative infinity to s)
$\Phi[k]$	confidence level for k standard deviations for a Gaussian distribution, expressed as a fraction
φ	phase, expressed in radians
A_{rms}	root-mean-square (rms) value of a fitted sine wave, rms signal
ac	alternating current
ADC	analog-to-digital converter
BW	bandwidth
cdf	cumulative distribution function
CMRR	common-mode rejection ratio
COMINT	communications intelligence
DAC	digital-to-analog converter
dBFS	decibels full scale
dc	direct current
DFT	discrete Fourier transform
DG	differential gain error
DNL	differential nonlinearity (when given as one number without a code bin specification, it is the maximum differential nonlinearity of the entire range.)
DNL_{rms}	root-mean-square (rms) value of $DNL[k]$
$DNL[k]$	differential nonlinearity of code k
DP	differential phase error
E_{avm}	residual spectrum after the bins at dc and test frequencies have all been set to zero (deleted from the spectrum)
ELINT	electronic intelligence
ENOB	effective number of bits
f	frequency, expressed in hertz

f_{eq}	equivalent sampling rate
f_h	subset of harmonics used in computing total harmonic energy
f_i	input signal reference frequency or input signal repetition rate
f_m	discrete Fourier transform (DFT) basis frequency
f_{ref}	reference frequency
f_s	sampling frequency
Δf	separation between two frequencies
Δf_{rms}	root-mean-square (rms) frequency deviation over one record length
FDHM	full duration at half of maximum
FM	frequency modulation
FSR	full-scale range (of the recorder)
G	gain of waveform recorder, ideally = 1
$G(f)$	dynamic gain of waveform recorder as a function of input frequency f
$H(f)$	frequency response of the waveform recorder
$h(t)$	impulse response
IDFT	inverse discrete Fourier transform
IMD	intermodulation distortion
INL	integral nonlinearity
K	number of data records
LSB	least significant bit
LTI	linear time invariant
M	number of sequential samples in a data record
MSE	maximum static error
N	number of bits digitized by the waveform recorder
$N_{CHG}[i]$	cumulative number of samples up to bin i in sine wave histogram code transition level test
N_H	number of the highest order harmonic used for computing total harmonic distortion (THD)
$N_{HG}[i]$	average number of histogram samples received in the two code bins that share the given transition level
N_s	total number of samples processed over several records
NAD	noise and distortion
NPR	noise power ratio
NPR_{max}	maximum noise power ratio
NNPG	normalized noise power gain
P	probability
pdf	probability density function
PSD	power spectral distribution

Q	ideal code bin width, expressed in input units
Q'	actual average code bin width
rms	root mean square
SFDR	spurious free dynamic range
SIGINT	signal intelligence
SINAD	ratio of signal to noise and distortion
SNR	signal-to-noise ratio
T	time between digitized data samples or sample period
T_R	record length of the recorder in seconds
T_{eq}	interval between equivalent-time samples
t	time, in seconds unless otherwise specified
t_r	transition duration
$T[k]$	code transition level between code k and code $k-1$
$T[k_{in}]$	measured level used to fine tune location of a code transition level
TDR	time domain reflectometer
THD	total harmonic distortion
$u(t)$	unit step function
$u'(t)$	non-ideal step-like function
$U'(f)$	Fourier transform of $u'(t)$
V_{in}	amplitude of the input signal to the waveform recorder
$V_{in\ rms}$	root-mean-square (rms) of the input signal to the waveform recorder
V_{os}	input offset of waveform recorder, ideally = 0
V_{out}	peak-to-peak amplitude of the signal in the digitized data
VSWR	voltage standing wave ratio
$W[k]$	code bin width of code bin k
$W[n]$	Fourier transform of the data window function $w[k]$
$w[k]$	window function [for a discrete Fourier transform (DFT)]
$X[k]$	frequency domain samples of the input sequence $x[i]$ computed via the discrete Fourier transform (DFT)
$X_{avg}[m]$	averaged spectral magnitude of the discrete Fourier transforms (DFTs) of multiple data records
X_{avgh}	averaged discrete Fourier transform (DFT) spectrum, X_{avg} , with the components (bins) at dc, the test frequencies f_i and $(f_s - f_i)$, and the specified harmonic frequencies $f_{h[n]}$, all set to zero (deleted)
$X_{wavg}[m]$	averaged spectral magnitude of the discrete Fourier transforms (DFTs) of multiple windowed data records
$X_w[i]$	Fourier transform of a windowed input sequence $w[i] x[i]$
$X_{w,k}[i]$	windowed Fourier transform of the k^{th} record when multiple records are taken

$x[i]$	sequence of digitized input time domain samples
$x'[i]$	best fit points to a data record
$x(t)$	continuous input signal as a function of time
$\bar{x}$	average value of the input signal over the record
$y[i]$	filtered input sequence of digitized time domain samples
Z_0	transmission line impedance
Z_i	waveform recorder input impedance

4. Test parameters and methods

4.1 Manufacturer supplied information

4.1.1 General information

Manufacturers shall supply the following general information:

- Model number
- Dimensions and weight
- Power requirements
- Environmental conditions: Safe operating, nonoperating, and specified performance temperature range; altitude limitations; humidity limits, operating and storage; vibration tolerance; and compliance with applicable electromagnetic interference specifications
- Any special or peculiar characteristics
- Compliance with military and other specifications
- Available options and accessories, including their impact on equipment performance
- Exceptions to the above parameters where applicable
- Calibration interval, if required

4.1.2 Performance specification

The manufacturer shall provide the minimum specifications defined in 4.1.2.1. Additional potentially useful specifications are given in 4.1.2.2. See 3.1 for definitions.

4.1.2.1 Minimum specification

The manufacturer shall provide minimum specifications for the following:

- Number of digitized bits
- Sample rates
- Memory lengths
- Input impedances
- Analog bandwidths
- Input signal ranges
- Power supply voltage ranges

- Power dissipation

4.1.2.2 Additional specifications

The following additional specifications shall potentially be provided:

- Gain
- Offset
- Differential nonlinearity (DNL)
- Integral nonlinearity (INL)
- Maximum static error
- Monotonicity
- Hysteresis
- Total harmonic distortion (THD) and spurious response
- Intermodulation distortion (IMD)
- Noise power ratio (NPR)
- Ratio of signal to noise and distortion (SINAD)
- Signal-to-noise ratio (SNR)
- Effective number of bits (ENOB)
- Random noise
- Gain error versus frequency
- Frequency response
- Transition settling duration
- Transition duration of step response (rise time)
- Overshoot and precursor
- Slew limit
- Crosstalk
- Aperture uncertainty (short-term time base stability)
- Long-term time base stability
- Fixed error in sample time
- Trigger delay and jitter
- Trigger sensitivity
- Trigger hysteresis band
- Trigger minimum rate of change
- Trigger coupling to signal
- Overvoltage recovery
- Word error rate
- Cycle time

- Common-mode rejection ratio (CMRR)
- Maximum common-mode signal level
- Differential input impedance
- Maximum operating common-mode signal level

4.1.3 Critical waveform recorder parameters

Table 1 is presented as a guide for many of the most common waveform recorder applications. The wide range of waveform recorder applications makes a comprehensive listing impossible. This table is intended to be a helpful starting point for users to apply this standard to their particular applications.

Table 1—Critical waveform recorder parameters

Typical applications	Critical waveform recorder parameters	Performance issues
Audio	SINAD, THD, noise, frequency response	Power consumption Crosstalk and gain matching
Data acquisition	DNL, INL, gain, offset, noise, out-of-range recovery, settling time, full-scale step response, channel-to-channel crosstalk	Channel-to-channel interaction Accuracy, traceability
Digital oscilloscope/ waveform recorder	SINAD, ENOB, noise, bandwidth, out-of-range recovery, word error rate	SINAD for wide bandwidth amplitude resolution Low thermal noise for repeatability Bit error rate
Geophysical	THD, SINAD, long-term stability, noise	Millihertz response
Imaging	DNL, INL, SINAD, ENOB, noise, out-of-range recovery, full-scale step response	DNL for sharp-edge detection High-resolution at switching rate Recovery from blooming
Radar and sonar	SINAD, IMD, ENOB, SFDR, out-of-range recovery, noise	SINAD and IMD for clutter cancellation and Doppler processing.
Spectrum analysis	SINAD, ENOB, SFDR, noise	SINAD and SFDR for high linear dynamic range measurements.
Spread spectrum communication	SINAD, IMD, ENOB, SFDR, NPR, noise-to-distortion ratio, noise	IMD for quantization of small signals in a strong interference environment SFDR for spatial filtering NPR for interchannel crosstalk
Telecommunication personal communications	SINAD, NPR, SFDR, IMD, bit error rate, word error rate, noise	Wide input bandwidth channel bank Interchannel crosstalk Compression Power consumption
Video	DNL, SINAD, SFDR, DG, DP, noise	Differential gain and phase errors Frequency response
DVR (Digital Video Recorder)	Data rate, compression algorithm, power consumption, input impedance	Video standards compliance
Wideband digital receivers SIGINT, ELINT, COMINT	SFDR, IMD, SINAD, noise	Linear dynamic range for detection of low-level signals in a strong interference environment Sampling frequency

4.2 Test selection

In general, it is not worthwhile to test every parameter defined in this standard for any given waveform recorder application, nor is it possible to test parameters under the infinitely many possible test conditions. Thus, the first step in testing any waveform recorder shall be to determine the set of parameters to be measured and the conditions under which they are to be tested. No single process for making this determination will fit every need, but the general procedure outlined below is a good place to start. Users may choose to modify this outline as they gain experience in the particulars of their applications.

Test selection is typically an iterative process. At first, a core set of basic tests is run. Then, depending on the results of these tests, additional tests can be selected to more carefully measure the most significant parameters, further clarify the cause and/or extent of an error, verify an error model, etc. The core set of tests is run at several waveform recorder settings (e.g., different sample rates, input signal ranges) that form a representative sample for the intended use of the recorder, as the results of the tests can vary with the control settings. In addition, some tests (e.g., ENOB) shall be run at several different input conditions (e.g., at several input frequencies). The subclauses below define a core set of tests and input conditions, along with methods to interpret the results, and from there to select additional tests.

Several basic test methods may be used to characterize a waveform recorder. Brief description of the step response test, sine fit and harmonic distortion tests, and dc tests follow.

4.2.1 Step response

Start by measuring the recorder's step response using an input step from about 10% to about 90% of full scale (see 4.8). From this, calculate the recorder's analog bandwidth, frequency response, transition duration, overshoot, transition settling duration, etc. (see Clause 9 and Clause 10). Measuring the frequency response from the step response assumes the waveform recorder is a linear, time-invariant system. Note that in many recorder architectures, the step response and the parameters derived from it vary with the sample rate and/or the input signal range. Note, also, that some recorders cannot be clocked at rates that can fully record the step response, and equivalent-time sampling can be required (see 4.4).

4.2.2 Sine fit and harmonic distortion tests

Perform sine fit tests using a large (e.g., 90% full scale) amplitude signal at several frequencies spanning the range of major expected frequency components in the intended use of the recorder (see 4.6). The residuals of the sine fits show deviations from the linear, time-invariant model assumed in the step response testing. Calculate the ENOB of the recorder at the different frequencies (see 8.5). Also, calculate the Fourier transform of the sine wave data records, and check for harmonic distortion and other spurious components (see 7.7 and 8.7). These tests are then repeated for sample rates and input signal ranges of interest.

In the following, the sine-fit frequencies are categorized as low, medium, and high. Low frequencies are low enough to not cause significant dynamic errors in the waveform recorder. Frequencies less than a few percent of the analog bandwidth are generally safe to consider low. Medium frequencies are those high enough to cause some dynamic effects, but still well below the analog bandwidth. These will typically be in the range of 10% to 60% of the analog bandwidth. High frequencies are near enough to the analog bandwidth limit that the amplitude roll-off is a significant factor.

Also in the following, the ENOB parameter is characterized as low or falling if it is more than one-half to one bit below the number of digitized bits or below the ENOB at a lower frequency. Likewise, total harmonic distortion (THD) (expressed in decibels relative to the carrier) is considered high or rising if it is more than 3 dB to 6 dB above quantization errors or above the harmonic distortion at a lower frequency.

The ENOB will typically decrease at high frequencies. A major reason for this decrease is that aperture uncertainty effects become greater at high frequencies (see Clause 12). THD can decrease at high frequencies if the harmonics are outside the upper bandwidth limit.

If the low-frequency ENOB is low and the low-frequency THD is high, the waveform recorder can have significant static nonlinearity. Measure the code transition levels (see 4.7) and calculate the integral nonlinearity (INL) and DNL (see Clause 7).

If the low-frequency ENOB is low and the low-frequency THD is low, the waveform recorder probably has fine-grained (locally varying) nonlinearity and/or additive noise. Measure the INL, DNL, and random noise (see Clause 8).

If the medium-frequency ENOB is falling and the THD is rising commensurately, the waveform recorder has dynamic nonlinearity. This dynamic nonlinearity can also cause a variation in the step response with input amplitude. Retest the step response at a low input amplitude to determine whether there is a significant difference between small-signal and large-signal frequency responses.

If the medium-frequency ENOB is falling, but the THD remains low, the waveform recorder probably has errors in sample timing. Measure the aperture uncertainty and fixed error in sample time (see Clause 12).

If the high-frequency ENOB is rising and the high-frequency THD is falling, it is likely that there is a distorting stage before the bandwidth-limiting stage and that THD is being filtered out as it exceeds the analog bandwidth. Although this reduction in THD for single-tone sine wave inputs is real, the distorting stage will likely produce distortion products at or below the input frequencies for other input signals, which would not be filtered out. Thus the reduction of THD at high frequencies does not necessarily imply an increase in accuracy in typical use. Check for other signs of high-frequency nonlinearity, such as slew limiting and intermodulation distortion (IMD).

If the Fourier transform shows spurious components at multiples of f_s/J and/or at the input frequency $\pm$ multiples of f_s/J (where f_s is the sample rate and J is a small integer), the waveform recorder is likely implemented as J interleaved slower digitizers.

Additional information about the error sources within a waveform recorder can be gleaned from a visual inspection of plots of the residuals from the sine-fit tests. See Annex G for details on the interpretation of residual plots.

4.2.3 Tests at dc

For applications where dc accuracy is of major importance, find the code transition levels, and from there calculate the gain and offset, INL and DNL, maximum static error, etc. For applications where such a detailed view of the dc transfer function is not needed, find the waveform recorder's low-frequency gain and offset using the dynamic test method shown in 6.1.2 instead, as this can be done using the same sine-fit data gathered above.

4.3 Test setup

4.3.1 Setting up the recorder

The recorder shall be allowed to warm up and is then tested at specific settings of interest. These settings include some or all of the following:

- Full-scale range (FSR)
- Recorder offset adjustment
- Sample rate
- Record length
- Number of pretrigger samples

- Triggering source, level, slope, bandwidth, impedance, etc.
- Input impedance
- Bandwidth/anti-aliasing filter

Notice that testing a waveform recorder inverts the order of operations associated with normal use. For testing, first set the instrument to the desired operating settings, and then adjust the input source to provide an appropriate signal.

Note that some test methods call for relatively specific record lengths. Unnecessarily large record lengths shall be avoided because they can add additional errors due to drift or other long-term phenomena. Longer records also require more computer processing time. However, if the user is interested in the performance of the recorder over a particular record length, tests shall be performed over the length of interest. For example, the results of an ENOB test can degrade with increasing record length due to accumulated time base error.

Pre-trigger may be useful for positioning a test sample at a convenient place in the record. Having the proper input impedance selected will be very important for many tests, e.g., step pulse response, gain, over-voltage recovery.

More sophisticated recorders can have more parameters that shall be set properly. Consult the manual for the recorder for more detail. Some recorders allow the operating parameter setting either by manual or by computer control. Others can be set up only by computer control.

4.3.2 Selecting signal sources

The sources used to generate the test signals are dependent on the characteristics of the waveform recorder under test. It is important, whenever possible, that the signals, particularly the test signal, be defined at the inputs to the waveform recorder with the recorder replaced by an ideal load. Using the signal at the input points instead of at the source output eliminates the effects of cables and any signal conditioning components between the source and the recorder.

Because the properties of sine waves are easy to establish and maintain, sine waves are commonly used as sources for test signals. More complex waveforms, such as triangular, rectangular, or Gaussian, are also used, but it is more difficult to define and maintain the required frequency, amplitude, and phase relationships with high fidelity.

Combining the output of two (or more) sine wave generators can produce two-tone (or multi-tone) test signals for IMD testing. Additionally, a noise generator's output can be combined with a signal to provide low-level dither (see Gray and Stockham [B20]).

If the waveform recorder supports an external clock signal, the frequency synthesizers used to generate the test and clock signals can often be phase-locked to maintain precise phase relationships between the signal and the sampling clock. Phase-locking of synthesizers facilitates testing and simplifies subsequent digital signal processing, by preventing clock/signal walkthrough (beat patterns) that can artificially increase or reduce measured spurious output. This is the primary advantage of testing with a user-supplied clock. Both the clock and the test signals shall be suitable for the test being performed. Filters can be required in either the clock or signal paths to reduce noise or harmonic distortion. For example, sub-harmonics in the clock path can degrade waveform recorder performance; therefore, the clock signal can require filtering to smooth edges that might otherwise feed through to the signal path. Also, low-pass or band-pass filters can be required in the signal path to eliminate noise or other undesirable signals (e.g., harmonics). The relative jitter between the clock and test signals shall be low enough to prevent jitter artifacts from affecting the measured noise as described in [Clause 8](#).

4.3.3 Connecting signal sources

When complex test signals are required, use of additional signal-combining circuits can be employed. These can take the form of resistive or inductive passive combiners or, in some cases, an active summing amplifier can be used. It is important to understand the effects these circuits can have on the noise, distortion, and amplitude of the newly created test signal. Impedance-matching circuits can also be inserted to accurately couple the input test signal to the waveform recorder input terminals. Back-terminating the input signal is another common method to achieve proper input connection impedance matching. It is important to carefully understand the return signal path so that a ground loop is not formed that would cause amplitude degradation and increased noise. As in all sampling systems, the appearance of sampling transients, otherwise known as “kickout” or “kickback” error signals, can be present at the waveform recorder input that shall be properly terminated to reduce their effect on the input test signal. Buffer amplifiers or attenuators with adequate bandwidth can be inserted between the signal source and the recorder to reduce these glitches.

4.3.4 Terminations

Several test methods call for terminating the input of the waveform recorder. This phrase means that an appropriate circuit shall be placed across the input terminals of the recorder under test. The circuit will often simply be a resistor of a specified value.

4.3.5 Reducing noise

Unwanted components or noise can be present on the test signal or connections to the waveform recorder input. Filters, usually low-pass, band-pass, or even high-pass, are commonly used to attenuate these undesirable frequencies and noise signals.

4.3.5.1 Grounding and shielding

Proper grounding and shielding techniques are important in making accurate measurements that are not corrupted by electrical noise. The user shall minimize ground loops and provide low impedance paths to ground. The actual grounding method chosen will often be limited by features of the test equipment used and by electrical safety codes. Electronic instruments often have coaxial connectors where the cable shield is connected to the chassis, which is, in turn, connected to the ac power ground. Under those conditions, ground loops are inevitable when multiple chassis or signal cables are used. In this case, cable lengths shall be minimized and routed to minimize loop areas. Connecting electrical chassis to a high-conductivity ground plane with heavy-duty braid also helps reduce ground-loop pickup.

All connections between the test signal source(s) shall make use of shielded transmission lines such as coaxial cable to shield the test signal from radiated noise. Triaxial cable can be required in some severe noise applications. For low-frequency applications, differential data transmission techniques using twisted shielded pair cable can be used. See Morrison [B33] for more information on grounding and shielding.

“Ground” represents the reference node for all waveform recorder measurements, and it shall be as stable and quiet as possible. There are different philosophies that claim to be the best way to ground a device. Any ground system that provides an acceptably low noise floor is a good one. In connecting and grounding a waveform recorder, determining where the currents flow is essential. Most precision waveform recorders have separate digital and analog returns. Grounding problems can be minimized if proper attention is paid to where the currents flow and to where voltage drops are being generated. A block diagram of the returns in a typical modular waveform recorder application is shown in [Figure 3](#).

Seven nodes that are nominally at “ground” potential are identified in [Figure 3](#). These nodes are described as follows:

- PS1 return, labeled (1). This node is the return of one of the power supplies that are powering the analog portion of the waveform recorder.

- Analog return of the waveform recorder, labeled (2). This node is usually considered the reference node. The internal workings of the waveform recorder try to measure the specified level of the recorder input with reference to this node. The digital return and the analog return are normally connected somewhere in the system to chassis ground.
- Digital return of the waveform recorder, labeled (3). Note that the digital output currents are nominally returned through this node.
- Load return, labeled (4). The load that is applied to the digital outputs returns current to this node.
- Source LO, labeled (5). The source being applied to the recorder input has its return at this node.
- PS2 return, labeled (6). This node is a second power supply return. Current drawn by the recorder's digital section returns through this line. This power supply powers the digital portion of the waveform recorder.
- Chassis ground, labeled (7). This node is the system return that is eventually connected to the ac power line return.

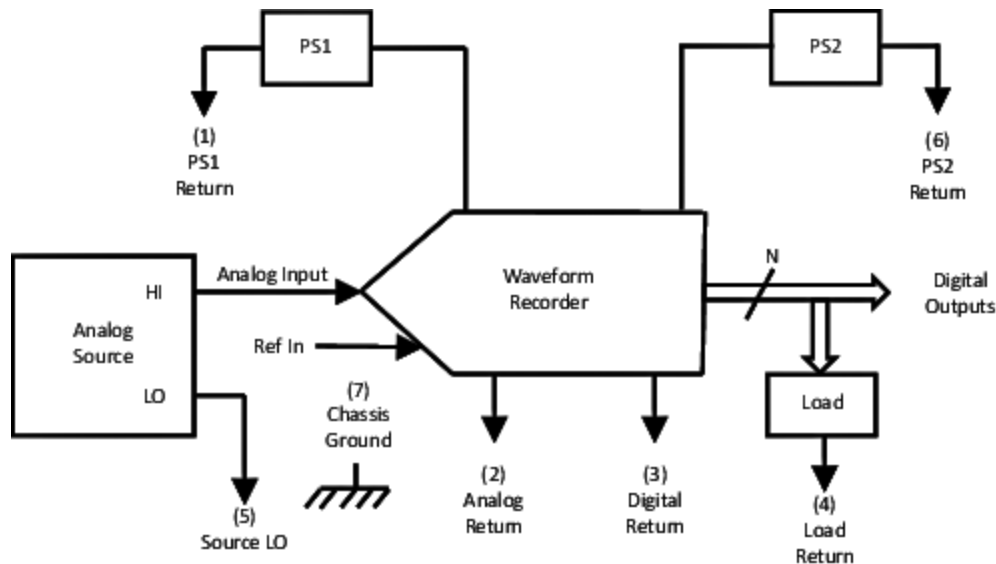


Figure 3—Block diagram of waveform recorder “ground” nodes

Some general rules, as follows, are vital for good measurement quality:

- Connect the source LO (5) to the analog return (2).
- Connect the chassis return (7) to the digital return (3) and to the digital power supply return (6).
- The reference input to the waveform recorder normally has its return referenced to the analog return (2).
- The analog power supply, PS1, normally has its return (6) connected through an isolated line to the analog return of the waveform recorder (2).
- The digital return (3) and the analog return (2) are normally strongly connected through a ground plane. Note that many waveform recorders have the analog and digital grounds internally connected or merged into a single pin.
- The load return (4) is normally connected with an isolated connection to the digital return (3).

Many of these rules can be judiciously violated, but the designer of the ground interconnect shall always be cognizant of the finite impedance of the ground interconnects and cognizant of the paths of the current flows—both static and dynamic.

4.3.6 Taking a record of data

A record of data is a sequential collection of samples acquired by the waveform recorder. The action of taking a record of data is defined as creating the record of data in the recorder by making a measurement and then transferring the data record to a computer for analysis.

Any test signal source(s) shall be connected to the recorder using proper cabling and auxiliary signal conditioning components. If the user has elected to use an external trigger, then the trigger signal shall also be connected, and the signal and trigger shall be synchronized. If a single-shot acquisition mode is being used, then the recorder shall be armed. After the digitizer records and possibly processes the data, the values can be read when the recorder indicates they are ready. Many recorders perform internal gain, offset, and time base corrections after the waveform is recorded.

The user can consult the recorder's manual for details in reading the data. Recorders output the data in different formats, and sometimes the user can select one of several formats. The user might need to convert the data into input units (e.g., volts) following the manufacturer's instructions.

4.3.7 General test setups

A few general test setups can be used to perform most of the waveform recorder tests presented in this standard. Test setups that use sine waves, arbitrary waveforms, and pulses are described in the following subclauses. Some tests, such as those for voltage standing wave ratio (VSWR) and out-of-range signals require setups other than those discussed in the following subclauses.

The following clauses describe general test setups for sine waves, arbitrary waveform generators, and pulses. Some waveform recorders have features such as clock and trigger outputs that facilitate these tests as shown in [Figure 4](#).

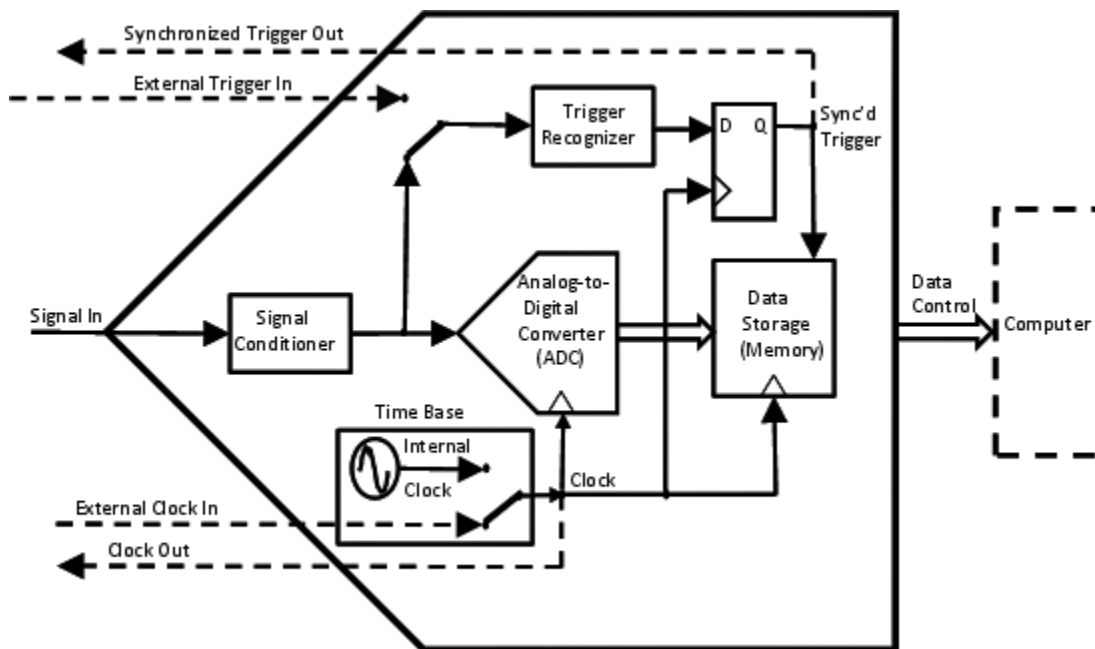


Figure 4—Generic waveform recorder with features that facilitate testing

It is assumed that the user/manufacturer has defined operating limits for the device under test. These limits are categorized as absolute (the limit beyond which the device will be destroyed) and operating (the limit beyond which the device will not operate properly). These limits will vary from device to device, depending on the design. It is not the intention of this document to describe the method of setting these limits, only to verify the operation within them. All test procedures described herein apply only to parameters of a device that is operated within its specified limits.

4.3.7.1 General test setup for sine waves

Figure 5 shows the sine wave test setup. Sine waves are commonly used in waveform recorder testing because appropriate sine wave sources are readily available and because it is relatively easy to establish the quality of the sine wave (e.g., with a spectrum analyzer). A sine wave generator provides the test signal while a clock generator provides the clock (or conversion) signal. Also, combining the output of two (or three) sine wave generators can produce two-tone (or three-tone) test signals for intermodulation distortion testing. Additionally, a noise generator's output can be combined with a signal to provide low-level dither (Gray and Stockham [B20]).

If frequency synthesizers are used to generate the test and clock signals, the synthesizers can often be phase-locked to maintain precise phase relationships between the signal and the sampling clock. Phase-locking of synthesizers facilitates testing and simplifies subsequent digital signal processing, by preventing clock/signal walkthrough (beat patterns) that may artificially increase or reduce measured spurious output.

Both the clock and the test signals shall be suitable for the test being performed. Filters may be required in either the clock or signal paths to reduce noise or harmonic distortion. For example, sub-harmonics in the clock path will degrade waveform recorder performance, so the clock signal may require filtering to smooth edges that might otherwise feed through to the signal path. Also, low-pass or band-pass filters may be required in the signal path to eliminate noise or other undesirable signals (e.g., harmonics). The relative jitter between the clock and test signals shall be low enough to prevent jitter artifacts from affecting the measured noise floor as described in 8.5.7.

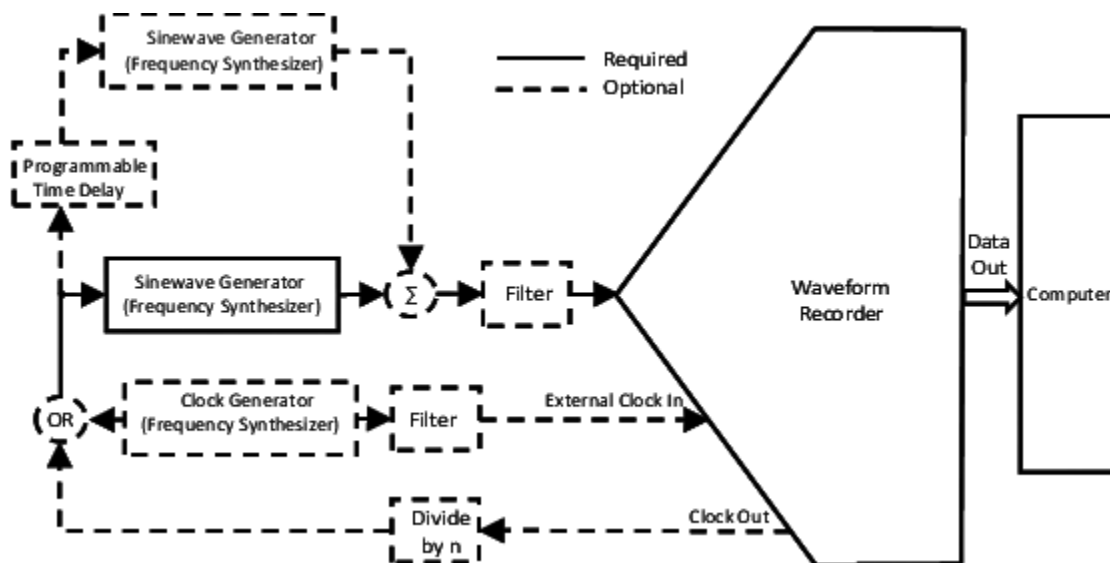


Figure 5—General sine wave test setup

4.3.7.2 General test setup for arbitrary signal test

The arbitrary waveform test setup of Figure 6 can be used for arbitrary test signals, such as ramps, chirps, and steps. In this setup, the test signal is generated digitally and then converted to analog. Care shall be taken to quantify the performance of the arbitrary waveform generator and filter in order to assess (or remove) its impact on the measured performance of the waveform recorder under test. See 4.3.7.1 for comments on filters and data capture. If an arbitrary waveform generator instrument is not available, the function can be performed with local memory and a digital-to-analog converter (DAC).

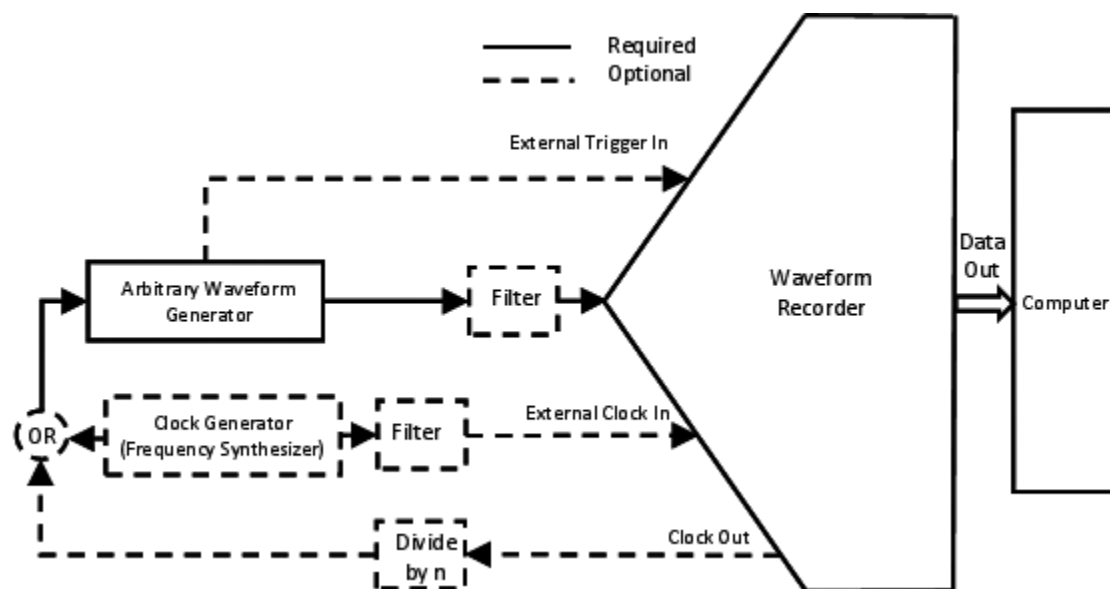


Figure 6—General setup for test using an arbitrary signal generator

4.3.7.3 General test setup for pulses

Figure 7 shows a step waveform test setup to be used for testing with precision step signals that are not digitally generated. Precision pulses and step signals can be used to measure both time domain parameters (such as impulse response, transition duration, overshoot, and settling time) and frequency domain parameters (such as frequency response amplitude and phase, bandwidth, and gain flatness). Equivalent-time sampling can be employed, and certain data analysis tasks can be simplified, if the optional step repetition generator is phase locked to the sampling clock. See 4.3.7.1 for comments on data capture and on filters. Careful attention shall be paid to the phase linearity of any filters placed in the pulse/step signal path.

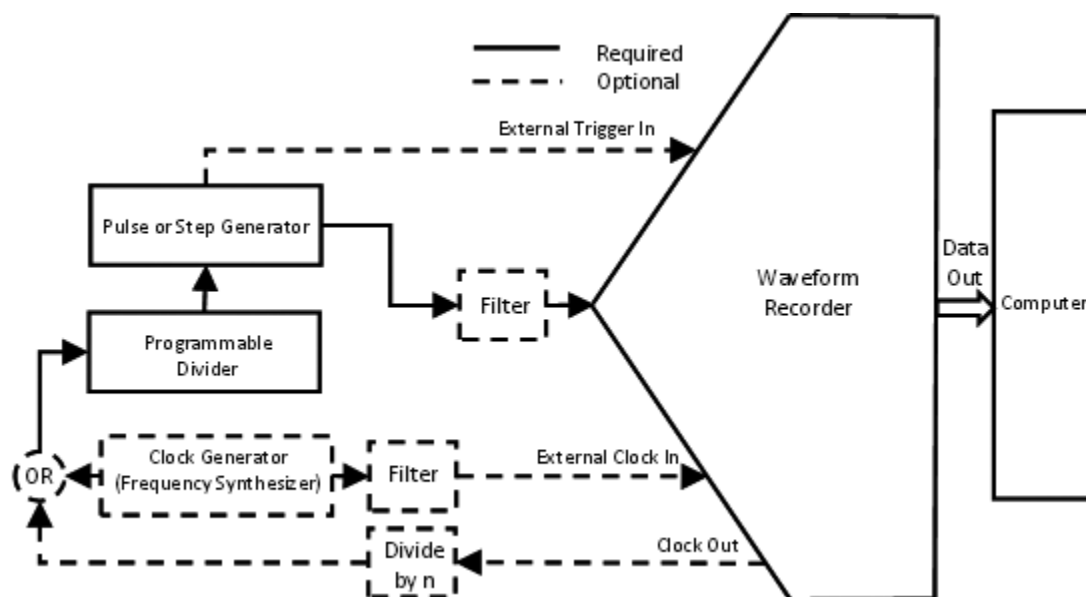


Figure 7—General setup for pulse and step signal testing

4.4 Equivalent-time sampling

The sampling rate of a real-time recorder can limit the accuracy of the measurement of the recorder's bandwidth and step pulse parameters. If the sampling rate is not at least twice the frequency of the highest frequency component of the input signal, then aliasing errors can result. If the input signal is repetitive, these limitations can be reduced by equivalent-time sampling. Several methods of equivalent-time sampling exist. Two methods are described here. The first method consists of extracting equivalent-time samples from a single record using the recorder's internal time base, provided that the input signal's repetition rate is selected appropriately. The second method uses random equivalent-time sampling. These methods are needed only if the recorder does not internally support equivalent-time sampling.

4.4.1 Test method for extracting data for single-record equivalent-time sampling method

If the input signal is repetitive, sampling rate limitations can be reduced by using the principle of equivalent-time sampling to effectively multiply the real-time sampling rate of the waveform recorder by an integer, D . By choosing the sampling rate of the input signal (f_i) appropriately, at least D periods of the input waveform are recorded in a single record; then, upon rearranging the samples with a simple algorithm, a single repetition of the input signal is obtained that is effectively sampled at D times the real-time sampling rate. This is illustrated in Figure 8 for $D = 5$. The steps in implementing the method are given below and illustrated with an example in which the sampling frequency of the recorder is 1 MHz and the desired equivalent-time sampling rate is 5 MHz. The number of data points is kept small so the example can be easily illustrated in Figure 8.

This method is implemented via the following steps:

- Choose the integer, D , based on the requirement that $D \geq f_{eq} / f_s$, where f_s is the maximum sample rate of the recorder. In this example, $D = 5$.
- Choose M , the number of samples in the record. $M = 20$ in this example.
- Using D and M , solve for $L = \text{INT}(M/D)$, the number of real-time samples taken during each repetition of the input waveform. In this example, $L = 4$.
- Determine the input sampling frequency using Equation (4).

$$f_i = f_s \frac{D}{LD - 1} \quad (4)$$

In this example, f_i evaluates as 263.158 kHz. In Figure 8, the horizontal axis for the upper plot can be interpreted as being in microseconds. The lower reconstructed plot can be interpreted as having units of 200 ns per sample.

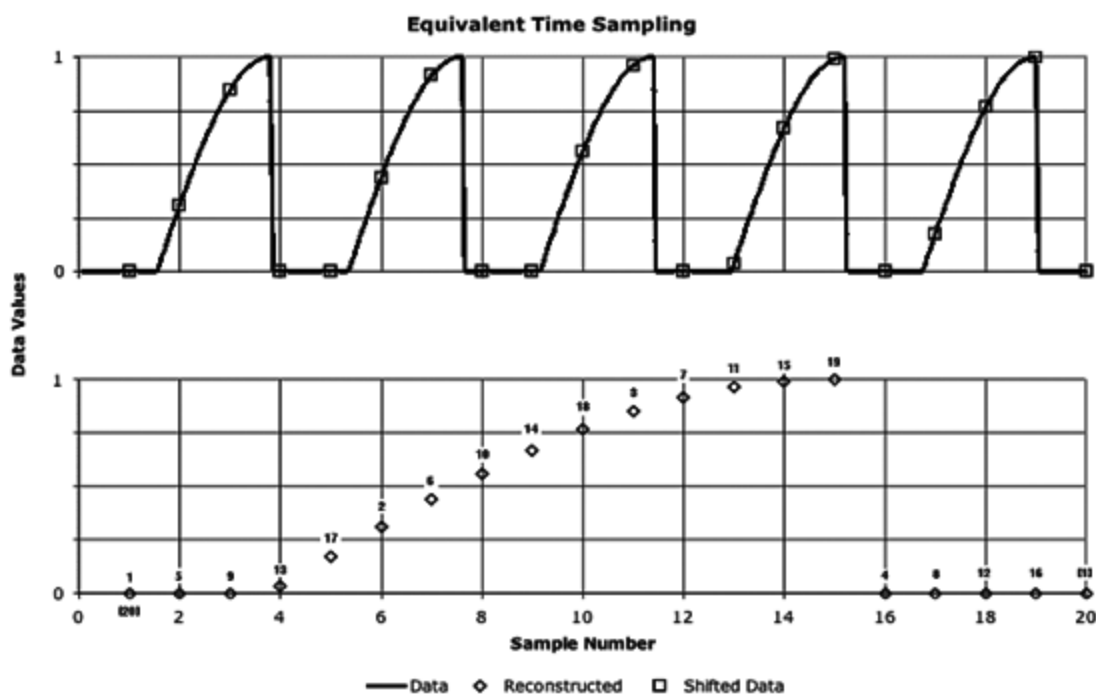


Figure 8—Equivalent-time sampling extraction example

The following pseudo-code program implements the algorithm to rearrange the samples in equivalent time:

```
for J = 1 to L*D
  E(J) = R(1 + MOD(L*(J-1), L*(D-1)))
next J
```

where

- D is the sample rate multiplier (5 in the sample case)
- M is the record length (20 in the sample case)

L	is $\text{INT}(M/D)$ where $\text{INT}(*)$ designates the integer part of $*$ (4 in the sample case)
$E(*)$	is the array containing $(L \times D)$ equivalent-time samples
j	is the equivalent-time sampling index
$R(*)$	is the array containing real-time samples
$\text{MOD}(A,B)$	is the remainder when A is divided by B

4.4.2 Comments on the extraction method for single-record equivalent-time sampling

This method of achieving higher equivalent sampling rates requires that the repetition rate, f_i , of the input signal and the sample rate, f_s , be precisely controlled. While the average equivalent-time sample rate is just Df_s , independent of f_i , the relative spacing of the equivalent-time samples becomes nonuniform when f_i deviates from the value given by Equation (4). If f_i is too great, $D-1$ out of D successive samples will occur too late, while one sample will be correctly placed; if f_i is too small, $D-1$ samples will occur too soon. In either case, the maximum sampling time error, ϵ_{Teq} , is given, to a good approximation, by Equation (5).

$$\epsilon_{Teq} \approx \frac{M(D-1)}{Df_s} \times \frac{\epsilon_{tr}}{t_r} \quad (5)$$

where

T_{eq}	is the average equivalent-time sampling interval, i.e., $1/(Df_s)$
ϵ_{Teq}	is the maximum sampling time error
ϵ_{tr} / t_r	is the proportional error in the repetition period (or repetition rate)

Note, however, that the errors are not cumulative; the average equivalent-time sampling period is still given by $1/(Df_s)$.

The assumption is made in Equation (4) that f_i is exactly known. If it is not exactly known, then the additional error given by an expression similar to Equation (5) will occur. As an example, if $D = 4$, $M = 1024$, and the equivalent sampling period is required to be known to 5%, then the repetition rate shall be set, and the sampling rate shall be known, each with an accuracy of $0.05/(1024 \times 3) = 16$ ppm. To achieve such accuracy, it is usually necessary to use a frequency synthesized source. It is sometimes necessary to measure the frequency of the input signal as well as the frequency of the waveform recorder's internal clock with an accurate frequency counter to assure that they are set with sufficient accuracy. If sufficient accuracy cannot be attained for a specific record length, the accuracy can be improved by decreasing the record length. However, since the lowest frequency component that is represented in a record of length M is given by f_{eq}/M , the range of frequencies that can be represented is limited.

4.4.3 Alternate extraction method for single record equivalent time sampling

Use the setup of 4.4.1 and the same parameters. Collect a record of length $L \times M + 1$ or larger. Use the following algorithm to extract the equivalent time samples:

```
for J = 1 to L*D
  E(J) = R(1 + (J-1)*L)
next J
```

All variables are the same as in 4.4.1.

4.4.4 Comments on the alternate extraction method for equivalent time sampling

This extraction method requires the same precision in the setting of the ratio of f_i to f_s as the extraction method of 4.4.1, but the consequence of errors in this ratio is different. In the previous method an error in the frequency

ratio produced non-uniform sampling with the correct average equivalent-time sampling rate. An error in the frequency ratio in the method of 4.4.3 produces uniform equivalent time sampling that has an error in the sampling frequency given by Equation (6), as follows:

$$\frac{\delta f_{eq}}{f_{eq}} \cong M \frac{\delta(f_i / f_s)}{f_i / f_s} \quad (6)$$

where

δf_{eq} is the error in f_{eq}
 $\delta(f_i / f_s)$ is the error in the frequency ratio

Also, when this extraction method is used, D is not restricted to being an integer.

4.4.5 Test method for random equivalent-time sampling

The random equivalent-time sampling test method has the advantage of not requiring accurate control of the pulse repetition frequency. Instead, it requires taking many, typically 100 or more, records of data. Application of the method requires that the input signal, as recorded on the waveform recorder, have a level, v_0 , with the property that the signal is increasing during a time interval T after crossing v_0 , where T is the reciprocal of the sampling frequency. The method also requires that the input pulse occur at times that are random relative to the clock of the waveform recorder. With these conditions met, do the following:

- Collect K records of data $x_k[i]$, for $k = 1 \dots K$ and $i = 1 \dots M$.
- Let i_0 be the first index in each record for which the recorded signal value is equal to or greater than v_0 . It is assumed that i_0 is the same for each record. If it is not, the beginnings of some records shall be truncated to make it the same.
- Reorder the numbering of the records so that $x_{k+1}[i_0] \geq x_k[i_0]$. Because the signal is increasing in this interval, the records are now ordered in time.
- Construct a single record of length $K \times M$ by taking the first point from each record (in the order of the records), followed by the second point of each record, then the third, and so on.
- Treat this larger record as if it had a sampling frequency of $K \times f_s$, where f_s is the sampling frequency of the individual records.

This approach eliminates the aliasing error and replaces it with a different error—the error due to the fact that the records are not truly uniformly distributed in time, but differ randomly from a uniform distribution. In [B12] Blair shows that the error in the calculated frequency response at any frequency is a random error with standard deviation, σ , given by Equation (7).

$$\sigma = \frac{1.1}{\sqrt{K}} e_{alias}(f) \quad (7)$$

where

$e_{alias}(f)$ is the aliasing error at frequency f . Methods for estimating the aliasing error are given in 10.3.2

4.5 Discrete Fourier transform (DFT)

The DFT is a mathematical operation that converts sampled data from the time domain to the frequency domain. (See Stearns and Hush [B41] or Oppenheim and Willsky [B35] for more information on the DFT.) In this standard, the DFT, $X[k]$, is defined as shown in Equation (8).

$$X[k] = \sum_{n=0}^{M-1} x[n] \exp(-j2\pi kn / M) \quad \text{for } 0 \leq k \leq M-1 \quad (8)$$

where

$x[n]$ is the samples in the data record
 M is the number of samples in the data record

The inverse discrete Fourier transform (IDFT) is the mathematical operation that converts sampled data from the frequency domain to the time domain. The formula is shown in Equation (9).

$$x[n] = \frac{1}{M} \sum_{k=0}^{M-1} X[k] \exp(+j2\pi kn / M) \quad (9)$$

Note that in this standard, the IDFT includes the requisite $(1/M)$ normalization. Other definitions of a DFT can use other normalization factors.

The frequency corresponding to $X[k]$, f_k , is shown in Equation (10).

$$\begin{aligned} f_k &= \frac{k}{M} f_s & \text{for } 0 \leq k \leq M/2 \\ f_k &= -\left(1 - \frac{k}{M}\right) f_s & \text{for } M/2 + 1 \leq k \leq M-1 \end{aligned} \quad (10)$$

where

f_k is the frequency corresponding to the k^{th} DFT component
 f_s is the sampling frequency

Note that the second half of the frequency components can be interpreted as negative frequencies, and for real signals their transform values are the complex conjugates of the values for the corresponding positive frequencies. Sometimes, to simplify mathematical expressions (such as the modulo function), the upper formula will be used for all k , mapping the negative frequencies to frequencies between the Nyquist frequency and twice the Nyquist frequency. The frequencies are separated by an interval of $\Delta f = f_s / M$. The interval of frequencies from $f_k - \Delta f/2$ to $f_k + \Delta f/2$ is called the k^{th} frequency bin. Associated with any frequency, f , is the bin number, $f/\Delta f$. If f is at the center of the k^{th} frequency bin, the bin number will be the integer, k . Noninteger values of the bin number correspond to frequencies that are not at the center of a bin.

$X[f_k]$ may be written in place of $X[k]$ to make the frequency dependence more obvious.

Example DFT plots are shown in Figure 9 and Figure 10. Both show the DFT of a signal sampled at 100 samples per second. The Nyquist frequency is 50 samples per second. The signal is a 21 Hz sine wave with amplitude 1, a 2nd harmonic distortion with amplitude 0.5, and a 3rd harmonic distortion with amplitude 0.25. Figure 9 shows the DFT spectrum using positive and negative frequencies. All signals are symmetrical and have the same magnitude for positive and negative frequencies. The 3rd harmonic, which is at 63 Hz, is aliased down to 37 Hz. Figure 10 shows the negative frequencies mapped up to the frequencies between the Nyquist frequency and twice the Nyquist frequency. They are moved to the right 100 frequency units (the sampling frequency). Note that one of the 3rd harmonic terms is at the correct frequency of 63 Hz. However, the other

frequencies that are mapped for the negative side are at locations that are not very intuitive. Figure 9 gives a better visual picture, while Figure 10 is often more convenient in mathematical calculations, specifically in the tracking of frequencies that are above the Nyquist frequency.

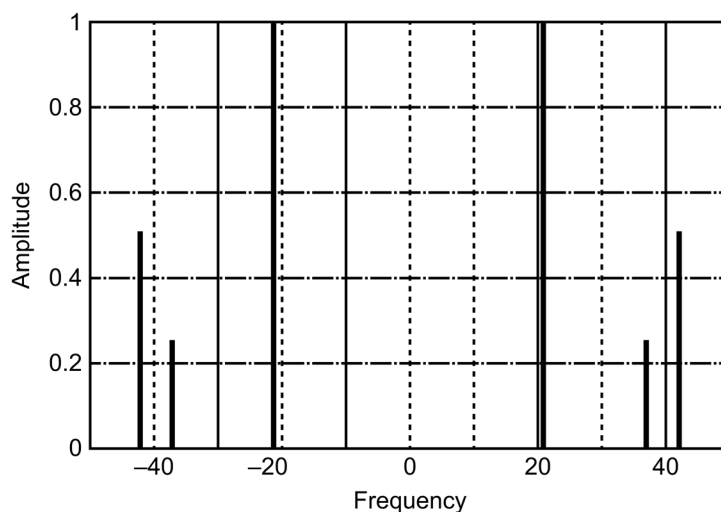


Figure 9—Example DFT spectrum using positive and negative frequencies

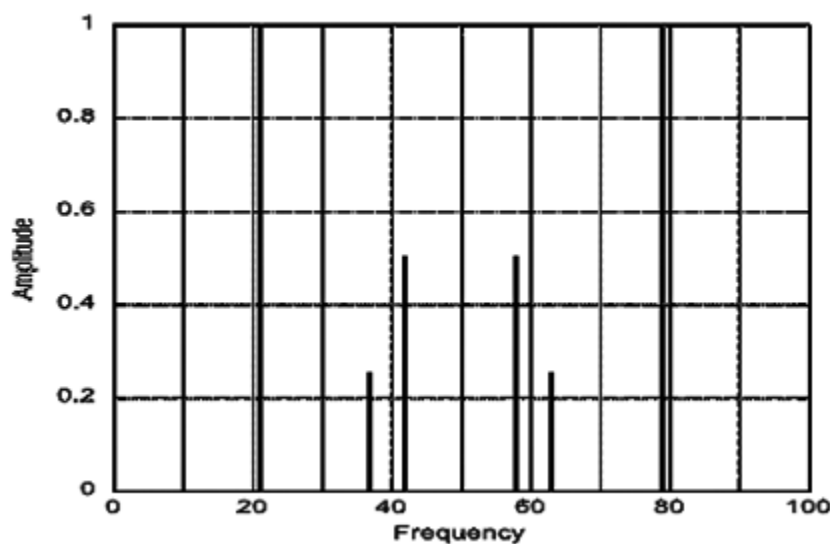


Figure 10—Example DFT showing the negative frequencies mapped up to the frequencies between the Nyquist frequency and twice the Nyquist frequency

The “energies” in the time and frequency representations of the data are related by the Parseval relation shown in Equation (11).

$$\sum_{n=0}^{M-1} |x[n]|^2 = \frac{1}{M} \sum_{k=0}^{M-1} |X[k]|^2 \quad (11)$$

4.5.1 Aliasing

Aliasing is the phenomenon in sampled data systems whereby signal components at input frequencies greater than half the sampling frequency are converted to frequencies at less than half the sampling frequency in the output data record (see Stearns and Hush [B41] or Oppenheim and Willsky [B35]).

Frequency aliasing is a phenomenon that can best be understood using the plot shown in Figure 11.

The horizontal axis represents the frequency being applied to the recorder. In the region of input frequencies that are less than half the sample frequency ($f/f_s < 0.5$), there is an exact correspondence between the frequencies identified by the DFT and the applied frequency. When the input frequency is greater than half the sampling frequency (the Nyquist frequency), the DFT moves the frequency to a frequency that is less than the Nyquist frequency. The value of the aliased frequency can be obtained either from Figure 11 or from mathematics that implement the results shown in Figure 11. Note that both the positive and the negative frequencies are generated by the DFT used in this standard, but for the sake of clarity, only the positive frequencies are shown in Figure 11.

The advantage of the DFT display shown in Figure 12 is apparent when it is noted that the mathematics described by Figure 11 become trivial when the function $\text{mod}(f/f_s, 1)$ is used to determine the location of the DFT frequencies that are shown in Figure 11.

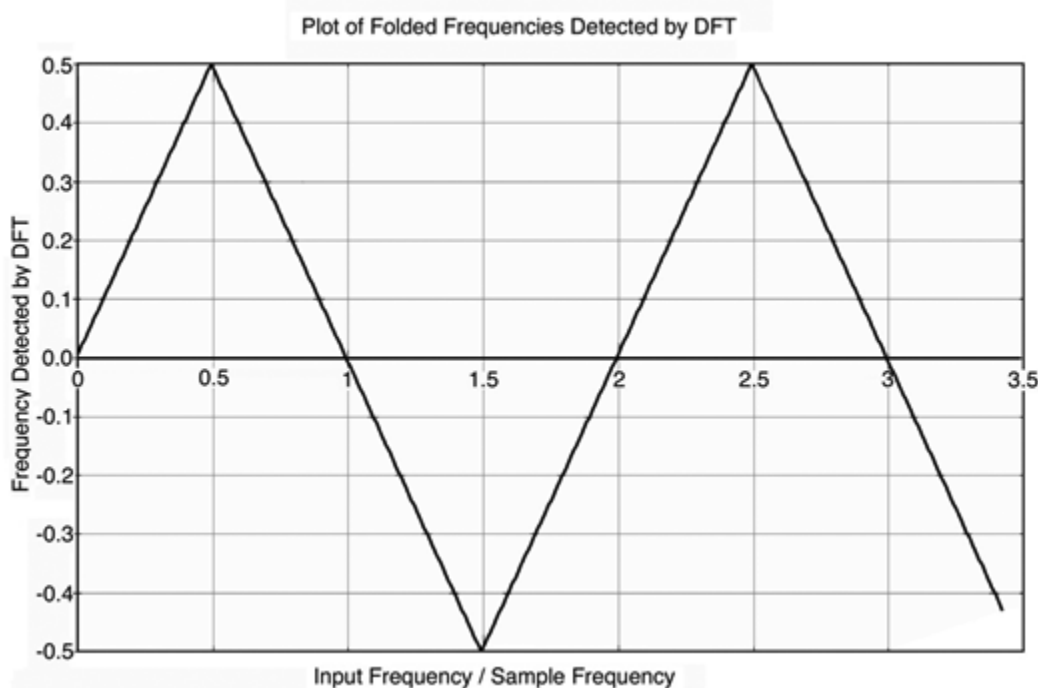


Figure 11—Plot of DFT detected frequencies versus input frequency for a waveform recorder

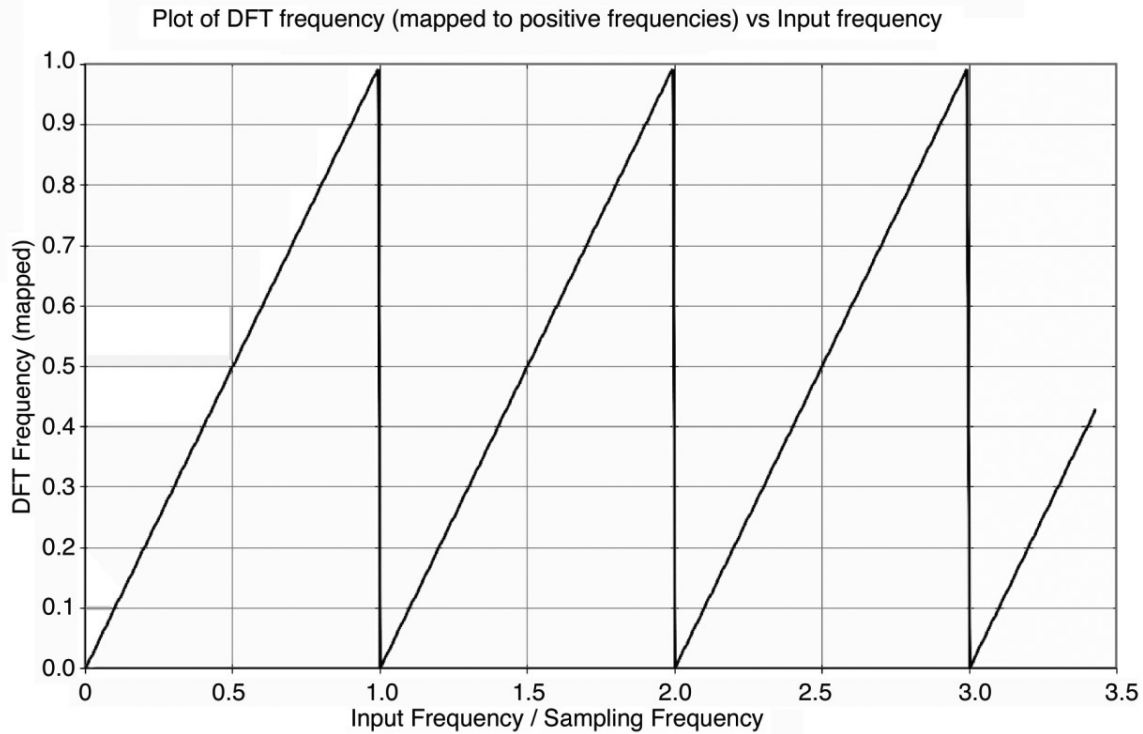


Figure 12—Plot showing how negative DFT frequencies are mapped up to the frequencies between the Nyquist frequency and twice the Nyquist frequency

4.5.2 Windowed DFT and spectral leakage

A windowed DFT is defined in Equation (12). The window factors, $w[n]$, are near one at the center of the data interval and approach zero at the ends of the interval.

$$X_w[k] = \sum_{n=0}^{M-1} w[n]x[n]\exp(-j2\pi nk / M) \quad (12)$$

Windowing is multiplication in the time domain and corresponds to convolution in the frequency domain. Some authors refer to the function $w[n]$ as the temporal weighting function and its Fourier transform, $W[k]$, as the window function; whereas some refer to $w[n]$ as the window function. When all the window factors are one, i.e., no windowing, the window is called the rectangular window. This rectangular window is also called nonweighting. Window functions other than the rectangular are used to reduce the effects of spectral leakage, which is described in 4.5.2.1.

4.5.2.1 Spectral leakage

The rectangular-windowed DFT of a sinusoid with a frequency at the center of a DFT bin produces a single spectral line. If the frequency of the sinusoid is not at the center of a bin, the DFT will produce lines in all frequency bins (see Stearns and Hush [B41] or Oppenheim and Willsky [B35]). The leakage is described by the continuous-time Fourier transform, $W[f]$, of the window function. Two examples are shown in Figure 13.

The dashed line is the Hann window (defined in 4.5.2.3), and the solid line is the rectangular window. The horizontal axis is the frequency converted to DFT bin number, and the vertical axis is $W[f]$ expressed in decibels. Since $W[-f] = W[f]$, the data are not shown for negative values. If the signal being analyzed is a

sinusoid with amplitude A and frequency with a bin number of x (not necessarily an integer), the k^{th} spectral line will have an amplitude of $A \times W[k-x]$.

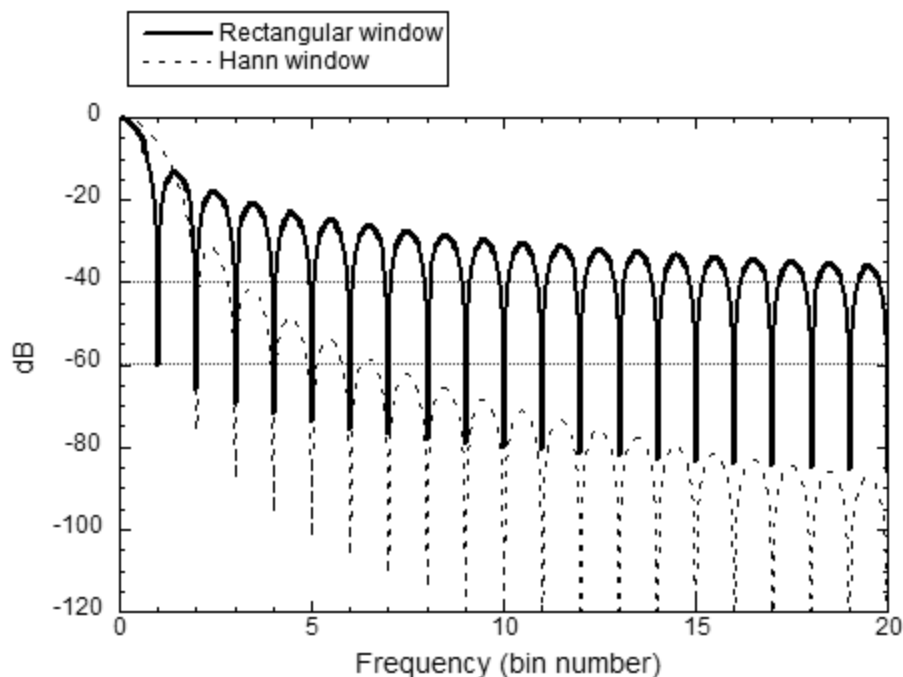


Figure 13—Fourier transform of the Hann and the rectangular windows

The negative spikes in the plot are where the function is zero. For the rectangular window, the function is zero at all integers except zero, while the function for the Hann window is zero at all integers except zero and ± 1 . Thus, for a coherently sampled sinusoid (one with an integer bin number), the rectangular window will produce a single line, while the Hann window will produce three lines. However, for incoherent sampling, both windows produce lines at all frequencies. The amplitudes of the lines fall off much more slowly for the rectangular window than for the Hann window.

Figure 14 shows the same information for the Hann window (dashed line) and the Blackman window (solid line; also defined in 4.5.2.3). Note that the difference is much smaller than the difference between the rectangular and Hann windows. This example illustrates that once the user chooses to use a good window other than the rectangular, it is often not critical which one is chosen.

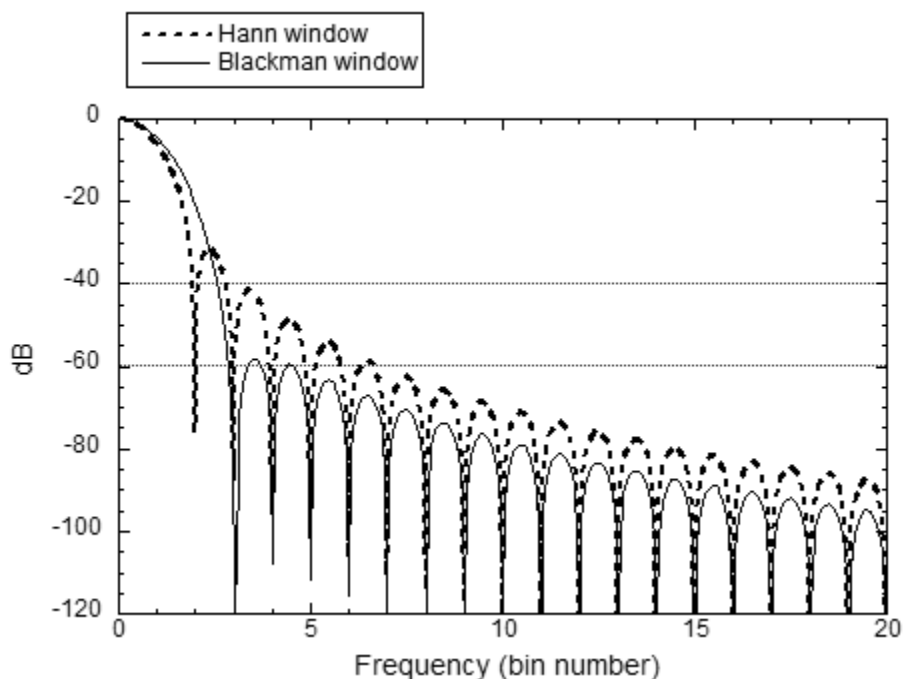


Figure 14—Fourier transform of the Hann and the Blackman windows

4.5.2.2 Coherent-sampling and sine fitting methods of reducing spectral leakage

There are several methods of dealing with spectral leakage other than the judicious choice of windows. The basic problem is that the energy of some large sinusoidal components of the signal leaks into spectral bins that have smaller components that need to be identified. This problem is often solved simply by choosing a sufficiently long record length. As seen from Figure 13 and Figure 14, the simplest windows (Hann and Blackman) are both down by more than 87 dB at an offset of 20 frequency bins. As the record becomes longer, the frequency deviation corresponding to 20 frequency bins becomes smaller. For example, with a record length of 4096, 20 frequency bins is less than 1% of the Nyquist frequency.

Coherent sampling, in which the bin number of the applied signal is an exact integer, is the usually the best approach for dealing with spectral leakage. It eliminates the leakage problem completely if exact coherence is obtained. Also, the recommended frequencies given elsewhere in this standard for best achieving other ends typically result in coherent sampling. However, sometimes the accuracy and/or precision of the frequency setting of available oscillators does not allow for coherent sampling.

Sine fitting is another approach to reduce leakage problems. Use the four-parameter sine-fitting algorithm described in 4.6 to determine the frequency, amplitude, and phase of large incoherent sinusoidal components of the data record. Then subtract these components from the data record, and perform the DFT on the result. This approach typically reduces leakage problems by 30 dB to 50 dB. The user can also first truncate the record to a length containing approximately an integer number of periods.

4.5.2.3 Useful windows and their characteristics

The only window parameter used in this standard is the normalized noise power gain (NNPG), which is given by Equation (13).

$$\text{NNPG} = \frac{1}{M} \sum_{n=0}^{M-1} w[n]^2 \quad (13)$$

where

$w[n]$ is the window coefficients
 M is the number of samples in the data record

The *NNPG* is a conversion factor that gives a Parseval-like relationship for the windowed DFT in Equation (14).

$$\frac{1}{M} \sum_{n=0}^{M-1} x[n]^2 = \frac{1}{M \times \text{NNPG}} \sum_{k=0}^{M-1} X_w[k]^2 \quad (14)$$

where

$X_w[i]$ is the Fourier transform of a windowed input sequence $w[i] x[i]$

The value of NNPG is 1 for the rectangular window and is less than 1 for all other windows.

Useful windows and their characteristics are described by Harris [B22] and Nuttall [B34]. These windows are of the form shown in Equation (15).

$$w[n] = \sum_{l=0}^L a_l (-1)^l \cos(2\pi n l / M) \quad (15)$$

where

a_l is the l^{th} coefficient used to define the window function (the sum of the coefficients equals 1)
 L is the order, or number, of coefficients in the window

These windows are commonly used and are discussed at length by Nuttall [B34]. Any window of this form is referred to as a cosine window of order L .

For these windows, the coefficients of the DFT of the window function are proportional to the coefficients multiplying the cosine terms. There is one nonzero coefficient at zero frequency, L nonzero coefficients at positive frequencies, and L nonzero coefficients at negative frequencies. Therefore, a coherently sampled sine wave produces L lines on each side of the central bin.

From Equation (14), the Parseval relation, and the fact that the a_i are the Fourier coefficients of the window function, Equation (16) can be derived, as follows:

$$\text{NNPG} \cong a_0^2 + \frac{1}{2} \sum_{l=1}^L a_l^2 \quad (16)$$

The windows shown earlier are described in Table 2.