

IEEE Standard Definitions of Physical Quantities for Fundamental Frequency and Time Metrology—Random Instabilities

Sponsor

**IEEE Standards Coordinating Committee 27
on Time and Frequency**

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Abstract: Methods of describing random instabilities of importance to frequency and time metrology is covered in this standard. Quantities covered include frequency, amplitude, and phase instabilities; spectral densities of frequency, amplitude, and phase fluctuations; and time-domain variances of frequency fluctuations. In addition, recommendations are made for the reporting of measurements of frequency, amplitude and phase instabilities, especially as regards the recording of experimental parameters, experimental conditions, and calculation techniques.

Keywords: AM noise, amplitude instability, FM noise, frequency domain, frequency instability, frequency metrology, frequency modulation, noise, phase instability, phase modulation, phase noise, PM noise, time domain, time metrology

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Introduction

(This introduction is not part of IEEE Std 1139-1999, IEEE Standard Definitions of Physical Quantities for Fundamental Frequency and Time Metrology—Random Instabilities.)

Techniques to characterize and to measure the frequency, phase, and amplitude instabilities in frequency and time devices and in received radio signals are of fundamental importance to all manufacturers and users of frequency and time technology.

In 1964, the Standards Coordinating Committee 14 and, in 1966, the Technical Committee on Frequency and Time were formed to prepare an IEEE standard on frequency stability. In 1969, these committees completed a document proposing definitions for measures of frequency and phase stabilities (Barnes [B12]^a). In 1988, an updated IEEE standard on frequency stability was published by the IEEE as IEEE Std 1139-1988, Standard Definitions of Physical Quantities for Fundamental Frequency and Time Metrology. The recommended measures of instabilities in frequency generators have gained acceptance among frequency and time users throughout the world.

This standard is a revision of IEEE Std 1139-1988, which had been prepared by a previous Standards Coordinating Committee 14 consisting of Helmut Hellwig, Chairman; David Allan; Peter Kartaschoff; Jacques Vanier; John Vig; Gernot M. R. Winkler; and Nicholas F. Yannoni. Some clauses of the 1988 standard remain unchanged.

Most of the major manufacturers now specify instability characteristics of their standards in terms of the recommended measures. This standard thus defines and formalizes the general practice.

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^aThe numbers in brackets correspond to those of the bibliography in Annex E.

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IEEE Standard Definitions of Physical Quantities for Fundamental Frequency and Time Metrology—Random Instabilities

1. Overview

1.1 Scope

This standard covers methods of describing random instabilities of importance to frequency and time metrology. Quantities covered include frequency, amplitude, and phase instabilities; spectral densities of frequency, amplitude, and phase fluctuations; and time-domain measures of frequency fluctuations. In addition, recommendations are made for the reporting of measurements of frequency, amplitude and phase instabilities, especially as regards the recording of experimental parameters, experimental conditions, and calculation techniques. Basic concepts and definitions, time prediction, confidence limits when estimating the variance from a finite data set, and confidence limits for frequency domain measures are covered in the annexes. The annexes also cover translation between the frequency-domain and time-domain measures, examples on how to calculate the time-domain measures of frequency fluctuations, and an extensive bibliography of the relevant literature. Systematic instabilities, such as environmental effects and aging, are discussed in IEEE Std 1193-1994 [B40]¹.

2. Summary of definitions

The standard definitions given in the main body of the text and the annexes are listed here in narrative form. If ambiguities are created between the narrative definition given here and the mathematical equation given in the text, the text has priority.

2.1 amplitude deviation $\varepsilon(t)$: Instantaneous amplitude departure from a nominal amplitude.

2.2 amplitude instability $S_a(f)$: One-sided spectral density of the fractional amplitude deviation.

¹The numbers in brackets correspond to those of the bibliography in Annex E.

2.3 confidence limit: The uncertainty associated with the estimate of a time- or frequency-domain instability measure from a finite number of measurements.

2.4 frequency deviation $y(t)$: Instantaneous, normalized, or fractional frequency departure from a nominal frequency.

2.5 frequency instability $S_y(f)$: One-sided spectral density of the fractional frequency deviation.

2.6 phase deviation $\phi(t)$: Instantaneous phase departure from a nominal phase.

2.7 phase instability $S_\phi(f)$: One-sided spectral density of the phase deviation.

2.8 phase noise $\mathcal{A}(f)$: One-half of the phase instability $S_\phi(f)$, as defined in 1.7.

2.9 time deviation $x(t)$: Instantaneous time departure from a nominal time.

2.10 time instability $S_x(f)$: One-sided spectral density of the time deviation.

2.11 time interval error (TIE): The variation of the time difference between a real clock and an ideal uniform time scale following a time period t after perfect synchronization.

2.12 two-sample deviation $\sigma_y(\tau)$: Also called the Allan deviation; the square root of the two-sample variance, as defined in 2.13.

2.13 two-sample variance $\sigma_y^2(\tau)$: Also called the Allan variance; time average over the sum of the squares of the differences between successive readings of the normalized frequency departure sampled over the sampling time τ , under the assumption that there is no dead time between the normalized frequency departure samples.

3. Standards for characterizing or reporting measurements of frequency, amplitude, and phase instabilities

The standard measure for characterizing frequency and phase instabilities in the frequency domain is the phase noise, $\mathcal{A}(f)$ (pronounced “script-ell of f”), defined as one half of the double-sideband spectral density of phase fluctuations,

$$\mathcal{A}(f) \equiv \frac{1}{2} S_\phi(f).$$

When expressed in decibels, the units of $\mathcal{A}(f)$ are dBc/Hz (dB below the carrier in a 1 Hz bandwidth). A device shall be characterized by a plot of $\mathcal{A}(f)$ versus Fourier frequency f . In some applications, providing $\mathcal{A}(f)$ versus discrete values of Fourier frequency is sufficient. (See Annexes A and B for further discussion). The standard measure for characterizing amplitude instability in the frequency domain is one half of the double-sideband spectral density of the fractional amplitude fluctuations, $1/2 S_a(f)$. When expressed in decibels the units of $1/2 S_a(f)$ are dBc/Hz (dB below the carrier in a 1 Hz bandwidth). See Annex A for a detailed discussion on spectral densities.

In the time domain, the standard measure of frequency and phase instabilities is the fully overlapped Allan deviation $\sigma_y(\tau)$. A device shall be characterized by a plot of $\sigma_y(\tau)$ versus sampling time τ . In some cases, providing discrete values of $\sigma_y(\tau)$ versus sampling time τ is sufficient. (See Annex A and Annex B for further discussion). The measurement system bandwidth (f_h) and the total measurement time shall be indicated.

In addition, the provisions in 3.1 and 3.2 are recommended when reporting measurements on frequency and phase instabilities.

3.1 Nonrandom phenomena should be recognized

Examples of nonrandom phenomena include

- a) Any observed time dependence of the statistical measures should be stated.
- b) The method of modeling systematic behavior should be specified (e.g., an estimate of the linear frequency drift was obtained from the coefficients of a linear least-squares regression to M frequency measurements, each with a specified averaging or sample time t and measurement bandwidth f_h).
- c) The environmental sensitivities should be stated (e.g., the dependence of frequency and/or phase on temperature, magnetic field, barometric pressure, and vibration).

3.2 Relevant measurement or specification parameters should be given

Relevant measurement or specification parameters include

- a) The nominal signal frequency ν_o .
- b) The method of measurements.
- c) The measurement system bandwidth f_h and the corresponding low-pass filter response.
- d) The total measurement time (data sample) and the number of measurements N .
- e) The characteristics of the reference signal (equal noise or much lower noise assumed).
- f) For sample averaging times that exceed 10% of the total measurement time, $\hat{\sigma}_{y, \text{TOTAL}}(\tau)$ should be used to estimate $\sigma_y(\tau)$ (see Howe [B35] and [B36] and Howe and Greenhall [B37]).
- g) The calculation techniques [e.g., details of the window function when estimating power spectral densities from time-domain data or the assumptions about effects of dead time when estimating the two-sample deviation $\sigma_y(t)$].
- h) The confidence of the estimate (or uncertainty) and its statistical probability (e.g., 1σ for 68%, 2σ for 95%). See Annex D.
- i) The environment during measurement.

Annex A

(informative)

Measures of frequency, amplitude, and phase instabilities

A.1 Introduction

The instantaneous output voltage of a precision oscillator can be expressed as

$$V(t) = (V_o + \varepsilon(t)) \sin(2\pi\nu_o t + \phi(t)), \quad (\text{A.1})$$

where

- V_o is the nominal peak voltage amplitude,
- $\varepsilon(t)$ is the deviation from the nominal amplitude,
- ν_o is the nominal frequency,
- $\phi(t)$ is the phase deviation from the nominal phase $2\pi\nu_o t$.

Figure A.1² illustrates a signal with frequency, amplitude, and phase instabilities. As shown, frequency instability is the result of fluctuations in the period of oscillation. Fluctuations in the phase result in instability of the zero crossing. Fluctuations in the peak value of the signal (V_{peak}) result in amplitude instability.

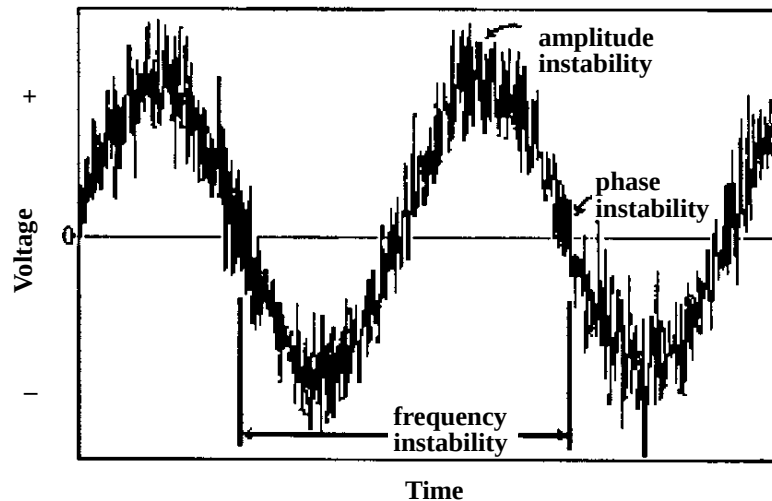


Figure A.1—Instantaneous output voltage of an oscillator

Frequency instability of a precision oscillator is defined in terms of the instantaneous, normalized frequency deviation, $y(t)$, as follows

$$y(t) \equiv \frac{\nu(t) - \nu_o}{\nu_o} = \frac{\dot{\phi}(t)}{2\pi\nu_o}, \quad (\text{A.2})$$

²In the signal shown in Figure A.1, the frequency components of the noise are higher than the carrier frequency. The higher frequency noise components are used for illustration purposes only. In general, this standard applies to the frequency components of amplitude, phase, and frequency instabilities that are lower in frequency than the carrier frequency.

where $\nu(t)$ is the instantaneous frequency (time derivative of the phase divided by 2π), and

$$\dot{\phi}(t) = \frac{d\phi(t)}{dt}. \quad (\text{A.3})$$

Amplitude instability is defined in terms of the instantaneous, normalized amplitude deviation

$$a(t) \equiv \varepsilon(t)/V_0. \quad (\text{A.4})$$

Phase instability, defined in terms of the instantaneous phase deviation $\phi(t)$, can also be expressed in units of time, as

$$x(t) = \phi(t)/2\pi\nu_0. \quad (\text{A.5})$$

With this definition, the instantaneous, normalized frequency deviation is

$$y(t) = dx(t)/dt. \quad (\text{A.6})$$

Other random phenomena observed in certain oscillators are frequency jumps, that is, discontinuities in the frequency of oscillation. These phenomena are not repetitive or well understood and thus cannot be characterized by standard statistical methods.

A.2 Frequency domain

In the frequency domain, frequency, amplitude, and phase instabilities can be defined or measured by one-sided spectral densities.

The measure of frequency instability is the spectral density of fractional frequency fluctuations, $S_y(f)$, given by

$$S_y(f) = y^2(f) \cdot \frac{1}{BW}, \quad (\text{A.7})$$

where

$y(f)$ is the root mean squared (rms) fractional frequency deviation as a function of Fourier frequency,
 BW is the measurement system bandwidth in Hz.

The units of $S_y(f)$ are 1/Hz.

The measure of amplitude instability is the spectral density of fractional amplitude fluctuations, $S_a(f)$, given by

$$S_a(f) = \left(\frac{\varepsilon(f)}{V_0} \right)^2 \frac{1}{BW}. \quad (\text{A.8})$$

The units of $S_a(f)$ are 1/Hz.

Phase instability can be characterized by the spectral density of phase fluctuations, $S_\phi(f)$, given by

$$S_{\phi}(f) = \phi^2(f) \frac{1}{BW}. \quad (\text{A.9})$$

The units of $S_{\phi}(f)$ are rad^2/Hz .

These spectral densities, $S_y(f)$, $S_a(f)$, and $S_{\phi}(f)$, are one-sided since the Fourier frequency f ranges from 0 to ∞ ; nevertheless, they include fluctuations from both the upper and the lower sidebands of the carrier.

$S_{\phi}(f)$ is the quantity that is generally measured in frequency metrology; however, $\mathcal{A}(f)$ has become the prevailing measure of phase noise among manufacturers and users of frequency standards. According to the old definition (see Kartaschoff [B42]), $\mathcal{A}(f)$ is the ratio of the power in one sideband due to phase modulation by noise (for a 1 Hz bandwidth) to the total signal power (carrier plus sidebands); that is,

$$\mathcal{A}(f) = \frac{\text{power density in one phase noise modulation sideband, per Hz}}{\text{total signal power}}. \quad (\text{A.10})$$

Usually $\mathcal{A}(f)$ is expressed in decibels (dB) as $10 \log(\mathcal{A}(f))$, and its units are dB below the carrier in a 1 Hz bandwidth, generally abbreviated as dBc/Hz.

The old definition of $\mathcal{A}(f)$ is related to $S_{\phi}(f)$ by

$$\mathcal{A}(f) \equiv \frac{S_{\phi}(f)}{2}. \quad (\text{A.11})$$

This definition breaks down when the mean squared phase deviation, $\langle \phi^2(t) \rangle =$ the integral of $S_{\phi}(f)$ from f to ∞ , exceeds about 0.1 rad^2 . To circumvent this difficulty, $\mathcal{A}(f)$ is redefined as

$$\mathcal{A}(f) \equiv \frac{S_{\phi}(f)}{2}. \quad (\text{A.12})$$

This redefinition is intended to avoid difficulties in the use of $\mathcal{A}(f)$ in situations where the small angle approximation is not valid. $\mathcal{A}(f)$, as defined by Equation (A.12), should be designated as the standard measure of phase instability in the frequency domain. The reasons are that

- It can always be measured unambiguously, and
- It conforms to the prevailing usage.

Phase instability has sometimes been expressed in units of time by $S_x(f)$, the one-sided spectral density of the phase fluctuations expressed in units of time ($x(t)$),

$$S_x(f) = x^2(f) \frac{1}{BW}. \quad (\text{A.13})$$

From Equation (A.5), $S_{\phi}(f)$ and $S_x(f)$ are related by

$$S_x(f) = \frac{1}{(2\pi\nu_0)^2} S_{\phi}(f). \quad (\text{A.14})$$

Since phase and frequency are directly related, that is, angular frequency is the time derivative of the phase, spectral densities of frequency and phase instabilities are also related: