



IEEE Standard Definitions of Physical Quantities for Fundamental Frequency and Time Metrology—Random Instabilities

IEEE Standards Coordinating Committee 27

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IEEE Standards Coordinating Committee 27 on
Time and Frequency



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IEEE Standard Definitions of Physical Quantities for Fundamental Frequency and Time Metrology—Random Instabilities

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**IEEE Standards Coordinating Committee 27 on
Time and Frequency**

Approved 26 September 2008

IEEE-SA Standards Board

Abstract: Methods of describing random instabilities of importance to frequency and time metrology are covered. Quantities covered include frequency, amplitude, and phase instabilities; spectral densities of frequency, amplitude, and phase fluctuations; and time-domain deviations of frequency fluctuations. In addition, recommendations are made for the reporting of measurements of frequency, amplitude, and phase instabilities, especially in regard to the recording of experimental parameters, experimental conditions, and calculation techniques.

Keywords: AM noise, amplitude instability, FM noise, frequency domain, frequency instability, frequency metrology, frequency modulation, noise, phase instability, phase modulation, phase noise, PM noise, time domain, time metrology

The Institute of Electrical and Electronics Engineers, Inc.
3 Park Avenue, New York, NY 10016-5997, USA

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Introduction

This introduction is not part of IEEE Std 1139-2008, IEEE Standard Definitions of Physical Quantities for Fundamental Frequency and Time Metrology—Random Instabilities.

Techniques to characterize and to measure the frequency, phase, and amplitude instabilities in frequency and time devices and in received radio signals are of fundamental importance to all manufacturers and users of frequency and time technology.

In 1964, Standards Coordinating Committee 14 and, in 1966, the Technical Committee on Frequency and Time were formed to prepare an IEEE standard on frequency stability. In 1969, these committees completed a document proposing definitions for measures of frequency and phase stabilities (Barnes [B14]).^a In 1988, an updated IEEE standard on frequency stability was published as IEEE Std 1139-1988, IEEE Standard Definitions of Physical Quantities for Fundamental Frequency and Time Metrology. Later on a revision of this standard was published as IEEE Std 1139-1999, IEEE Standard Definitions of Physical Quantities for Fundamental Frequency and Time Metrology—Random Instabilities. The recommended measures of instabilities in frequency generators have gained acceptance among frequency and time users throughout the world.

This standard is a revision of IEEE Std 1139-1999, which had been prepared by a previous Standards Coordinating Committee 27, consisting of John R. Vig, Chair; Eva S. Ferre-Pikal, Vice Chair; James C. Camparo, Leonard S. Cutler, Lute Maleki, William J. Riley, Samuel R. Stein, Claudine Thomas, Fred L. Walls, and Joseph D. White. Some clauses of the 1999 standard remain unchanged.

Most of the major manufacturers now specify instability characteristics of their standards in terms of the recommended measures. This standard thus defines and formalizes the general practice.

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^a The numbers in brackets correspond to those of the bibliography in Annex F.

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Participants

At the time this standard was submitted to the IEEE-SA Standards Board for approval, the 1139 Working Group had the following membership:

Eva S. Ferre-Pikal, *Chair*

John R. Vig, *Vice Chair*

James C. Camparo
Christopher R. Ekstrom
Charles A. Greenhall

Lute Maleki
Victor Reinhardt

William J. Riley
Fred L. Walls
Joseph D. White

The following members of the individual balloting committee voted on this standard. Balloters may have voted for approval, disapproval, or abstention.

William J. Ackerman
Ali Al Awazi
Martin J. Bishop
Keith Chow
Kevin Coggins
Eva S. Ferre-Pikal
Robert Graham

Randall Groves
Werner Hoelzl
Piotr Karocki
G. Luri
Bruce Muschlitz
Michael S. Newman

Charles Ngethe
Ulrich Pohl
Michael Roberts
Bartien Sayogo
James E. Smith
Walter Struppler
John Willis

When the IEEE-SA Standards Board approved this standard on 26 September 2008, it had the following membership:

Robert M. Grow, *Chair*

Thomas Prevost, *Vice Chair*

Steve M. Mills, *Past Chair*

Judith Gorman, *Secretary*

Victor Berman
Richard DeBlasio
Andy Drozd
Mark Epstein
Alexander Gelman
William Goldbach
Arnie Greenspan
Ken Hanus

Jim Hughes
Richard Hulett
Young Kyun Kim
Joseph L. Koepfinger*
John Kulick
David J. Law
Glenn Parsons
Ron Petersen

Chuck Powers
Thomas Prevost
Narayanan Ramachandran
Jon Walter Rosdahl
Anne-Marie Sahazizian
Malcolm Thaden
Howard Wolfman
Don Wright

*Member Emeritus

Also included are the following nonvoting IEEE-SA Standards Board liaisons:

Satish K. Aggarwal, *NRC Representative*

Michael Janezic, *NIST Representative*

Lorraine Patsco

IEEE Standards Program Manager, Document Development

Soo H. Kim

IEEE Standards Program Manager, Technical Program Development

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4. Overview

1. Scope

This standard covers the fundamental ~~methods of metrology for~~ describing random instabilities of importance to frequency and time metrology. Quantities covered include frequency, amplitude, and phase instabilities; spectral densities of frequency, amplitude, and phase fluctuations; and time-domain ~~measures~~ deviations of frequency fluctuations. In addition, recommendations are made for the reporting of measurements of frequency, amplitude, and phase instabilities, especially in regard to the recording of experimental parameters, experimental conditions, and calculation techniques. The annexes cover basic concepts and definitions, time prediction, and confidence limits when estimating ~~variance~~ deviations and spectral densities from a finite data set. The annexes also cover translation between the frequency domain and time domain instability measures, examples on how to calculate the time-domain measures of frequency fluctuations, and an extensive bibliography of the relevant literature. Systematic instabilities, such as environmental effects and aging, are discussed in IEEE Std 1193-~~1994~~ [B40]¹-~~TM-2003~~ [B54].¹

¹ The numbers in brackets correspond to those of the bibliography in Annex F.

2. Summary of Definitions

For the purposes of this standard, the ~~text~~ following terms and definitions apply. *The Authoritative Dictionary of IEEE Standards Terms* [B51] should be referenced for terms not defined in this clause. ~~the annexes are listed here in narrative form. definitions given in the main body of~~ If ambiguities are created between the narrative definition given here and the mathematical equation given in the text, the equation has priority.

2.1 amplitude spectrum $S_a(f)$: One-sided spectral density of the normalized amplitude fluctuations, as defined in normalized amplitude fluctuations $a(t)$.

~~2.2 amplitude instability fractional amplitude deviation~~

2.2 bias: The difference between the expected value of an estimator of a statistic and its correct value.

¹~~The numbers in brackets correspond to those of the bibliography in Annex E.~~

2.3 confidence interval: limit: ~~An interval of~~ uncertainty associated with an estimate of ~~a time or frequency domain~~ an instability measure from a finite number of measurements. The endpoints of a confidence interval are called confidence limits.

~~deviation $c(t)$: Instantaneous amplitude departure from a nominal amplitude.~~

2.4 frequency spectrum $S_y(f)$: One-sided spectral density of the ~~fractional frequency deviation~~ normalized frequency fluctuations, as defined in normalized frequency fluctuations $y(t)$.

~~deviation $y(t)$: Instantaneous, normalized, or fractional frequency departure from a nominal frequency.~~

~~2.5 frequency instability: One-sided spectral density of the fractional frequency deviation.~~

~~2.6 phase deviation ϕ : Instantaneous phase departure from a nominal phase.~~

2.5 normalized amplitude fluctuations $a(t)$: Instantaneous normalized amplitude departure from a nominal amplitude.

2.6 normalized frequency fluctuations $y(t)$: Instantaneous, normalized frequency departure from a nominal frequency.

2.7 phase fluctuations $\phi(t)$: Instantaneous phase departure from a nominal phase.

2.8 phase noise $L(f)$: One-half of the phase spectrum $S_\phi(f)$, as ~~as defined in 1.7~~ defined in phase spectrum $S_\phi(f)$.

2.9 phase spectrum $S_\phi(f)$: One-sided spectral density of the phase ~~deviation~~ fluctuations.

2.10 time fluctuations $x(t)$: Instantaneous time departure from a nominal time

~~2.10 time instability $S_x(f)$: One-sided spectral density of the time deviation.~~

2.11 time interval error (TIE): The ~~variation of the~~ time difference between a real clock and an ideal uniform time scale following a time period t after perfect synchronization.

2.12 time spectrum $S_x(f)$: One-sided spectral density of the time fluctuations.

2.13 two-sample deviation $\sigma_y(\tau)$: Also called the *Allan deviation*; the square root of the two sample variance, as defined in **two-sample variance $\sigma_y^2(\tau)$** .

2.14 two-sample variance $\sigma_y^2(\tau)$: Also called the *Allan variance*; ~~one-half the~~ time average ~~over the sum~~ of the squares of the differences between ~~successive readings of the~~ averages of normalized frequency ~~departure sampled over the sampling~~ fluctuations over two adjacent time, ~~under the assumption that there is~~ intervals of length τ , with no dead time between the ~~normalized frequency departure samples~~ two averaging intervals.

3. Standards for characterizing or reporting measurements of frequency, amplitude, and phase instabilities

The standard measure for characterizing frequency and phase instabilities in the frequency domain is ~~the~~ phase noise, $L(f)$, (pronounced “script-ell of f”), defined as one half of the ~~double sideband~~ one-sided spectral density of phase fluctuations:

$$L(f) = \frac{1}{2} S_\phi(f)$$

When expressed in decibels, the units of $L(f)$ are dBc/Hz (dB below the carrier in a 1 Hz bandwidth). A device shall be characterized by a plot of $L(f)$ versus Fourier frequency f . In some applications, providing $L(f)$ versus discrete values of Fourier frequency is sufficient. (See Annexes A and Annex B for further discussion). The standard measure for characterizing amplitude instability in the frequency domain is one half of the one-sided spectral density of the ~~fractional~~ normalized amplitude fluctuations, $1/2 S_a(f)$. When expressed in decibels the units of $1/2 S_a(f)$ are dBc/Hz (dB below the carrier in a 1 Hz bandwidth). See Annex A for a detailed discussion on spectral densities.

In the time domain, the standard measure of frequency and phase instabilities is the ~~fully overlapped~~ Allan deviation $\sigma_y(\tau)$. A device shall be characterized by a plot of $\sigma_y(\tau)$ versus sampling averaging time τ . In some cases, providing discrete values of $\sigma_y(\tau)$ versus ~~sampling time~~ τ is sufficient. (See Annex A and Annex B for further discussion.) The measurement system bandwidth (f_h) and the total measurement time shall be indicated.

In addition the provisions in 3.1 and 3.2 are recommended when reporting measurements on frequency and phase instabilities.

3.1 Nonrandom phenomena should be recognized

In particular:

~~Examples of nonrandom phenomena include~~

- a) Any observed time dependence of the statistical measures should be stated.
- b) The method of modeling systematic behavior should be specified (e.g., an estimate of the linear frequency drift was obtained from the coefficients of a linear least-squares regression to M frequency measurements, each with a specified averaging or sample time t and measurement bandwidth f_h).
- c) The environmental sensitivities should be stated (for example, the dependence of frequency and/or phase on temperature, magnetic field, barometric pressure, and vibration).

3.2 Relevant measurement or specification parameters should be given

Relevant measurement or specification parameters include the following:

- a) The nominal signal frequency ν_o .
- b) The method of measurements.
- c) The measurement system bandwidth f_h and the corresponding low-pass filter response.
- d) The total measurement (data sample) time and the number of measurements N .
- e) The characteristics of the reference signal (equal noise or much lower noise assumed).
- f) For ~~sample~~-averaging times which exceed 10% of the total measurement time using $\hat{\sigma}_{y,TOTAL}(\tau)$ to estimate $\sigma_y(\tau)$ [is recommended](#) (see Howe [B41], [B42] and Howe and Greenhall [B44]).
- g) The calculation techniques [e.g., details of the window function when estimating power spectral densities from time-domain data, or the assumptions about effects of dead time when estimating the two-sample deviation $\sigma_y(\tau)$].
- h) The confidence [interval](#) (or uncertainty) of the estimate and its statistical probability (e.g., 1σ for 68%, 2σ for 95%). See Annex E.
- i) The environment during measurement.